The visibility complex made visibly simple
an introduction to 2D structures of visibility

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Spatial coherence is a very important topic in image Synthesis, particularly in the popular ray-tracing algorithm and all its expensive intersection computations, or the radiosity light modeling which involves huge visibility calculations to determine the *form factors*. Motion planning where objects are seen as obstacles to straight line trajectories deals too with visibility problems. In order to solve them, a data structure called the *visibility graph* has been widely used. In polygonal scene it links the vertices that can see each other, in a scene of convex objects, it is made of the bitangents which don't intersect any object (see [GM91], [OW88]).

But its discrete structure does not allow taking efficiently into account the coherence between the intersections of two "very near" lines, typically like for the computation of a view from a point (visibility polygon). That's why the *visibility complex* of a set of a set of n pairwise disjoint convex objects in the plane was introduced by [PV93]. Working in a dual space that transforms lines into points allows to group the lines sharing the same visibility in faces, and the main computational structure is now the complex, instead of the scene as before with the graph.

The aim of the video is to give a better understanding of various notions involved in the definition and use of the visibility complex. It starts with basic considerations on the duality in the plane, and shows visually the correspondance between the real and the dual space. Then it introduces the dual arrangement of a set of convex objects. It gives a graphical idea of what the visibility complex is by first using lines with origins, which then helps to introduce the notion of *maximal free segments* which are used for the visibility complex. We insist on the structure of a typical face of the complex. Then a sweeping algorithm that can build the complex in $O(mlog(n))$ where n is the size of the visibility graph is shown quickly.

The video illustrates also the use of the complex to compute the view "around" a point. It consists in going through the complex along the curve defined by the set of segments originated at this point, which is a sine curve in the dual space. The object seen changes only where the segment leaves a face ; it is calculated in $O(klog(n))$ where k is the size of the view.

Time coherence is also taken into account, since when an object is moved, the modifications of the complex takes only place along its curve of tangency, and the topology of the complex is modified only when a line becomes tangent to three objects. This is shown in the video.

The complex is build using an algorithm by Michel Pocchiola and Gert Vegter [PV93] implemented by Stéphane Rivière (see [SR93]) and us. The implementation of the interactive representation of the complex was written in C++ using the Inventor library on a Silicon Graphics Indigo2 workstation. The images computed were then transfered on a Macintosh where the editing was realized under Adobe Premiere associated to a Radius VideoVision card. Some schema features were also added with Premiere, such as some arrows, lines or crosses.

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[PV93] Michel Pocchiola and Gert Vegter. The visibility complex In *Proceedings of the 9th ACM Symposium on Computational Geometry*, San Diego, June 1993.

[RS93] Stéphane Rivière Comparaison d'algorithmes de calcul de graphes de visibilité *Rapport de stage du DEA IMA*

[CG89] B. Chazelle and L. Guibas. Visibility and intersection problems in plane geometry *Discrete and Computational Geometry*, 4:551-581, 1989.

[CGL85] B.M. Chazelle, L.J. Guibas and D.T. Lee. The power of geometric duality *BIT*, 25:76-90, 1985.

[GM91] S.K. Ghosh and D. Mount. An output sensitive algorithm for computing visibility graphs. *Siam J. Comput.*, 20:888-910, 1991.

[OW88] M.H. Overmars and E. Welzl. New methods for computing visibility graphs In *Proceedings of the fourth ACM Symposium on Computational Geometry*, pages 164-171, 1988.

[PO90] Michel Pocchiola. Graphics in flatland revisited. In *Proceedings of the 2nd Scandinavian Workshop on Algorithm Theory, Bergen*, pages 85-96. Springer Lecture Notes in Comp. Sc., vol. 447, July 1990.

[VE90] Gert Vegter. The visibility diagram : a data structure for visibility and motion planning problems in the plane. In *Proceedings of the 2nd Scandinavian Workshop on Algorithm Theory, Bergen*, pages 425-436. Springer Lecture Notes in Comp. Sc., vol 519, July 1990.

[VE91] Gert Vegter. Dynamically maintaining the visibility graph. In *Proceeding of the 2nd Workshop WADS'91, Ottawa, Canada*, pages 425-436. Springer Lecture Notes in Comp. Sc., vol 519, August 1991.