6.S899 Distributed Graph Algorithms

Problem Set 1

Due date: 10/03/2014

1 Distributed Hypergraph Coloring

Design a single-round distributed vertex coloring algorithm for r-uniform hypergraphs with the smallest number of colors possible.

More precisely, the network is modeled as an r-uniform hypergraph $H = (V, E)$ where $n = |V|$ and for each hyperedge $e \in E$, we have $e \subset V$ and $|e| = r$. Nodes have ids 1 to n. The maximum node-degree is at most Δ, i.e., $\forall v \in V, |\{e \in E | v \in e\}| \leq \Delta$. A legal k-coloring $\mathcal{C}$ of H is a mapping from V to $\{1, 2, \ldots, k\}$ such that no hyperedge e is monochromatic, that is,

$$\forall e \in E, |\{x | x \in \{1, 2, \ldots, k\}, u \in e, \mathcal{C}(u) = x\}| \geq 2.$$

Design a deterministic coloring function that for any r-uniform hypergraph H with n nodes and maximum degree Δ, receives the id of each node v and also (the ids of the other nodes in each of) the hyperedges incident on v in H and outputs the color of v in a way that the colors of all nodes together form a legal k-coloring of H. Try to find the smallest possible k. Note that k will be a function of n, r, and Δ.

Bonus Points Prove a non-trivial lower bound on k as a function of n, r and Δ.