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A Frequency Analysis of Light Transport

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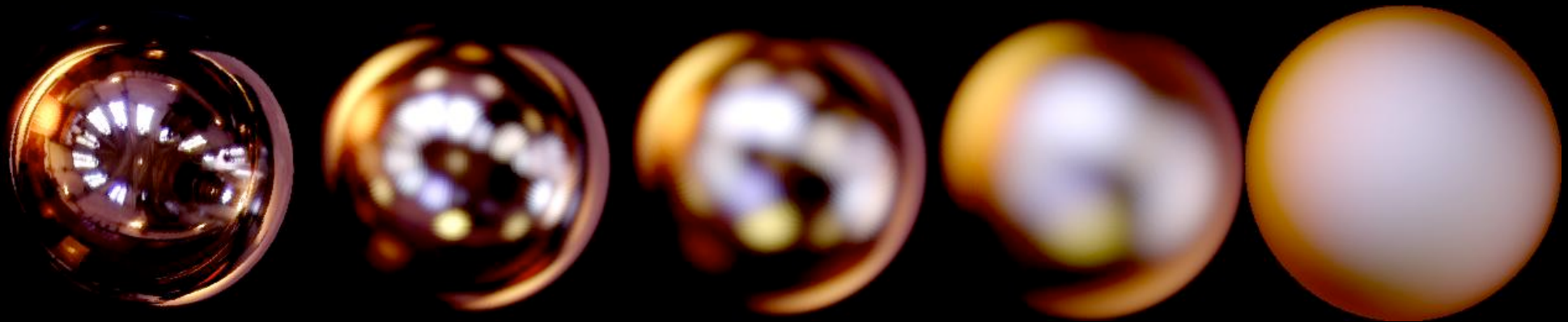
F. Sillion, ARTIS/GRAVIR-IMAG INRIA



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Illumination effects

- Blurry reflections:



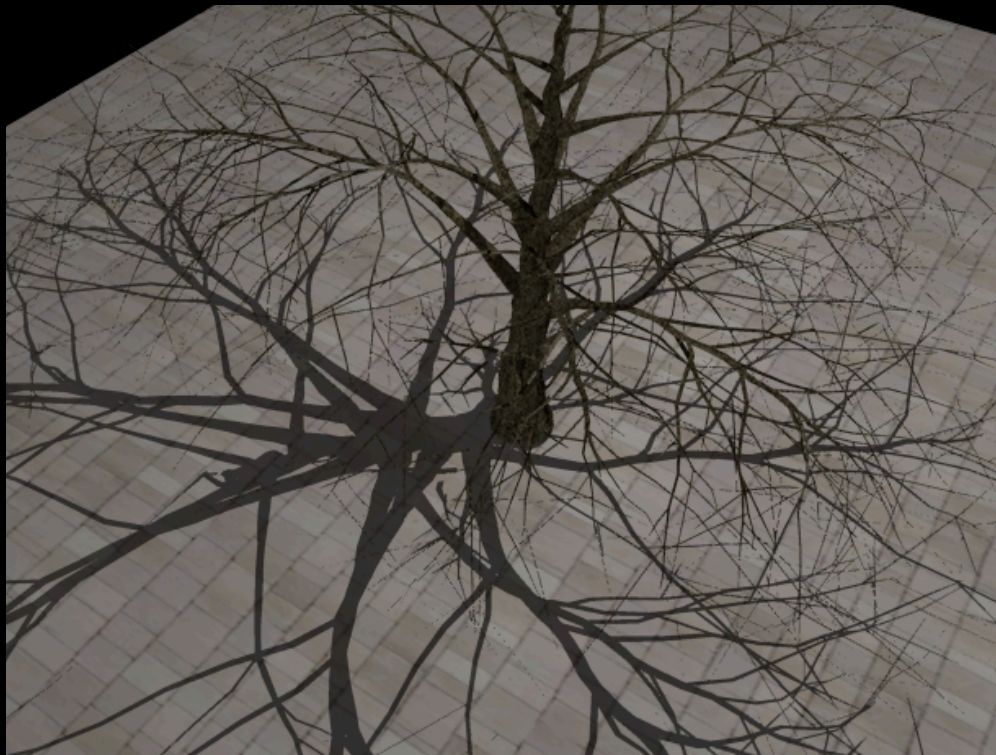
From [Ramamoorthi and Hanrahan 2001]



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Illumination effects

- Shadow boundaries:



Point light source



Area light source



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Illumination effects

- Indirect lighting is usually blurry:



Complete lighting



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Illumination effects

- Indirect lighting is usually blurry:



Direct lighting only



Indirect lighting only

Frequency aspects of light transport



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- Blurriness = frequency content
 - Sharp variations: high frequency
 - Smooth variations: low frequency
- All effects are expressed as frequency content:
 - Diffuse shading: low frequency
 - Shadows: introduce high frequencies
 - Indirect lighting: tends to be low frequency



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Problem statement

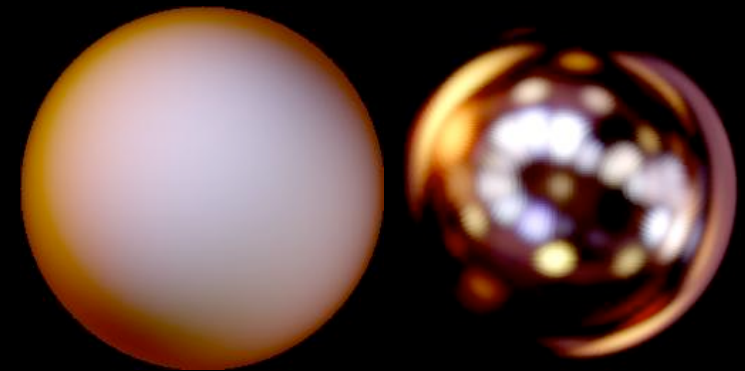
- How does light interaction in a scene explain the frequency content?
- Theoretical framework:
 - Understand the frequency spectrum of the radiance function
 - From equations of light transport



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Unified framework:

- Spatial frequency
(e.g. shadows, textures)
- Angular frequency
(e.g. blurry highlight)



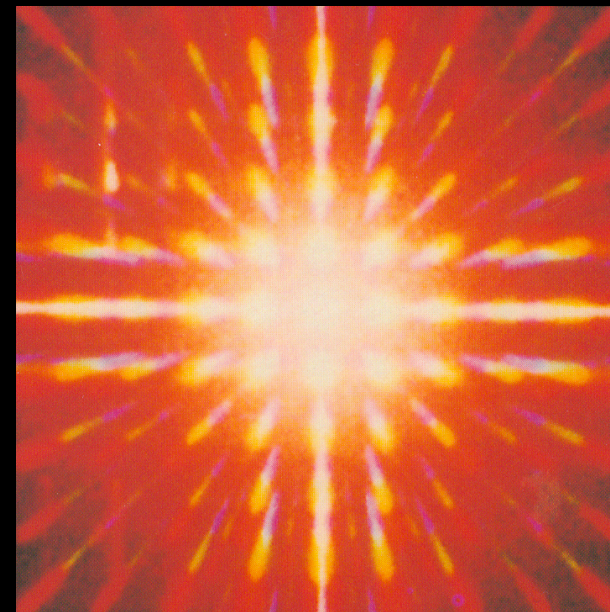
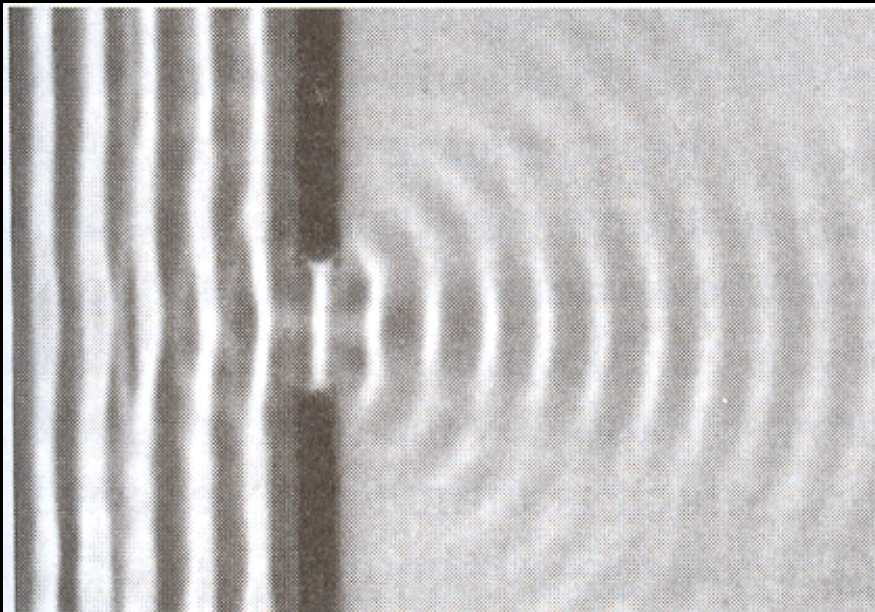
Disclaimer: not Fourier optics



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- We do not consider wave optics, interference, diffraction
- Only geometrical optics

From [Hecht]



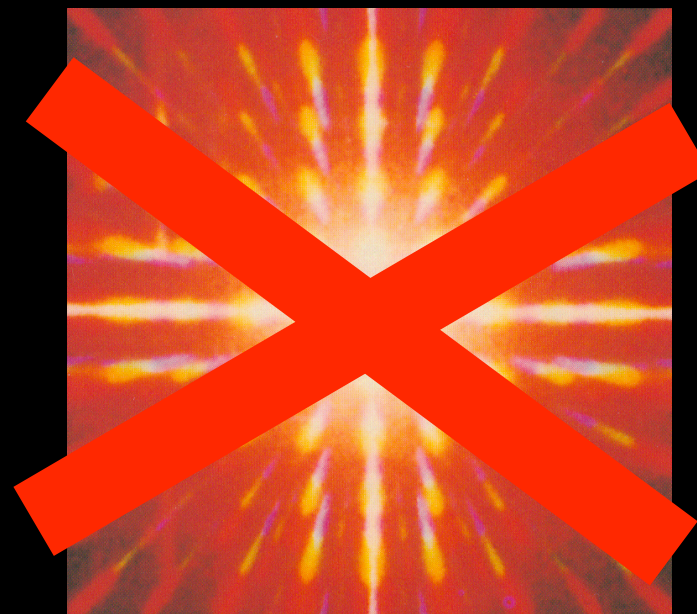
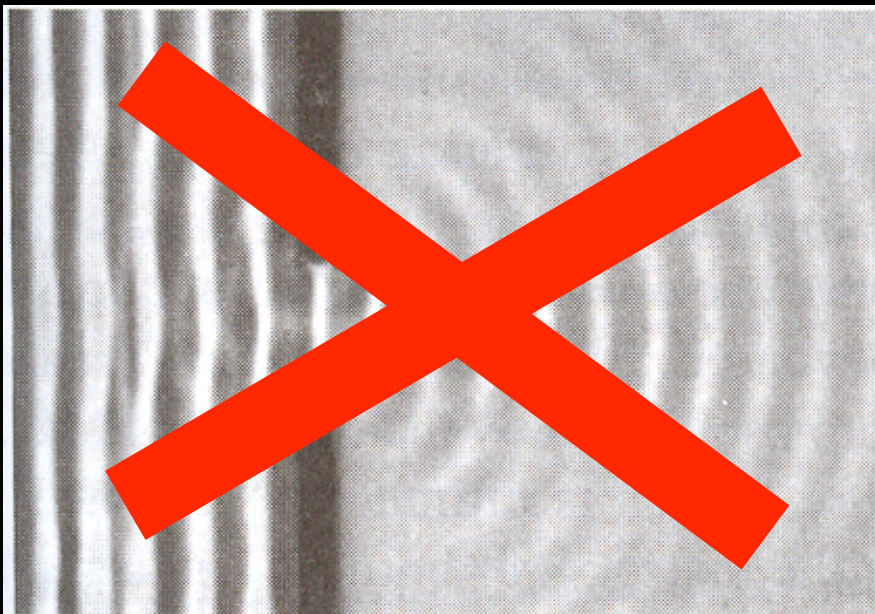
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Overview

- Previous work
- Our approach:
 - Local light field
 - Transformations on local light field
- Case studies:
 - Diffuse soft shadows
 - Adaptive shading sampling
- Conclusions and future directions



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Previous work

- Vast body of literature:
 - Light field sampling
 - Perceptually-based rendering
 - Wavelets for Computer Graphics
 - Irradiance caching
 - Photon mapping
 - ...
- We focus on frequency analysis in graphics:
 - Light field sampling
 - Reflection as a convolution

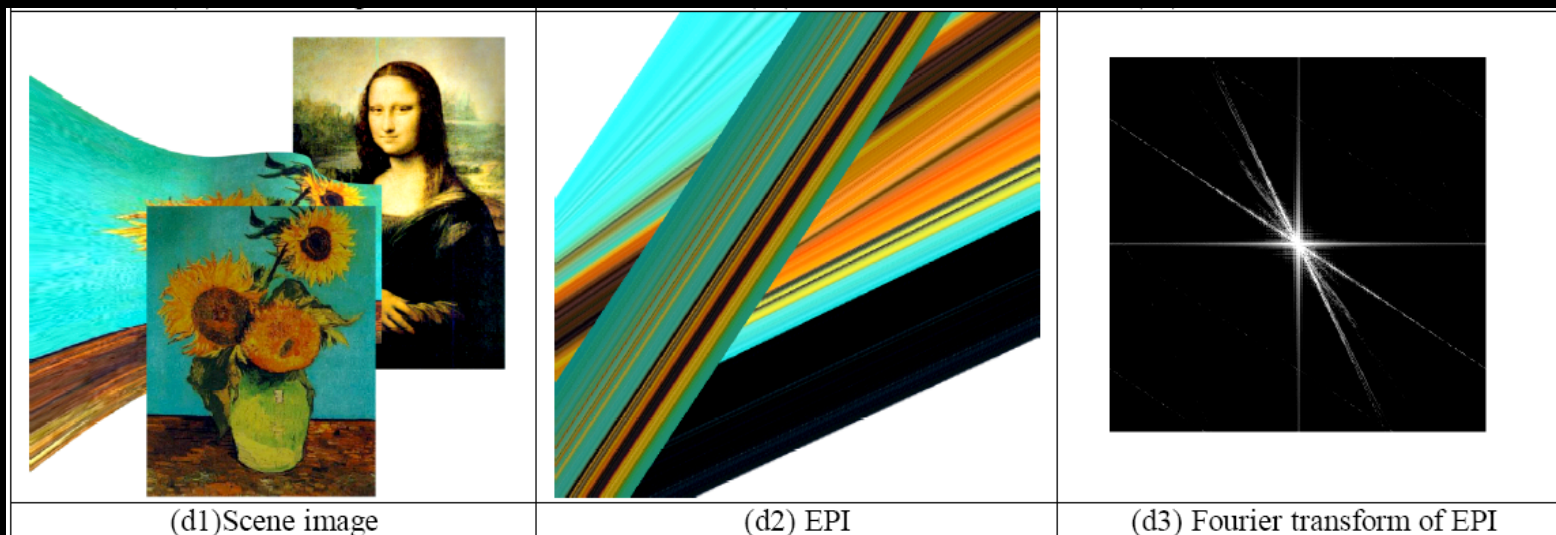


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Light field sampling

[Chai et al. 00, Isaksen et al. 00, Stewart et al. 03]

- Light field spectrum as a function of object distance
- No BRDF, occlusion ignored



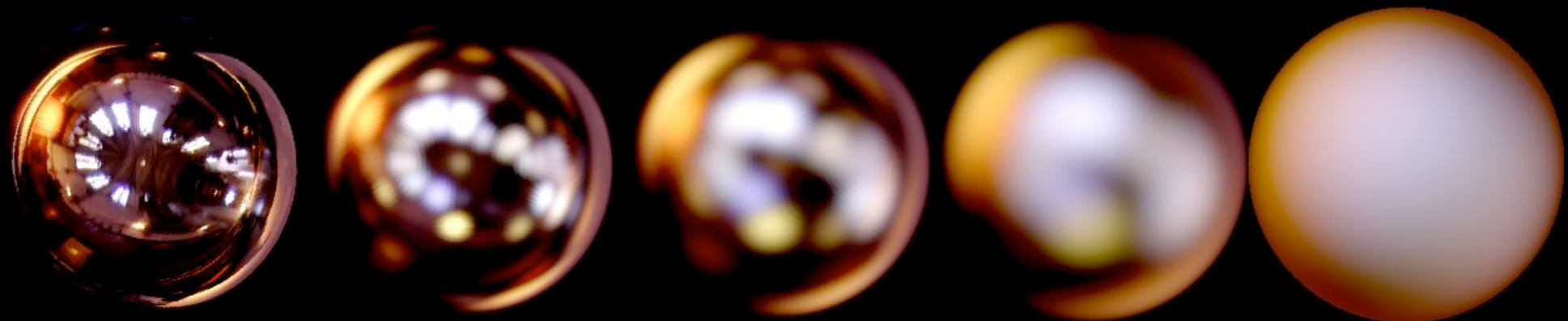
Signal processing for reflection



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[Ramamoorthi & Hanrahan 01, Basri & Jacobs 03]

- Reflection on a curved surface is a convolution
- Direction only



From [Ramamoorthi and Hanrahan 2001]



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Our approach

- Light sources are input signal
- Interactions are filters/transforms
 - Transport
 - Visibility
 - BRDF
 - Etc.



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Light source signal



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Light source signal



Transport



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Our approach

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Light source signal



Transport



Signal 1



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Our approach

- Light sources are input signal
- Interactions are filters/transforms
 - Transport
 - Visibility
 - BRDF
 - Etc.

Light source signal



Transport

Occlusion



Signal 2



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Our approach

- Light sources are input signal
- Interactions are filters/transforms
 - Transport
 - Visibility
 - BRDF
 - Etc.

Light source signal



Transport

Occlusion

Transport



Signal 3



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Our approach

- Light sources are input signal
- Interactions are filters/transforms
 - Transport
 - Visibility
 - BRDF
 - Etc.

Light source signal



Transport

Occlusion

Transport

BRDF

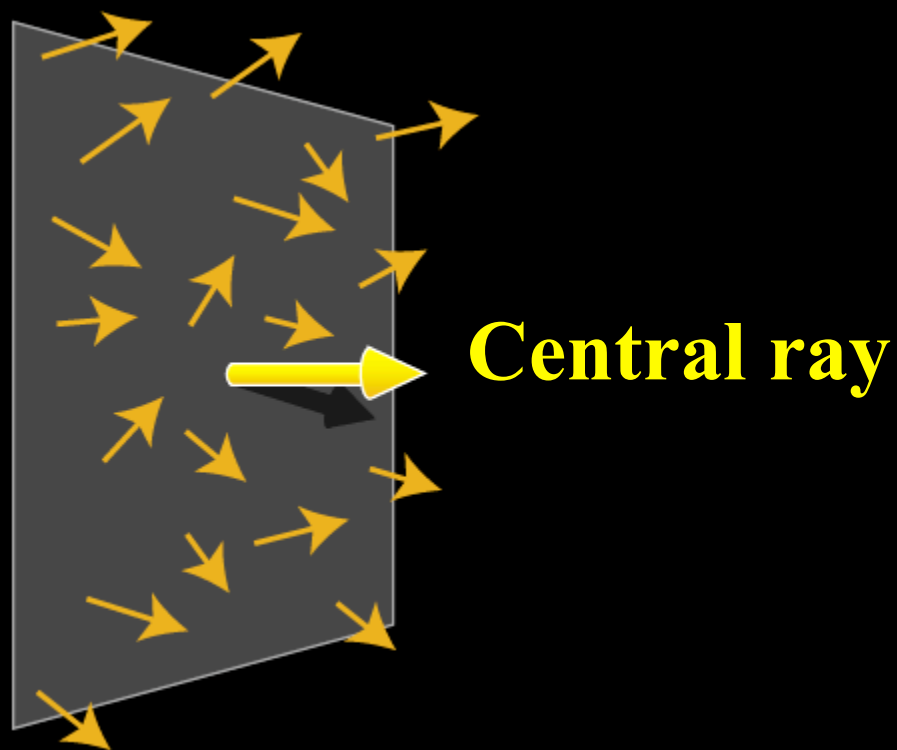


Signal 4



Local light field

- 4D light field, around a *central ray*

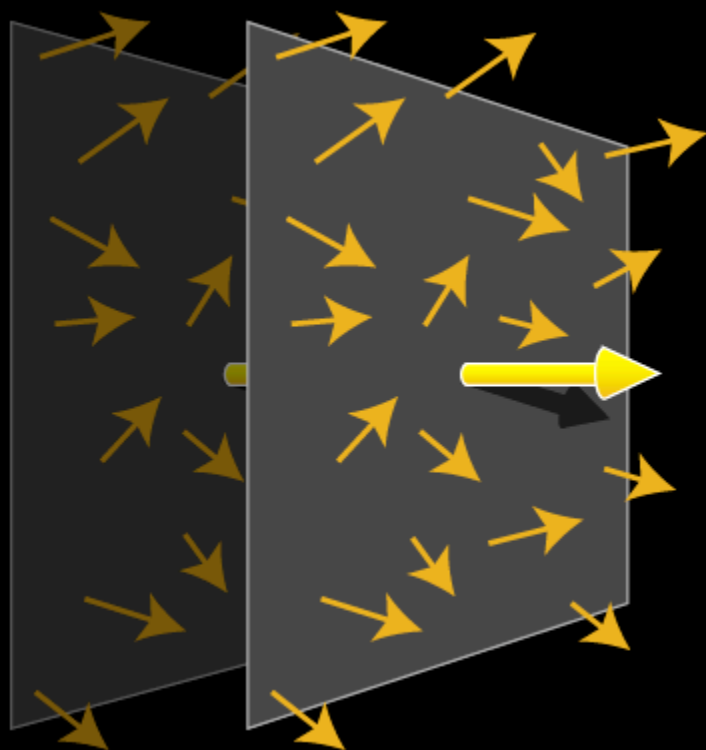




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Local light field

- 4D light field, around a *central ray*
- We study its spectrum during transport

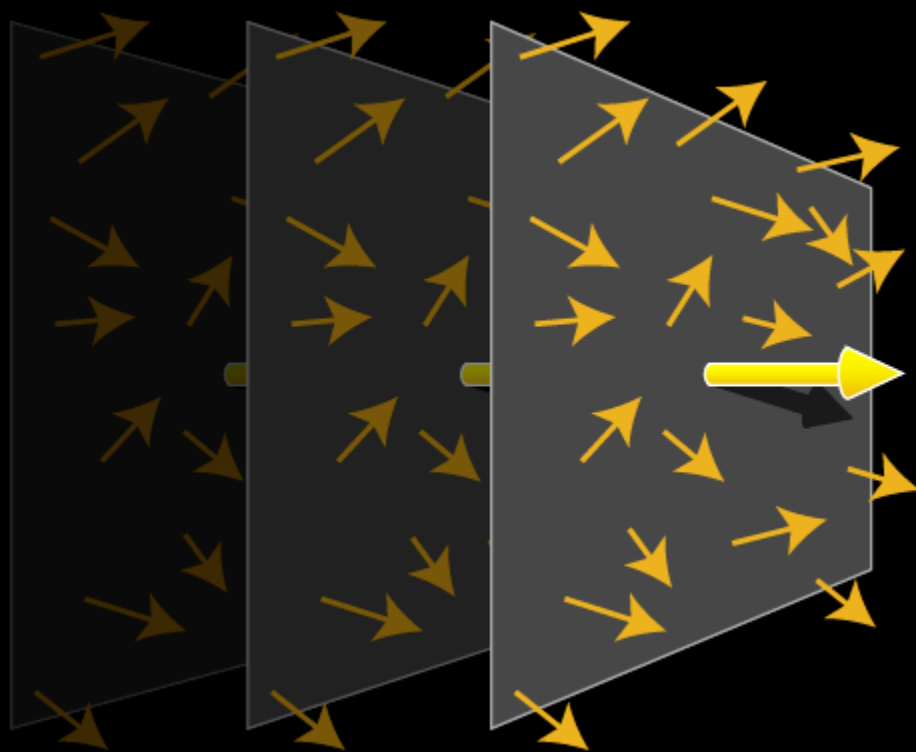




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Local light field

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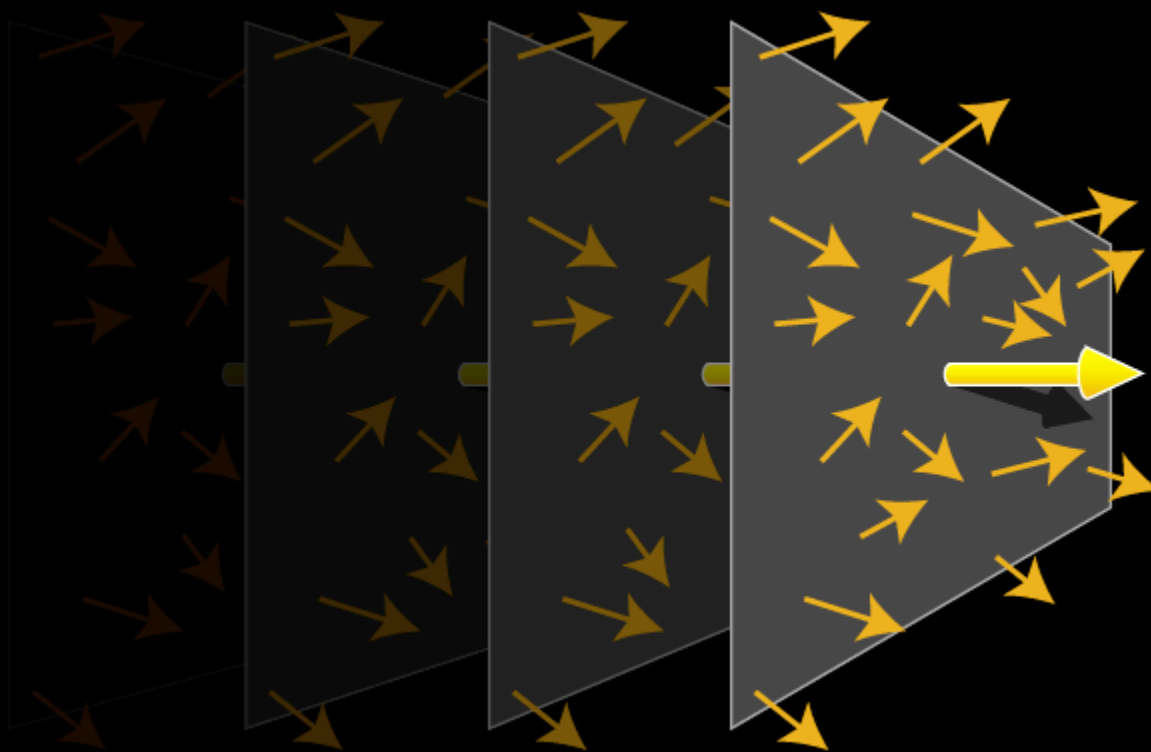




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Local light field

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Local light field

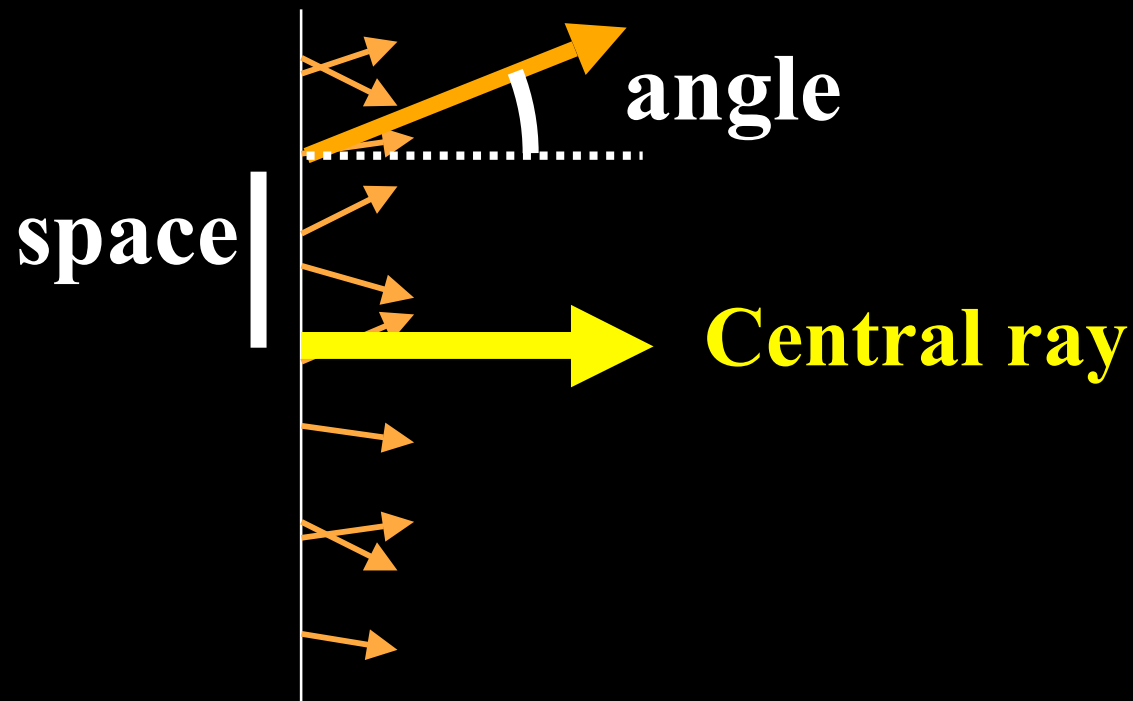
- We give explanations in 2D
 - Local light field is therefore 2D
- See paper for extension to 3D

Local light field parameterization



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- Space and angle



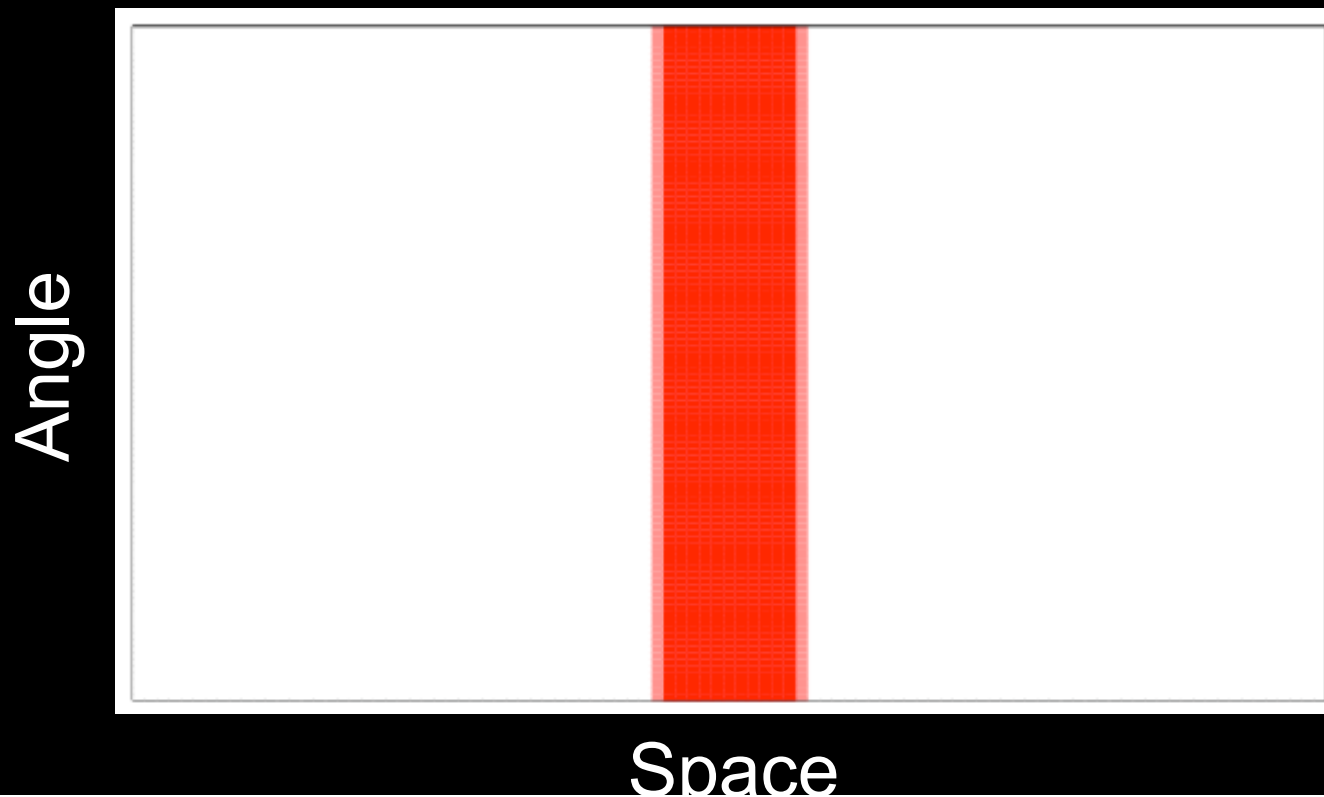
Local light field representation



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- Density plot:

Area light source



Local light field Fourier spectrum



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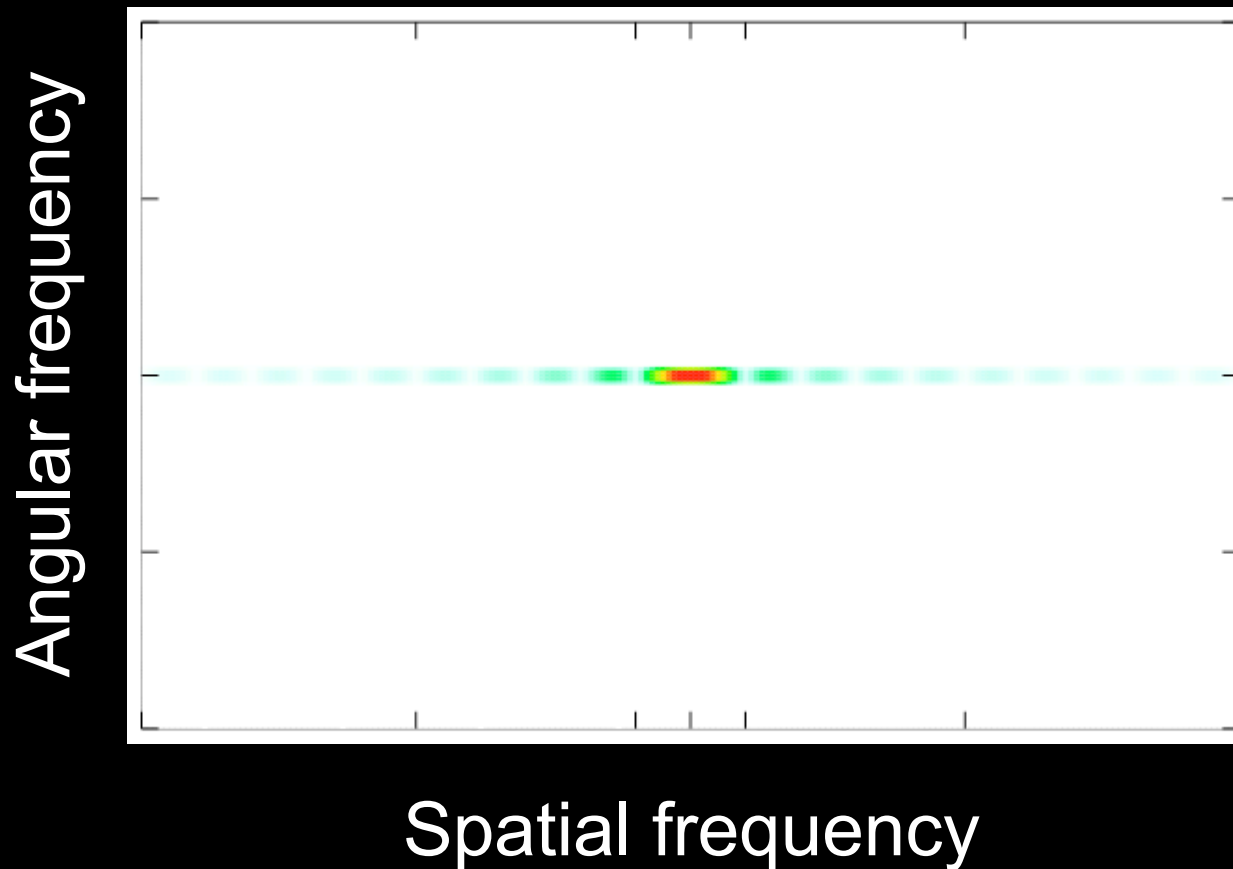
- We are interested in the Fourier spectrum of the local light field
- Also represented as a density plot

Local light field Fourier spectrum



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Spectrum of an area light source:





Fourier analysis 101

- Spectrum corresponds to blurriness:
 - Sharpest feature has size $1/F_{\max}$
- Convolution theorem:
 - Multiplication \leftrightarrow convolution
- Classical spectra:
 - Box \leftrightarrow sinc
 - Dirac \leftrightarrow constant



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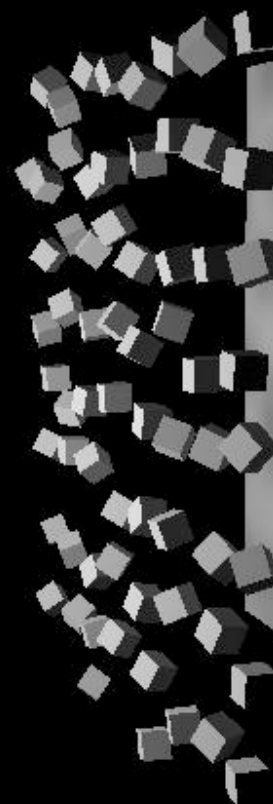
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Example scene

Light source



Blockers



Receiver





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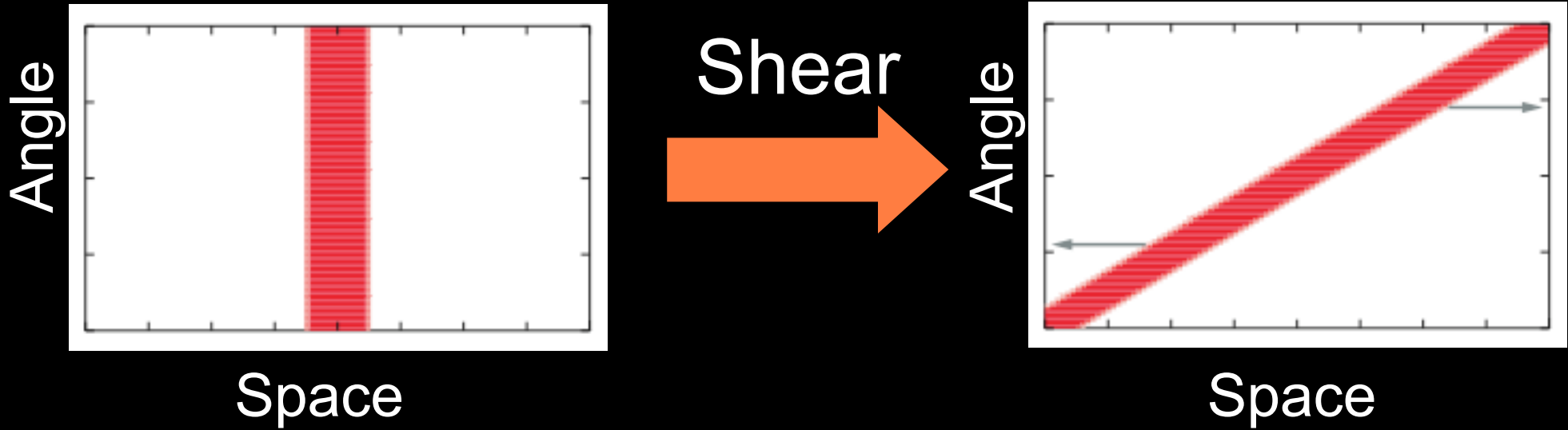
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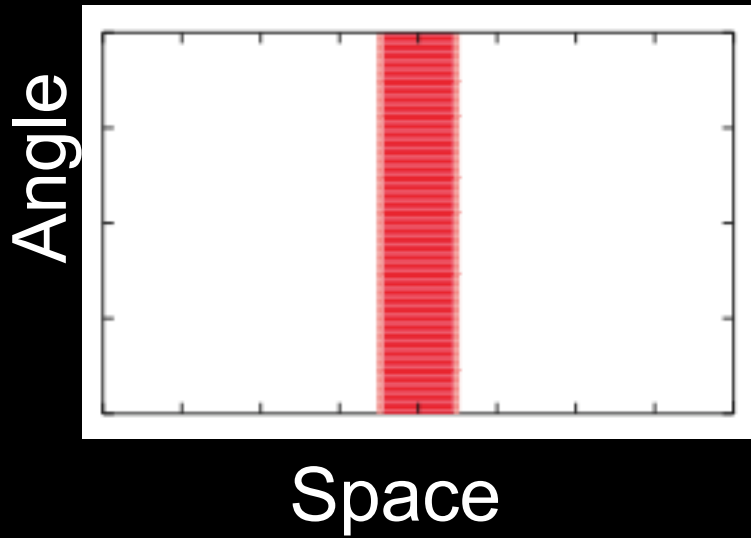
Transport in free space



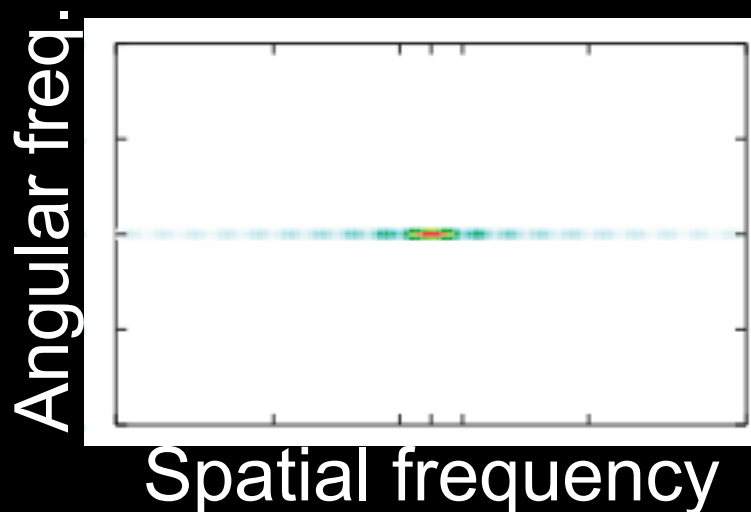
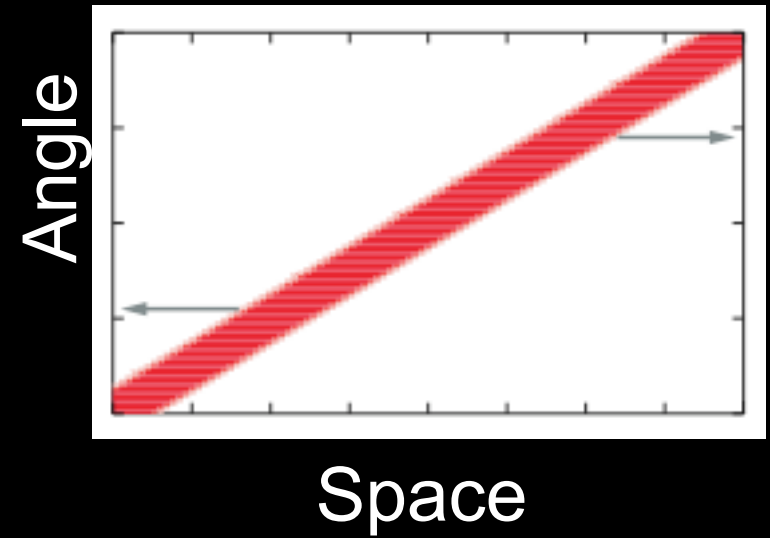


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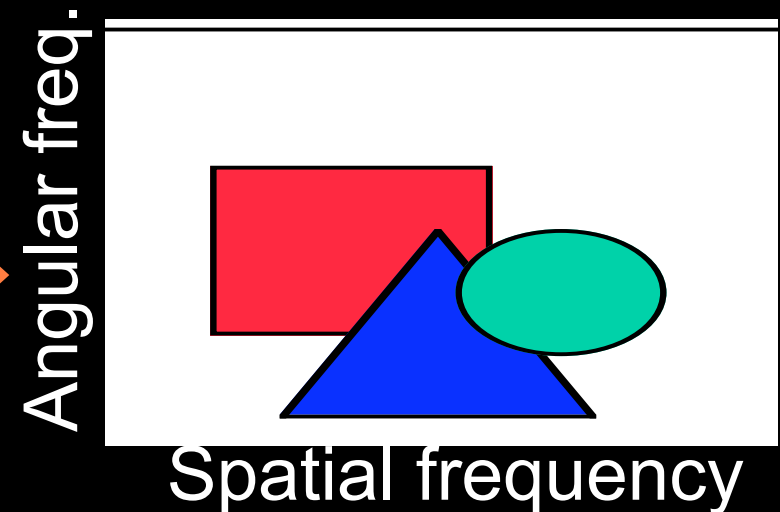
Transport in free space



Shear



Shear



propagation

stopagation



propagation

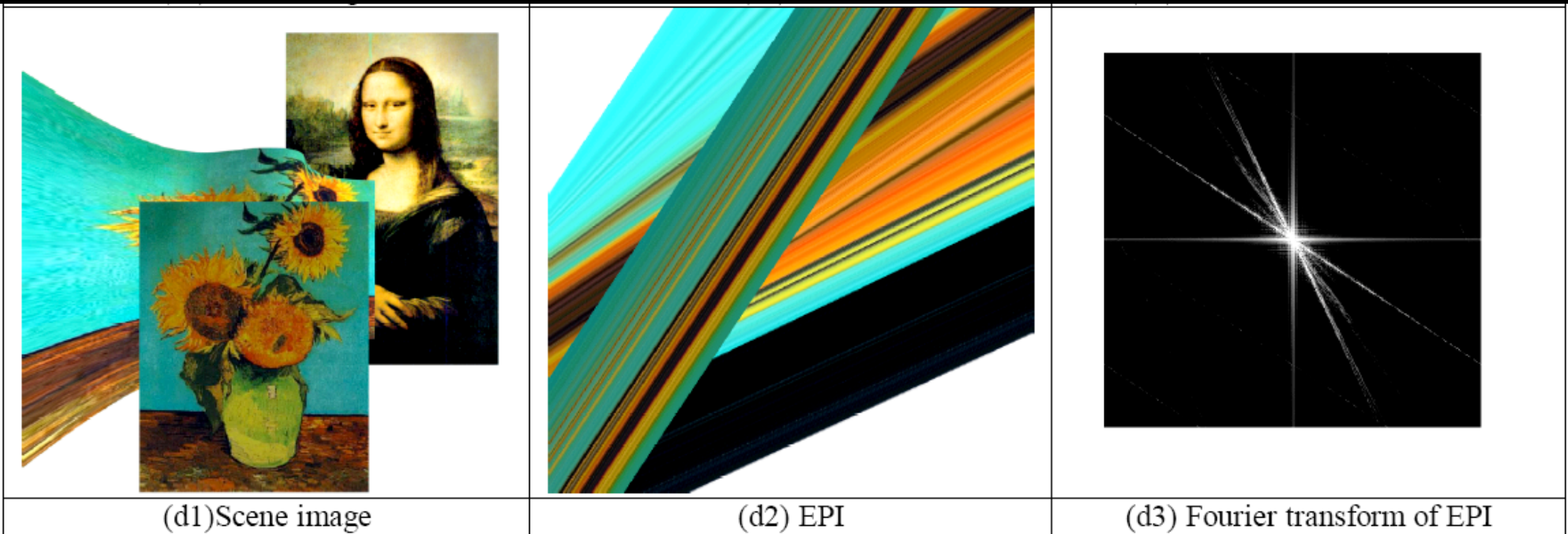
stopagation



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Transport → Shear

- This is consistent with light field spectra [Chai et al. 00, Isaksen et al. 00]





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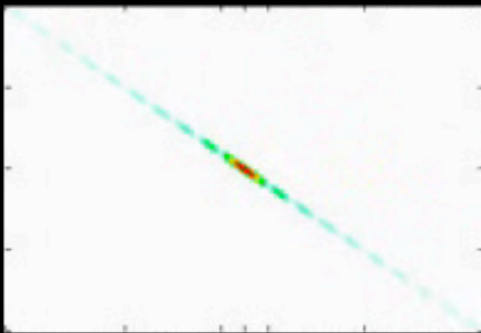
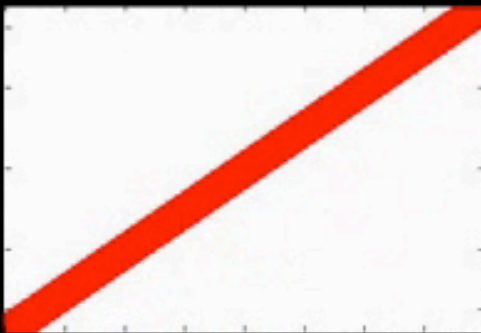


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Occlusion: multiplication

- Occlusion is a multiplication in ray space
 - Convolution in Fourier space
- Creates new spatial frequency content
 - Related to the spectrum of the blockers

Occlusion



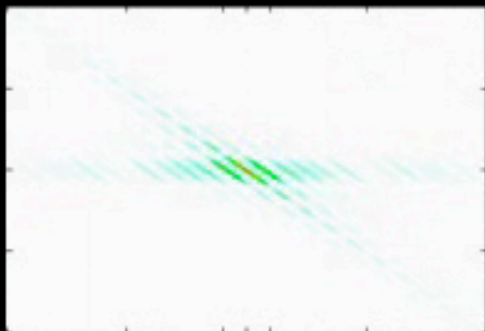
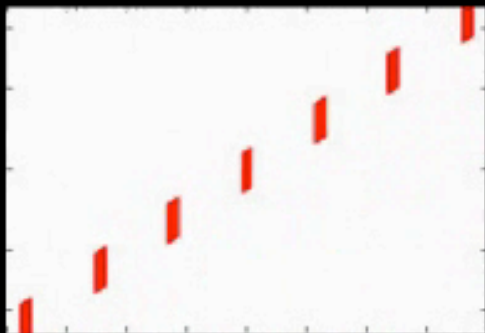
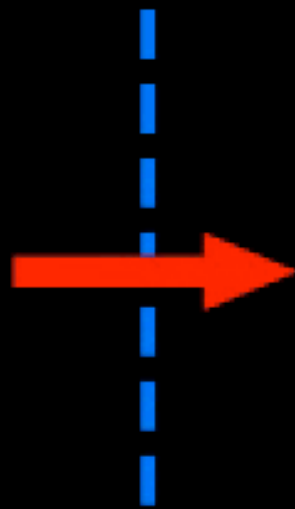


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Light Propagation





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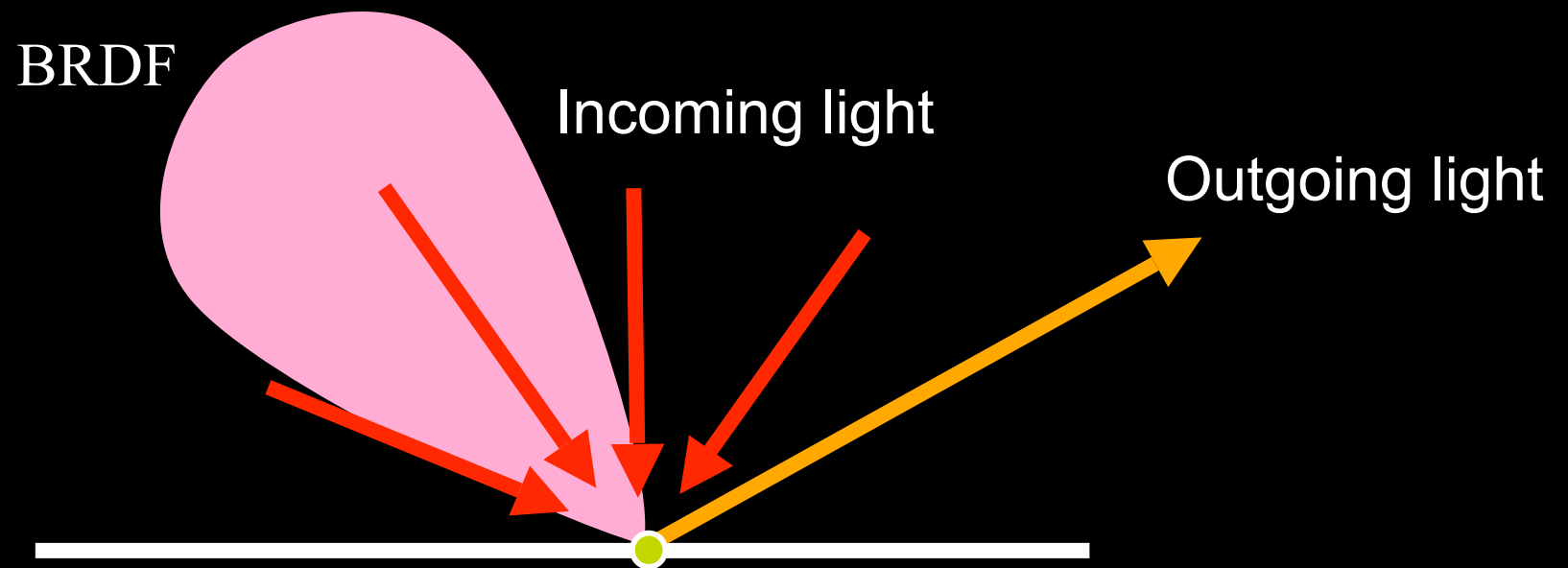
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BRDF integration

- Outgoing light:
 - Integration of incoming light times BRDF





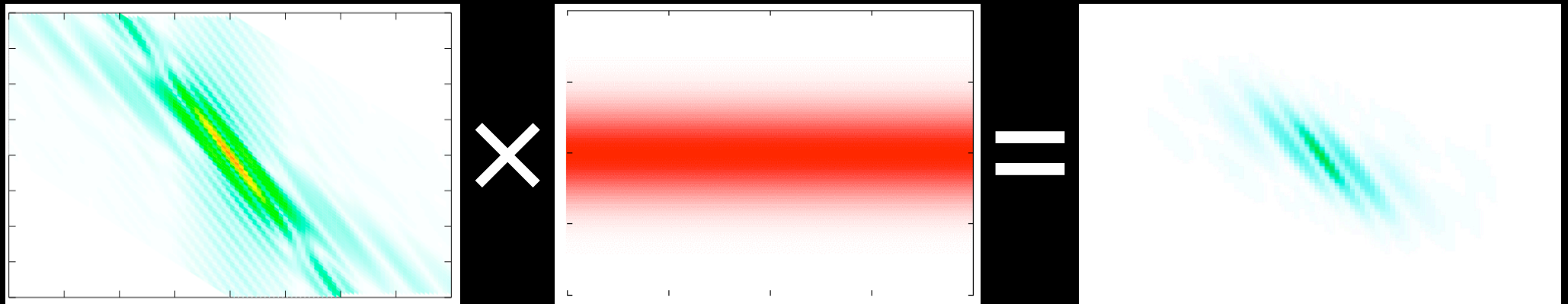
BRDF integration

- **Ray-space: convolution**
 - Outgoing light: convolution of incoming light and BRDF
 - For rotationally-invariant BRDFs
- **Fourier domain: multiplication**
 - Outgoing spectrum: multiplication of incoming spectrum and BRDF spectrum

BRDF in Fourier: multiplication



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- BRDF is bandwidth-limiting in angle



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Example: diffuse BRDF

- Convolve by constant:
 - multiply by Dirac
 - Only spatial frequencies remain





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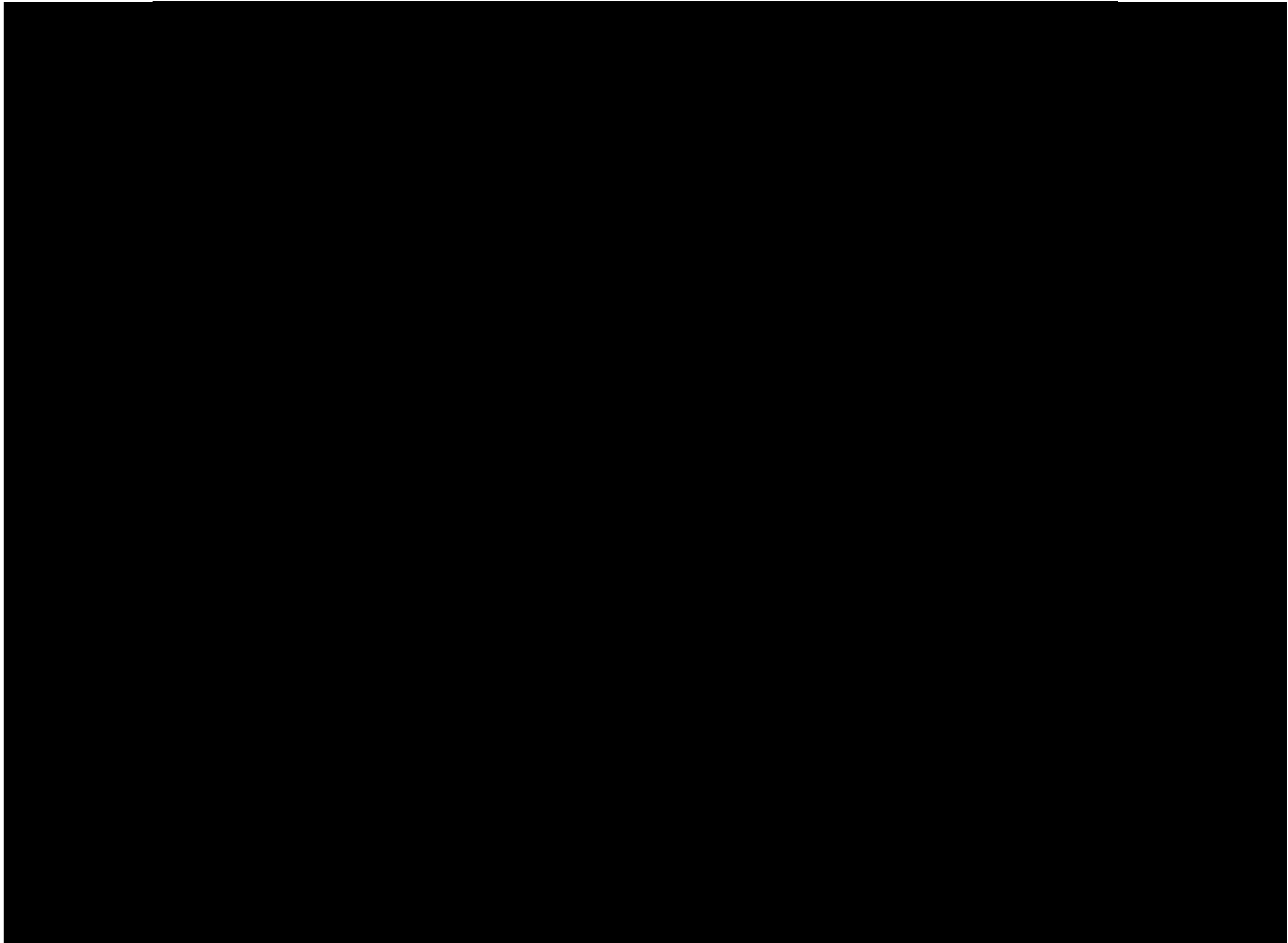
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Curved receiver

- Reduce to the case of a planar surface:
 - “Unroll” the curved receiver
- Equivalent to changing angular content:
 - Linear effect on angular dimension
 - No effect on spatial dimension
- Shear in the angular dimension





Transforms: summary

	Radiance/Fourier	Effect
Transport	Shear	
Occlusion	Multiplication/Convolution	Adds spatial frequencies
BRDF	Convolution/Multiplication	Removes angular frequencies
Curvature	Shear	



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More effects in paper

- Cosine term (multiplication/convolution)
- Fresnel term (multiplication/convolution)
- Texture mapping (multiplication/convolution)
- Central incidence angle (scaling)
- Separable BRDF
- Spatially varying BRDF (semi-convolution)

- ...and extension to 3D



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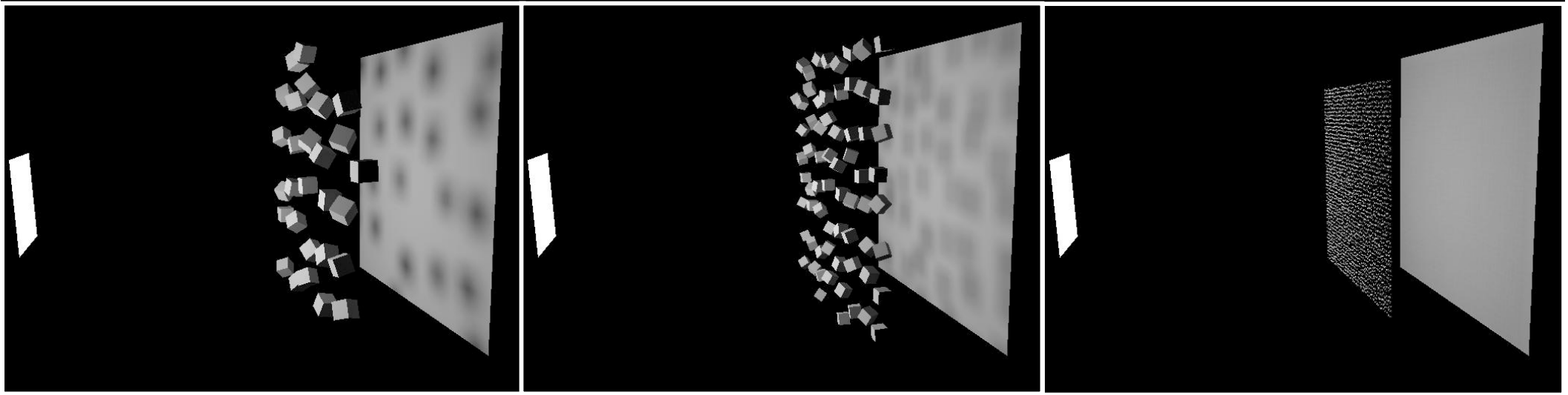
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Diffuse soft shadows

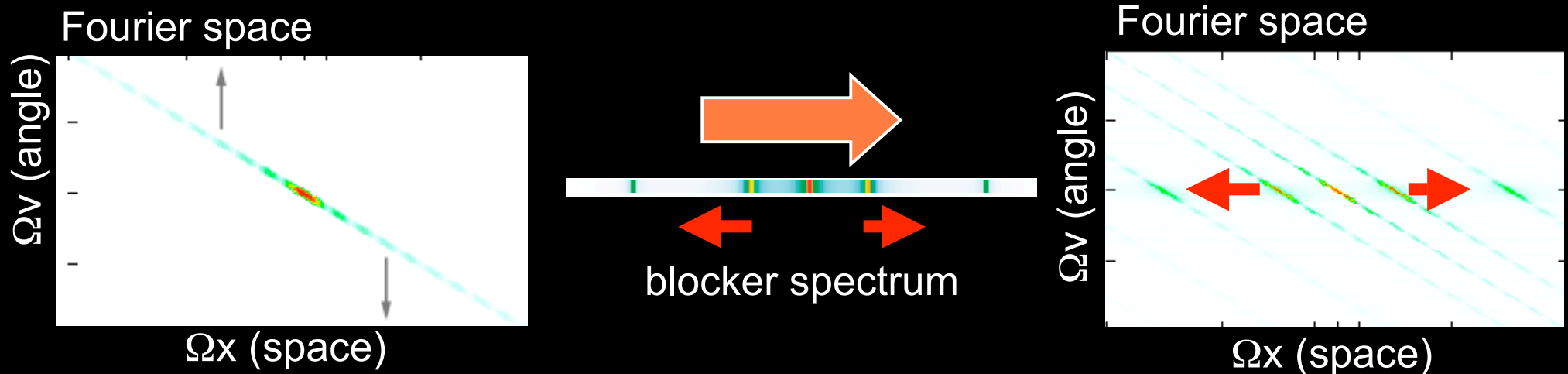
- Decreasing blockers size:
 - First high-frequencies increase
 - Then only low frequency
 - Non-monotonic behavior





Diffuse soft shadows (2)

- Occlusion : convolution in Fourier
 - creates high frequencies
 - Blockers scaled down \rightarrow spectrum scaled up

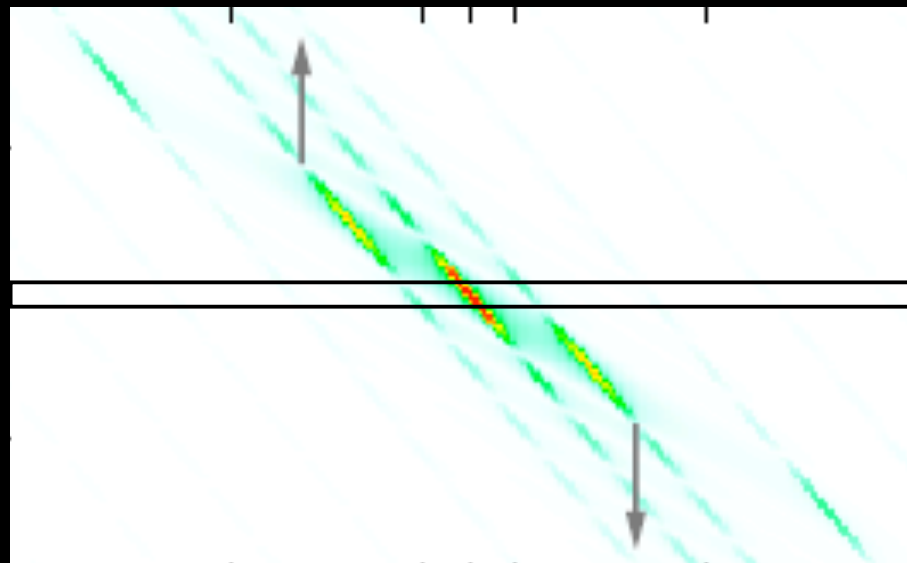




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Diffuse soft shadows (3)

- Transport after occlusion:
 - Spatial frequencies moved to angular dimension
- Diffuse reflector:
 - Angular frequencies are cancelled

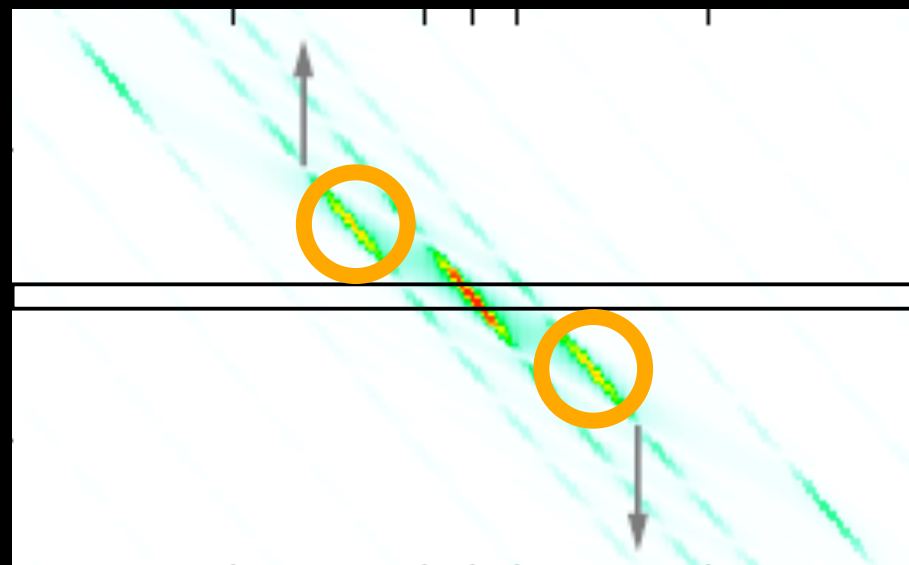




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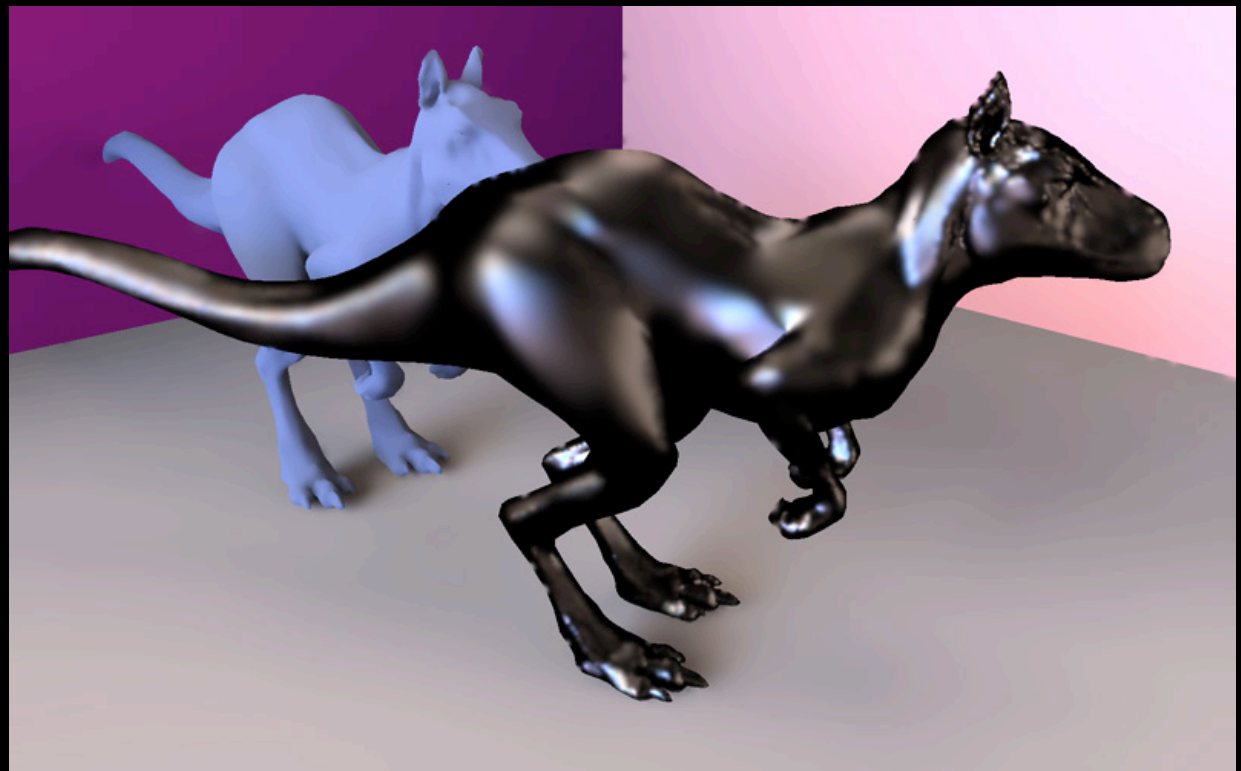
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Adaptive shading sampling

- Monte-Carlo ray tracing
- Blurry regions need fewer shading samples

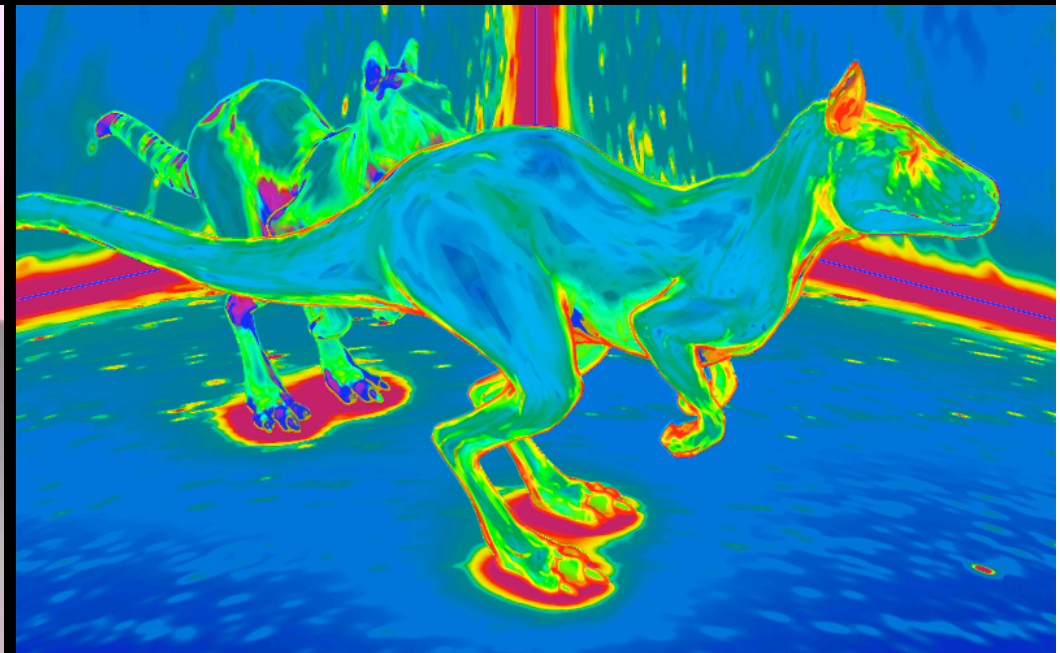
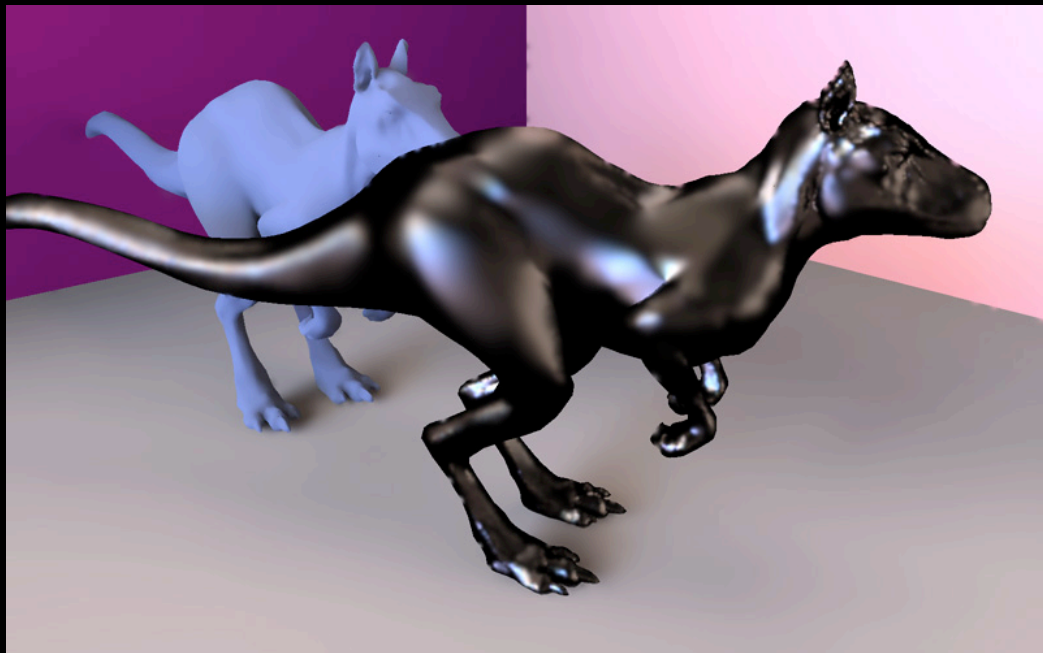




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Adaptive shading sampling

- Per-pixel prediction of max. frequency (bandwidth)
 - Based on curvature, BRDF, distance to occluder, etc.
 - No spectrum computed, just estimate max frequency



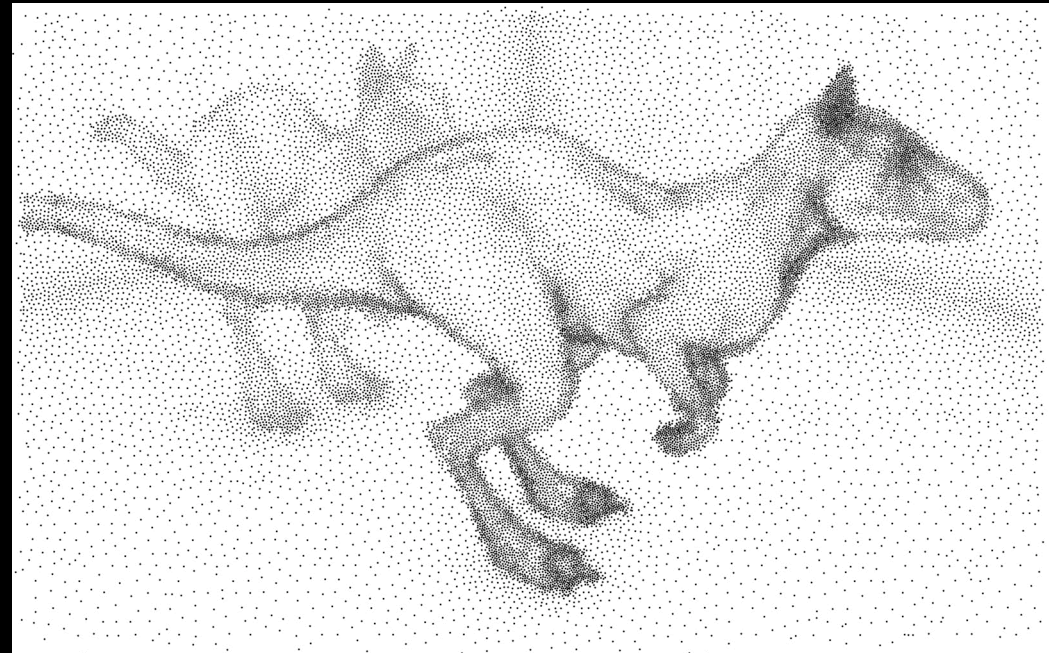
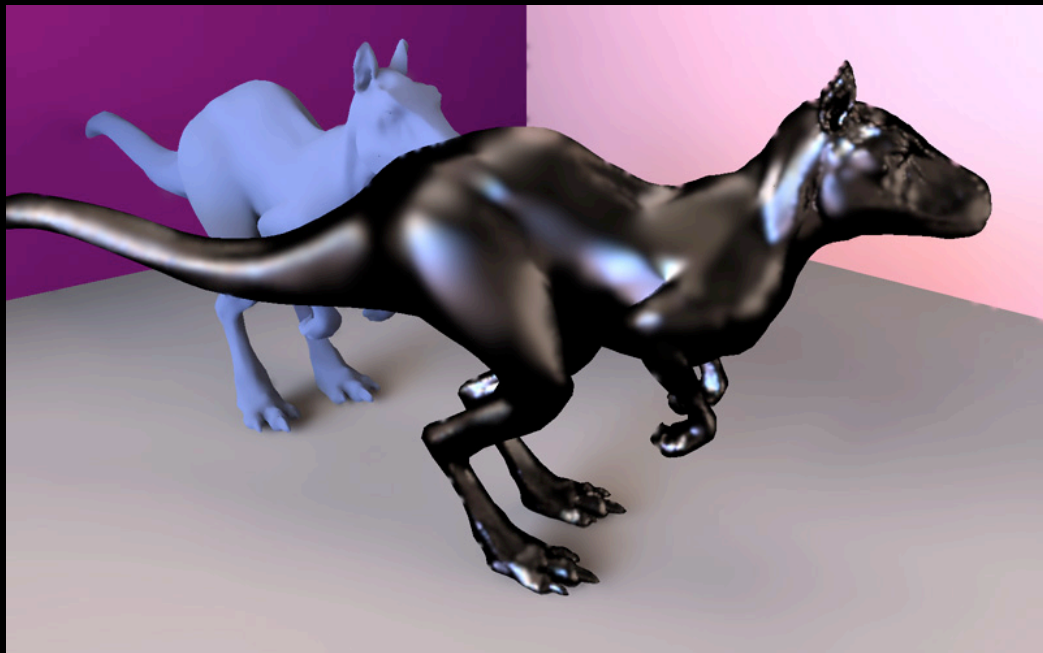
Per-pixel bandwidth criterion



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Adaptive shading sampling

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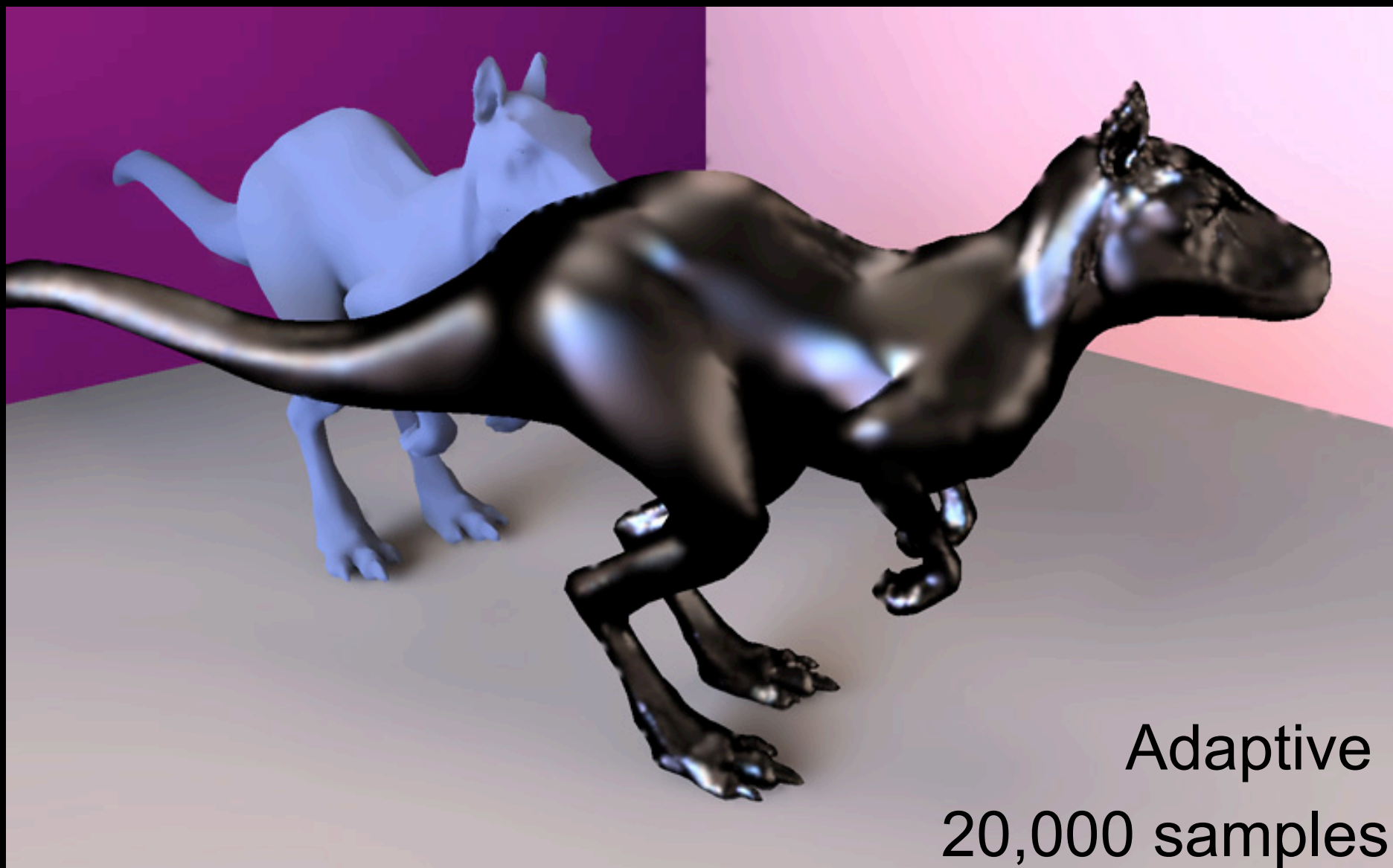


Shading samples

Adaptive sampling



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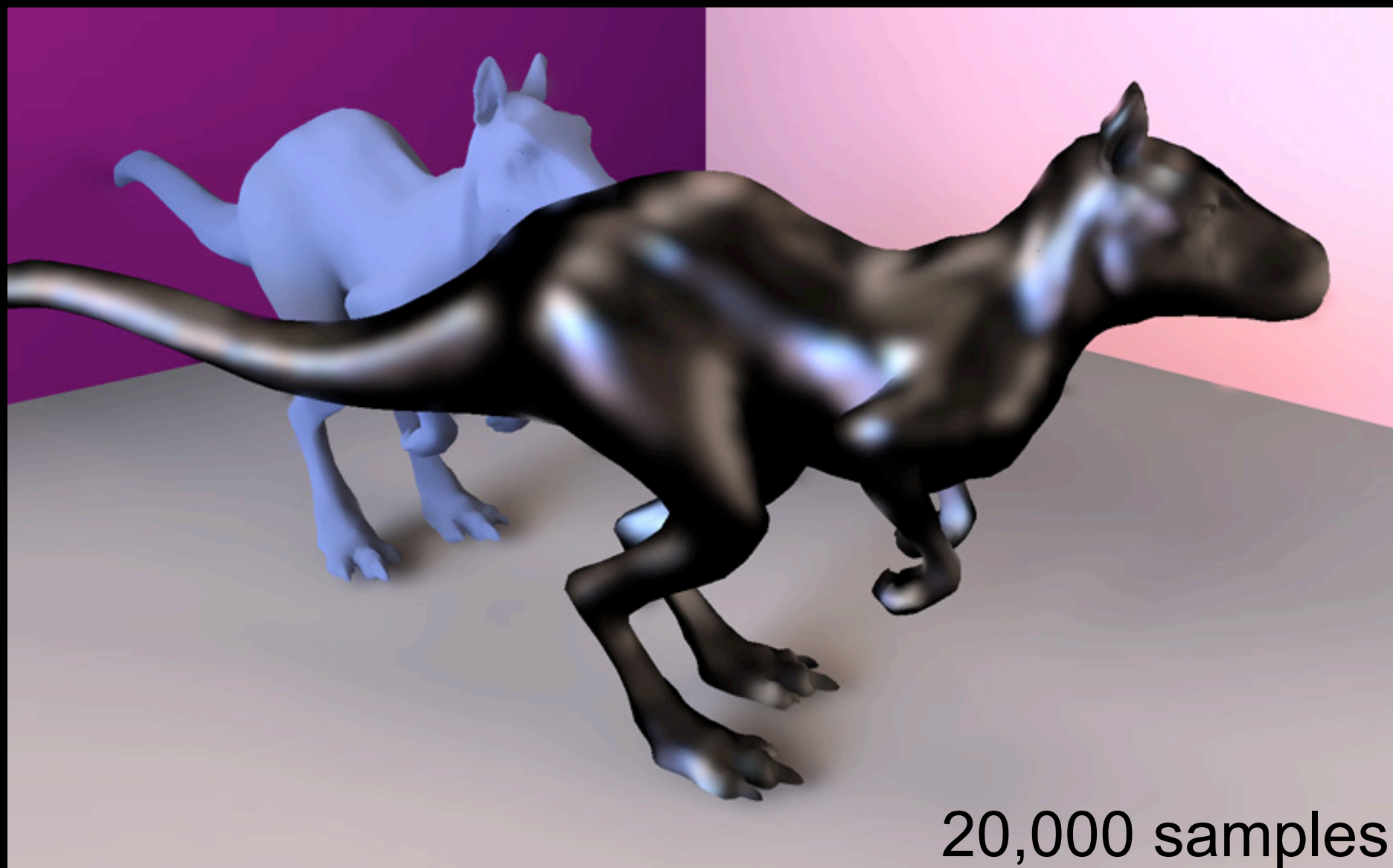


Adaptive
20,000 samples

Uniform sampling



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20,000 samples



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Conclusions

- Unified framework:
 - For frequency analysis of radiance
 - In both space and angle
 - Simple mathematical operators
 - Extends previous analyses
- Explains interesting lighting effects:
 - Soft shadows, caustics
- Proof-of-concept:
 - Adaptive sampling



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Future work

- More experimental validation on synthetic scenes
- Extend the theory:
 - Bump mapping, microfacet BRDFs, sub-surface scattering...
 - Participating media
- Applications to rendering:
 - Photon mapping
 - Spatial sampling for PRT
 - Revisit traditional techniques
- Applications to vision and shape from shading



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Acknowledgments

- Jaakko Lehtinen
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 - NSF CAREER award 0447561 Transient Signal Processing for Realistic Imagery
 - NSF CISE Research Infrastructure Award (EIA9802220)
 - ASEE National Defense Science and Engineering Graduate fellowship
 - INRIA Équipe associée
 - Realreflect EU IST project
 - MIT-France

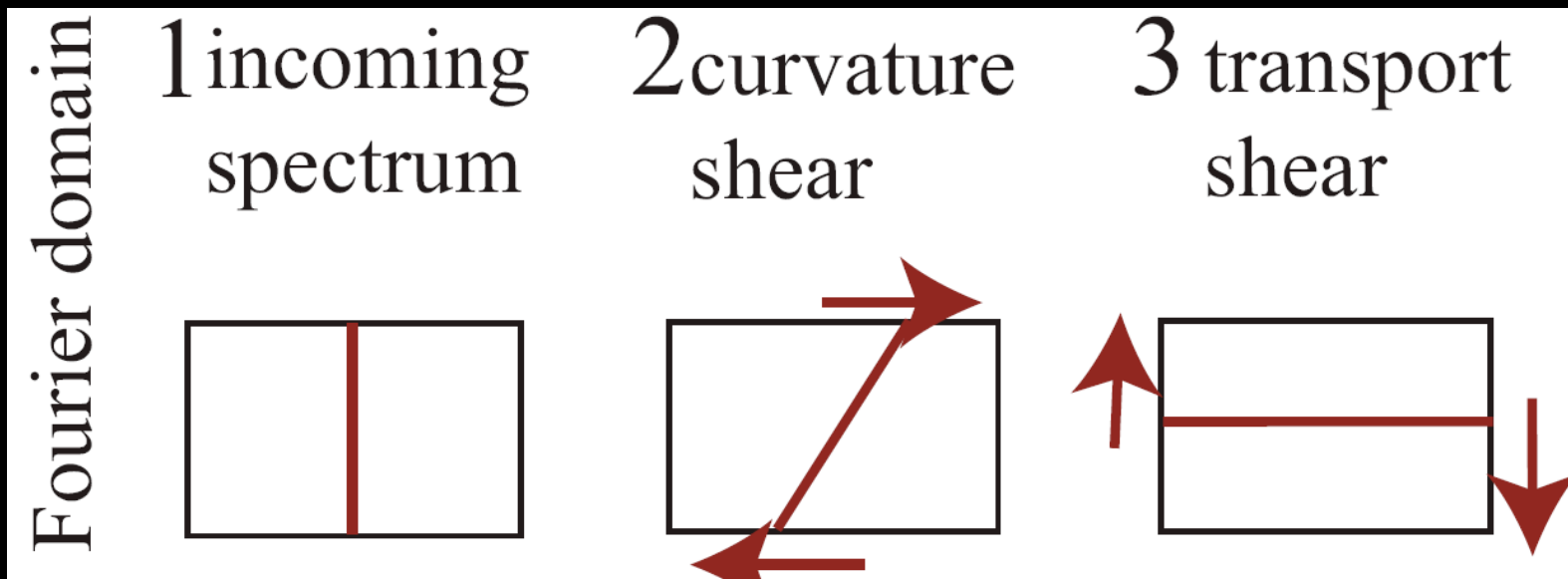
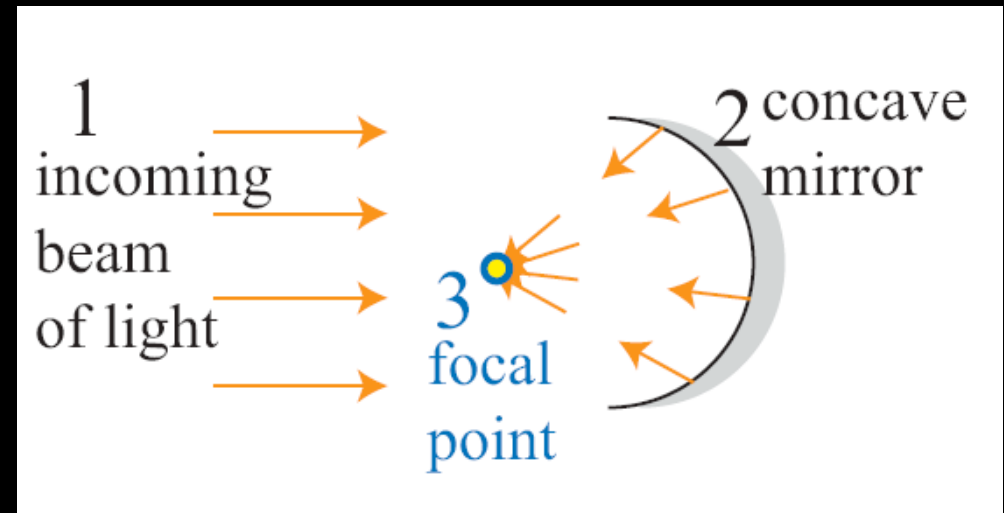


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Solar oven

- Curved surface
- In: parallel light rays
- Out: focal point





Other bases?

- We're not using Fourier as a function basis
 - Don't recommend it, actually
 - Just used for analysis, understanding, predictions
- Results are useable with any other base:
 - Wavelets, Spherical Harmonics, point sampling, etc
 - Max. frequency translates in sampling rate
- Analysis relies on Fourier properties:
 - Especially the convolution theorem



Why *Local* Light Field?

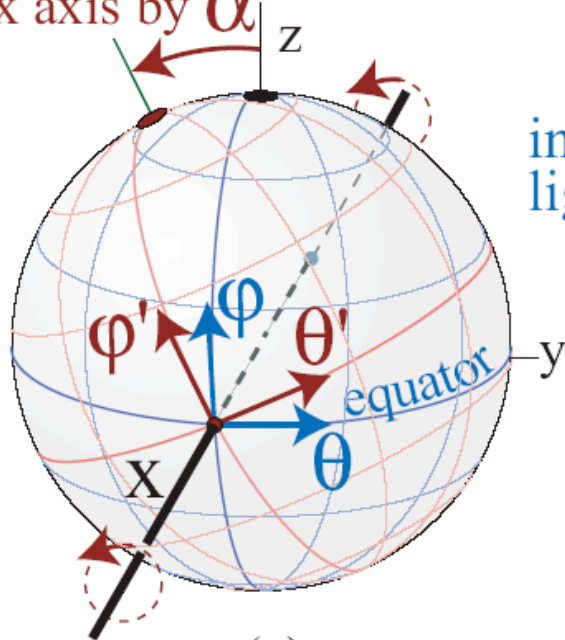
- Linearization:
 - $\theta \approx \tan \theta$
 - Curvature
- Local information is what we need:
 - Local frequency content, for local sampling
 - Local properties of the scene (occluders, curv.)



Extension to 3D

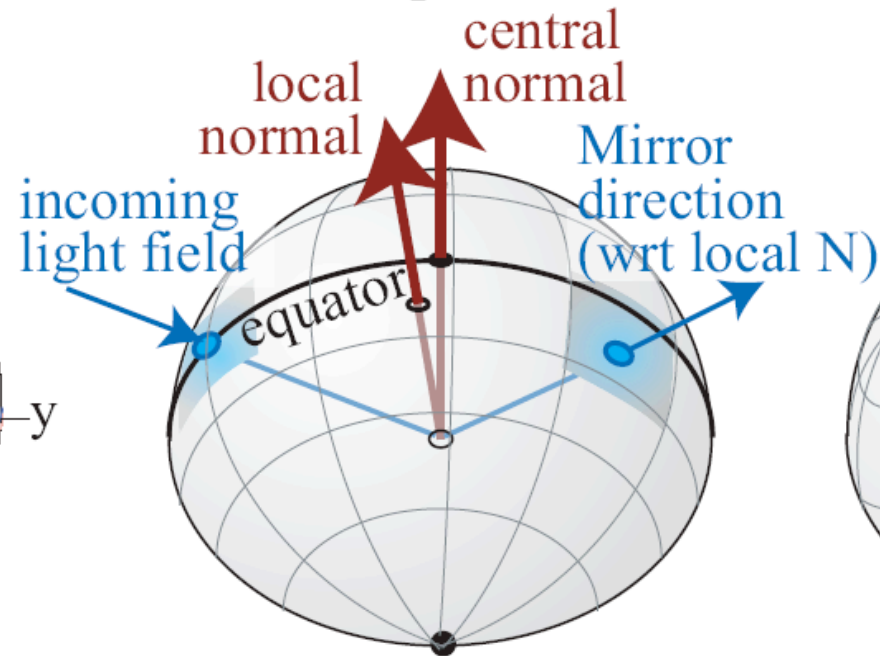
- It works. See paper:

rotation around
the x axis by α



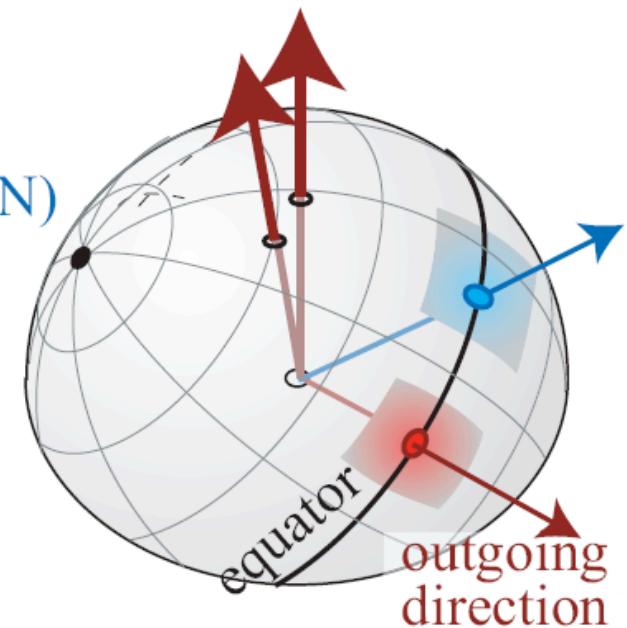
(a)

step 1, 2, 3, 4



(b)

step 5



(c)

Reflection on a surface: Full summary



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- Angle of incidence
- Curvature
- Cosine/Fresnel term
- Mirror re-parameterization
- BRDF
- Curvature



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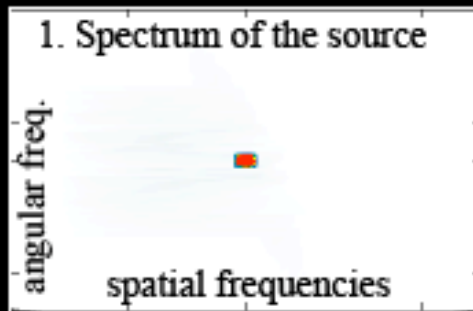
Reflection on a surface: Full summary

- Angle of incidence: scaling
- Curvature: shear in angle
- Cosine/Fresnel term: multiplication/convolution
- Mirror re-parameterization
- BRDF: convolution/multiplication
- Curvature: shear in angle



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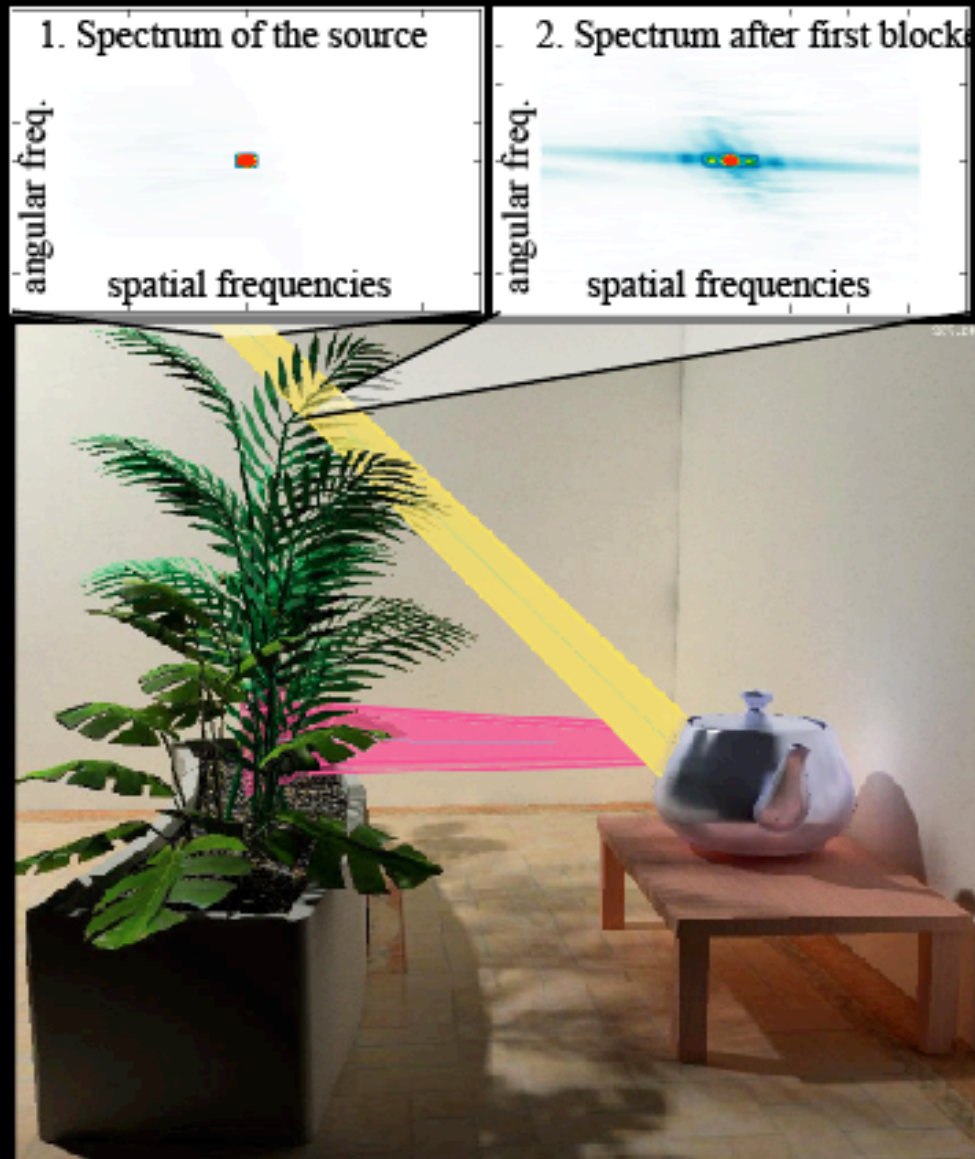
Application: 3D scene





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Application: 3D scene





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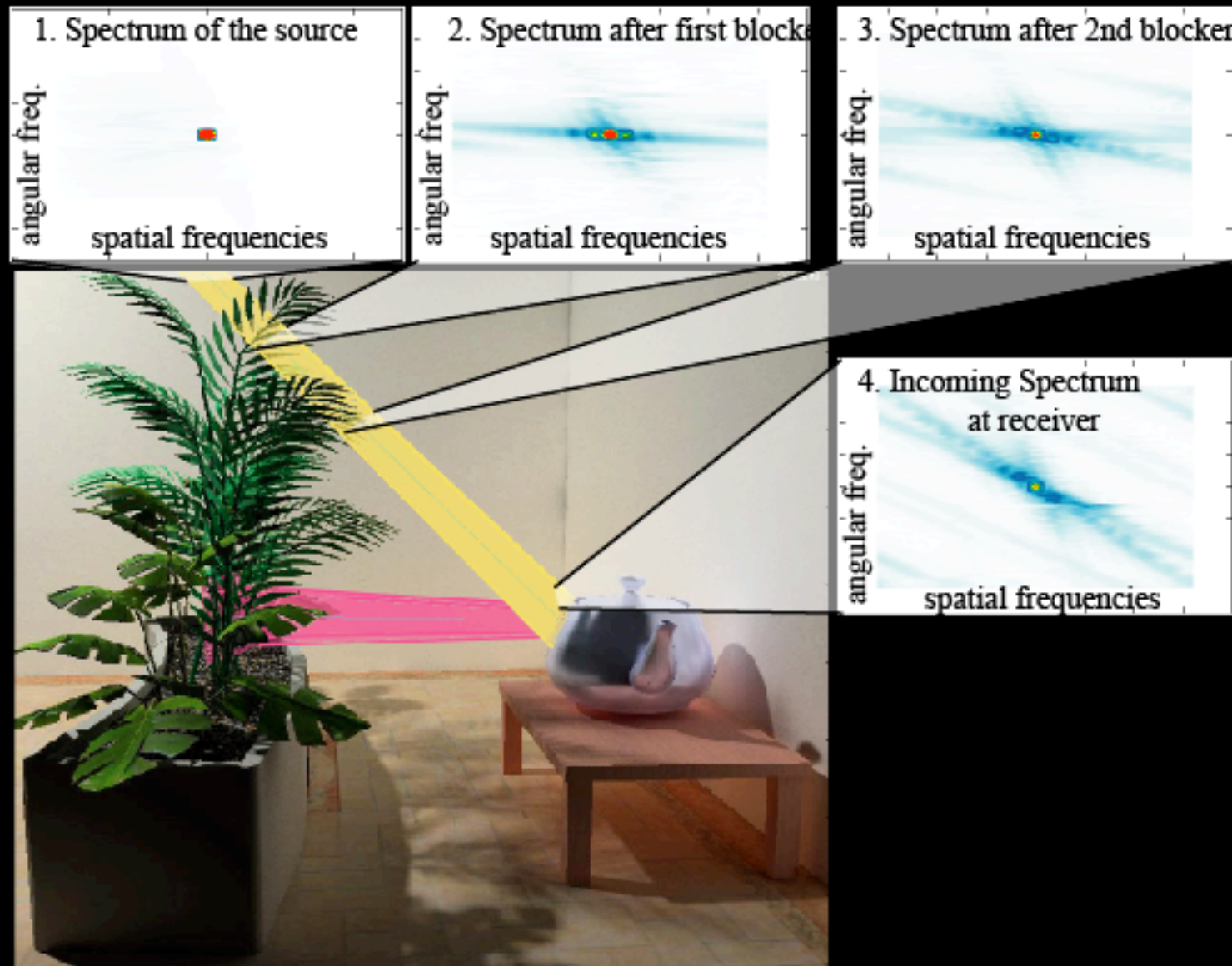
Application: 3D scene





SIGGRAPH2005

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