Exercise 05

Lecturer: Mohsen Ghaffari *Teaching Assistant: Michal Dory*

1 Shorter Sketches for Connectivity

In the lectures, we saw an algorithm for building a maximal spanning forest using $O(\log^4 n)$-bit sketches per vertex. Show that the same task can be solved using shorter sketches of $O(\log^3 n)$ bits. More concretely, design an algorithm in the coordinator model, in which each vertex sends an $O(\log^3 n)$ bit message to the coordinator, and based on these messages the coordinator can determine if the graph is connected with high probability.

2 Streaming a Minimum Spanning Tree

In this question, we will see algorithms for computing a minimum spanning tree (MST) in the streaming model. Throughout the question, you can assume that the weights of the edges in the input graphs are non-negative integers, and that they are polynomially bounded.

1. Design a streaming algorithm for computing a 2-approximation for the MST of the graph, using $\tilde{O}(n)$ total memory.
2. Design a streaming algorithm for computing the MST of the graph, using $\tilde{O}(n)$ total memory and $O(\log^2 n)$ passes. In a k-pass streaming algorithm, the algorithm is allowed to have k passes over the input graph.

3 Streaming 3-Connectivity

In the class, we saw an algorithm with memory $n \cdot \text{poly}(\log n)$ that solves the connectivity problem in the streaming setting where we have a stream edge of arrivals and departures on a set V of n vertices. Devise an algorithm with $n{\cdot}\text{poly}(\log n)$ memory that solves the 3-connectivity problem in the same setting. That is, if the graph at the end of the stream is 3-edge-connected your algorithm should say YES with probability at least $1 - 1/n^2$, and otherwise it should say NO with probability at least $1 - 1/n^2$. Recall that a graph is 3-edge-connected if the graph remains connected after removing any set of at most 2 edges.

4 Communication with a Coordinator

Consider n-players numbered $1, 2, \ldots, n$. A set $S \subseteq \{1, 2, \ldots, n\}$ of players is selected and revealed, where each player gets to know whether itself is in S or not, but it does not get to know about the other members. Each player can send a B-bit message to the coordinator, but unfortunately, the coordinate receives the bit-wise AND of the sent messages, instead of receiving each of them. Devise a scheme with $B = O(\log^2 n)$ so that the coordinator can still approximate $s = |S|$ up to a 2-factor, with probability $1 - 1/n^2$.

More formally, you should describe the B-bit message $m^i = (m^i_1, m^i_2, \ldots, m^i_B)$ that each player $i \in \{1, \ldots, n\}$ sends, as well as how the coordinator infers a 2-factor approximation $\tilde{s}$ of $s = |S|$ by receiving just the B-bit bitwise AND of these messages. That is, the coordinator will receive only $\big((m_1^1 \wedge m_1^2 \wedge m_1^3 \wedge \cdots \wedge m_1^n), (m_2^1 \wedge m_2^2 \wedge m_2^3 \wedge \cdots \wedge m_2^n), \ldots, (m_B^1 \wedge m_B^2 \wedge m_B^3 \wedge \cdots \wedge m_B^n)\big)$. Your algorithm should guarantee that $\Pr[\tilde{s} \in [s, 2s)] \geq 1 - 1/n^2$.