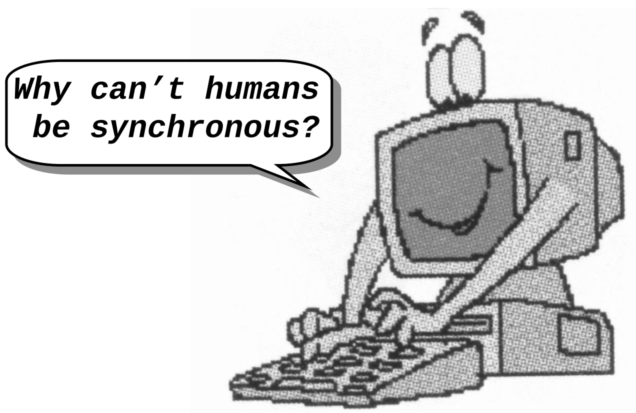
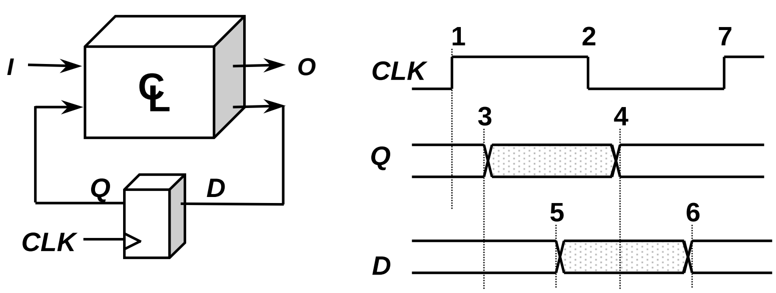
Asynchrony and Arbitration



1

Recap: Flip-Flop Timing Rules



$t_{13} \geq t_{cd\ C\text{-}Q}$ $t_{35} \geq t_{cd\ C.L.}$ $t_{14} \leq t_{pd\ C\text{-}Q}$ $t_{46} \leq t_{pd\ C.L}$

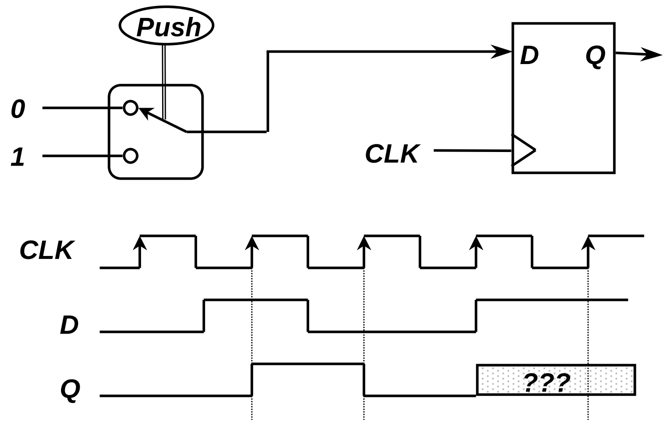
Timing rules for flip-flop dictate:

$t_{67} \geq t_s$ $t_{15} \geq t_h$

2

Asynchronous Inputs

The specifications of a system may <u>require</u> sampling an asynchronous input!



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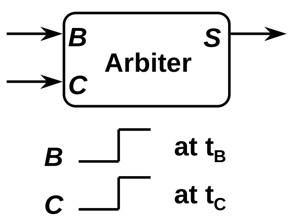
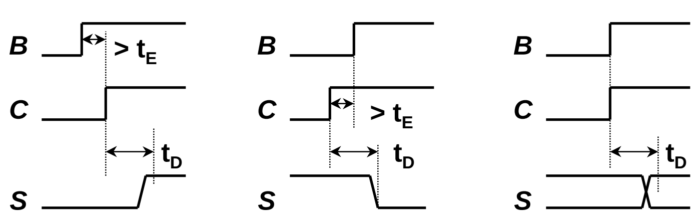
Asynchronous Bounded-Time Arbiter



Arbiter Specifications:
- finite t_D (decision time)
- finite t_E (allowable error)
- Value of S at time $t_C + t_D$
  - 1 if $t_B < t_C - t_E$
  - 0 if $t_B > t_C + t_E$
  - 0 or 1 otherwise



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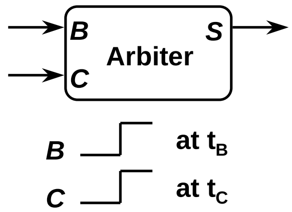
Asynchronous Bounded-Time Arbiter - 2



Arbiter Specifications:
- **finite t_D (decision time)**
- **finite t_E (allowable error)**
- **Value of S at time $t_C + t_D$**
  - **1 if $t_B < t_C - t_E$**
  - **0 if $t_B > t_C + t_E$**
  - **0 or 1 otherwise**

For NO finite values of t_E and t_D is this specification realizable!

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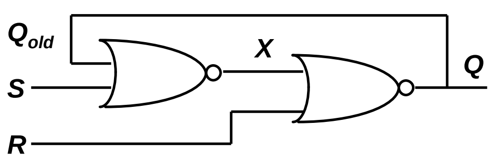
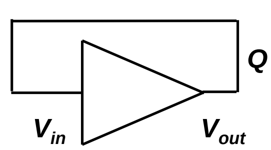
R-S Latch in Flip-Flop



S	*R*	*Q*
0	*0*	*Q_{old}*
0	*1*	*0*
1	*0*	*1*
1	*1*	*0*

Can be 0, 1 or ________

Model:



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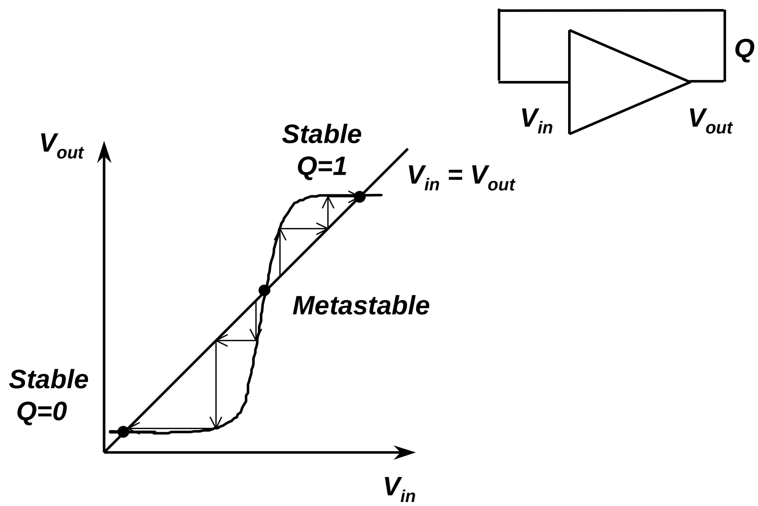
Possible States of R-S Latch



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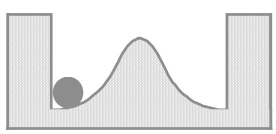
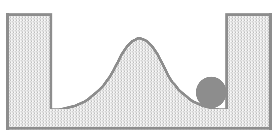
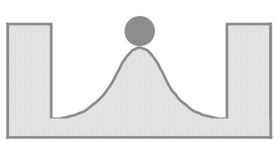
Mechanical Analog



Stable Equilibrium 0



Stable Equilibrium 1



Metastable Equilibrium

- **Probability of staying metastable falls off as $e^{-\alpha t}$**
- **Recovery time <u>unbounded</u>, but influenced by gain, noise etc**

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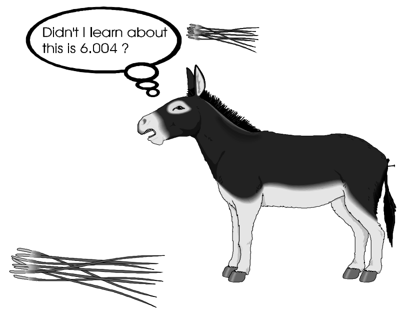
Buridan's Ass



Buridan, Jean (1300-58), French Scholastic philosopher, who held a theory of determinism, contending that the will must choose the greater good. Born in Bethune, he was educated at the University of Paris, where he studied with the English Scholastic philosopher William of Ockham. After his studies were completed, he was appointed professor of philosophy, and later rector, at the same university. Buridan is traditionally but probably incorrectly associated with a philosophical dilemma of moral choice called "Buridan's ass." In the problem an ass starves to death between two alluring bundles of hay because it does not have the will to decide which one to eat.

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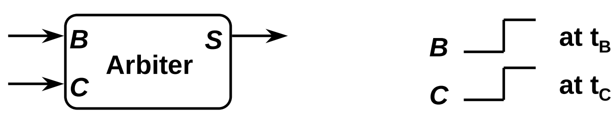
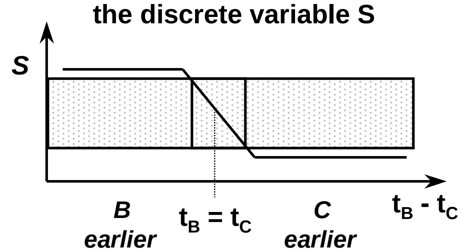
Metastability and Arbitration



Mapping the continuous variable (t_B - t_C) to the discrete variable S



Need a forbidden zone in the (t_B - t_C) time domain to ensure bistable S!

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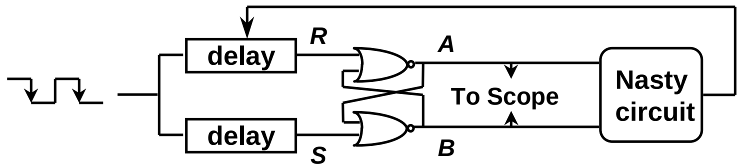
Demonstration Circuit



- **Nasty Circuit:**
  - Apply nearly simultaneous *R*, *S* transitions
  - If *A* ends up 0, speed up *R* transition slightly
  - If *A* ends up 1, delay *R* transition slightly
- **Result:**
  - Gravitate to timing which yields 50% 1's for A
  - Force flip-flop to make "tough" arbitrations
  - Maximize metastable behavior

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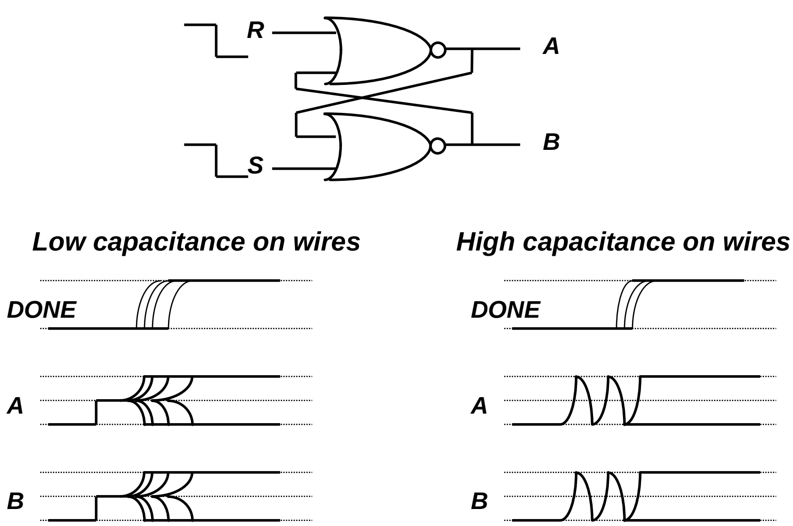
Observed Behavior



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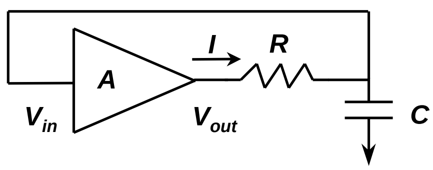
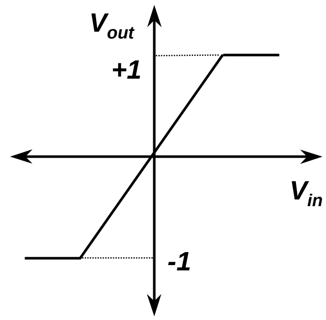
A Simple Model of Metastability





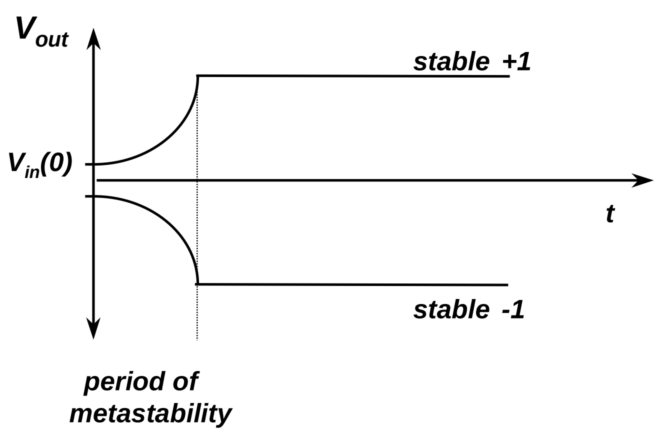
Assume $V_{out} = A\, V_{in}$ (not saturated)

$$I = \frac{(V_{out} - V_{in})}{R} = \frac{V_{in}\,(A - 1)}{R} = C\,\frac{dV_{in}}{dt}$$

$$V_{in} = V_{in}(0)\, e^{\,t\,(A-1)/RC}$$

Evolution of V_{out}

$$V_{out} = A\, V_{in}(0)\, e^{\,t\,(A-1)/RC}$$ **before saturation**



Probability of Staying Metastable

$$V_{out} = A\ V_{in}(0)\ e^{t(A-1)/RC} = A\ V_{in}(0)\ e^{t/\tau}$$

How small must $V_{in}(0)$ be for saturation to take longer than time T ?

$$|V_{in}(0)| < \frac{1}{A} e^{-T/\tau}$$

Assume $V_{in}(0)$ is uniformly distributed over *{-1, 1}*

Probability of staying metastable after T is: ______
(probability of failure)

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Suppose we wait 100 ns...

$$\text{Probability of failure} = p\left(|V_{in}(0)| < \frac{e^{-T/\tau}}{A}\right) = \frac{e^{-T/\tau}}{A}$$

$T = 100$ ns

$\tau = 1$ ns

$A = 10$

$$\text{Probability of failure} = \frac{e^{-100}}{10} = 3.72 \times 10^{-45}$$

At 10^8 input samples per second,
one failure every 10^{30} years!

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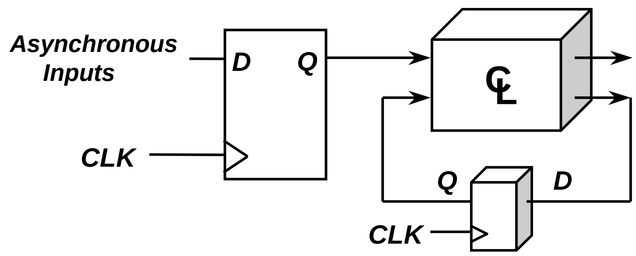
Simple and Practical Solution



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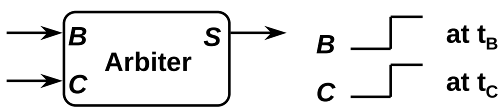
What we can't and can build

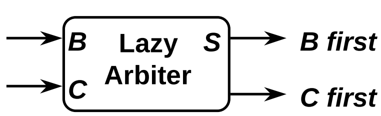
Cannot build a Bounded-Time Asynchronous Arbiter



$S = 1$ if $t_B < t_C - t_E$
$S = 0$ if $t_B > t_C + t_E$
else $S = 0$ or 1

S valid at time T after MAX(t_B, t_C)

Can build an Unbounded-Time Asynchronous Arbiter



Exactly one output becomes 1 eventually, other remains at stable 0

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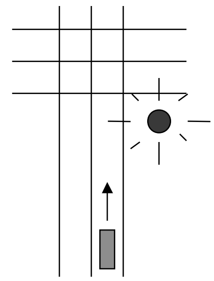
Everyday Metastability: Traffic Light Problem

GIVEN:
- **Traffic light: GREEN, ORANGE, RED sequence**
- **55 MPH Speed Limit**
- **Sufficiently long ORANGE, GREEN periods**



IS IT POSSIBLE TO RELIABLY AVOID RUNNING A RED LIGHT ...

1. **Under ANY circumstances**
2. **Without blocking intersection**
3. **Assuming you don't slow down when the light is continuously GREEN**

PLAUSIBLE STRATEGIES:

A. Move at 55. At calculated distance D from light, sample color (using an unbounded-time arbiter). GO ONLY WHEN stable GREEN.

B. GO on GREEN; STOP otherwise.

C. Stop 1 foot before light. On GREEN, gun it.

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