LHCCRR SUMMARY

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Linear Homogeneous Constant Coefficient Recurrence Sequences are the nicest class of recurrent sequence (they are equivalent to those with rational generating functions or those that can be represented as a generalized power sum). I am interested in several topics concerning these sequences:

(1) Decomposition of LHCCRR space: Viewing $\mathbb{C}^{\mathbb{N}}$ as a $\mathbb{C}[x]$ module under the action of the successor operator E, we can realize the space of LHCCRR as a torsion submodule of this space. Unfortunately it is not finitely generated so many of our favourite theorems about modules over a PID no longer apply. However, not all hope is lost. There are two approaches to viewing this module structure that are valuable:
   (a) Firstly, we can consider the finite dimensional subspaces corresponding to the set of elements annihilated by a specific polynomial $f(E)$. This approach is analagous to the ad hoc studies in the classical theory of sequences [1]. Here the approach is to identify which choice of decompositions reconstructs the three standard bases for the space: sequences, generating functions, GPS.
   (b) Secondly, we can work with the global module structure. I have shown that the torsion submodule is injective and has many properties similar to $\mathbb{Q}/\mathbb{Z}$ as a $\mathbb{Z}$–module. The next step is to find a natural decomposition similar to the p–primary decomposition of $\mathbb{Q}/\mathbb{Z}$.

(2) Another question that I am interested in is the generalization of Lucas Bases for the spaces considered in 1(a) [2, 3]. I have show that for restricted sets of eigenvalues such bases always exists for any order, so the next step is to remove the eigenvalue dependence. (This is my NSF project).

(3) Finally, for cryptographic purposes it would be valuable to know the full class of LHCCRR with the property that the corresponding Zeckendorff decomposition is unique under the greedy algorithm. I believe this problem can be solved with matroid theory. I am interested in learning more about this approach to try to solve this problem.

References

[1] D. DeTemple and W. Webb: *Combinatorial Reasoning: An Introduction to the Art of Counting*, Wiley, 2014.

[2] E. Roettger: *A Cubic Extension of the Lucas Functions*, (2009), Ph.D. Dissertation, University of Calgary.

[3] H. C. Williams and R. K. Guy: *Some Fourth–Order Linear Divisibility Sequences*, International Journal of Number Theory, (2011) **07(05)**, 1255–1277.

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