Introduction to Machine Learning, Fall 2012

Problem Set 6: Hidden Markov Models

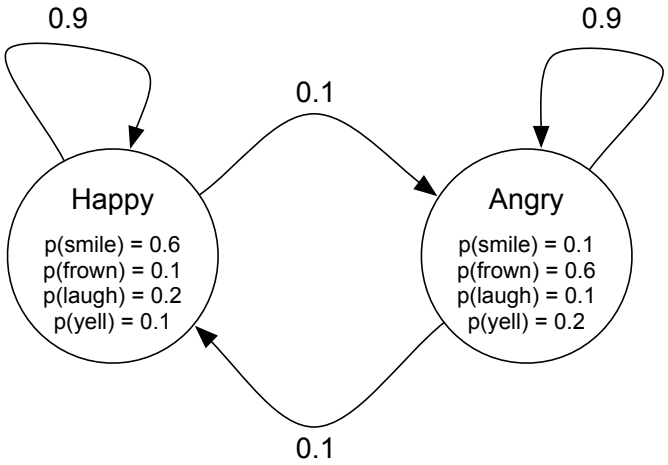
Due: Tuesday, December 11, 2012 by 11am (in class, *before* class begins)

Important: *See problem set policy on the course web site.* You must show **all** of your work and be rigorous in your writeups to obtain full credit.

Amy lives a simple life. Some days she is Angry and some days she is Happy. But she hides her emotional state, and so all we can observe is whether she smiles, frowns, laughs, or yells. Amy's best friend is utterly confused about whether Amy is actually happy or angry and decides to model her emotional state using a hidden Markov model.

Let $X_d \in \{\text{Happy, Angry}\}$ denote Amy's emotional state on day d, and let $Y_d \in \{\text{smile, frown, laugh, yell}\}$ denote the observation made about Amy on day d. **Assume that on day 1 Amy is in the Happy state**, i.e. $X_1 = \text{Happy}$. Furthermore, assume that Amy transitions between states exactly once per day (staying in the same state is an option) according to the following distribution: $p(X_{d+1} = \text{Happy} \mid X_d = \text{Angry}) = 0.1$, $p(X_{d+1} = \text{Angry} \mid X_d = \text{Happy}) = 0.1$, $p(X_{d+1} = \text{Angry} \mid X_d = \text{Angry}) = 0.9$, and $p(X_{d+1} = \text{Happy} \mid X_d = \text{Happy}) = 0.9$.

The observation distribution for Amy's Happy state is given by $p(Y_d = \text{smile} \mid X_d = \text{Happy}) = 0.6, p(Y_d = \text{frown} \mid X_d = \text{Happy}) = 0.1, p(Y_d = \text{laugh} \mid X_d = \text{Happy}) = 0.2$, and $p(Y_d = \text{yell} \mid X_d = \text{Happy}) = 0.1$. The observation distribution for Amy's Angry state is $p(Y_d = \text{smile} \mid X_d = \text{Angry}) = 0.1, p(Y_d = \text{frown} \mid X_d = \text{Angry}) = 0.6, p(Y_d = \text{laugh} \mid X_d = \text{Angry}) = 0.1$, and $p(Y_d = \text{yell} \mid X_d = \text{Angry}) = 0.2$. **All of this is summarized in the following figure**:



Each question below is worth 5 points. Be sure to show all of your work!

1. What is $p(X_2 = \text{Happy})$?
2. What is $p(Y_2 = \text{frown})$?
3. What is $p(X_2 = \text{Happy} \mid Y_2 = \text{frown})$?
4. What is $p(Y_{100} = \text{yell})$?
5. Assume that $Y_1 = Y_2 = Y_3 = Y_4 = Y_5 = \text{frown}$. What is the most likely sequence of the states? That is, compute the MAP assignment $\arg\max_{x_1,\ldots,x_5} p(X_1 = x_1, \ldots, X_5 = x_5 \mid Y_1 = Y_2 = Y_3 = Y_4 = Y_5 = \text{frown})$.