Fun Problems

1. Some number of circles whose circumferences sum to 10 are placed in the unit square. Prove that there exists a line passing through at least 4 of them.

2. Consider an $m \times n$ grid of unit squares. An $\overbrace{\underbrace{(m,n)}}$–admissible path is a path along adjacent (up, down, left, or right) squares from the lowest left square to the top right corner that does not repeat any squares and no four squares in the path share a vertex (even non–consecutively). How many $\overbrace{\underbrace{(5,5)}}$–admissible paths exist?

Distributions over Networks

Daryl DeFord

Dartmouth College
Department of Mathematics

Department of Mathematics
Graduate Open House
March, 31 2017

Outline

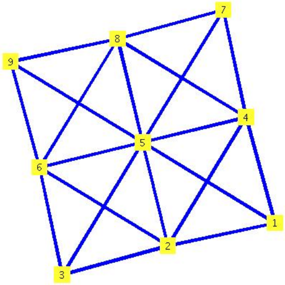
What are Complex Networks?

What are Complex Networks?

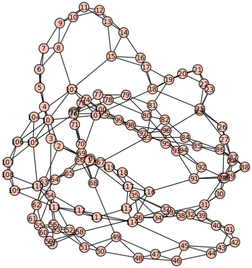
Definition (Complex Network)

???

Examples



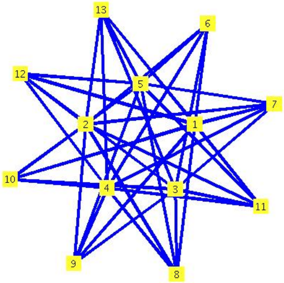
(a) Graph[1]



(b) Network

1 D. DeFord Enumerating Tilings of Rectangles by Squares, Journal of Combinatorics, 6(3), 339-351, (2015).

Examples



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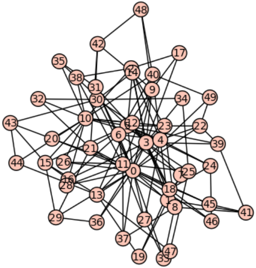
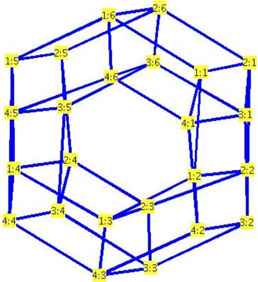


(b) Network

Examples



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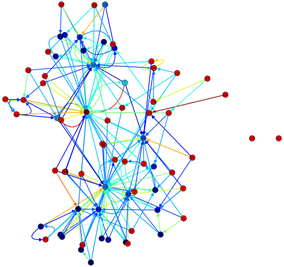

(b) Network [1]

1 Data: Feenstra, R., Lipsey, R., Deng, H., Ma, A. and Mo, H. (2005) World Trade Flows: 1962-2000. NBER Working Papers

Networks Basics (Centrality)

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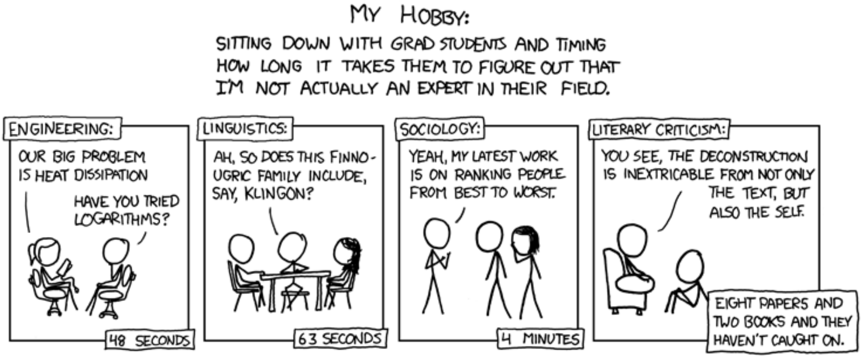

Figure: Impostor[1]

[1] https://xkcd.com/451/

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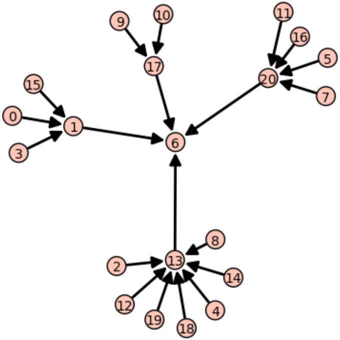

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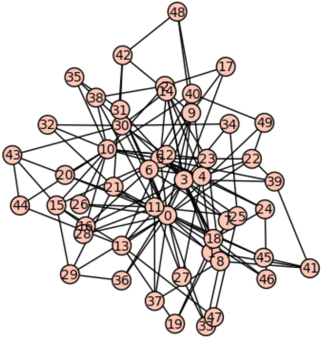

Networks Basics (Clustering)

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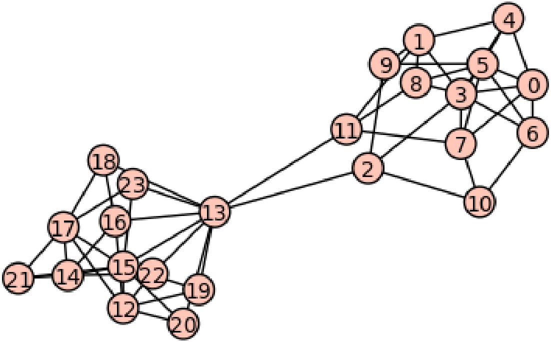

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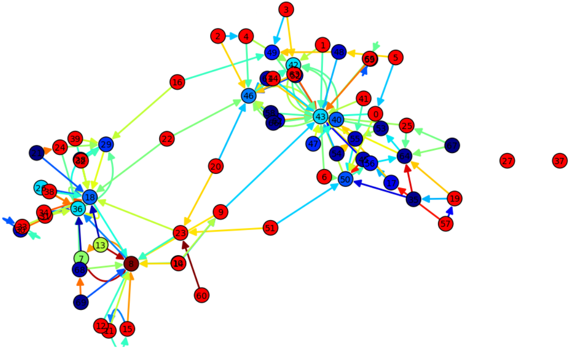

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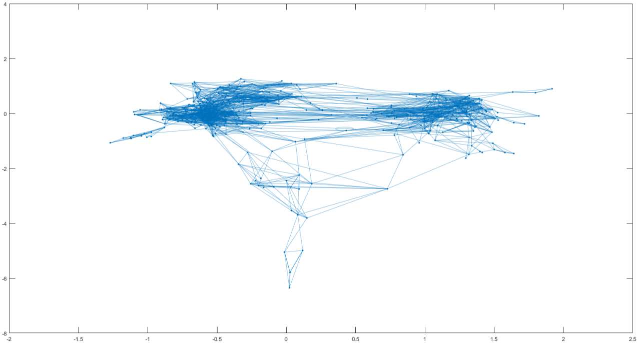

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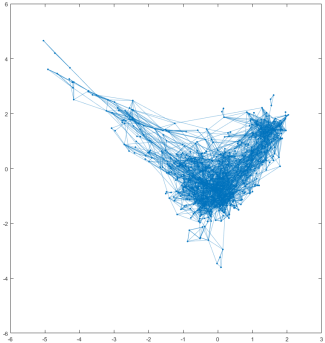

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Fuzzy Networks

- Seems pretty straightforward...

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- Noisy data/reality

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Fuzzy Networks

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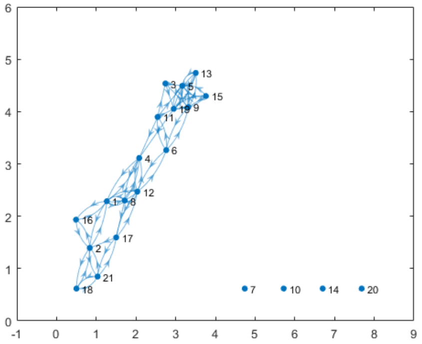
  - Network Evolution
  - Multiple interaction types

Different Perspectives on "Friendship"



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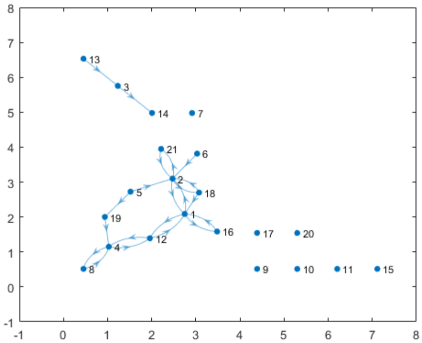

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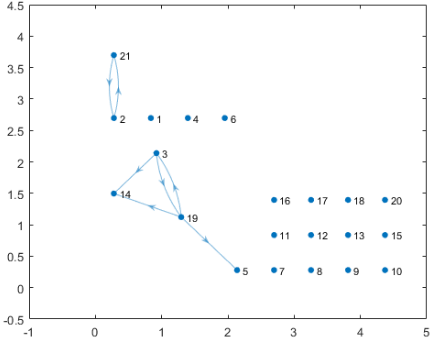

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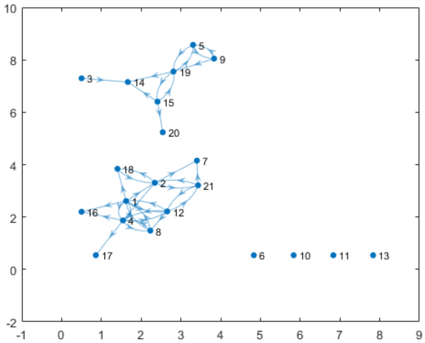

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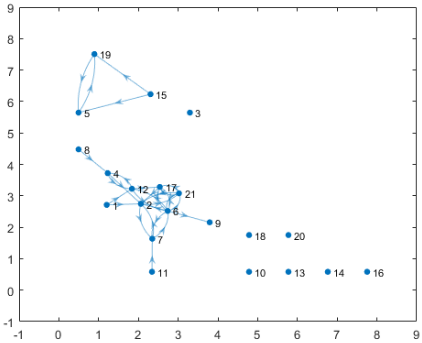

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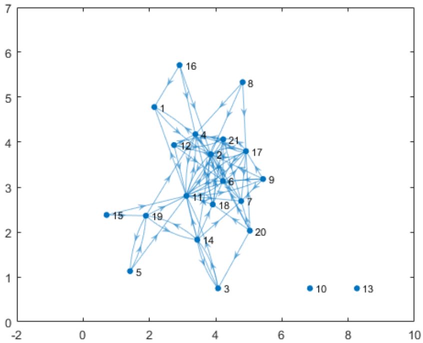

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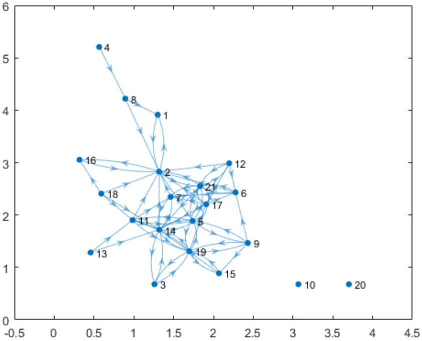

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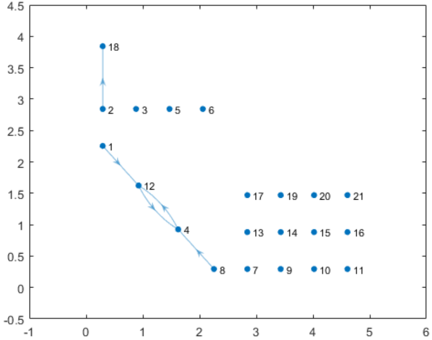

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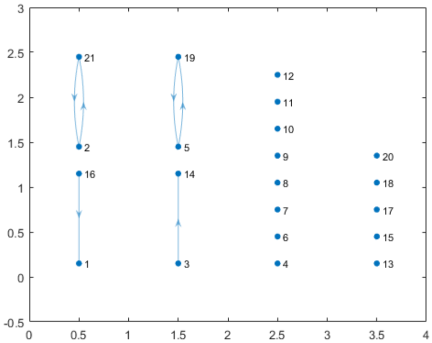

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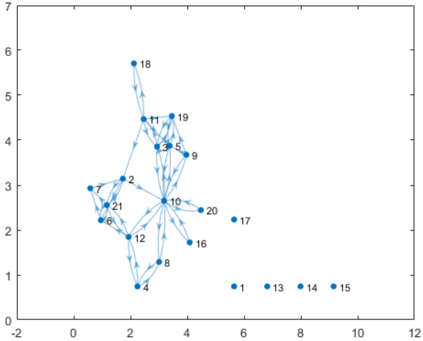

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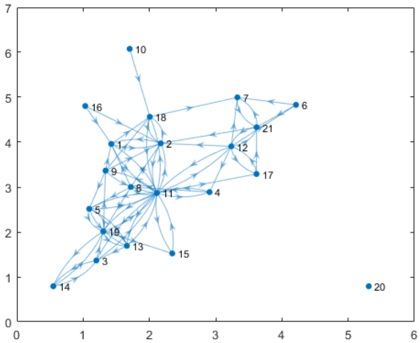

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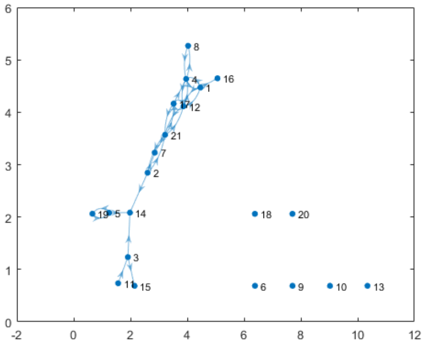

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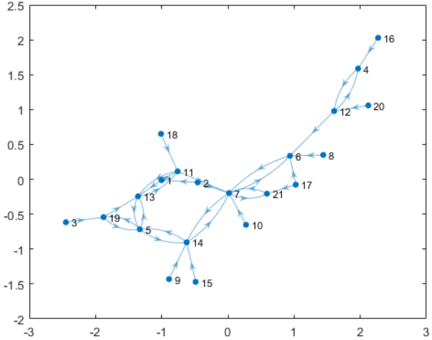

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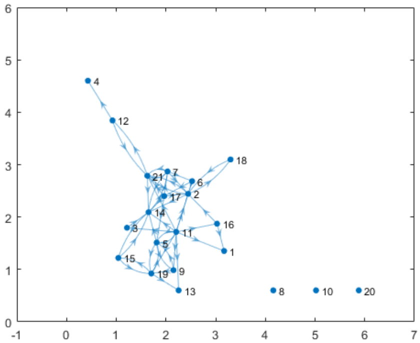

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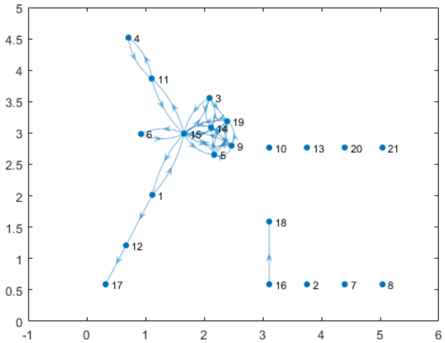

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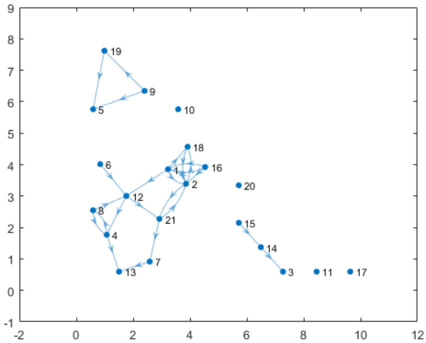

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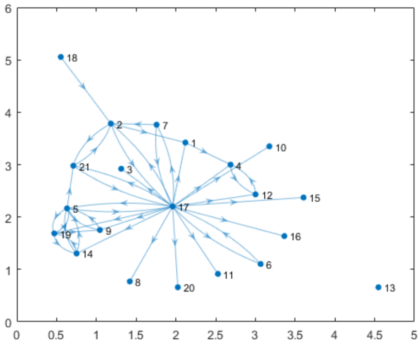

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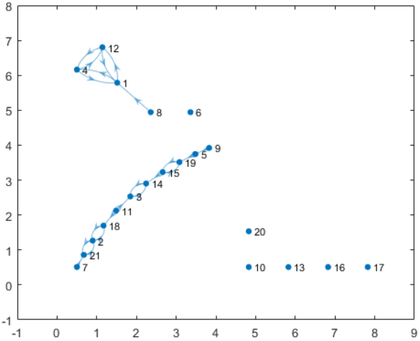

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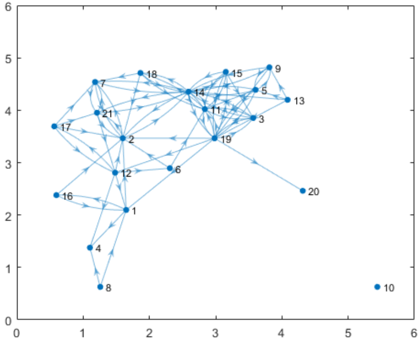

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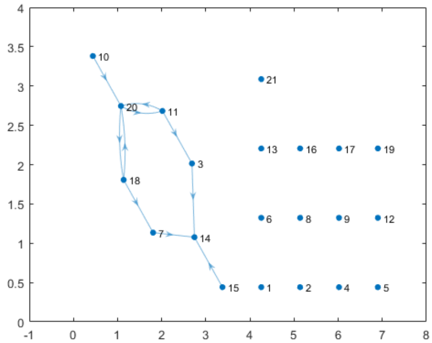

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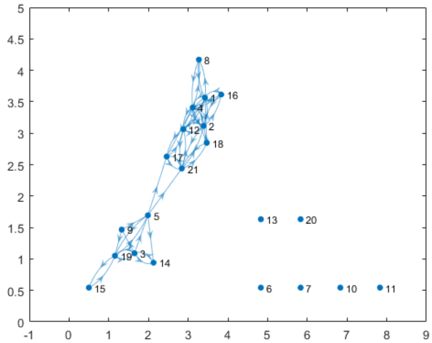

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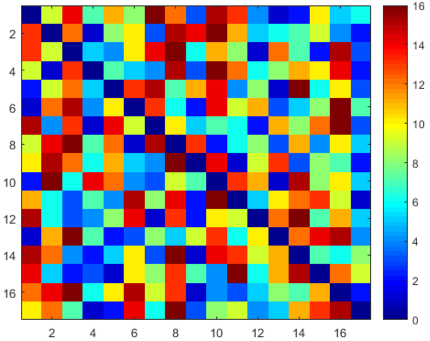
"Friendship" over Time



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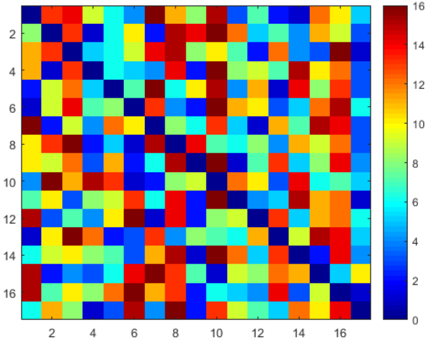

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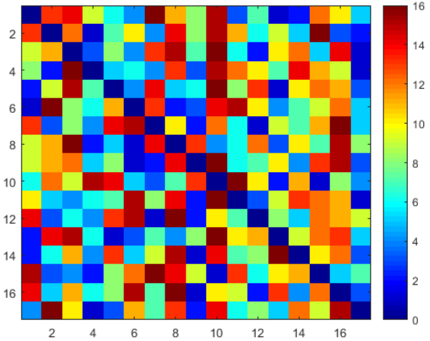

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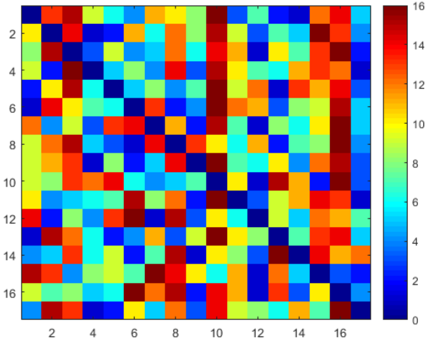

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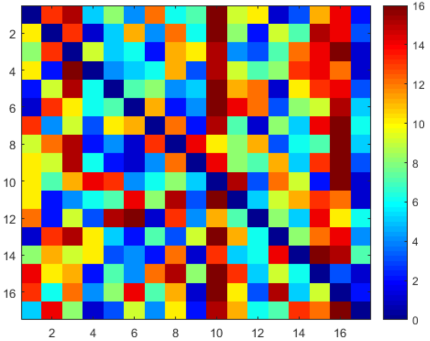

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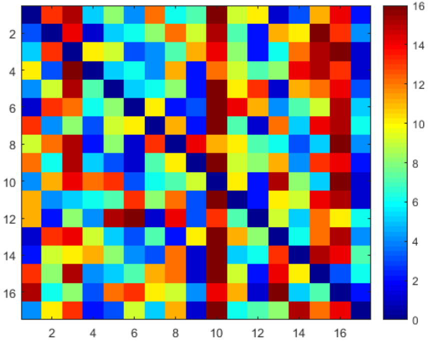

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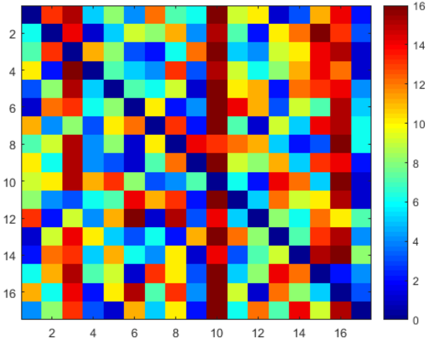

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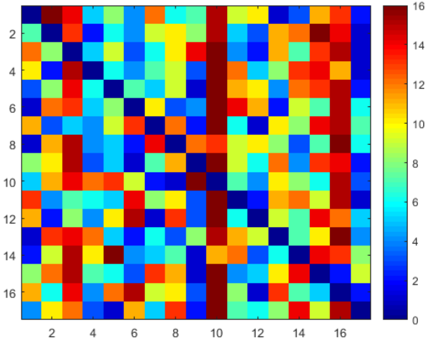

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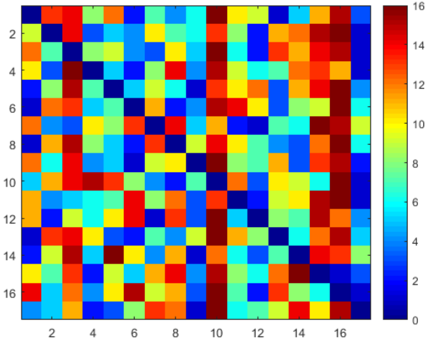

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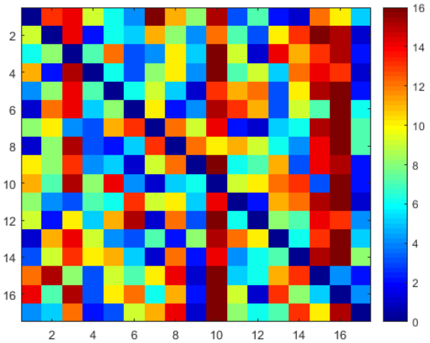

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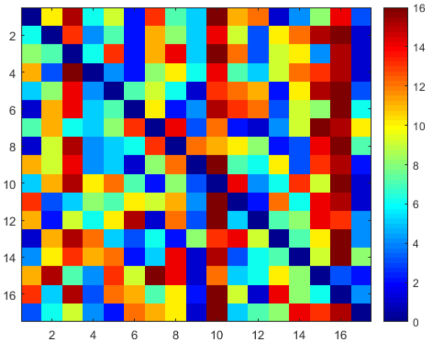

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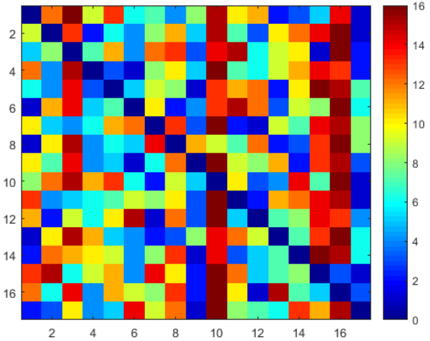

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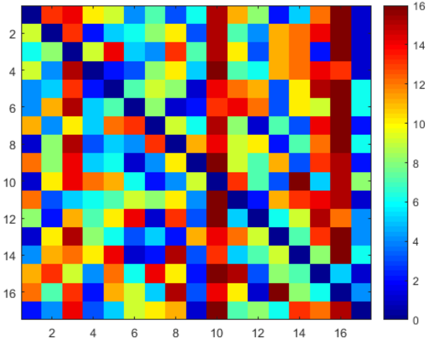

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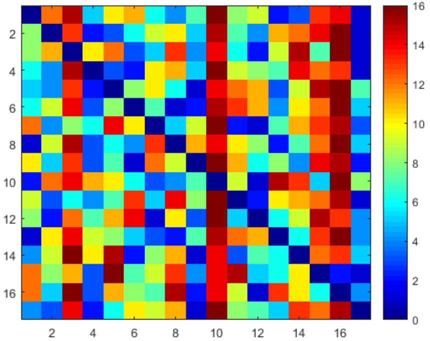

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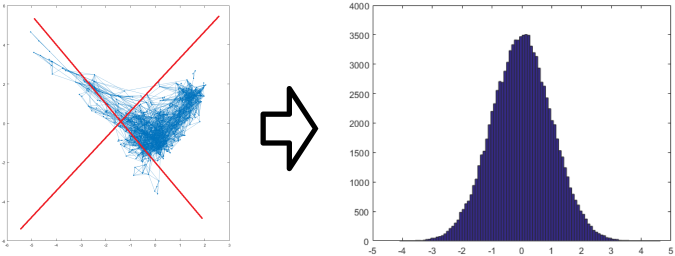
Solution

Solution



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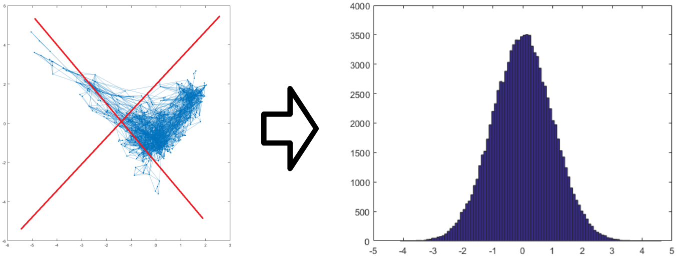

Definition (Null Model)

A random network, parameterized to match some features of a given network, used to compare "expected" network measures.

Erdos–Renyi

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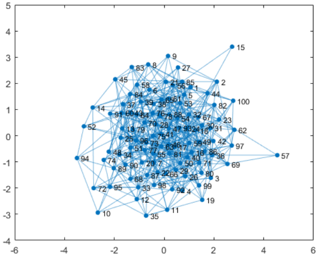
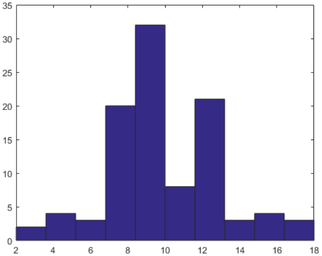


(c) Graph Example



(d) Degree Distribution

Barabasi–Albert (Centrality)

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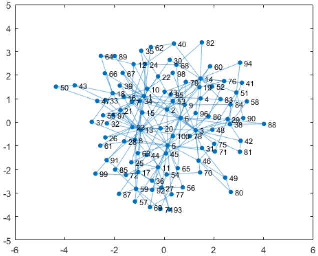

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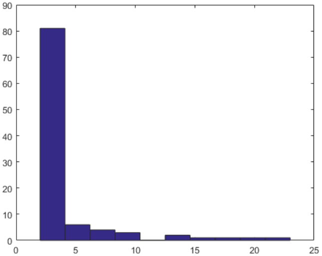

(d) Degree Distribution

Watts–Strogatz (Local Clustering)

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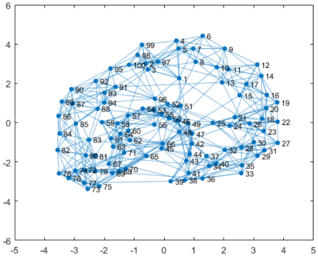

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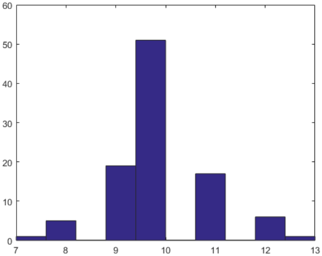

(d) Degree Distribution

Stochastic Block Model (Global Clustering)

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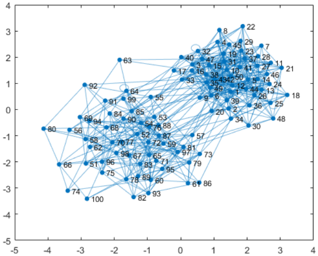

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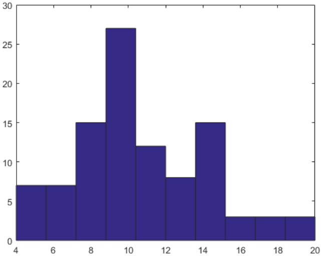

(d) Degree Distribution

Graphons

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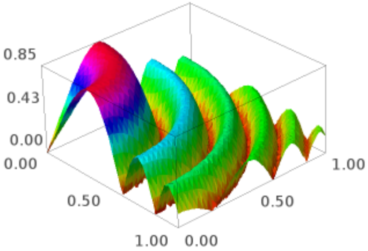

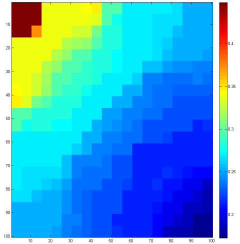

Advantages

- Flexibility
- Parametric models:

$$W(x, y) = a + b(1 - \max\{x, y\}) + c\delta(|x - y| < d)$$

- Compact metric space
- Contains structural models as special cases
- Replace (hard) discrete computations with (easier) continuous ones

RDPM

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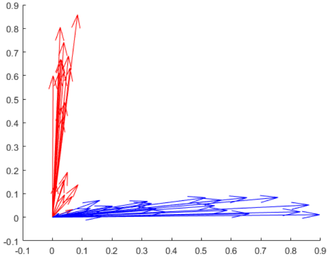
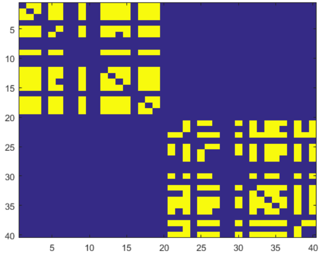
- Angle – Community assignment
- Magnitude – Centrality

Angle – Community Assignment

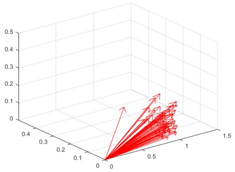


(a) Vectors

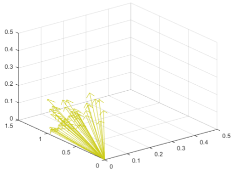


(b) Graph

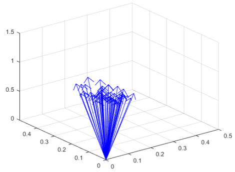
Example: Uniform Noise



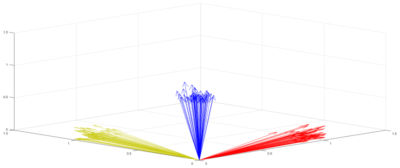
(a) Community 1 Vectors



(b) Community 2 Vectors



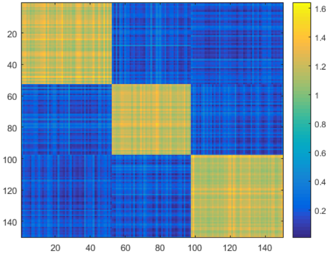
(c) Community 3 Vectors



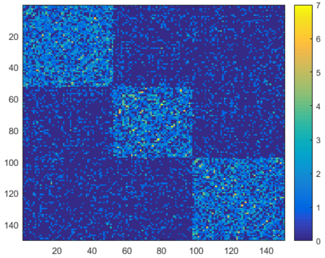
(d) All Vectors

Figure

Example: Uniform Noise

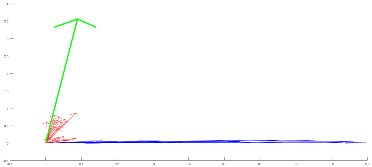


(a) Dot Products



(b) WRDPM Network

Magnitude – Centrality



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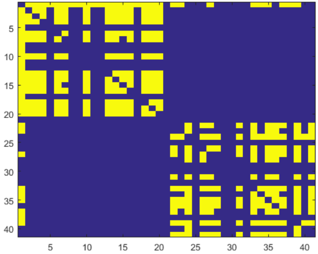
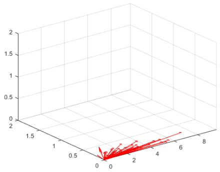


(b) Graph

Example: Multiresolution Communities



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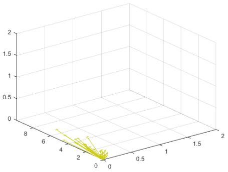
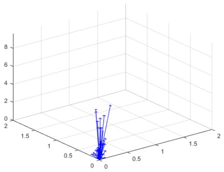
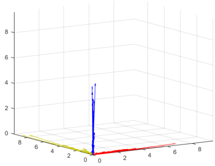


(b) Community 2 Vectors

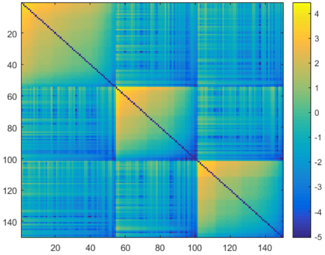


(c) Community 3 Vectors

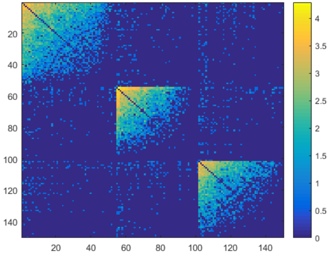


(d) All Vectors

Example: Multiresolution Communities



(a) Dot Products



(b) WRDPM Network

Edge Parameterized Models

Theorem

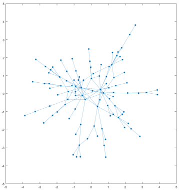
Let n be a fixed positive integer. For each pair (i, j) with $1 \leq i < j \leq n$ let $a_{i,j} = a_{j,i} \in \mathbb{R}$. Then there exist n real numbers $a_{\ell,\ell}$ for $1 \leq \ell \leq n$ such that the matrix $A_{i,j} = a_{i,j}$ is positive definite.

Corollary

Any generative network model, on a fixed number of nodes n, where the edge weight between each pair of nodes is drawn independently from a fixed probability distribution, possibly with different parameters for each pair, can be realized under the WRDPM.

D. DeFord and D. Rockmore, A Random Dot Product Model for Weighted Networks, with D. Rockmore, arXiv:1611.02530, (2016).

Unweighted Collaboration Network



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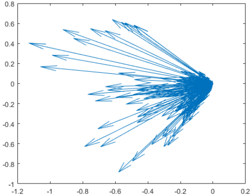
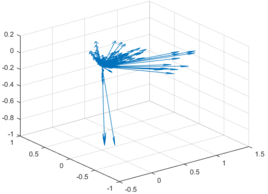


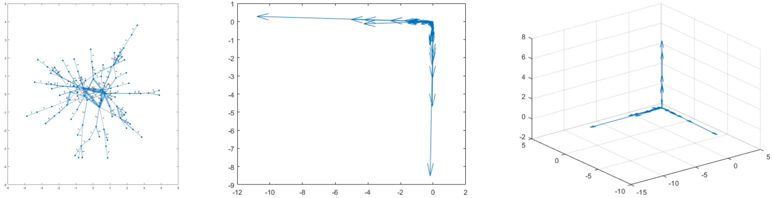
(b) Unweighted 2–Embedding



(c) Unweighted 3–Embedding

V. Batagelj and A. Mrvar: *Pajek datasets*, (2006).

Weighted Collaboration Network



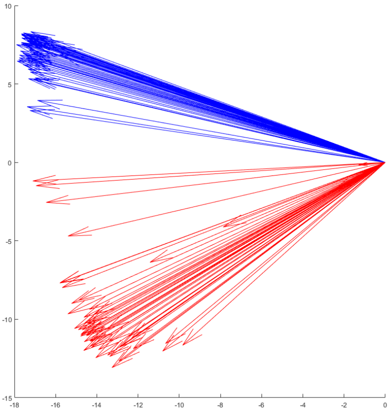
(a) Collaboration Network

(b) Weighted 2–Embedding

(c) Weighted 3–Embedding

V. Batagelj and A. Mrvar: *Pajek datasets*, (2006).

Voting Data



J. Lewis and K. Poole: *Roll Call Data*,

voteview.com/dwnl.html.

That's all...

Thank You!

Dimension Selection

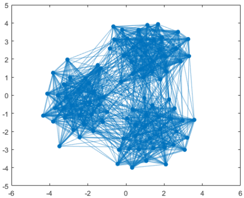
Since the dimension of the embedding is intrinsically related to the realized community structure it is natural to try and make use of this relationship to determine the right choice of d. Motivated by the case of disjoint communities, where if we have an effective, normalized embedding we should have

$$\langle X_i, X_j \rangle = \begin{cases} 1 & \text{i and j belong to the same community} \\ 0 & \text{i and j belong to different communities} \end{cases}$$

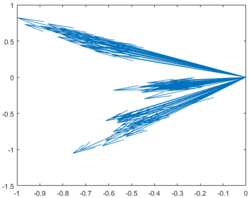
Thus, the sum of intra–community dot products should be $\sum_{i=1}^{\ell} \binom{z_\ell}{2}$. Similarly, the sum of the inter–community dot products should be 0. we define a stress function s depending on the community assignments after embedding.

$$s(d) = \sum_{i=1}^{d} \binom{z_i}{2} - \mathrm{s}_{\text{intra}}(d) + \mathrm{s}_{\text{inter}}(d)$$

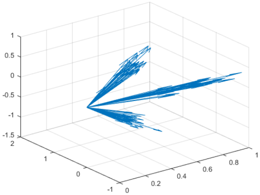
Dimension Example



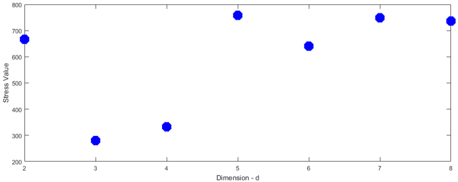
(a) Weighted Network



(b) 2-Embedding



(c) 3-Embedding



(d) Stress Function

Coauthorship Revisited

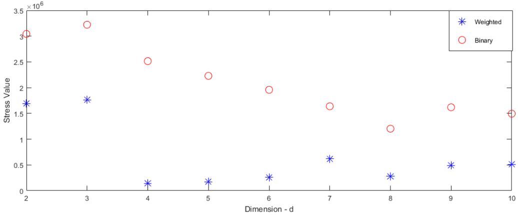


Figure: Comparison of stress values for the computational geometry coauthorship network between the weighted and unweighted realizations. The weighted embedding significantly outperforms the binarized model.