SPLITTING OF QUADARATIC FORMS VIA SPHERICAL POLYNOMIALS SUMMARY

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Every quadratic form Q has an associated theta series, a modular form of given weight, level, and character, which can be computed from invariants of Q. This form contains information about the representation numbers of integers by Q. A homogeneous polynomial P is called spherical with respect to Q if the Q–Laplacian annihilates P. Then, the product PQ gives rise to a new modular form of same level and character but higher weight.

Dimension formulas for the space of Q–spherical polynomials are well known, but it is not known how many linearly indepedent modular forms of a given weight can be reached from Q by multiplying various Ps. The binary case was considered and proved by Frederic Gooding in his Ph.D. thesis [2]. He also presents some bounds for the quaternary case. Evans produced a simpler proof of the main theorem a few years later [1]. Outside of some numerical results for specific 4th order forms, no other results appear in the literature.

One of the key ingredients in Gooding's approach is a theorem that characterizes the kernel of the map lifting the forms to higher dimensions. Gooding conjectures that this theorem extends to arbitrary dimensions. This conjecture is false because there exist forms with trivial automorphism group that have an arbitrarily large number of variables. This contradicts the result because it enforces the lifting map to be injective but by dimension counting the domain space grows exponentially while the target space grows linearly.

References

[1] R. Evans: *Polynomial Sums over Automorphs of a Positive Definite Binary Quadratic Form*, Journal of Number Theory, 9, (1977), 61–62.

[2] F. Gooding: *Modular Forms Arising from Spherical Polynomials and Positive Definite Quadratic Forms*, Ph.D. Thesis, University of Wisconsin, 1971.

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