The Discrete Logarithm Problem with Preprocessing

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Stanford University

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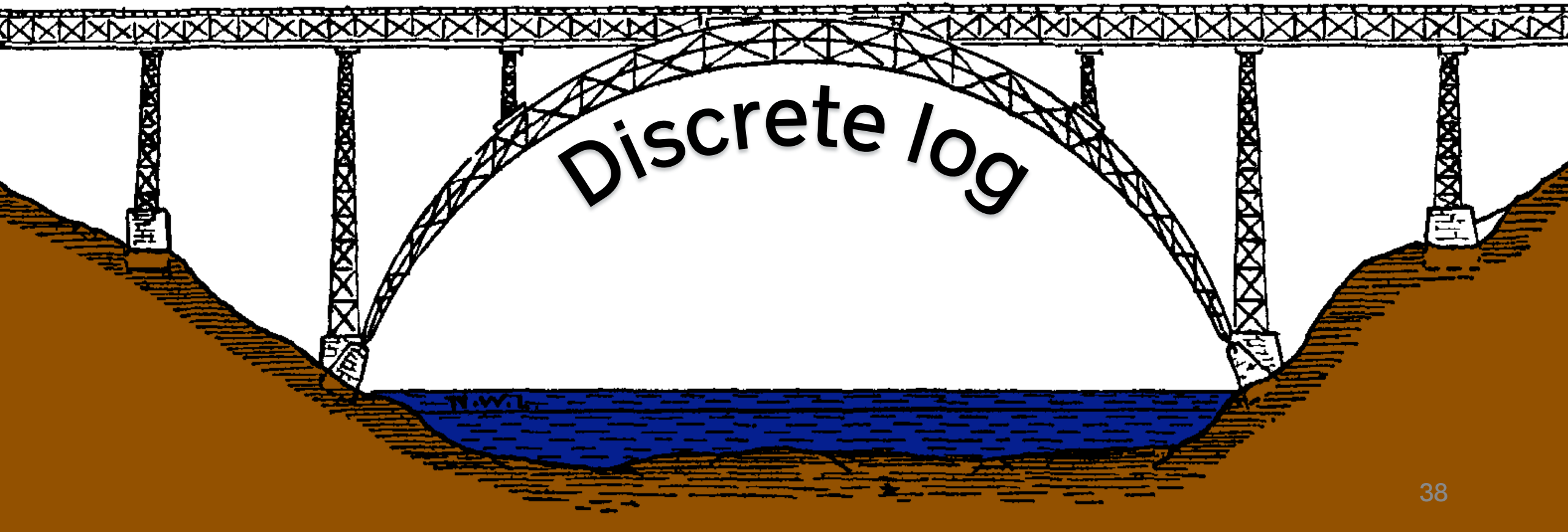
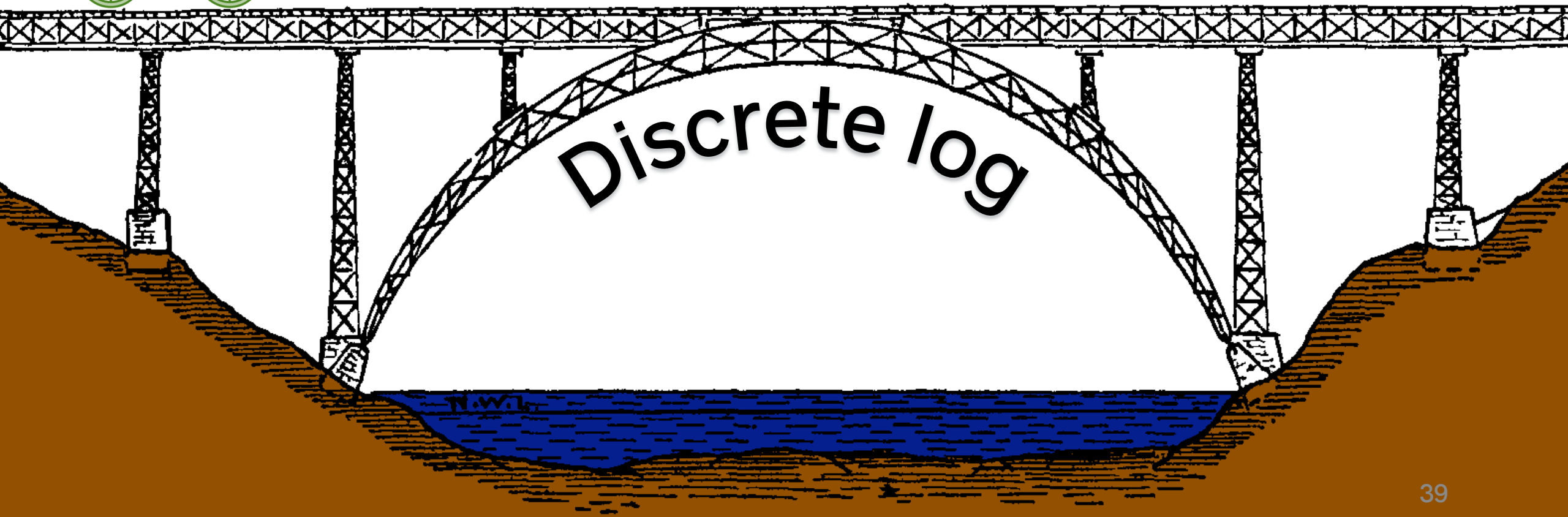
Tel Aviv, Israel


Discrete log

Signatures
(DSA and Schnorr)



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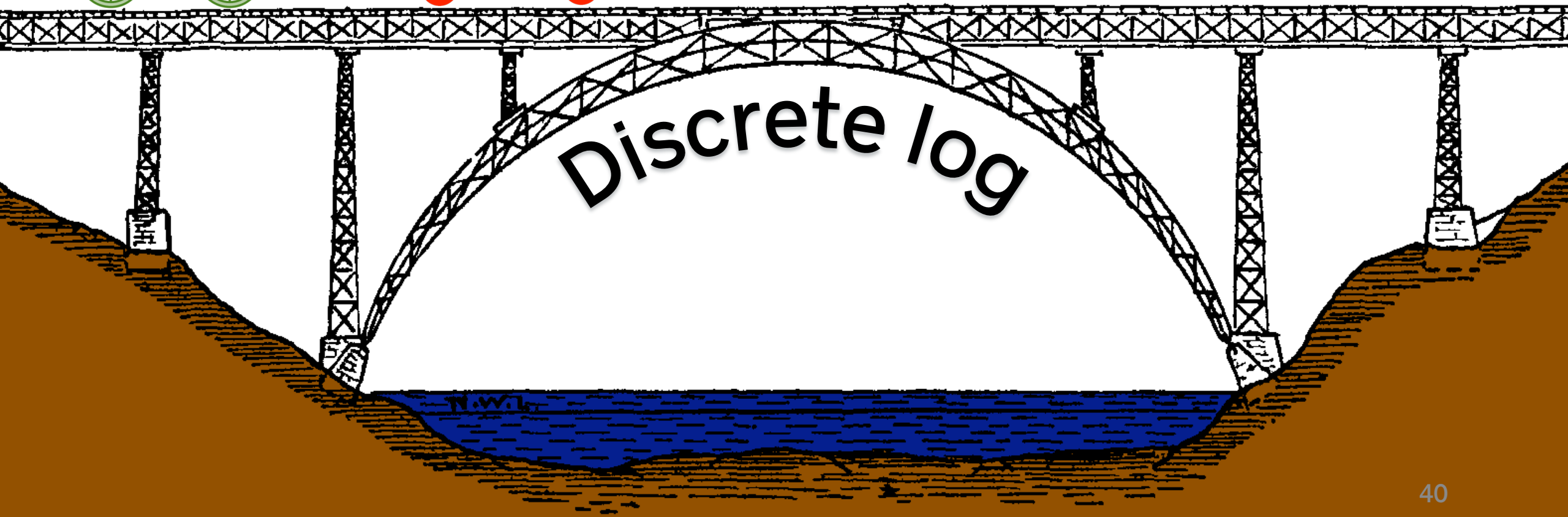

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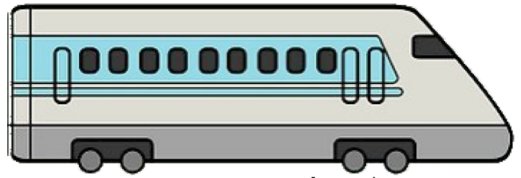

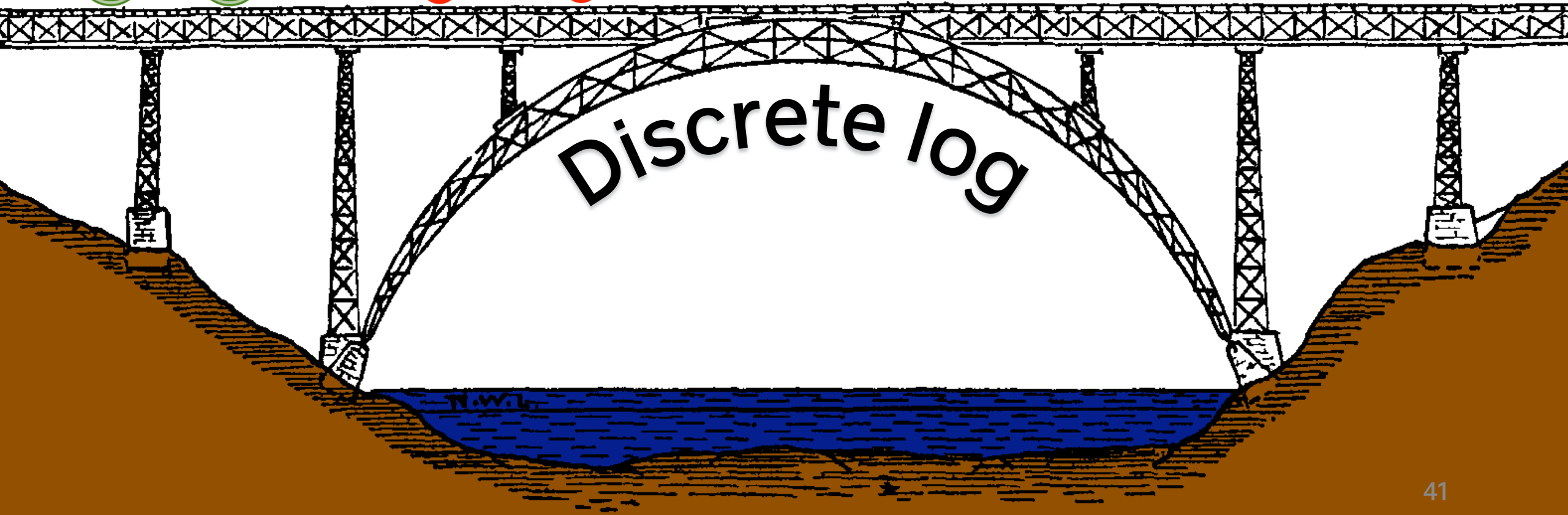

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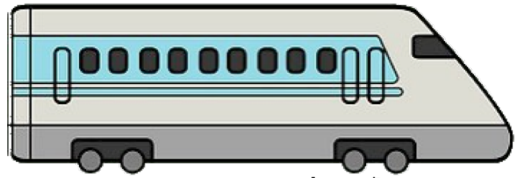
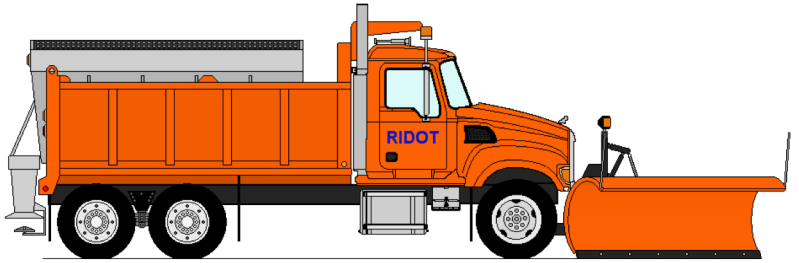


Pairings



Discrete log

The discrete-log problem

Group: $\mathbb{G} = \langle g \rangle$
of prime order N

Instance: $g^x \in \mathbb{G}$

Adversary $\mathcal{A}$

Solution:
$x \in \mathbb{Z}_N$

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Why do we believe this problem is hard?

Generic lower bounds give us confidence

Theorem. [Shoup'97] Every **generic** discrete-log algorithm that

- operates in a group of prime order N and
- succeeds with probability at least ½

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Best attacks on standard EC groups are generic

Generic algorithms can only make "black-box" use of the group operation

Generic-group model:

- Group is defined by an injective **"labeling" function**

$$\sigma\colon\ \mathbb{Z}_N \to \{0,1\}^*$$

- Algorithm has access to a **group-operation oracle:**

$$\mathcal{O}_\sigma\big(\sigma(i),\sigma(j)\big) \ \mapsto\ \ \sigma(i+j)$$

[Nechaev'94], [Shoup'97], [Maurer'05]

Generic dlog algorithm takes as input $\big(\sigma(1),\sigma(x)\big)$, representing (g, g^x), make queries to $\mathcal{O}_\sigma$, outputs x.

[Measure running time by query complexity]

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Very useful way to understand hardness

[BB04,B05,M05,D06, B08,Y15,...]

Existing generic lower bounds do not account for preprocessing

- Premise of generic-group model: the adversary **knows nothing** about the structure of the group $\mathbb{G}$ in advance
- In reality: the adversary knows a lot about $\mathbb{G}$!
  - $\mathbb{G}$ is one of a small number of groups: NIST P-256, Curve25519, …
- A realistic adversary can perform $\mathbb{G}$-specific preprocessing!
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Preprocessing phase

Group: $\mathbb{G} = \langle g \rangle$

$\mathcal{A}_0$

Advice: $st_{\mathbb{G}}$

Online phase

Instance: $g^x \in \mathbb{G}$

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Solution: $x \in \mathbb{Z}_N$

Initiated by Hellman (1980) in context of OWFs

Preprocessing phase

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Both algorithms are generic!

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Success prob. ϵ

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Rest of this talk

Background: Preprocessing attacks are relevant

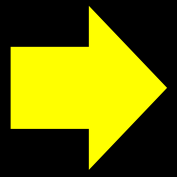
- Preexisting $S = T = \tilde{O}(N^{1/3})$ generic attack on discrete log

Our results: Preprocessing lower-bounds and attacks

- The $\tilde{O}(N^{1/3})$ generic dlog attack is optimal
- Any such attack must use lots of preprocessing: $\Omega(N^{2/3})$
- New $\tilde{O}(N^{1/5})$ preprocessing attack on DDH-like problem

Open questions

Rest of this talk



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A preexisting result...

Theorem. [Mihalcik 2010] [Lee, Cheon, Hong 2011] [Bernstein and Lange 2013]

There is a generic dlog algorithm with preprocessing that:

- uses S bits of group-specific advice,
- uses T online time, and
- succeeds with probability ϵ,

such that:

$$ST^2 = \tilde{O}(\epsilon N)$$

.... building on prior work on multiple-discrete-log algorithms [ESST99,KS01,HMCD04,BL12]

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Will sketch the algorithm for $S = T = N^{1/3}$, constant ϵ.

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Preliminaries

Define a pseudo-random walk on $\mathbb{G}$:

$$g^x \mapsto g^{x+\alpha} \quad \text{where} \quad \alpha = \mathsf{Hash}(g^x)$$

is a random function

[M10, LCH11, BL13]

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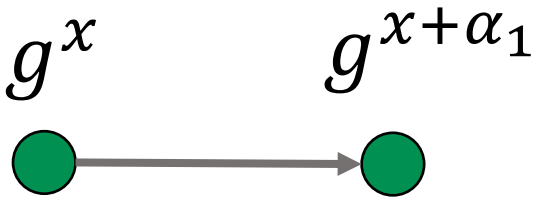

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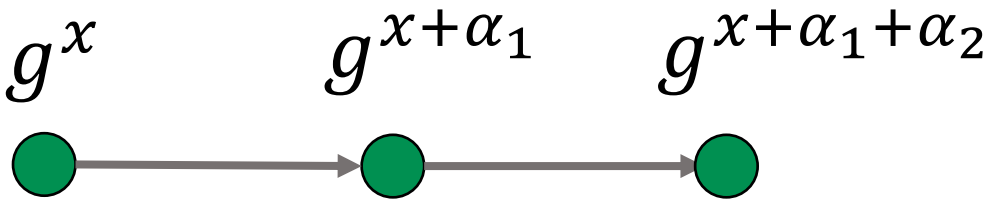

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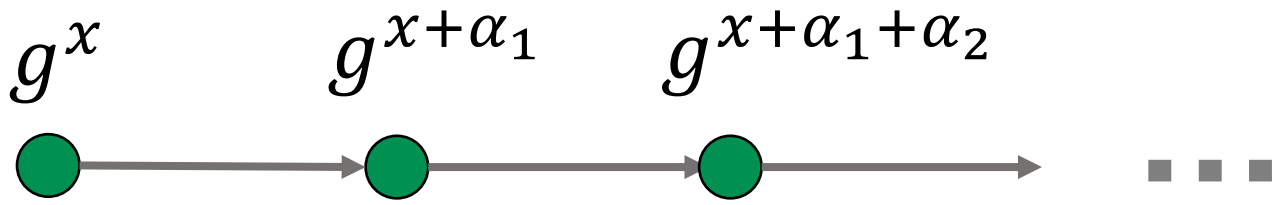

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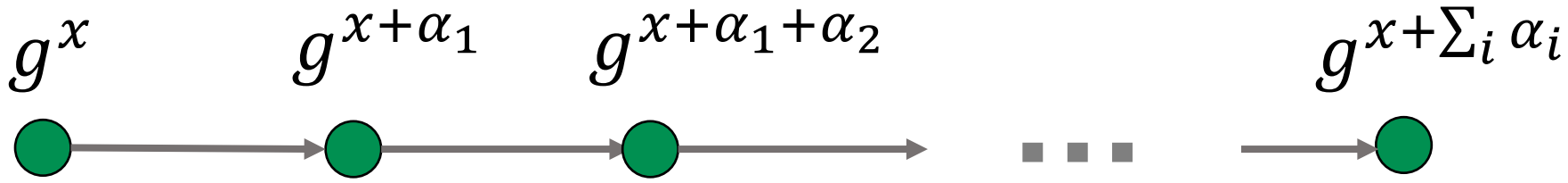

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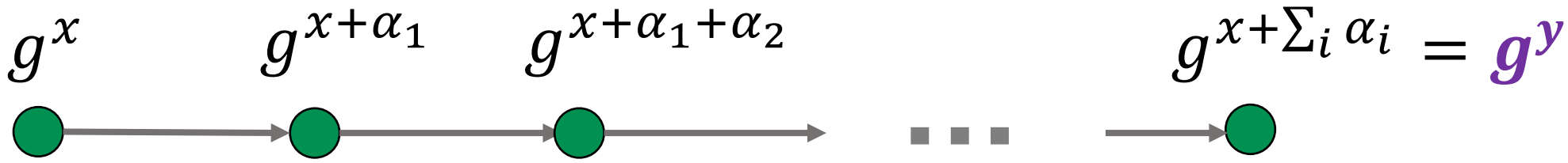

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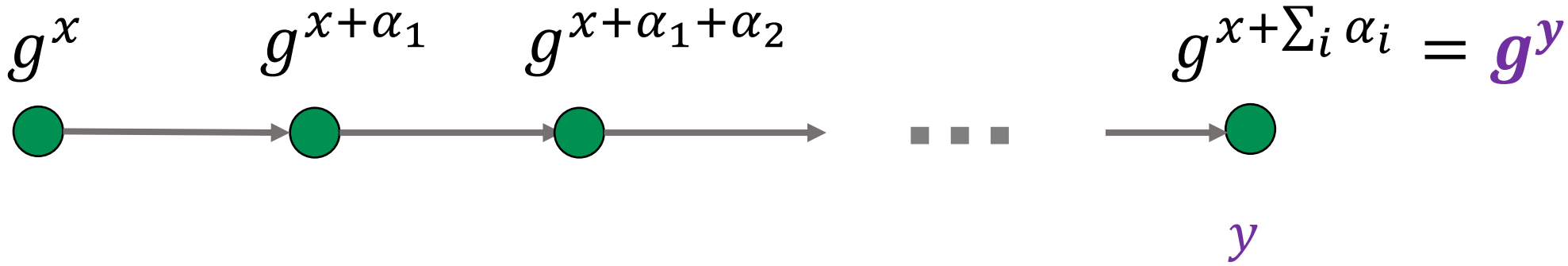

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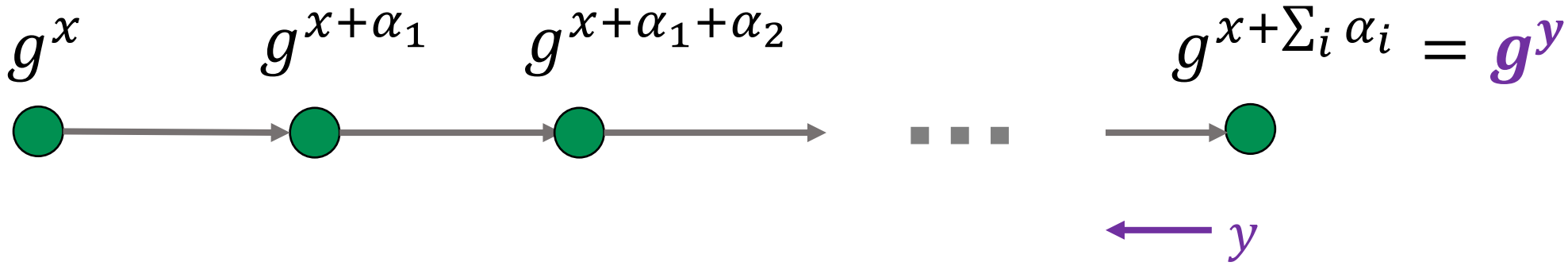

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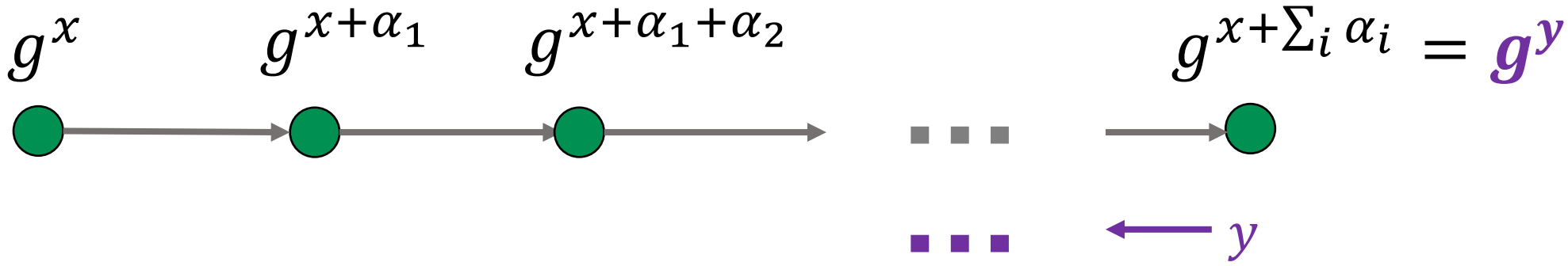

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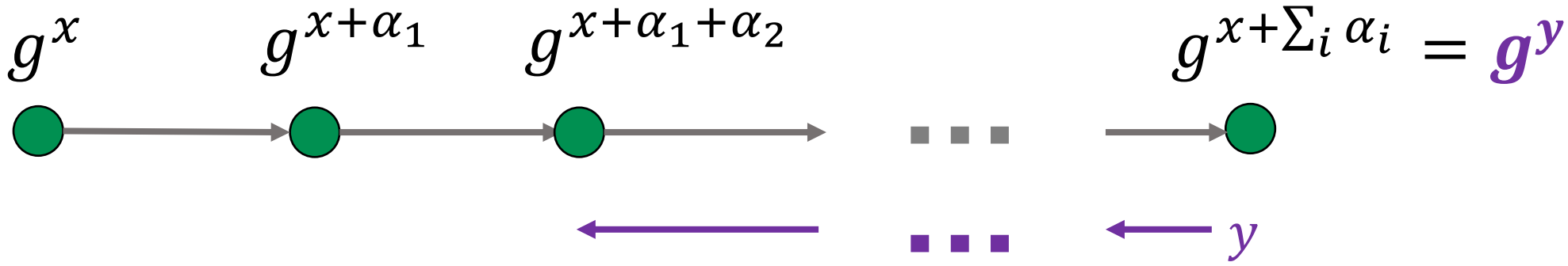

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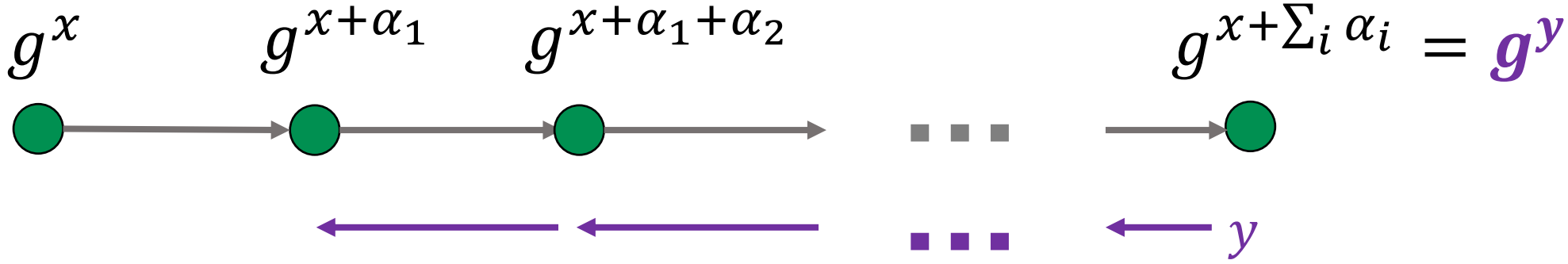

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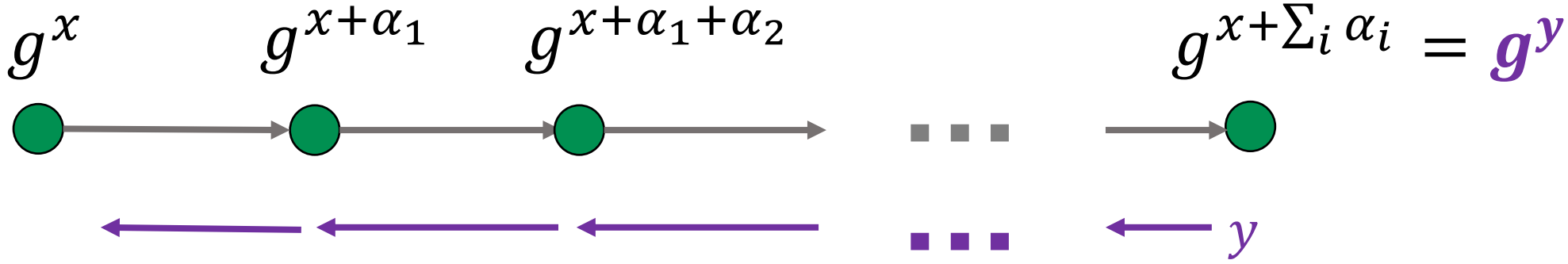

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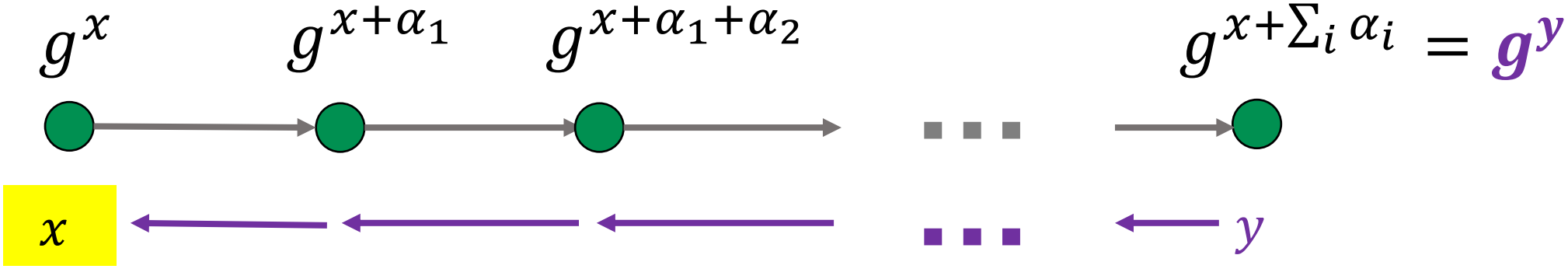

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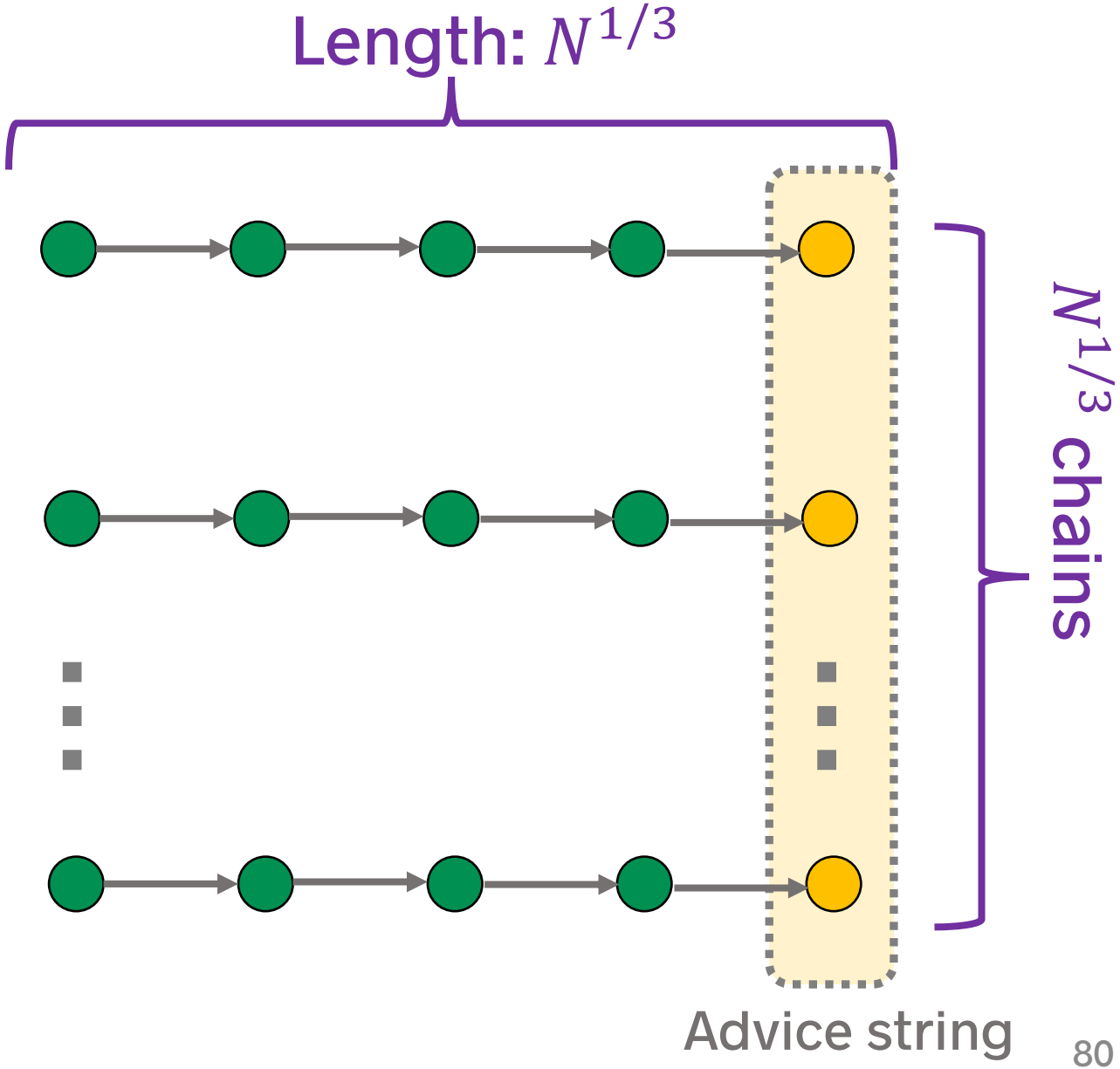
[M10, LCH11, BL13]

Preprocessing phase

- Build $N^{1/3}$ chains of length $N^{1/3}$
- Store dlogs of chain endpoints

Online phase

- Walk $O(N^{1/3})$ steps
- When you hit a stored point, output the discrete log



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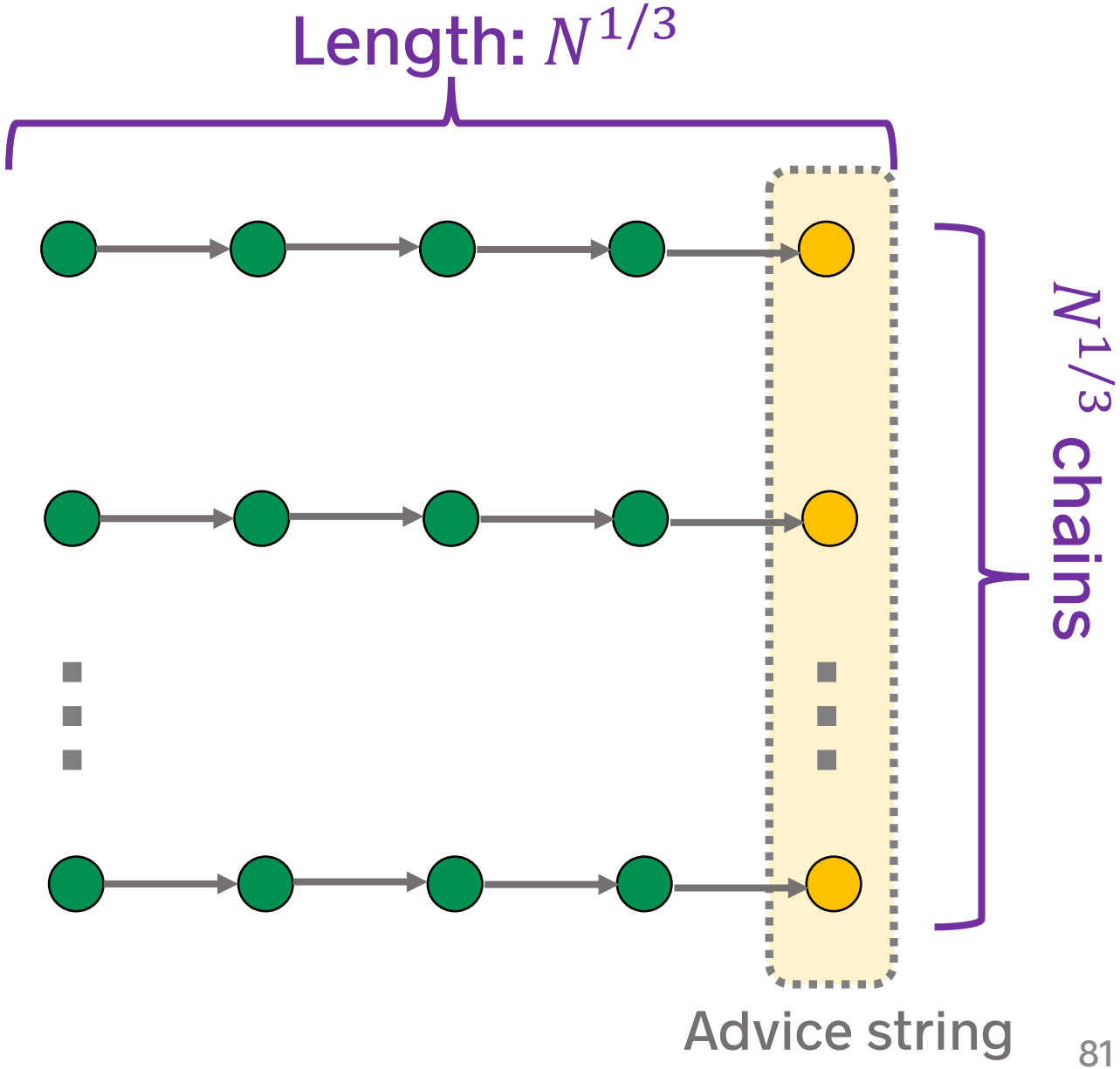

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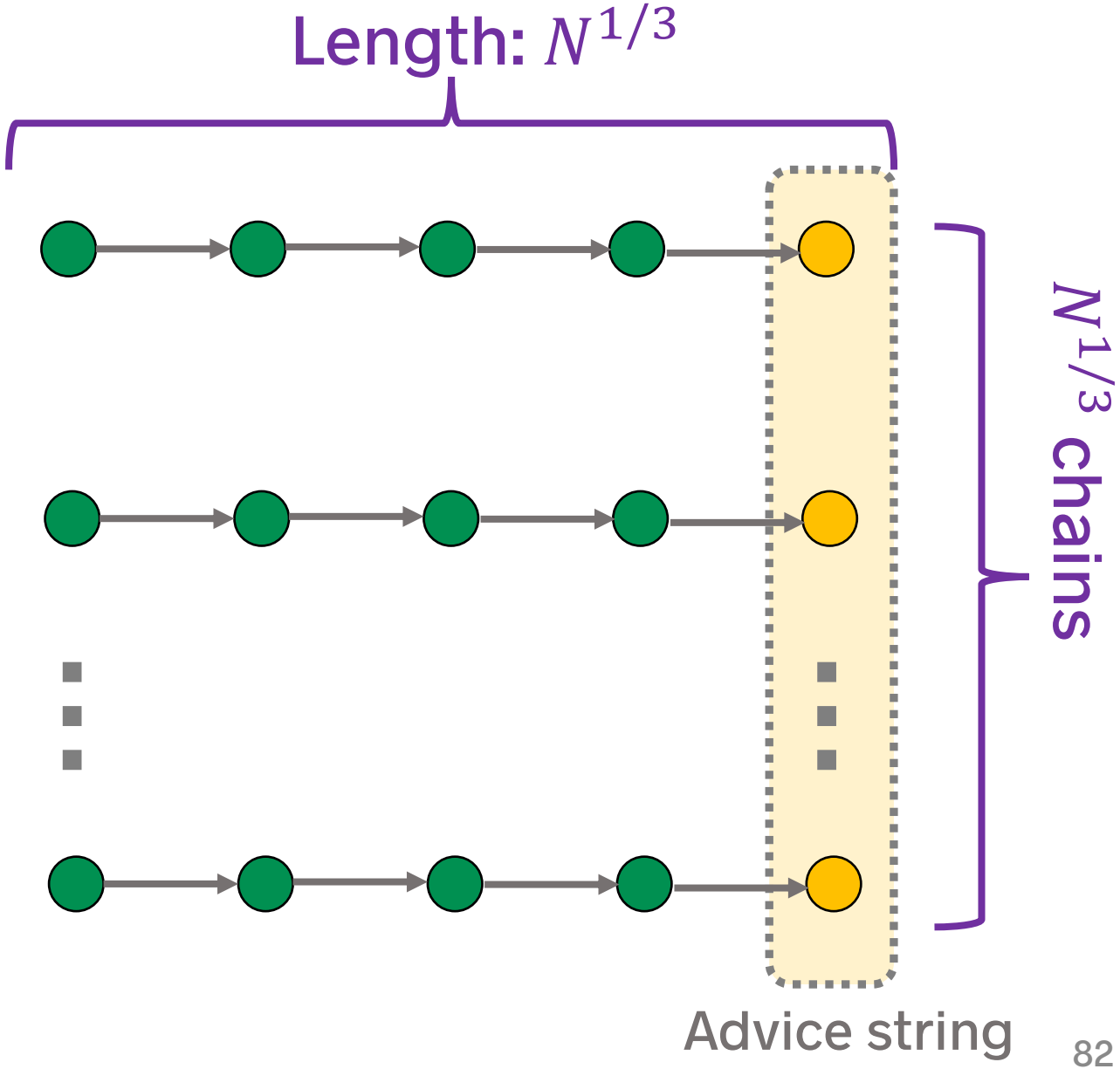

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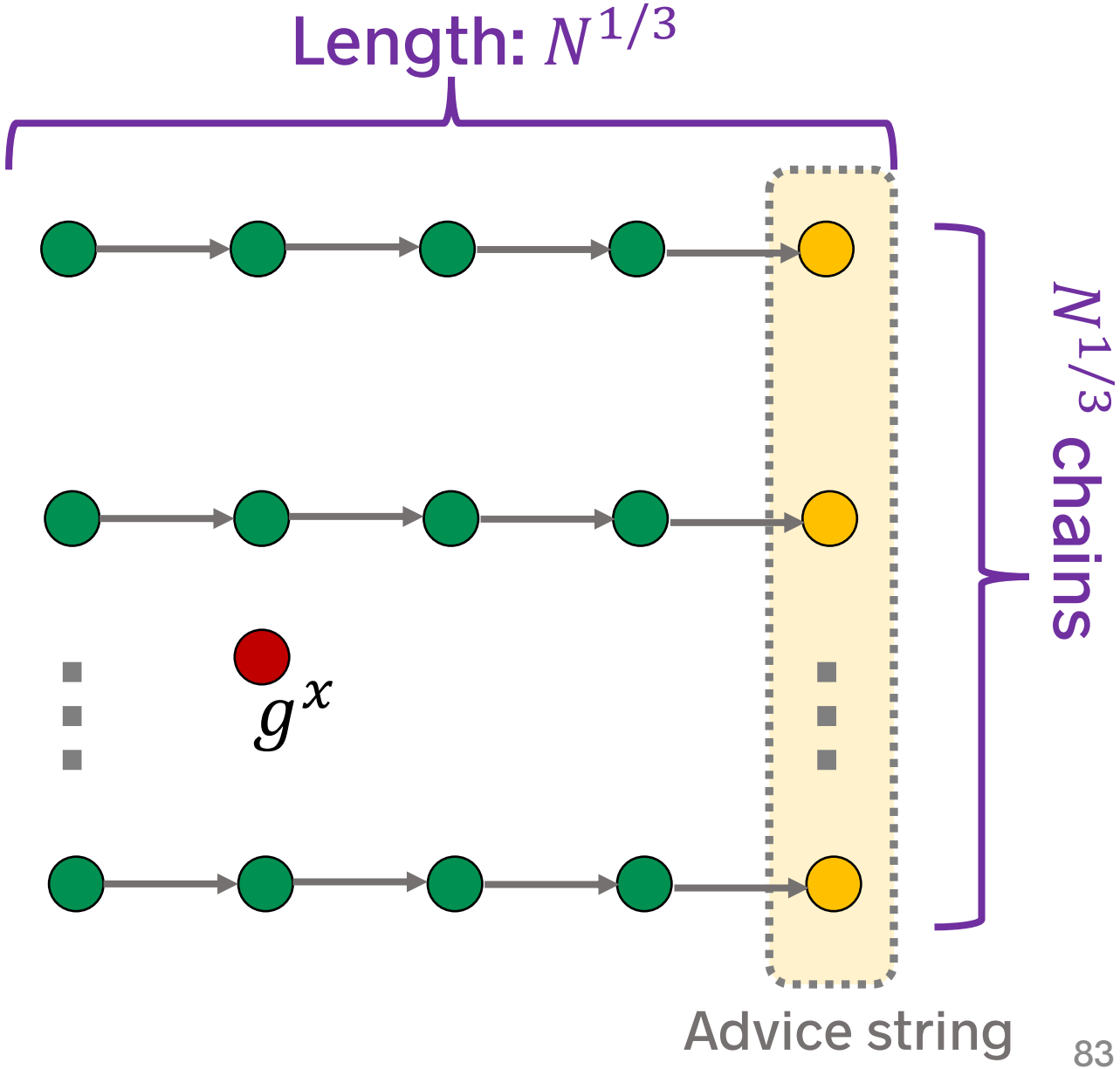

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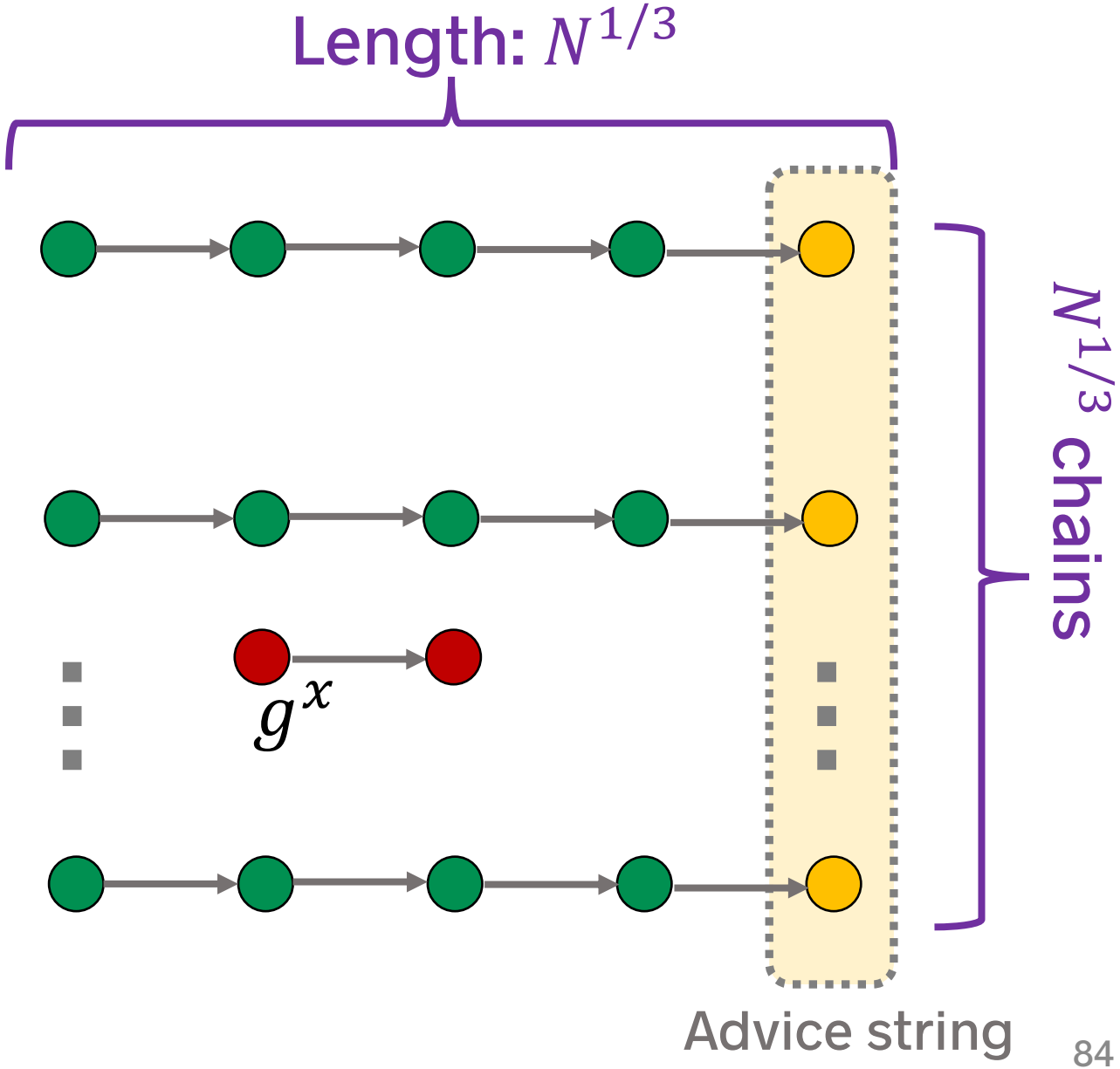

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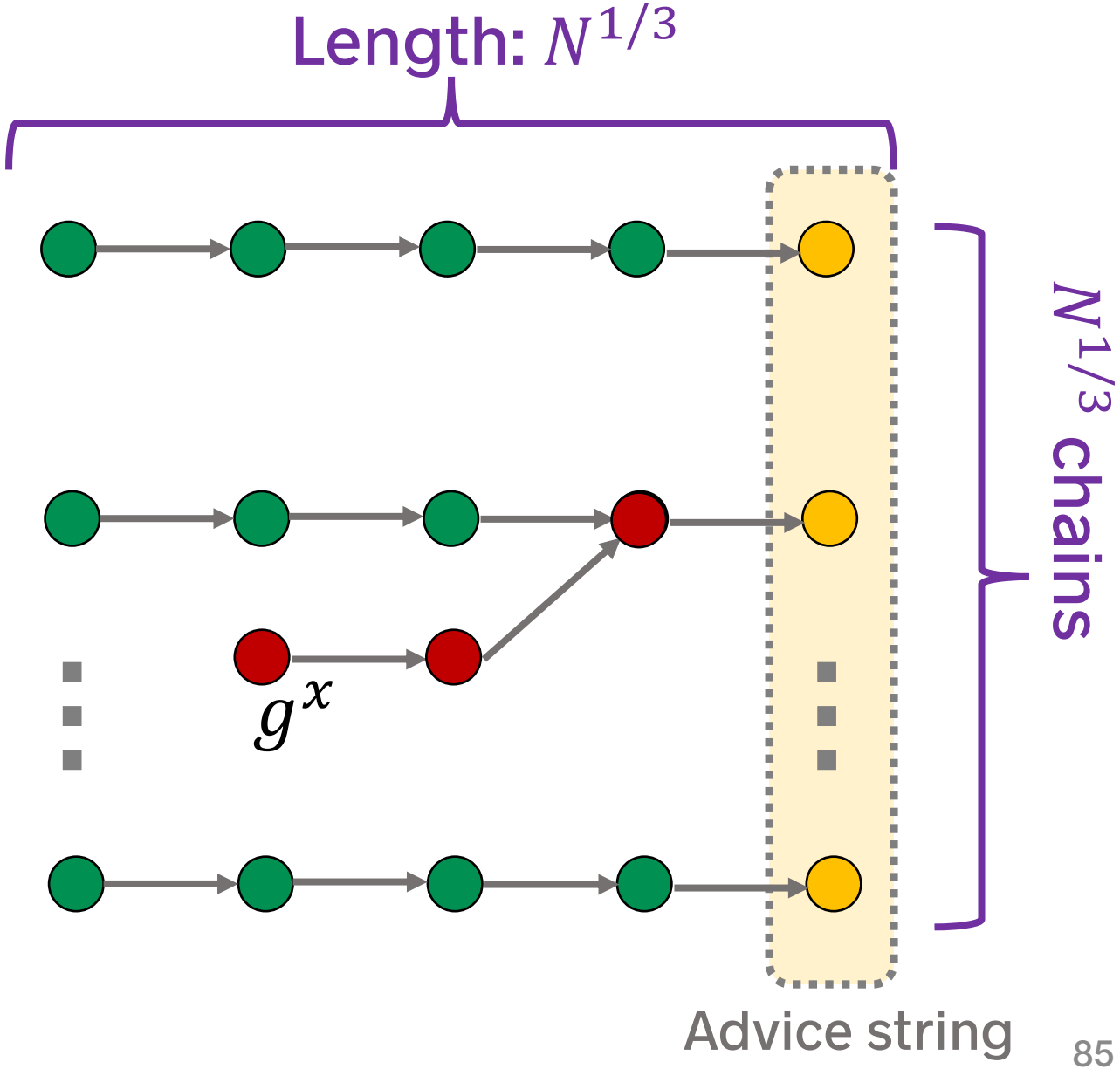

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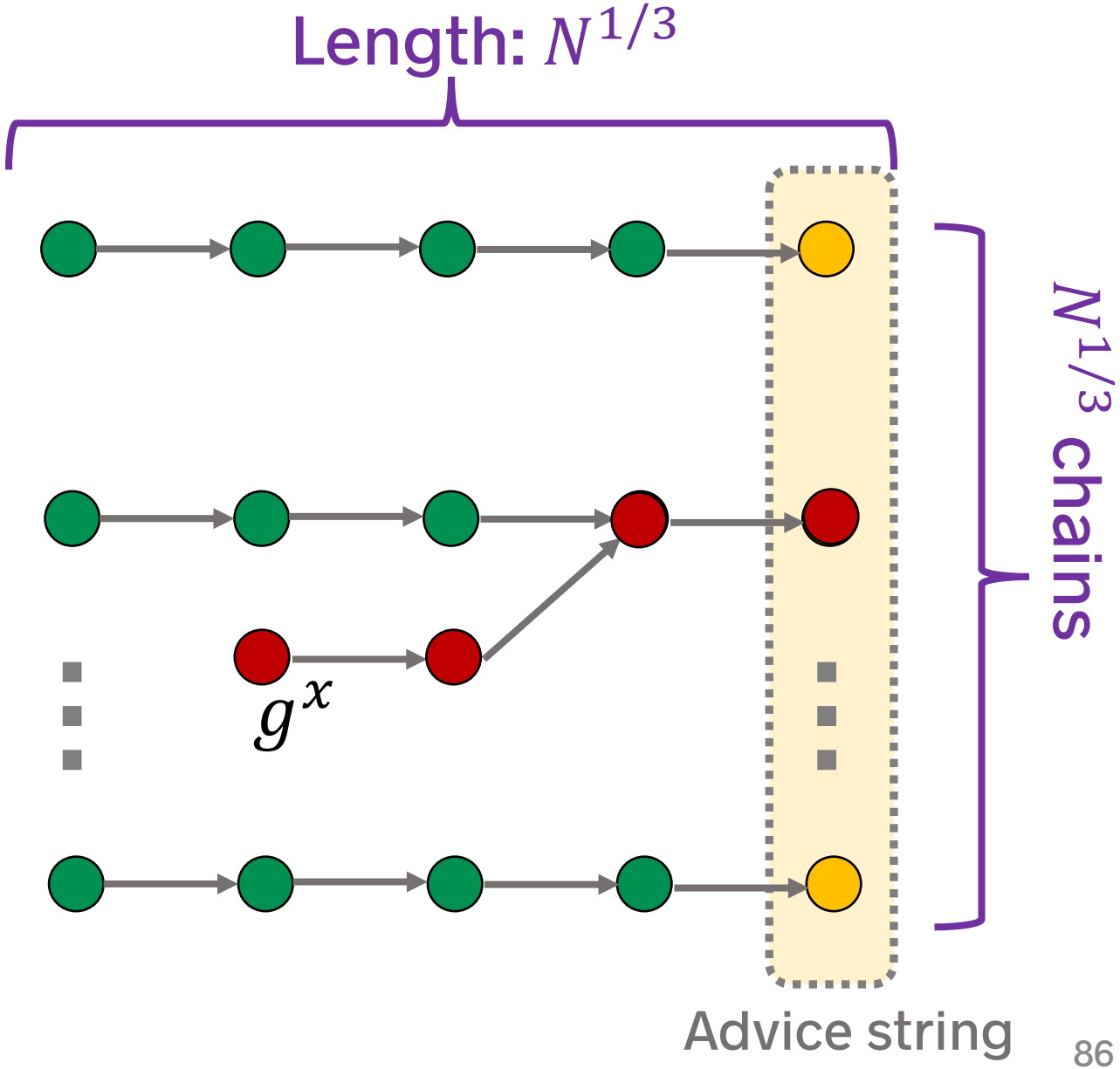

[M10, LCH11, BL13]

Preprocessing phase

- Build $N^{1/3}$ chains of length $N^{1/3}$
- Store dlogs of chain endpoints

Advice: $\tilde{O}(N^{1/3})$ bits

Online phase

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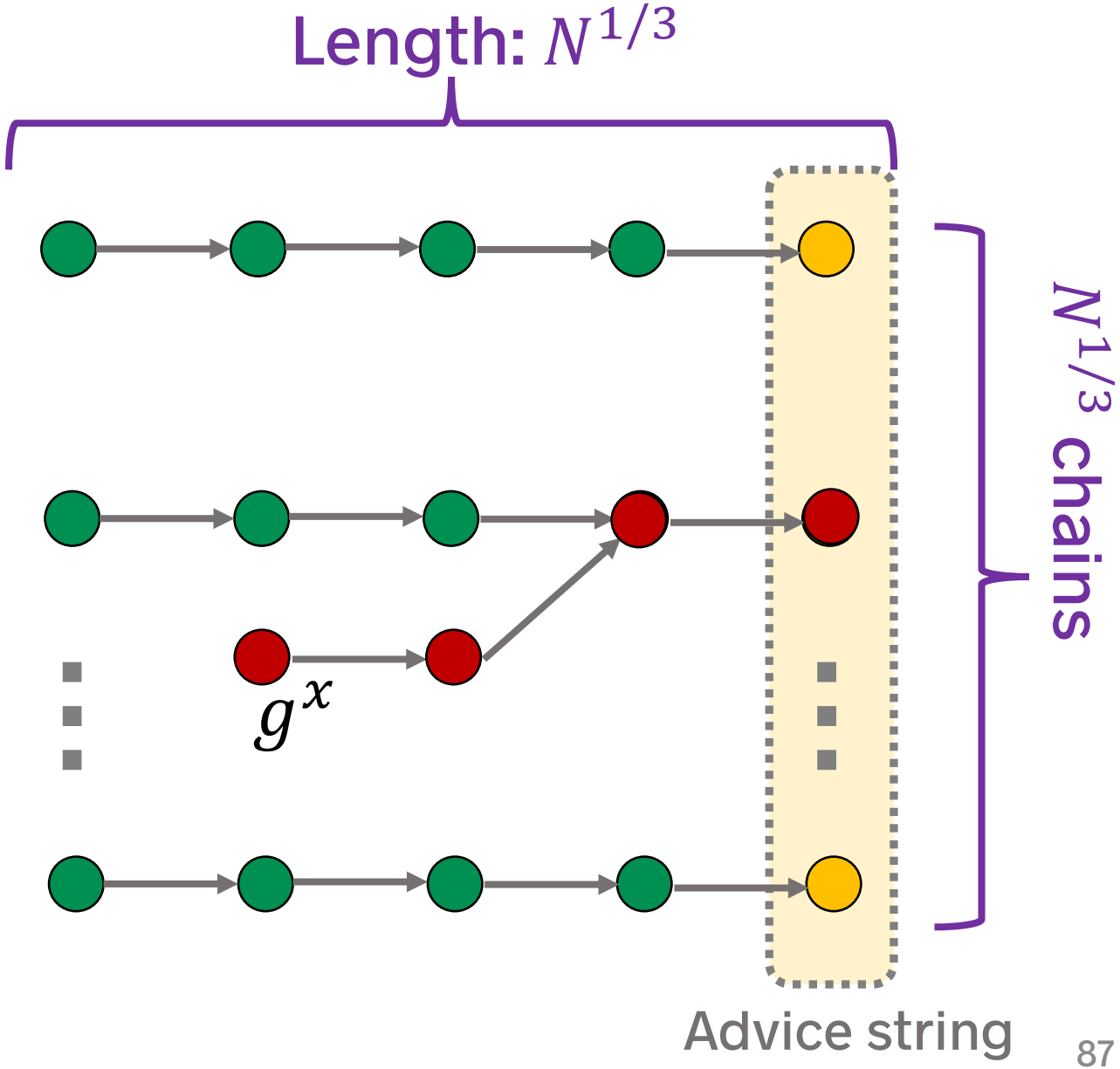
Time: $\tilde{O}(N^{1/3})$ steps



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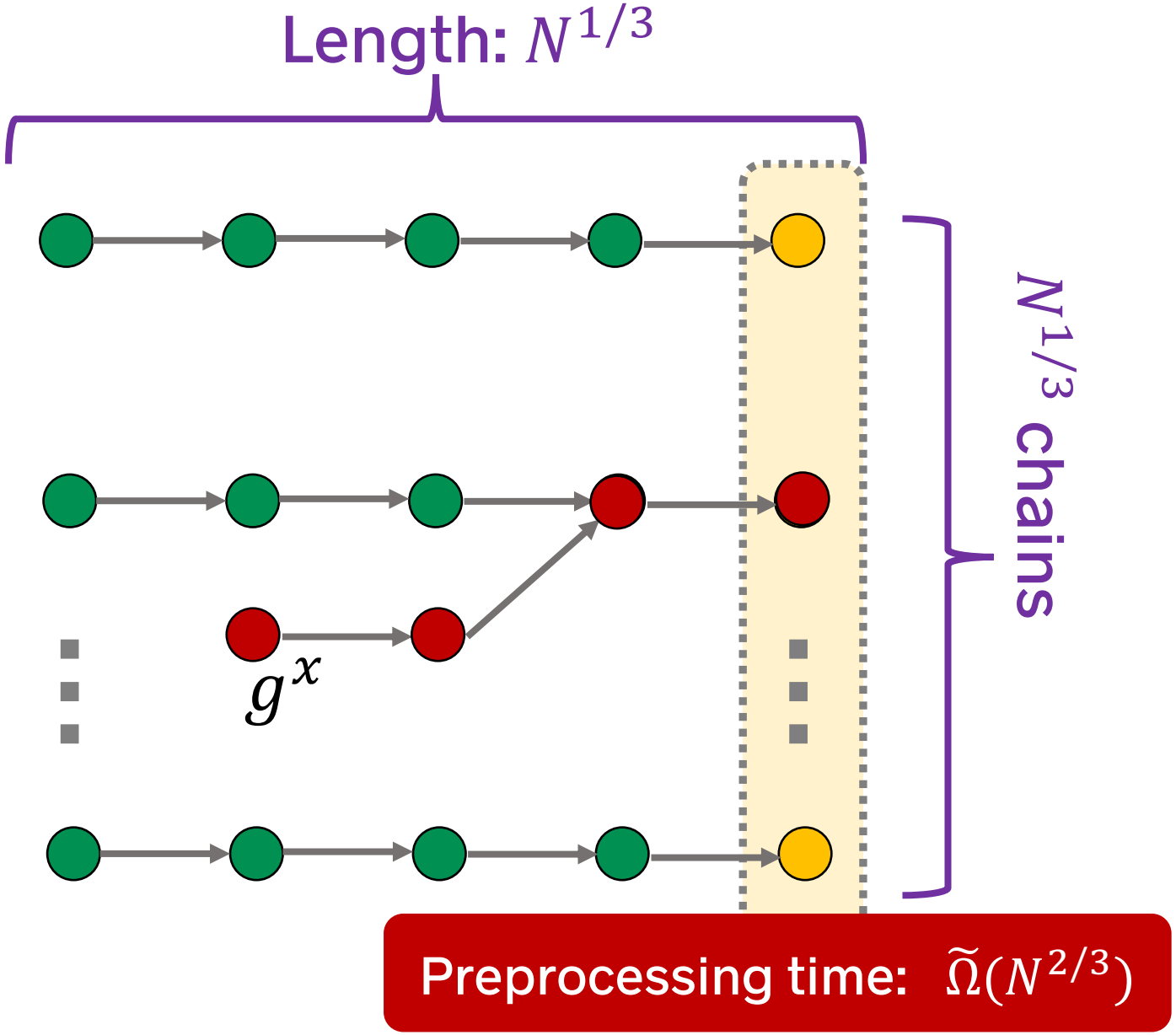

Generic discrete log

		256-bit ECDL
→ Without preprocessing:	$\Omega(N^{1/2})$	2^{128} time
→ With preprocessing:	$\tilde{O}(N^{1/3})$	2^{86} time

Related preprocessing attacks break:

- Multiple discrete log problem [This paper]
- One-round Even-Mansour cipher [FJM14]
- Merkle-Damgård hash with random IV [CDGS17]

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Is this dlog attack the best possible?!

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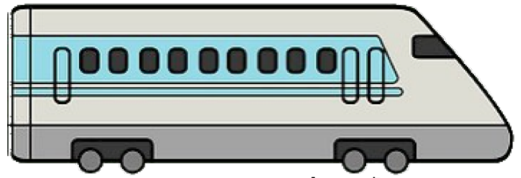
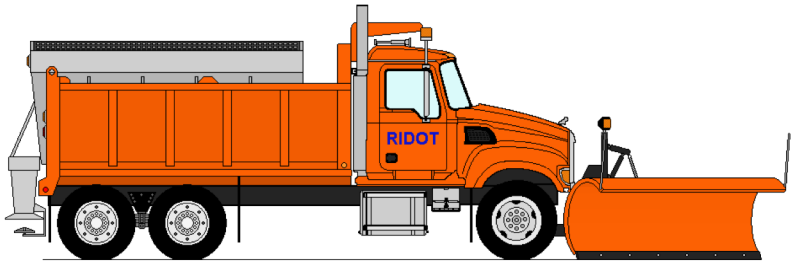
Signatures
(DSA and Schnorr)

DH key exchange

DDH



Pairings



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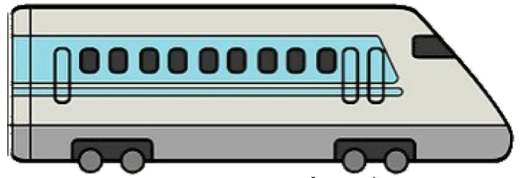

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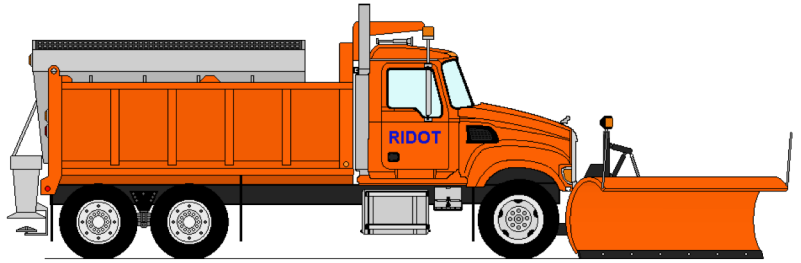

Discrete log

Could there exist a generic dlog preprocessing attack with $S = T = N^{1/10}$?

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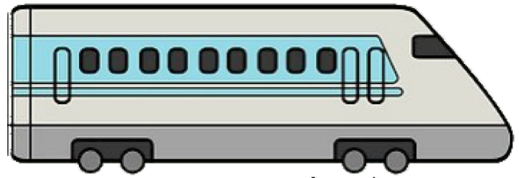

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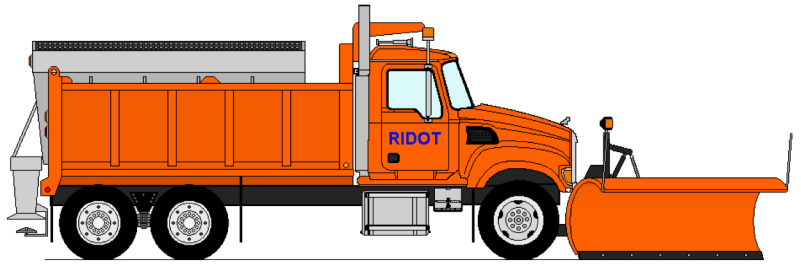
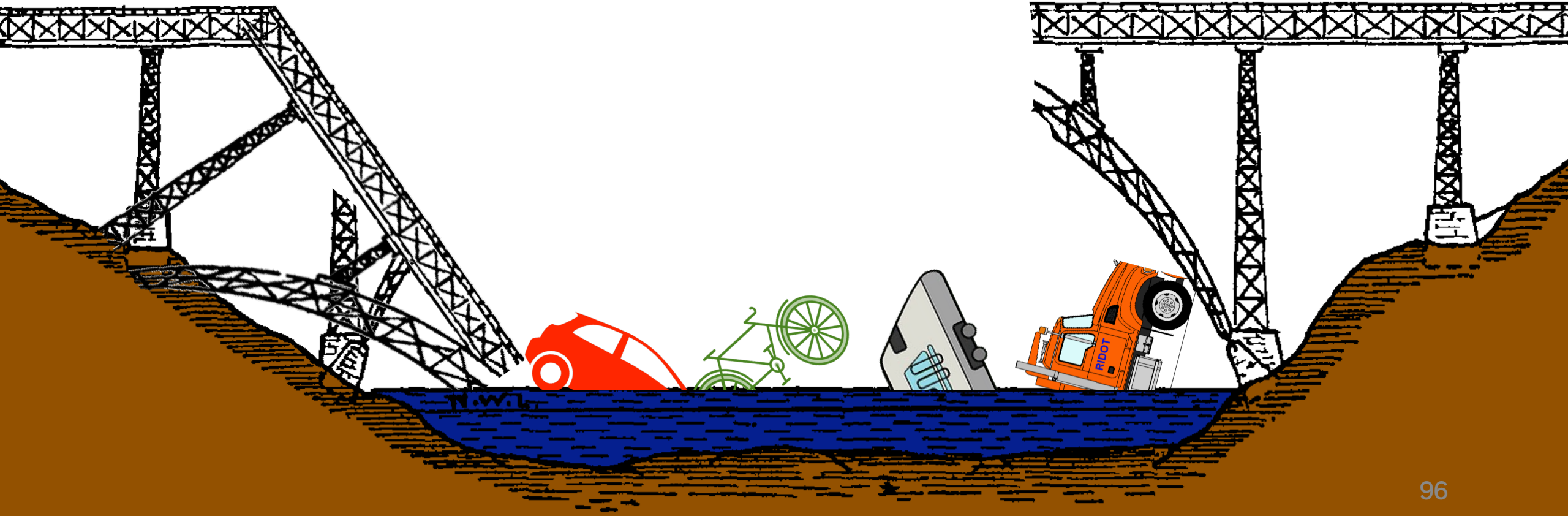


Discrete log

Could there exist a generic dlog preprocessing attack with $S = T = N^{1/10}$?

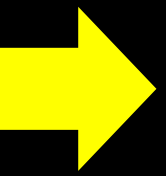
Preprocessing attacks might make us worry about 256-bit EC groups


T.W.L.
RIDOT

This talk

Background: Preprocessing attacks are relevant

- Preexisting $S = T = \tilde{O}(N^{1/3})$ generic attack on discrete log



Our results: Preprocessing lower-bounds and attacks

- The $\tilde{O}(N^{1/3})$ generic dlog attack is optimal
- Any such attack must use lots of preprocessing: $\Omega(N^{2/3})$
- New $\tilde{O}(N^{1/5})$ preprocessing attack on DDH-like problem

Open questions

Theorem. [Our paper]

Every generic dlog algorithm with preprocessing that:

- uses S bits of group-specific advice,
- uses T online time, and
- succeeds with probability ϵ,

must satisfy:

$$ST^2 = \widetilde{\Omega}(\epsilon N)$$

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This bound is tight for the full range of parameters (up to log factors)

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Shoup's proof technique (1997) relies on $\mathcal{A}$ having no information about the group $\mathbb{G}$ when it starts running

→ Need different proof technique

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Online time $N^{1/3}$ implies $\Omega(N^{2/3})$ preprocessing

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Reminder: Generic-group model

- A group is defined by an injective **"labeling" function**

$$\sigma: \ \mathbb{Z}_N \to \{0,1\}^*$$

- Algorithm has access to a **group-operation oracle:**

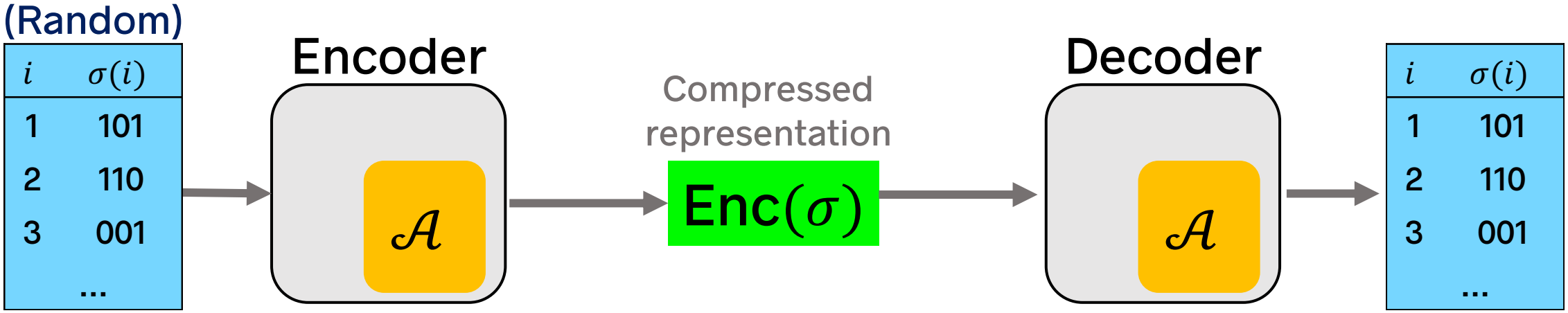
$$\mathcal{O}_\sigma\big(\sigma(i), \sigma(j)\big) \ \mapsto \ \sigma(i+j)$$

E.g., A dlog algorithm takes as input $\big(\sigma(1), \sigma(x)\big)$, representing (g, g^x), make queries to $\mathcal{O}_\sigma$, outputs x.

We prove the lower bound using an incompressibility argument [Yao90, GT00, DTT10, DHT12, DGK17...]

Use $\mathcal{A}$ to compress the mapping σ: $\mathbb{Z}_N \to \{0,1\}^*$ that defines the group

Similar technique used in [DHT12]



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- Random string is incompressible ⇒ Lower bound on S and T

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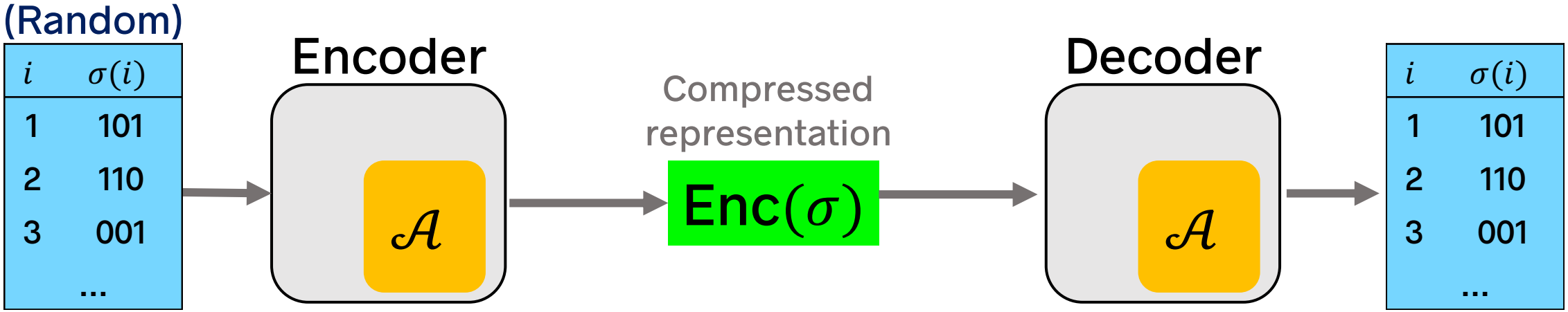

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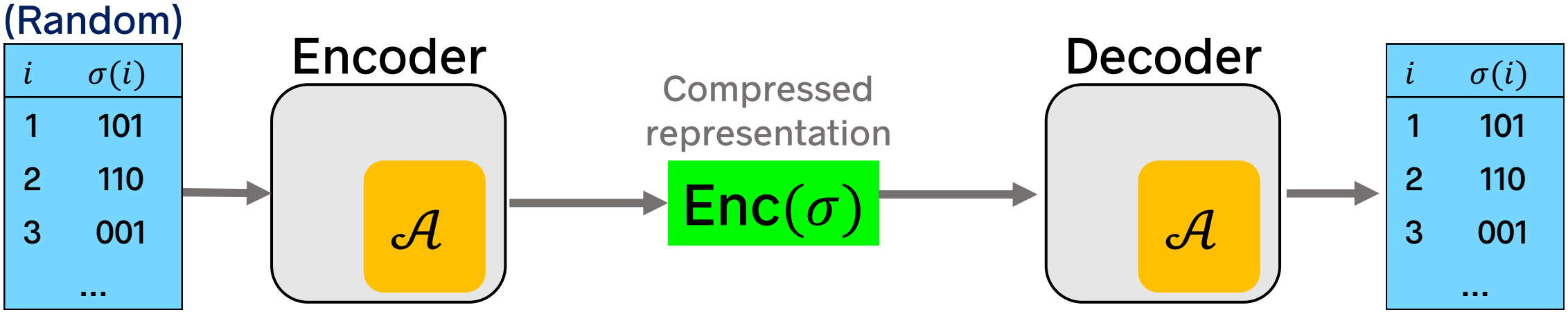

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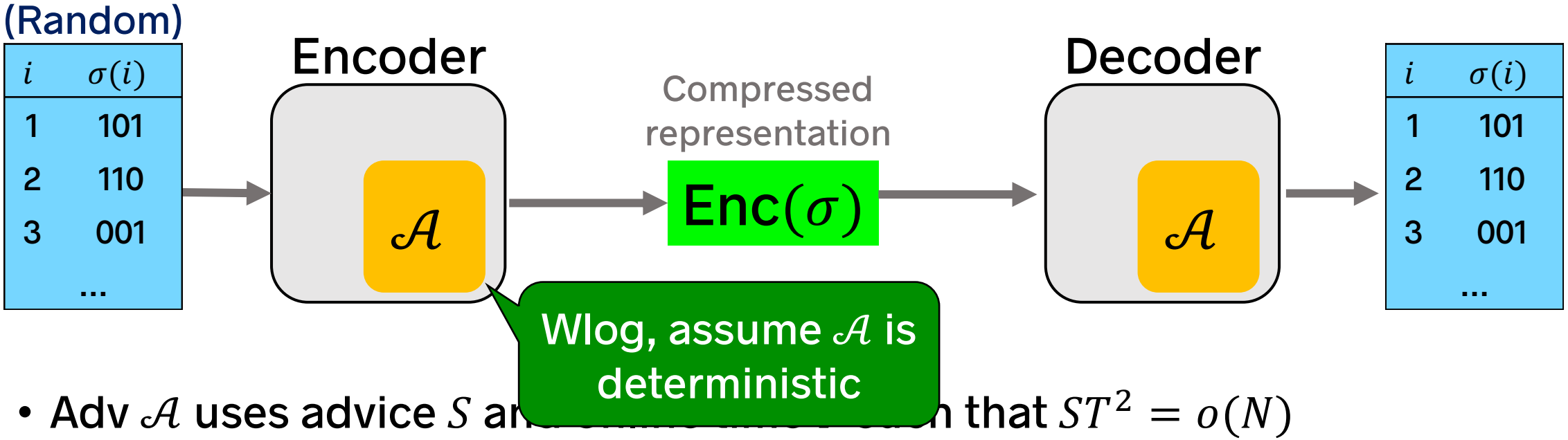

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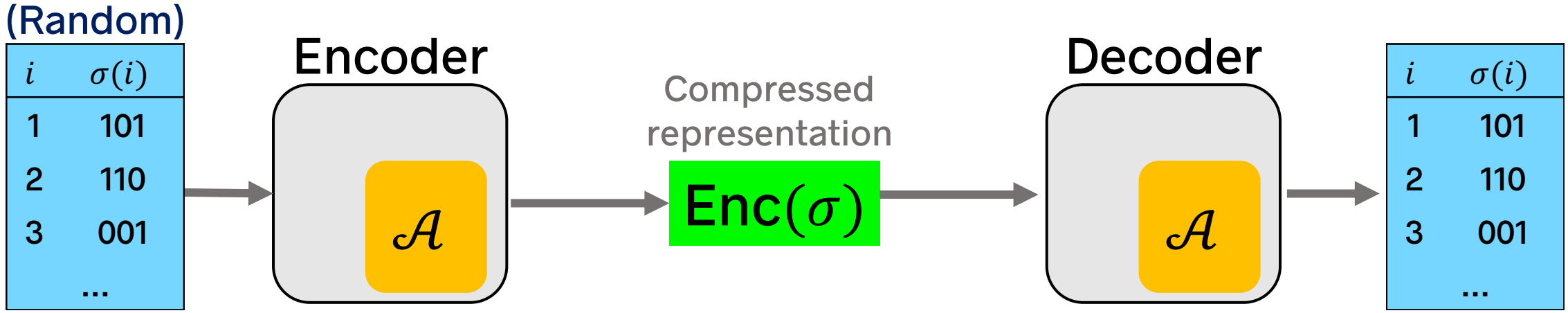

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Encoder

i	$\sigma(i)$
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4	000
5	1111
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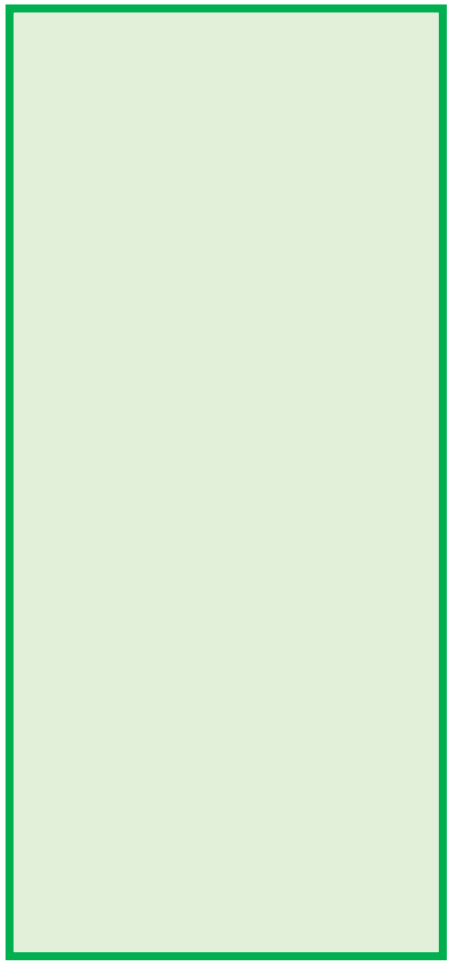
Compressed representation of σ



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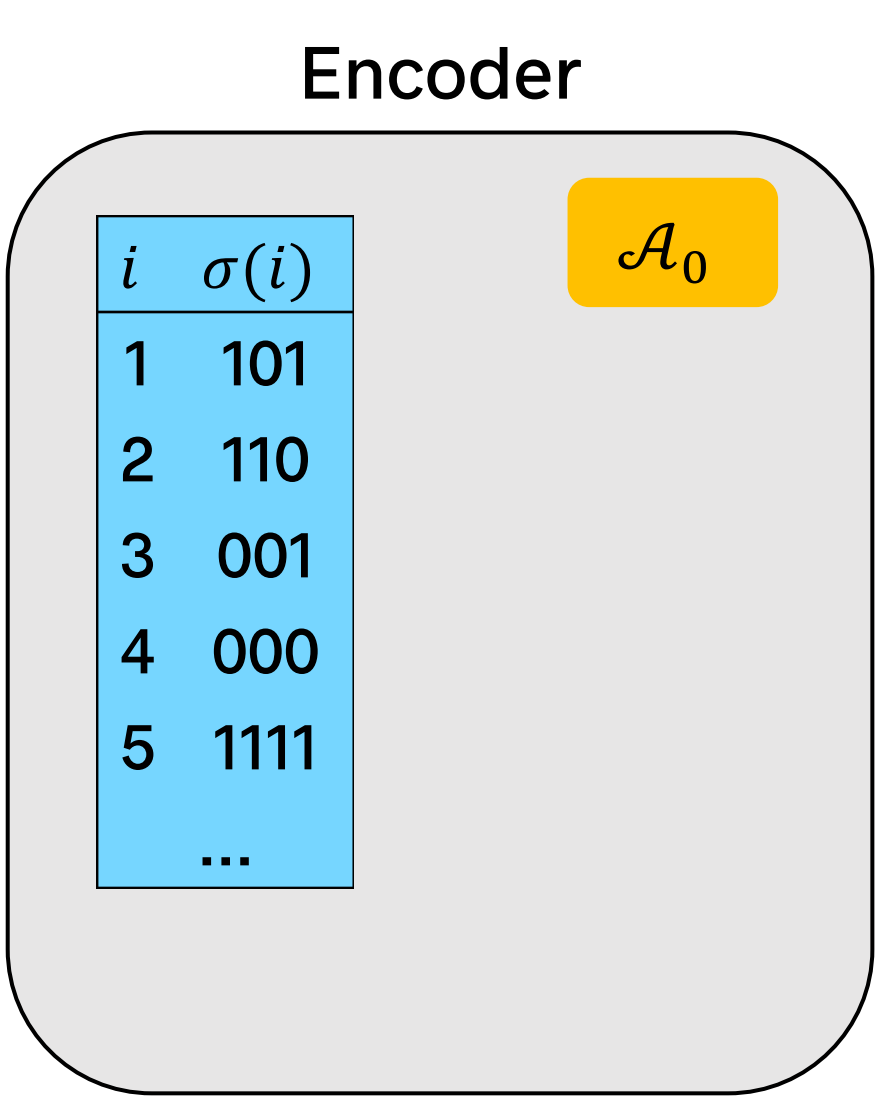

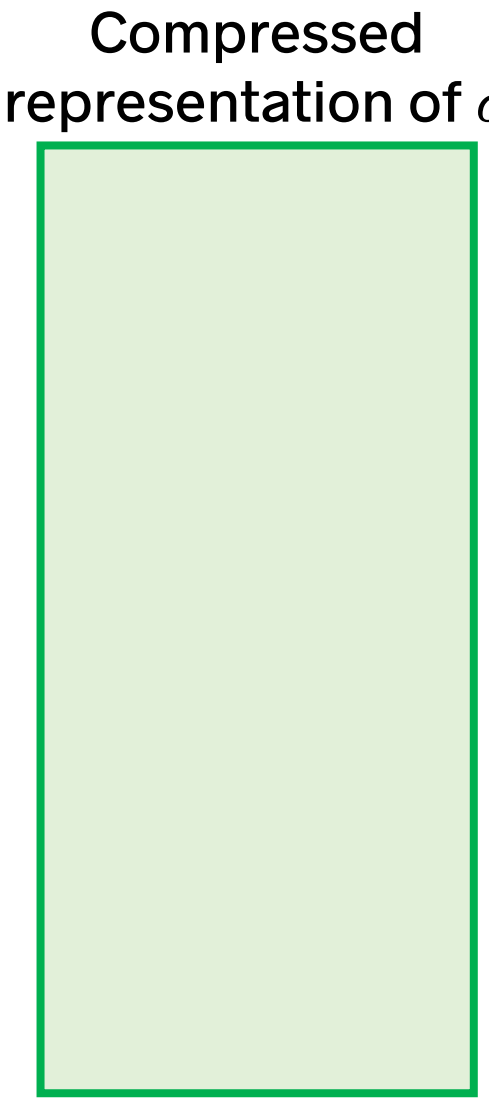

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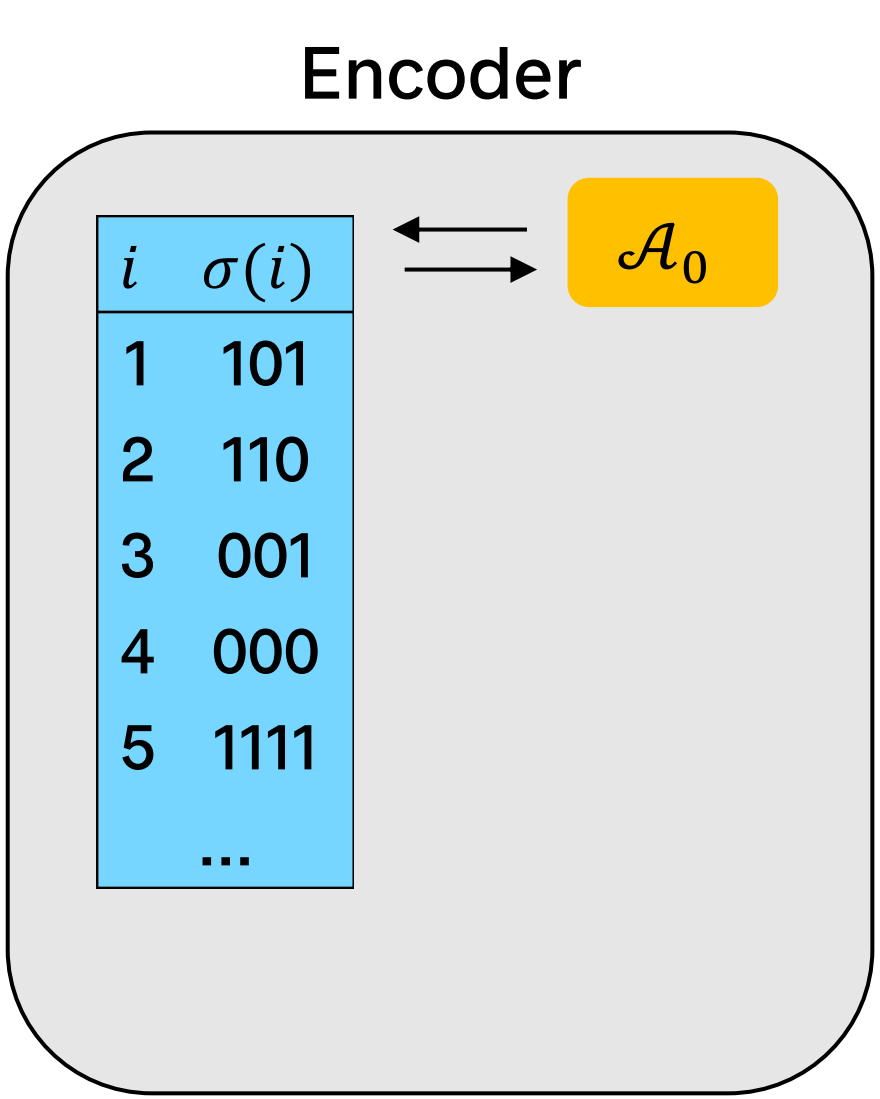

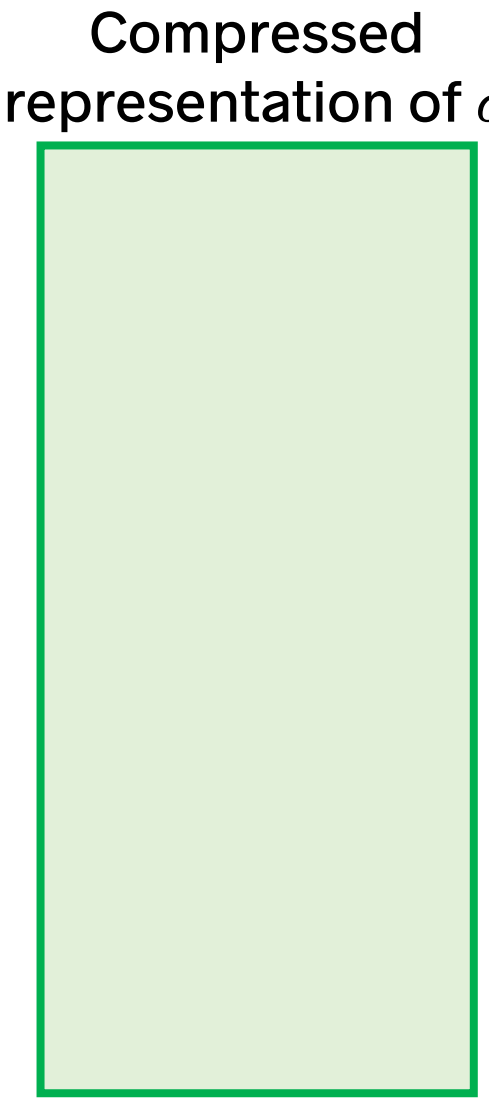

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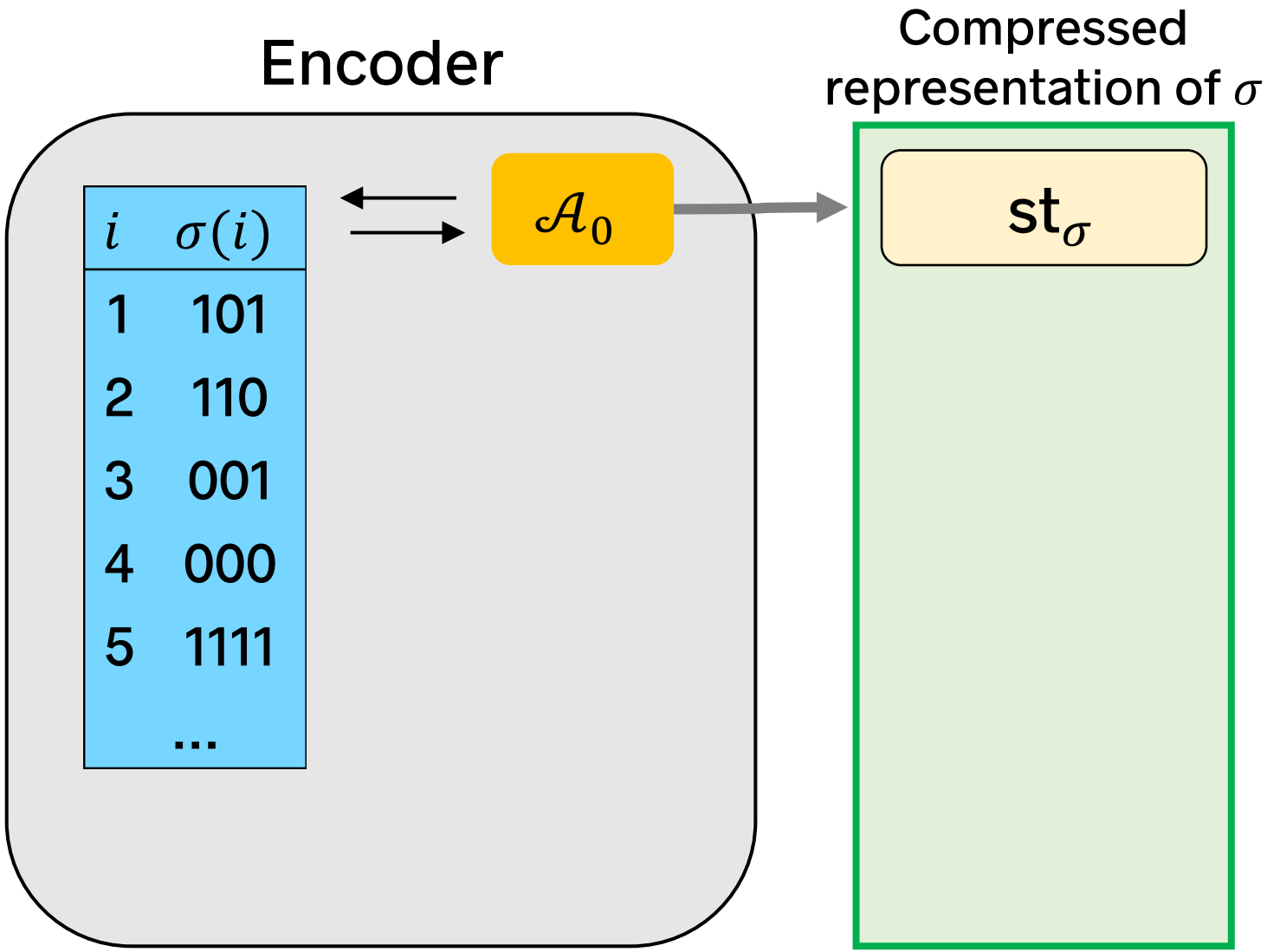

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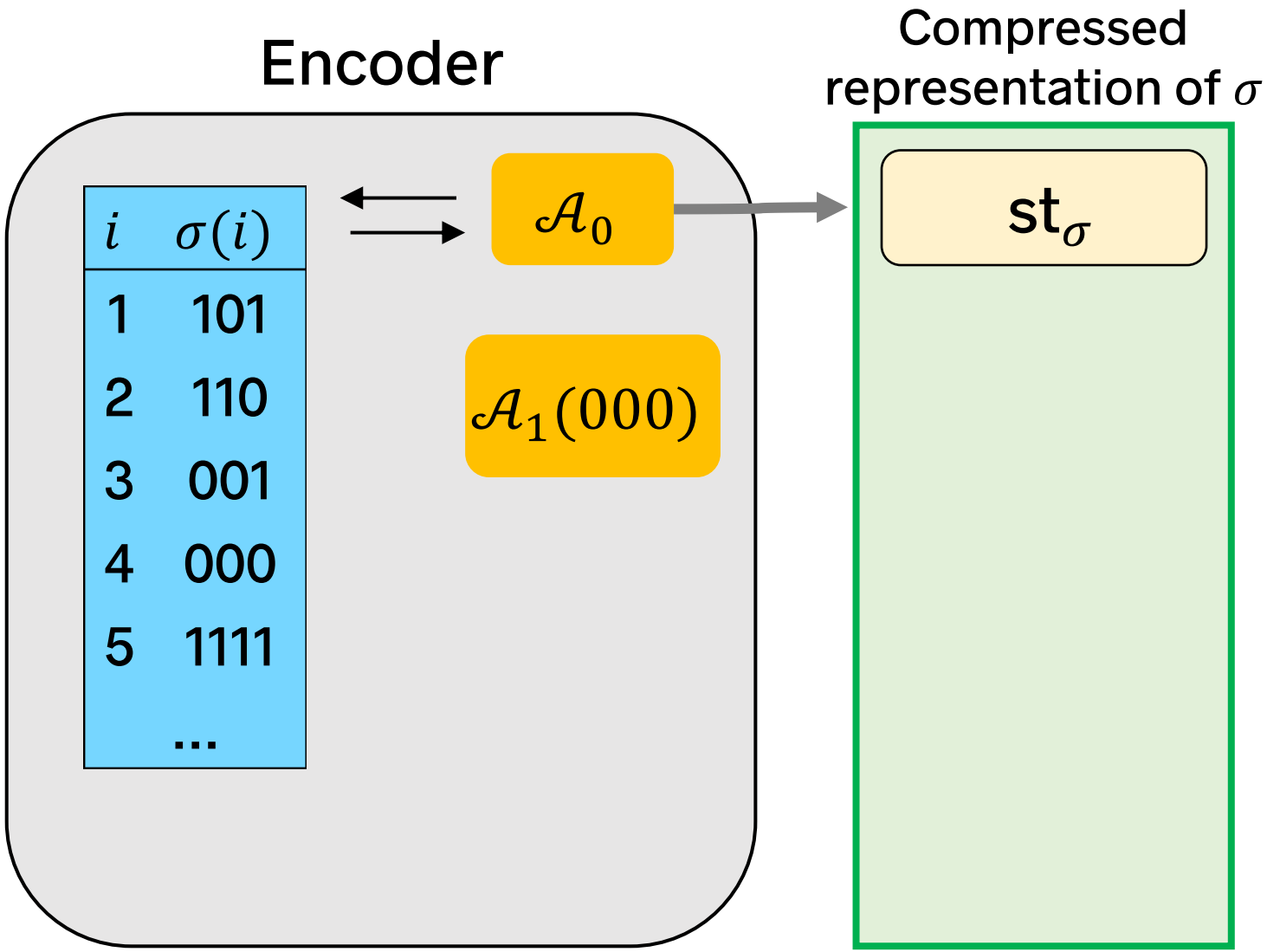

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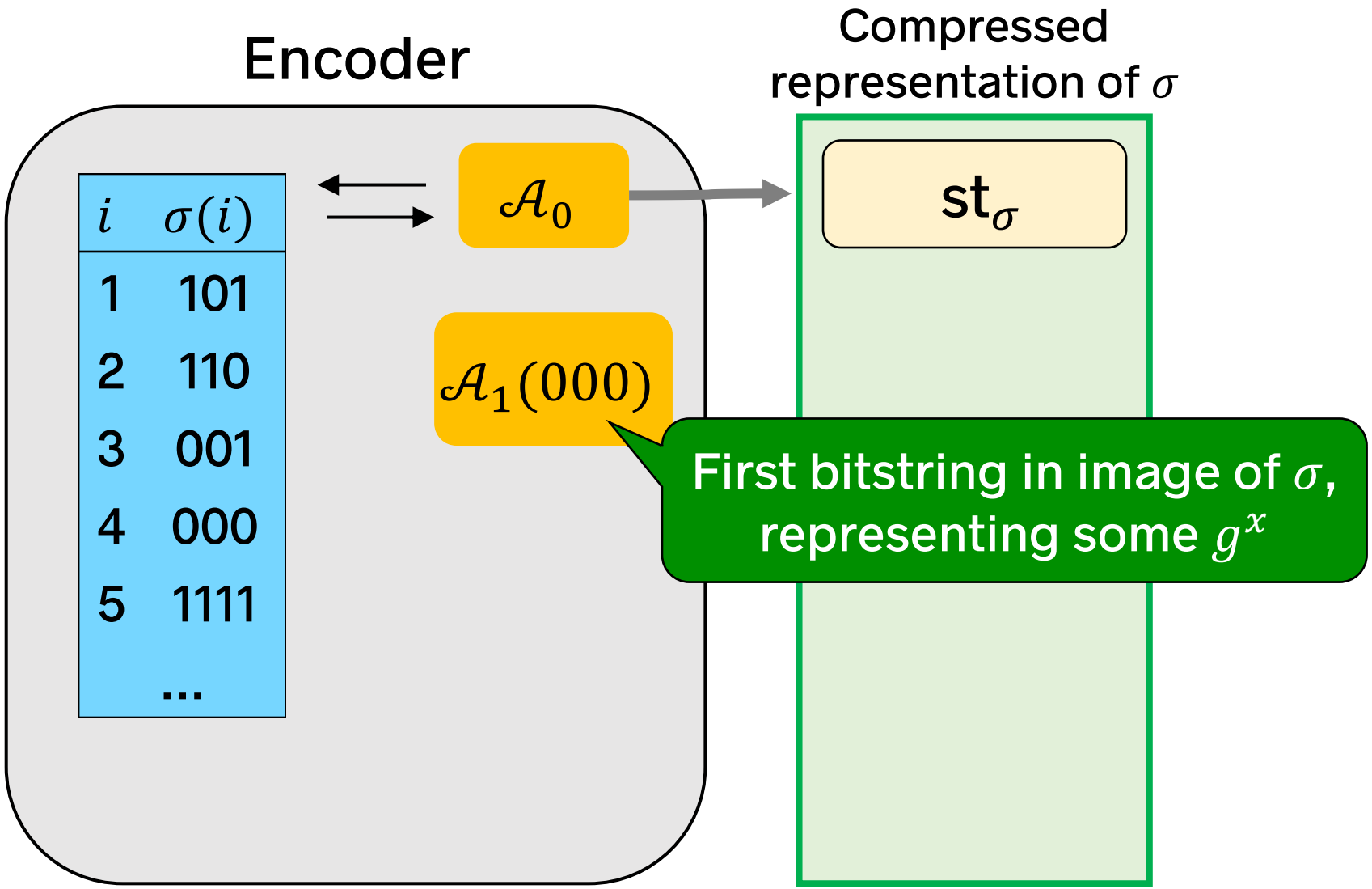

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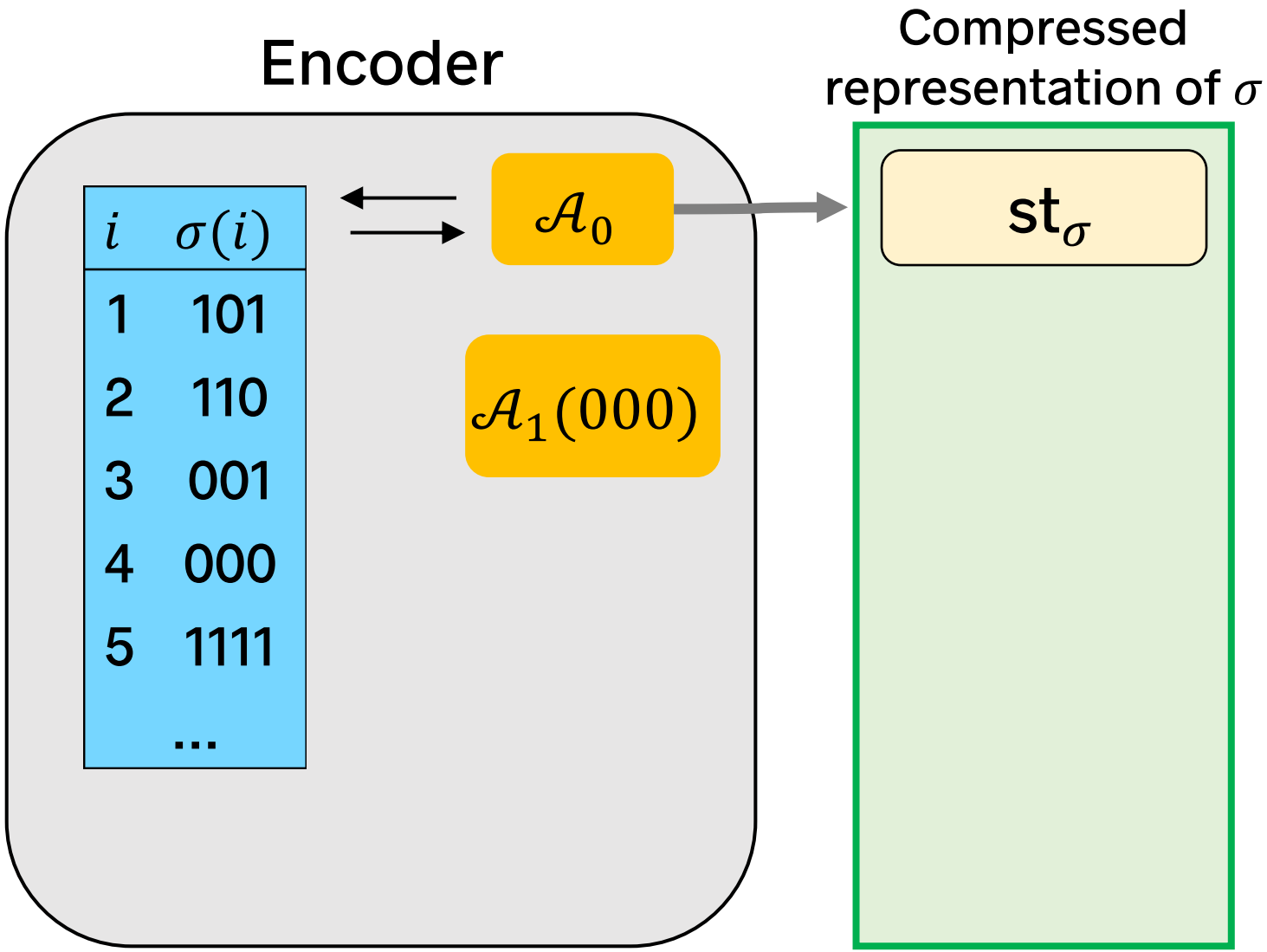

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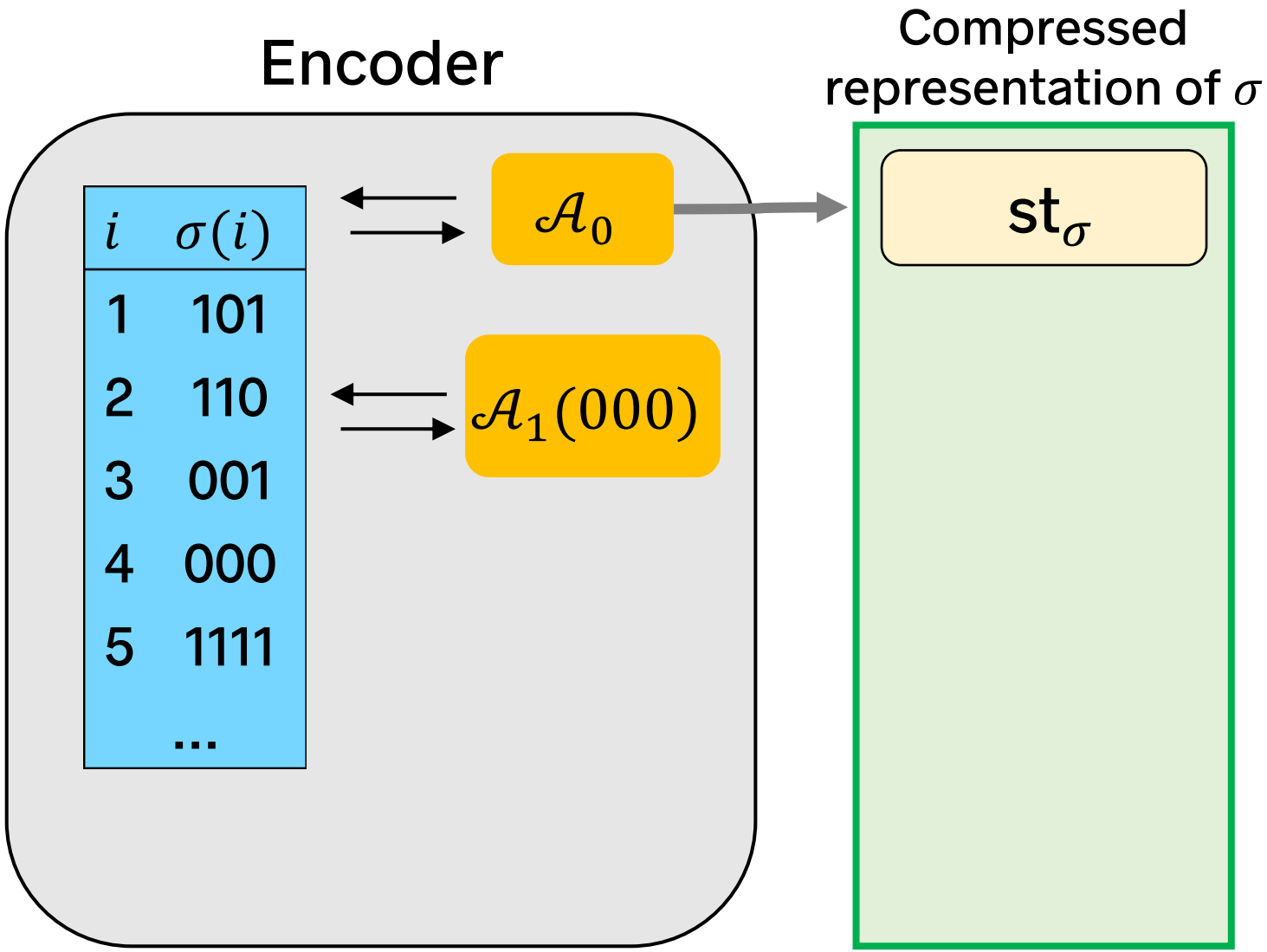

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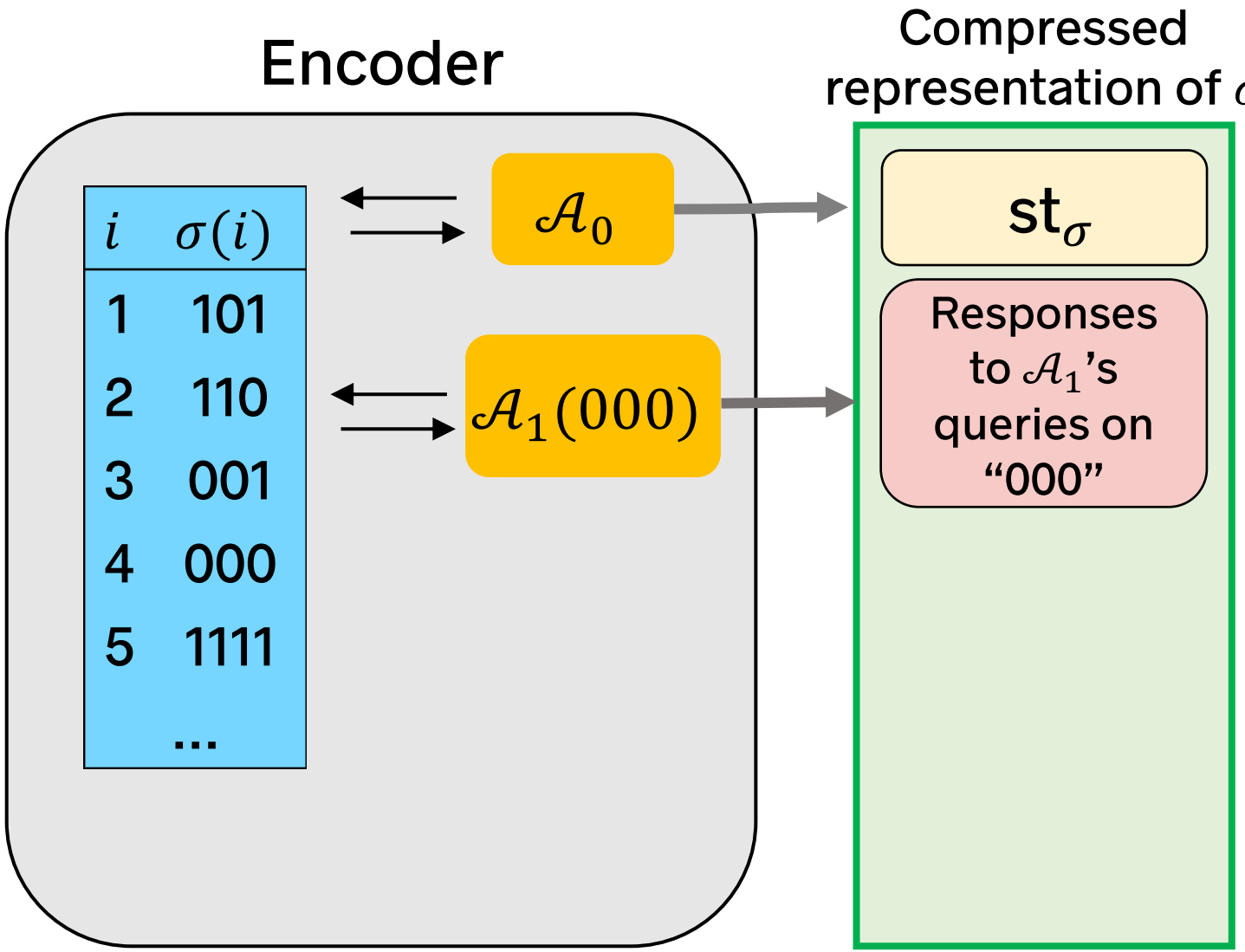

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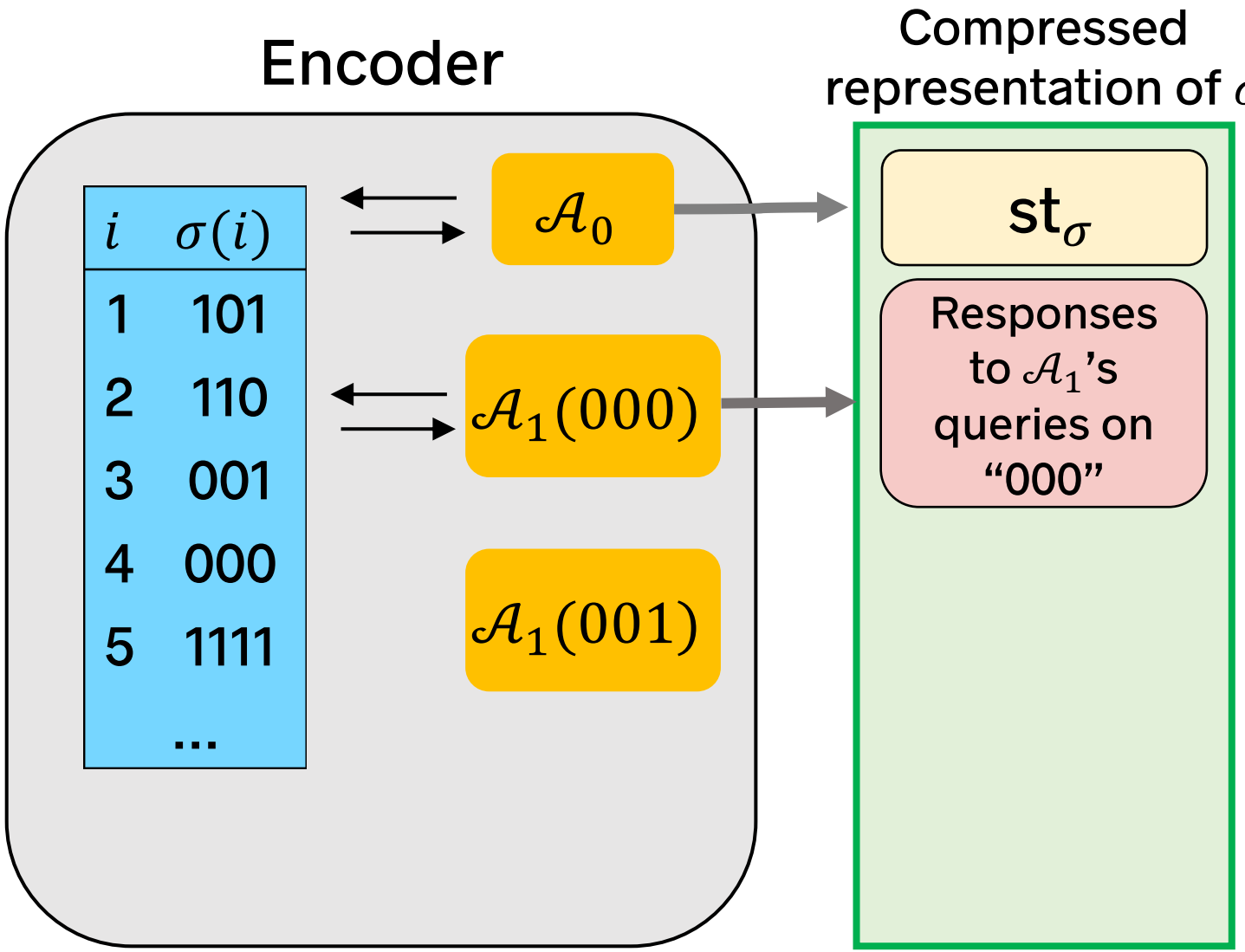

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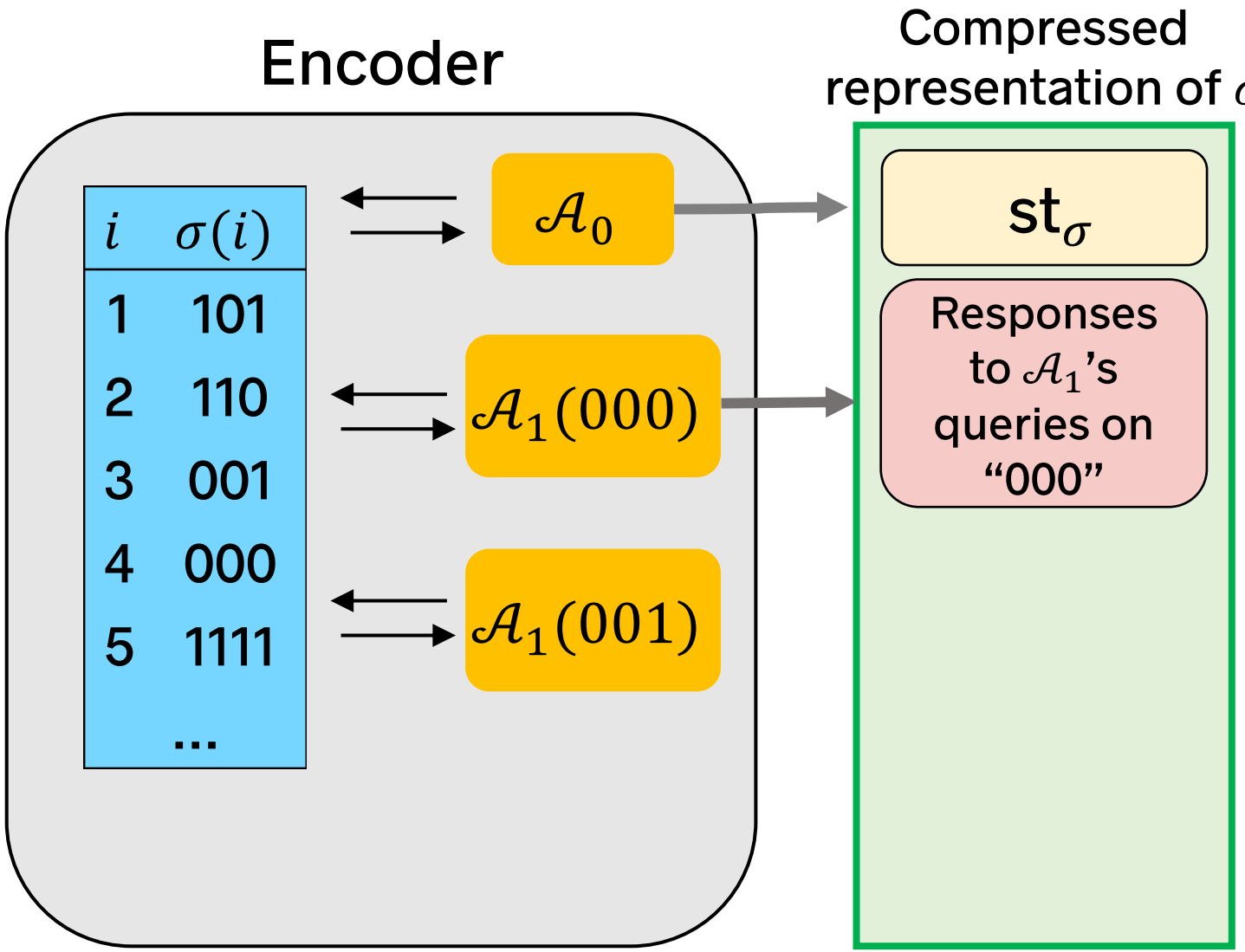

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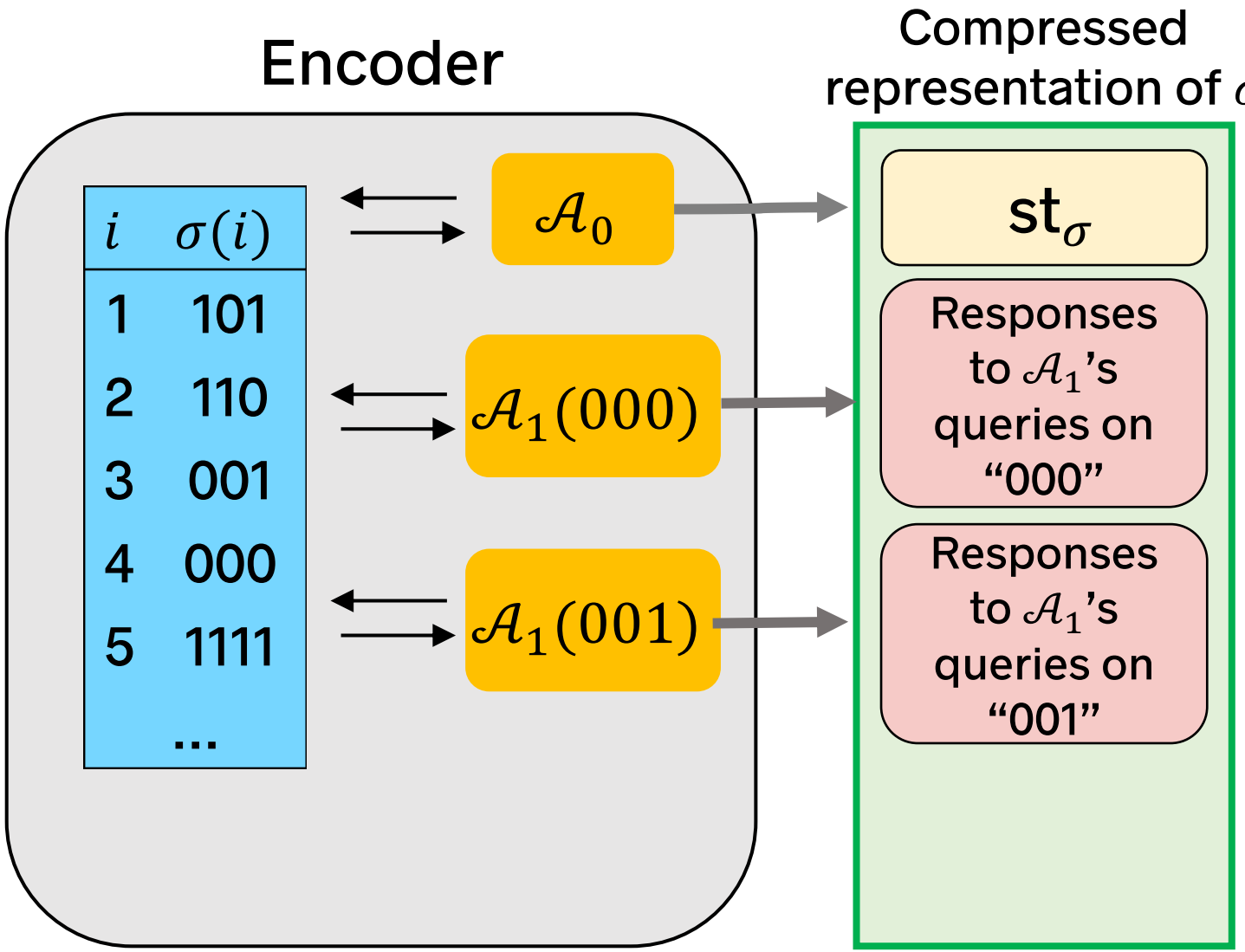

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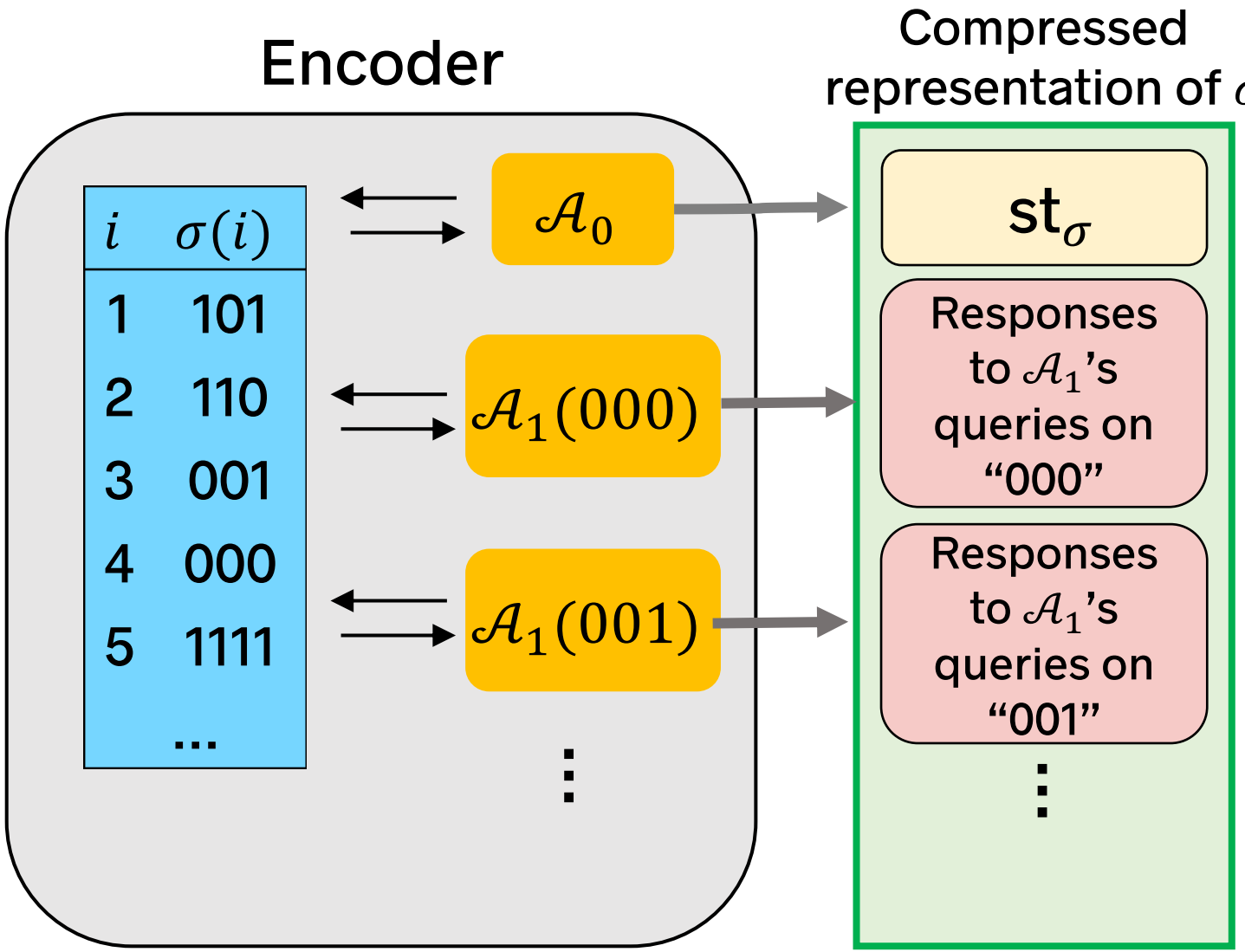

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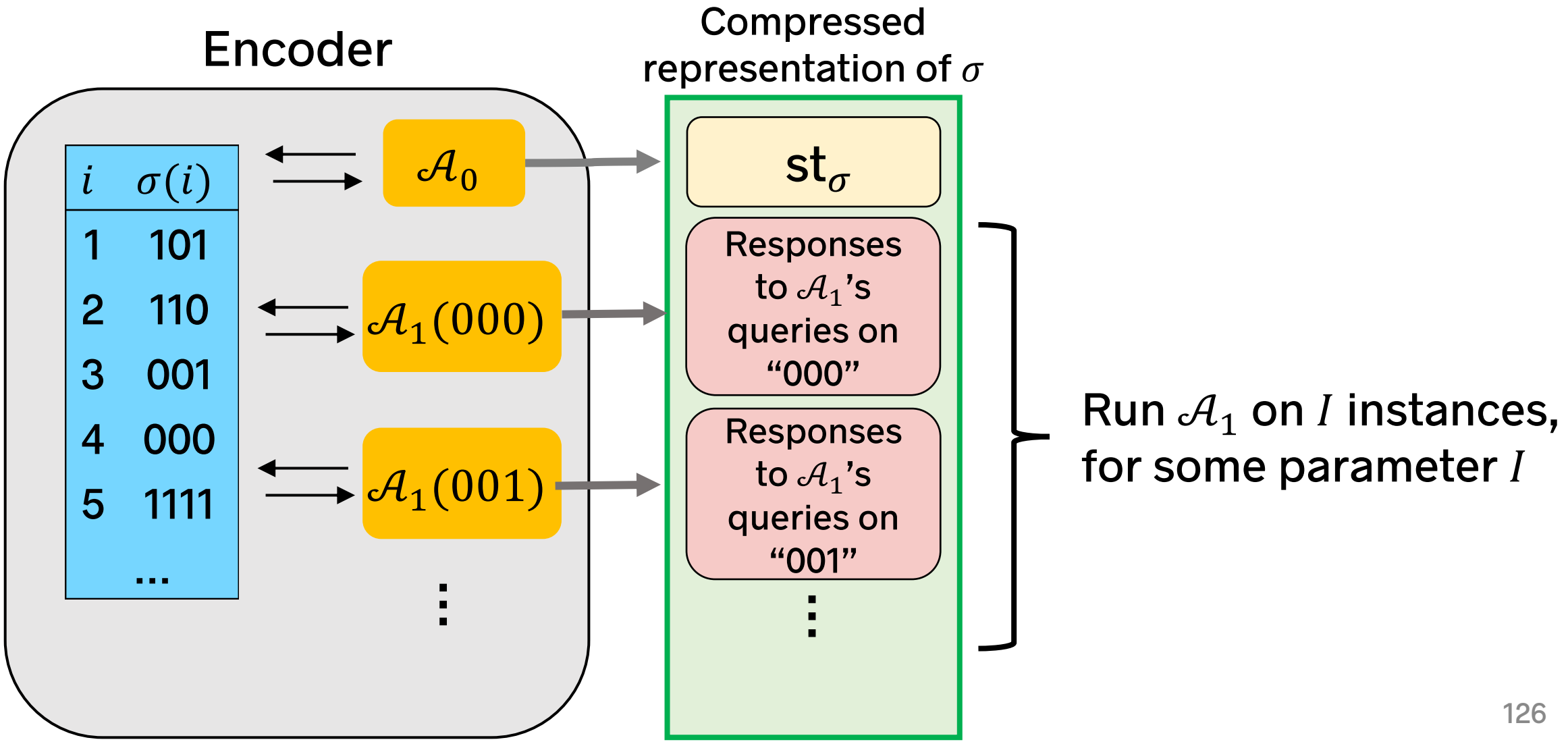

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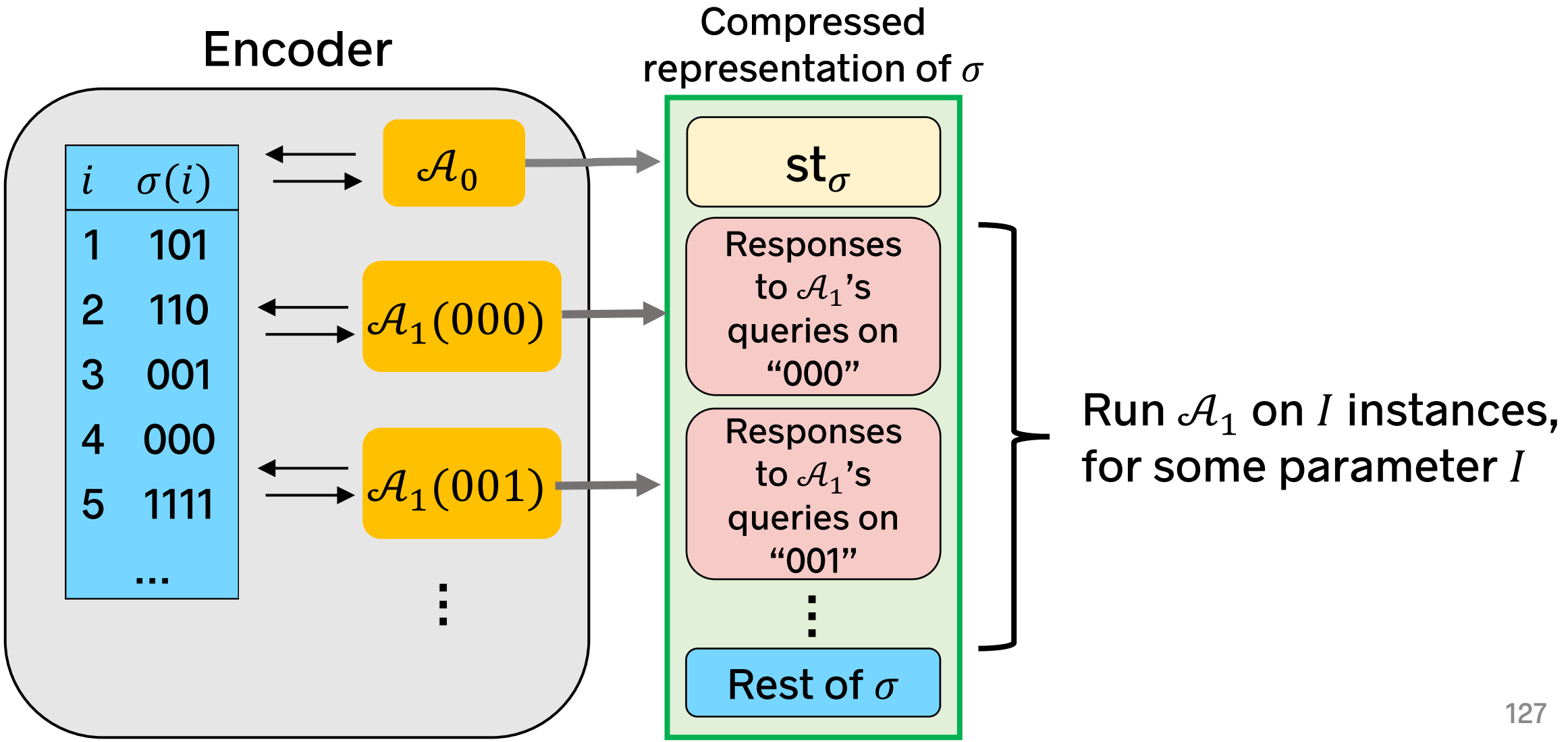

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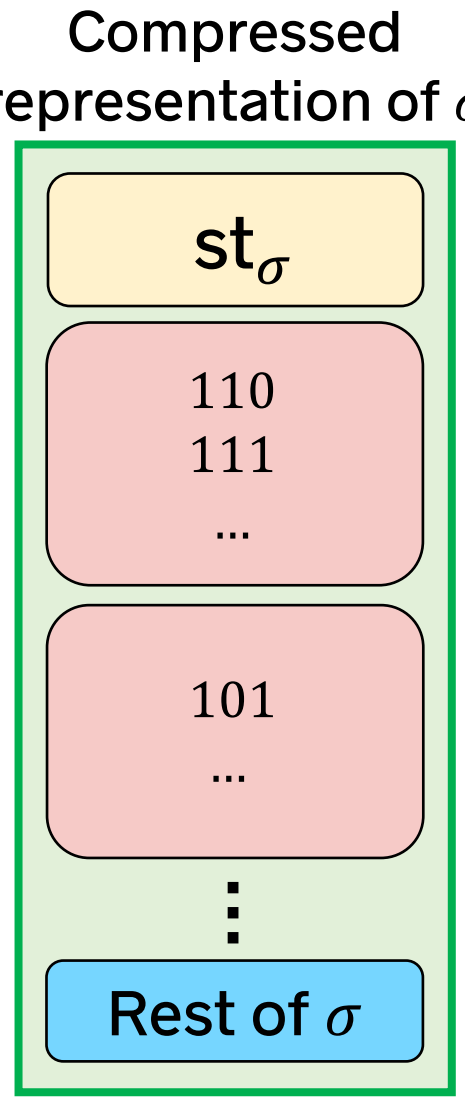
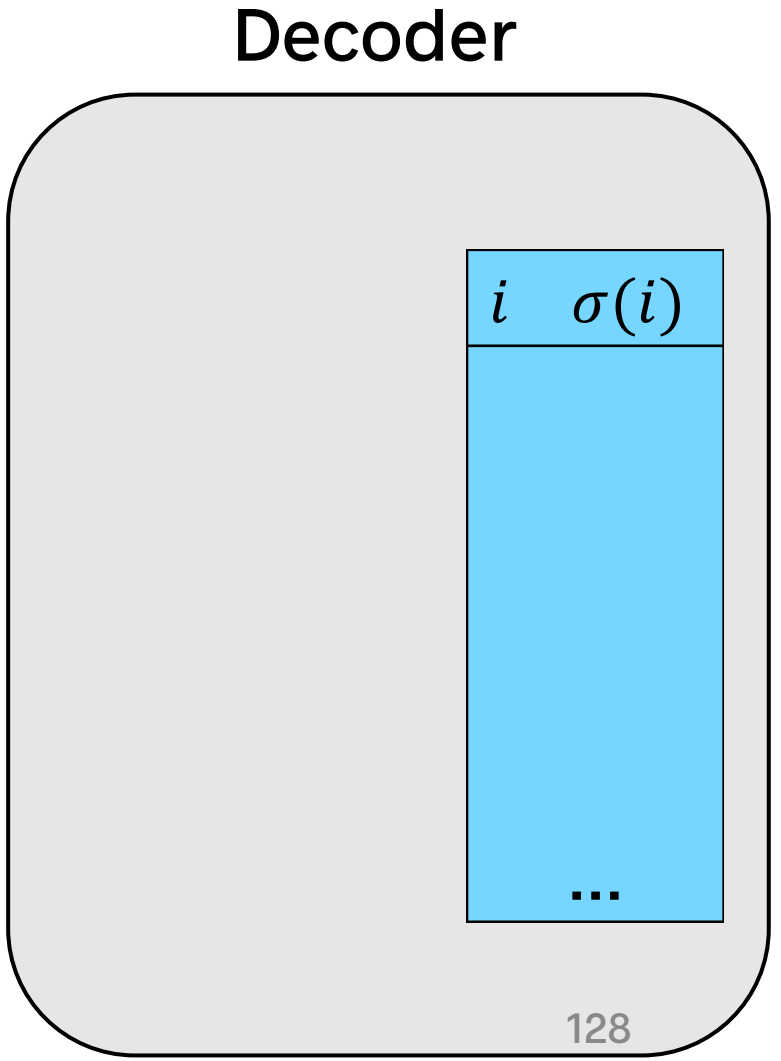
- Whenever $\mathcal{A}_1$ outputs a dlog, we get one value $\sigma(i)$ "for free"





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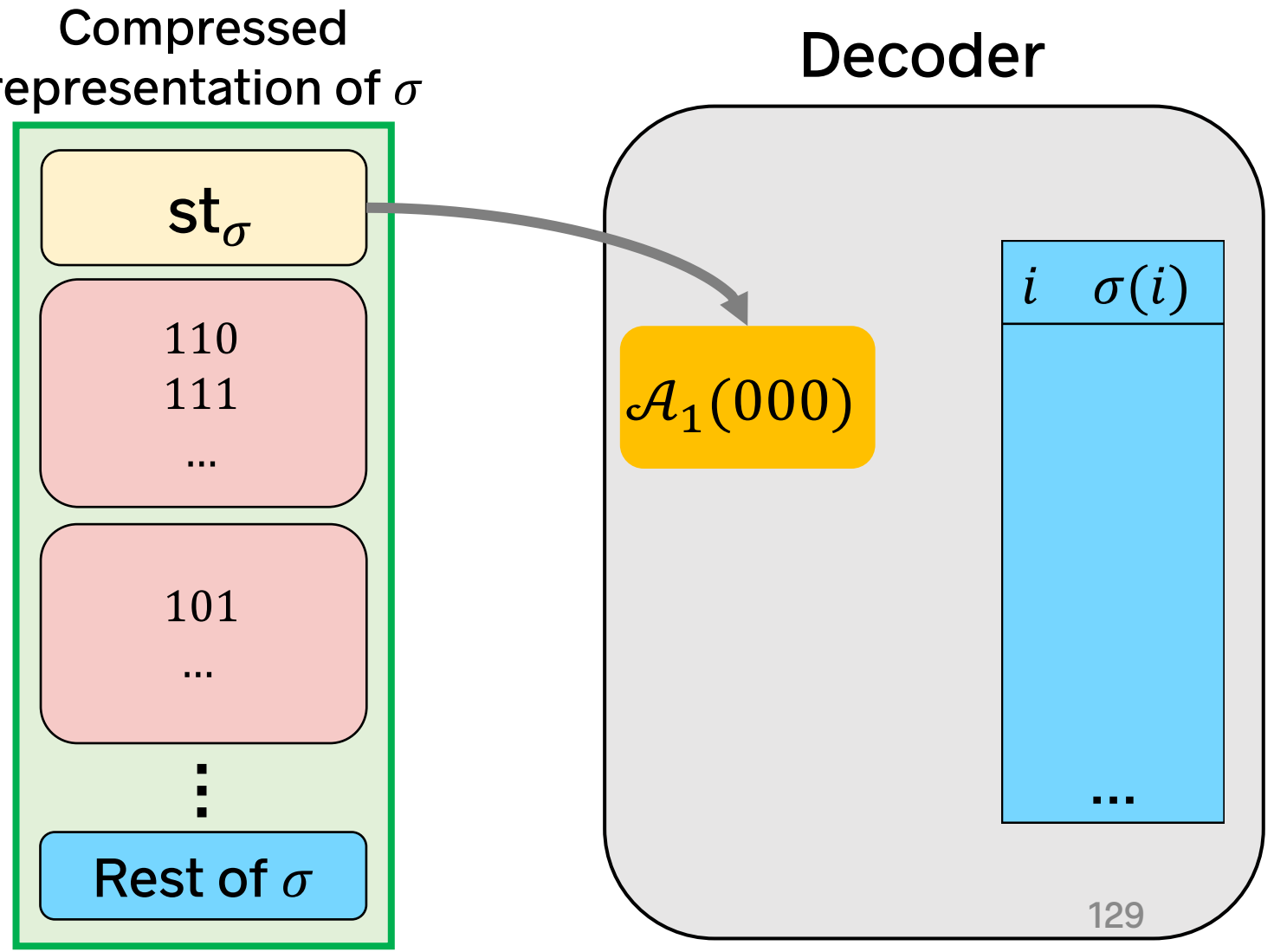

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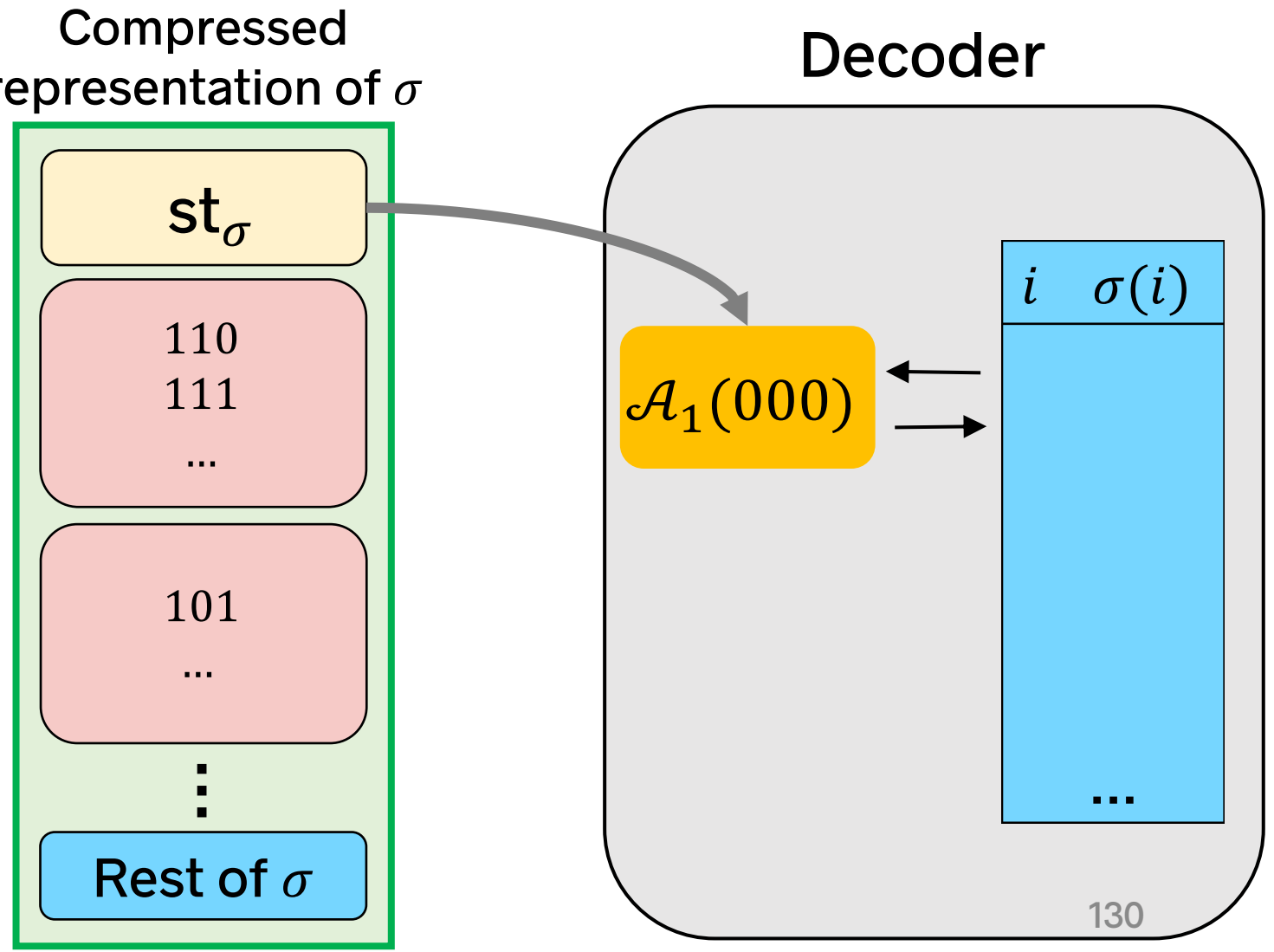

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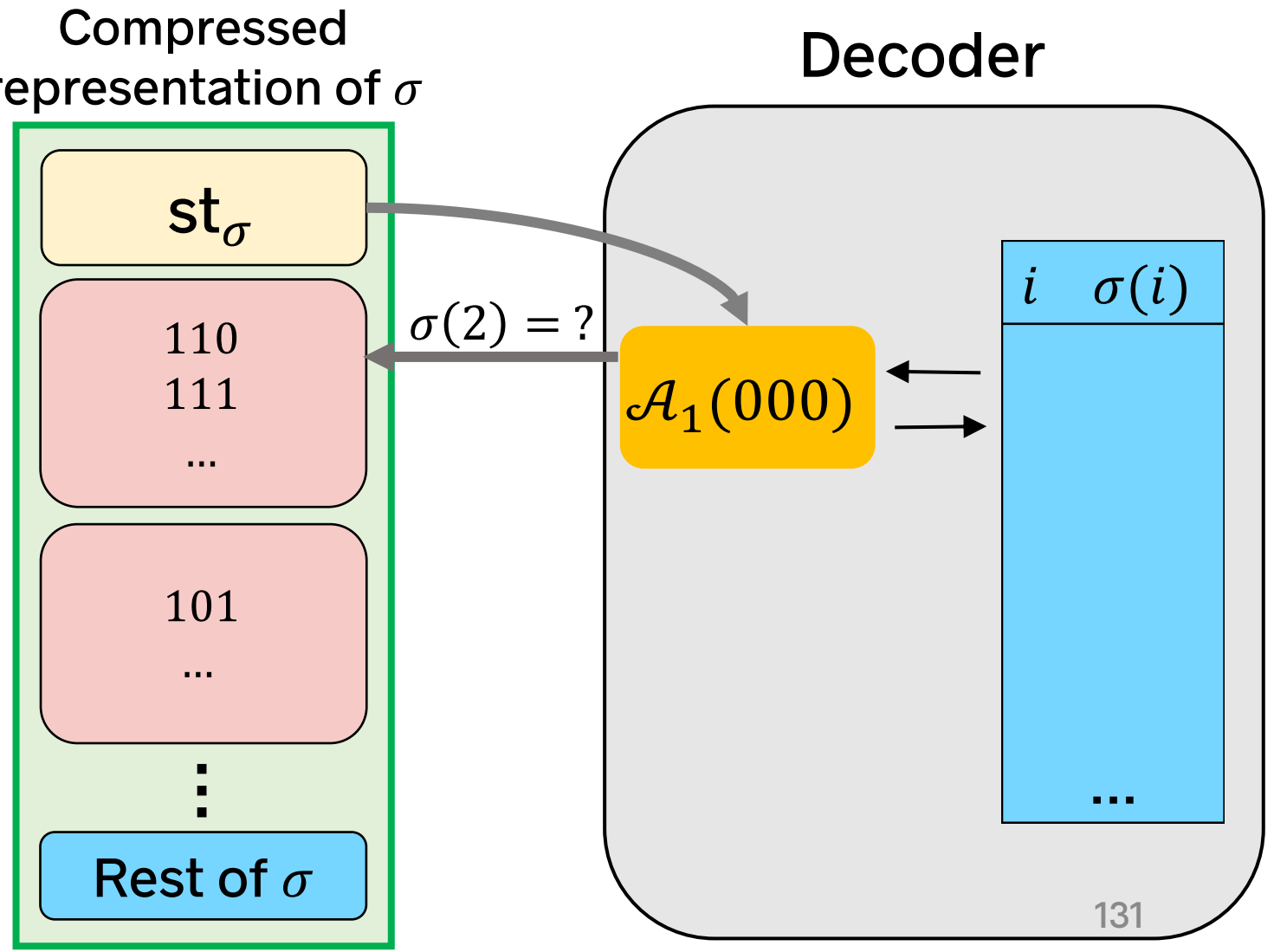

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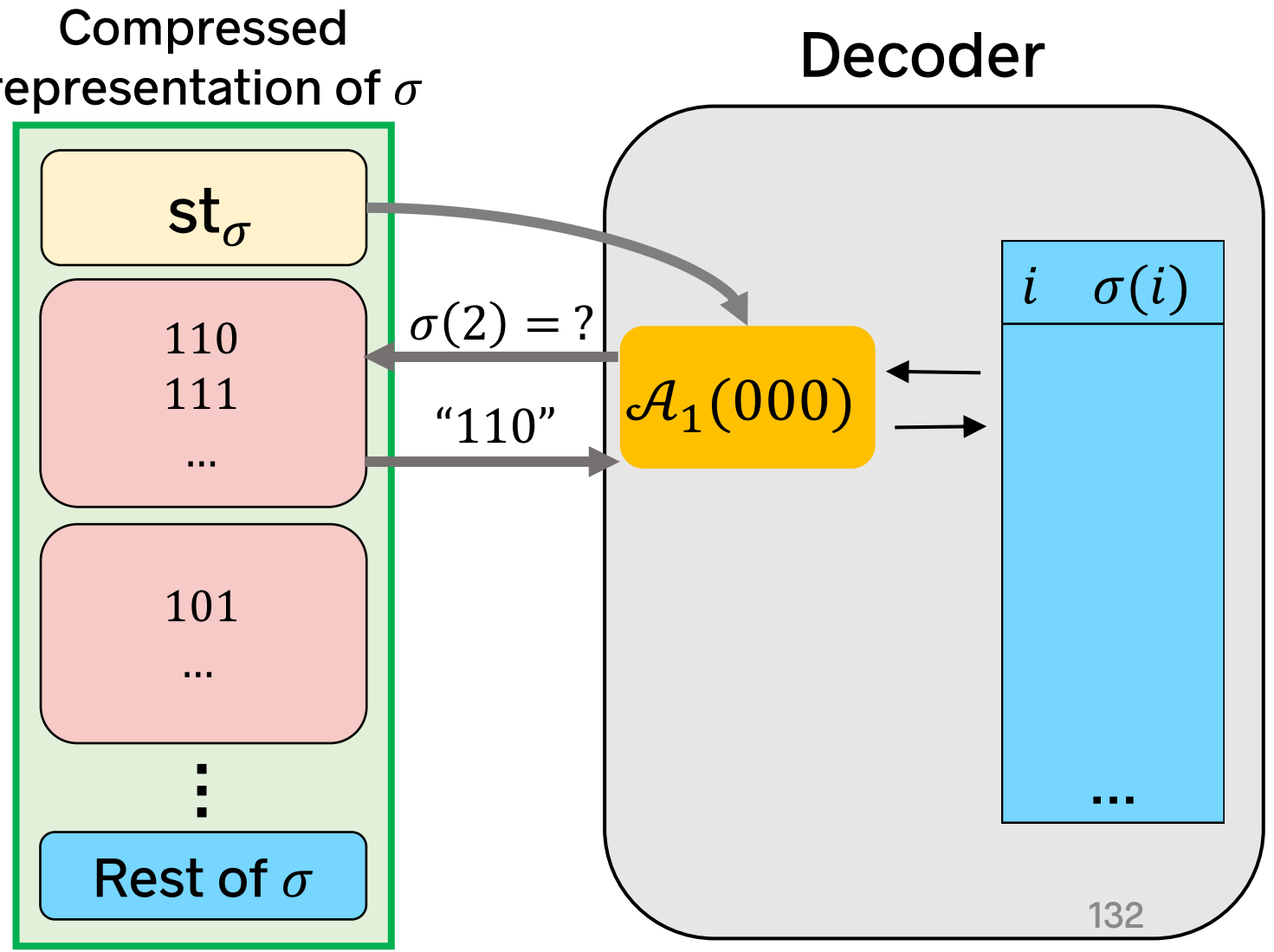

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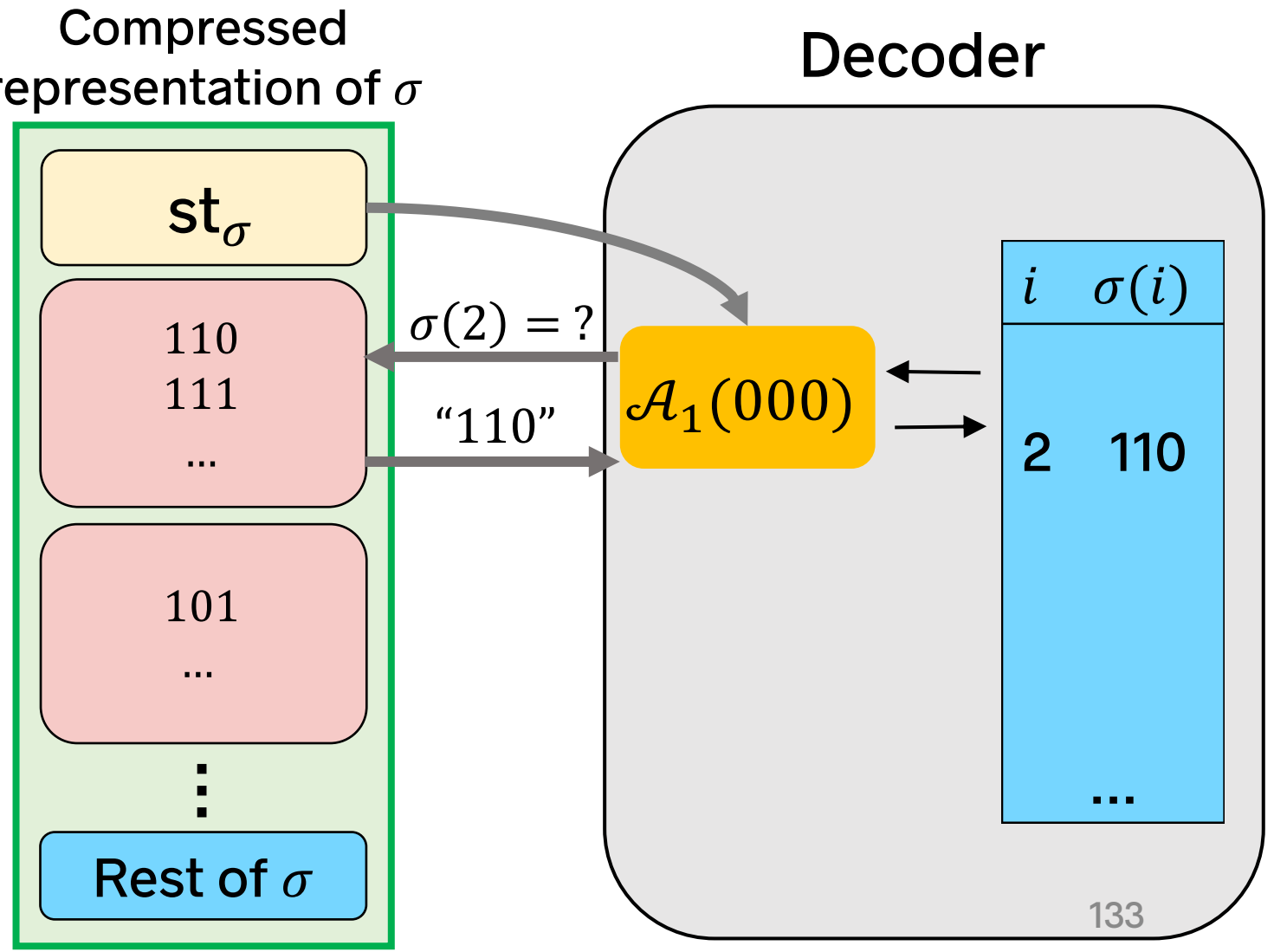

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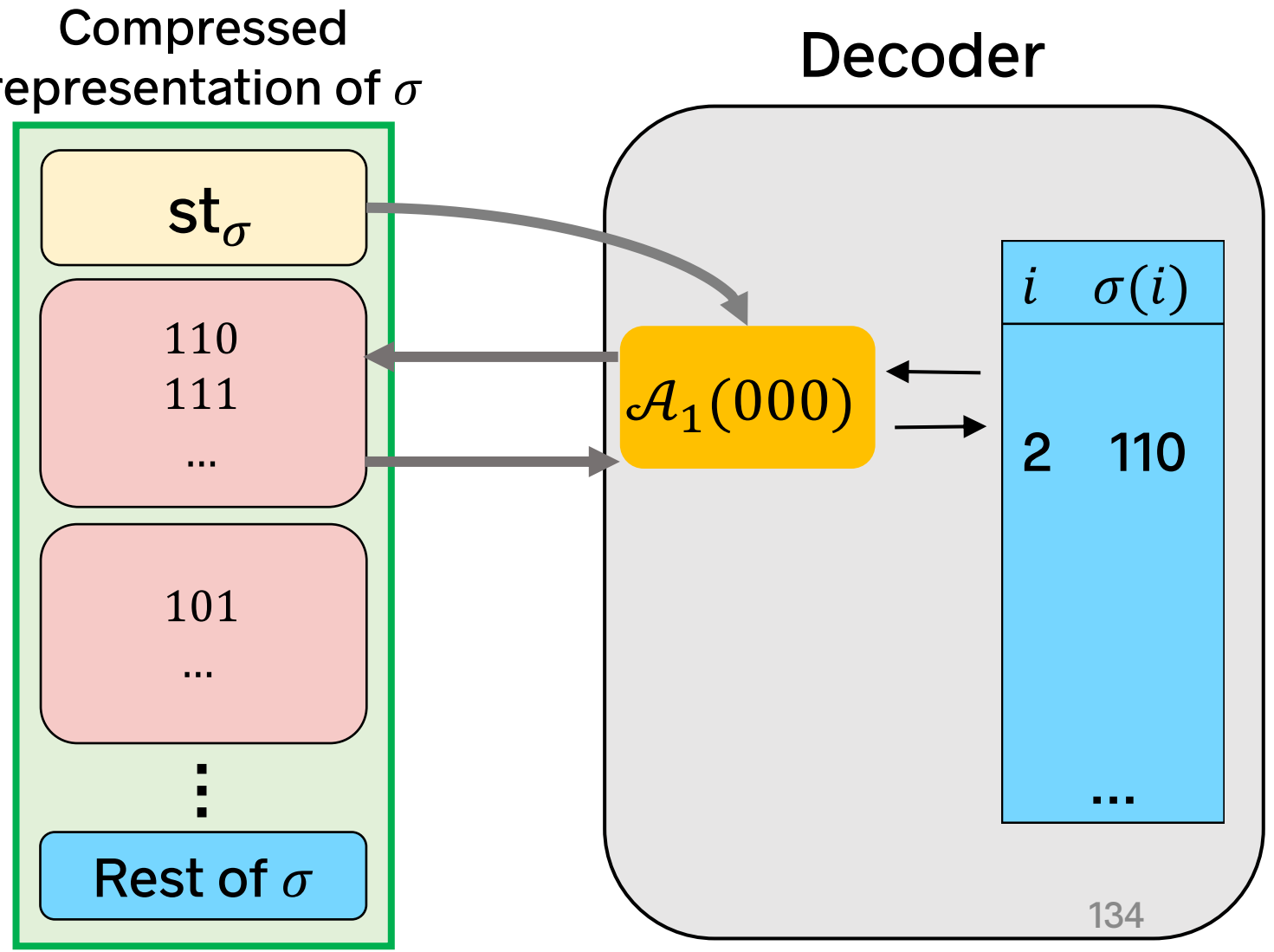

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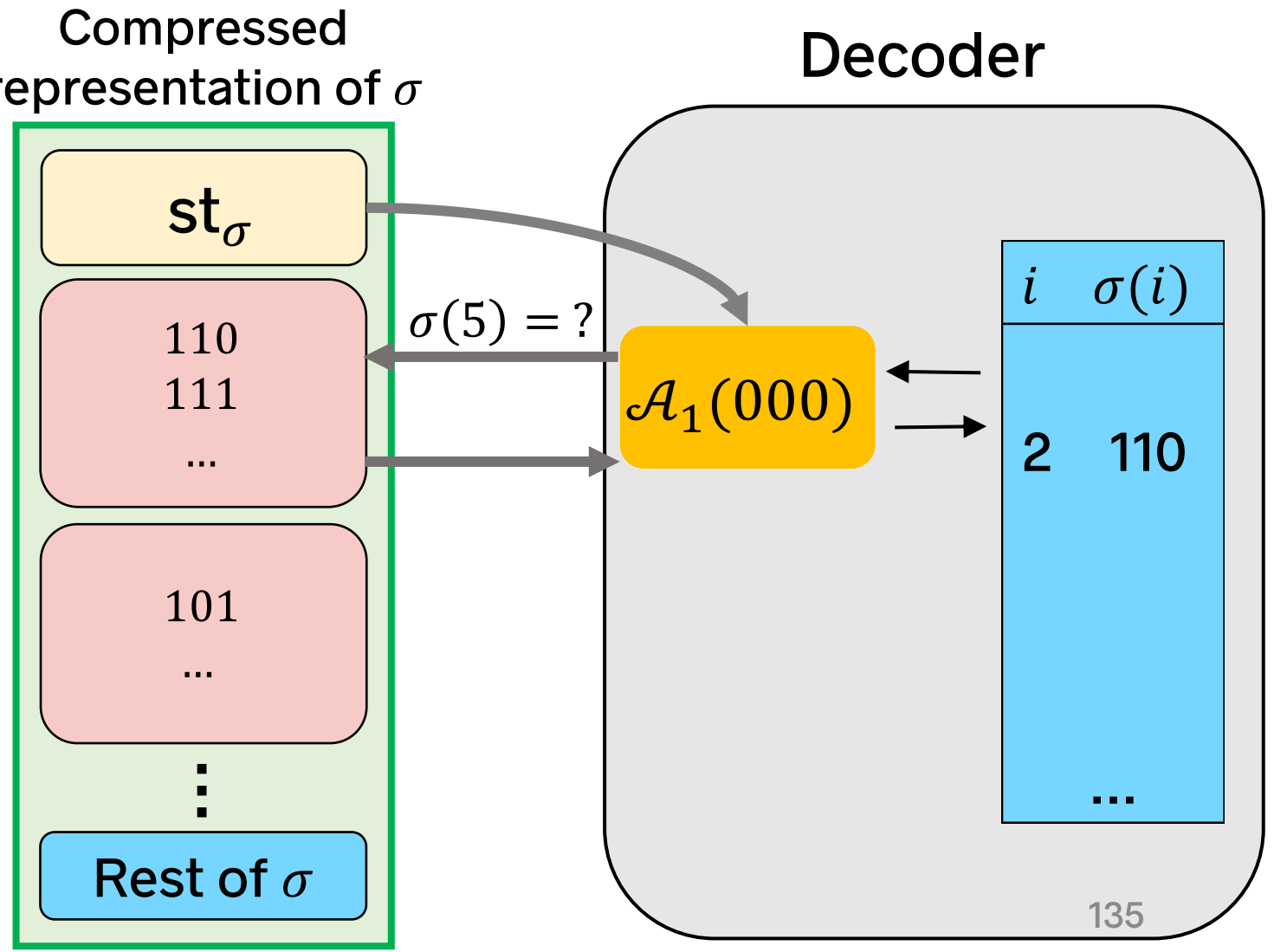

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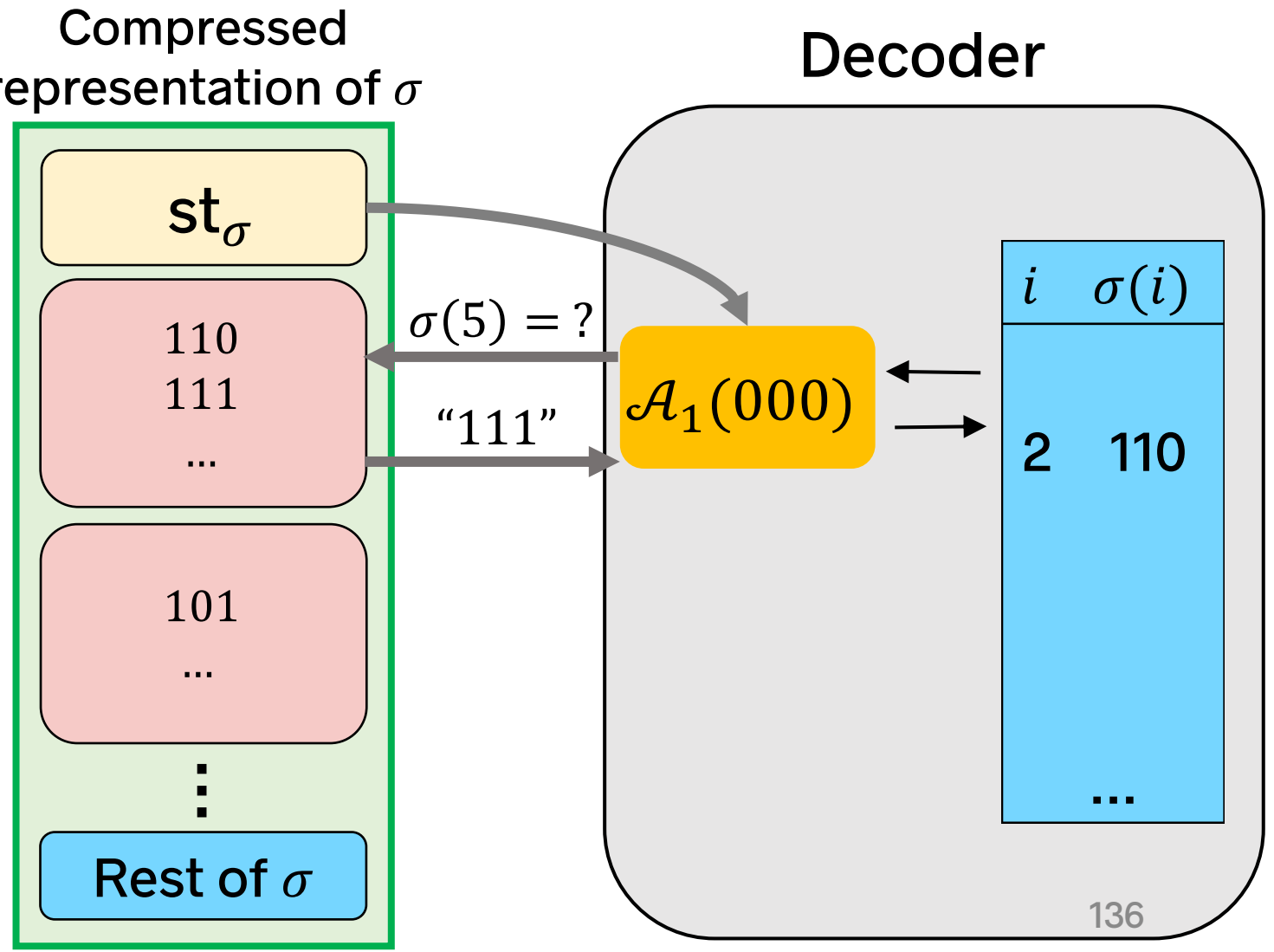

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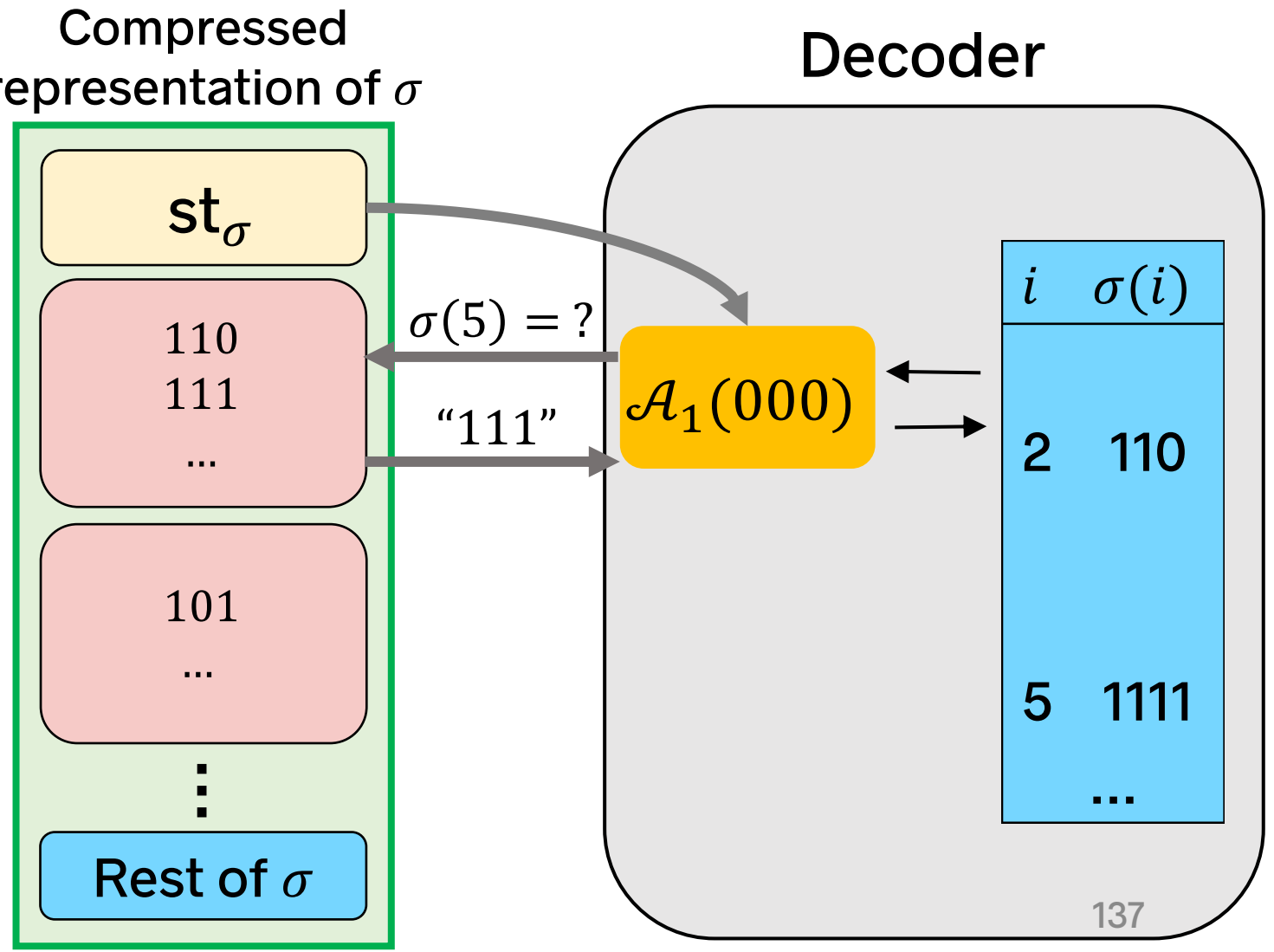

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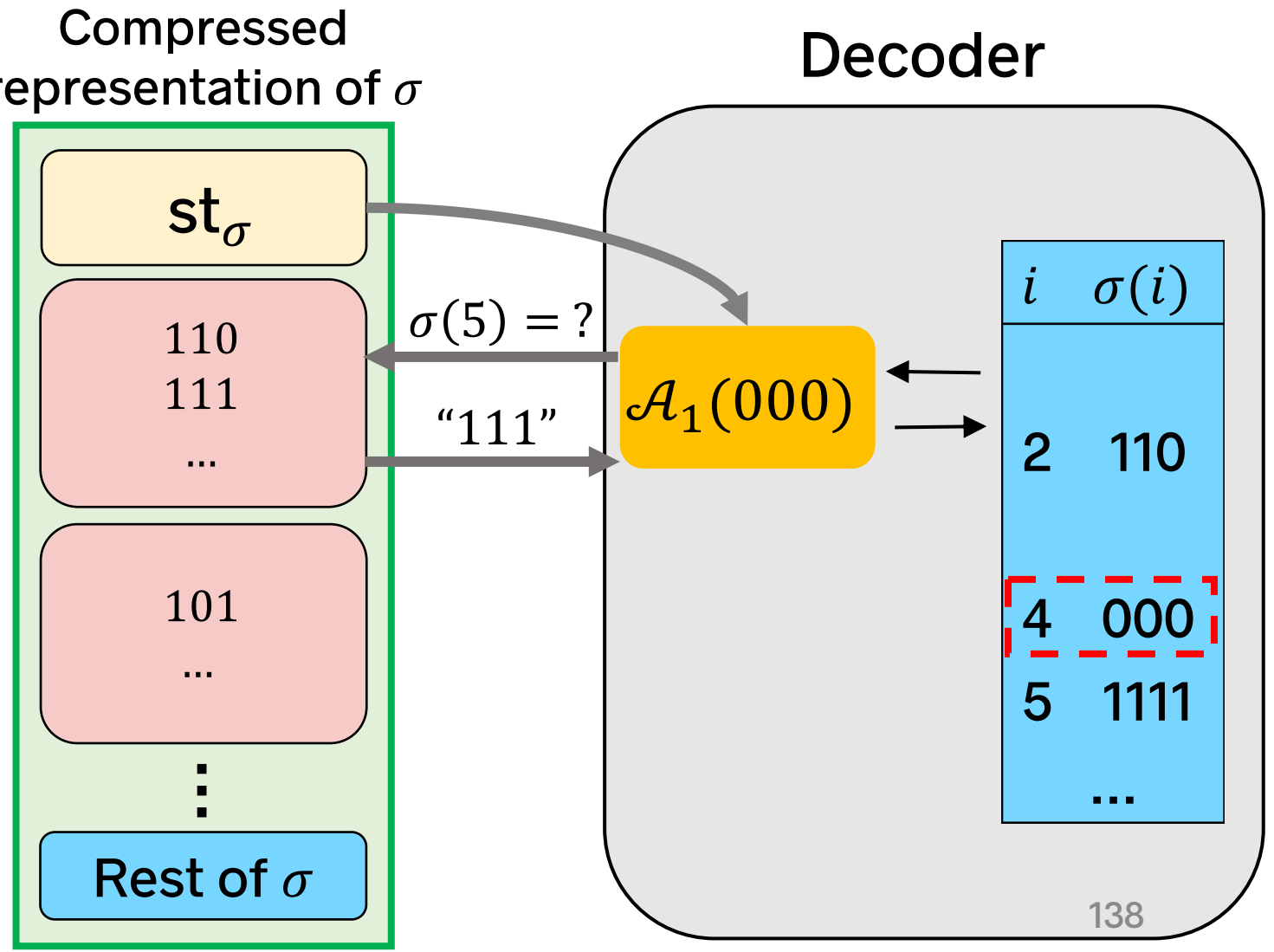

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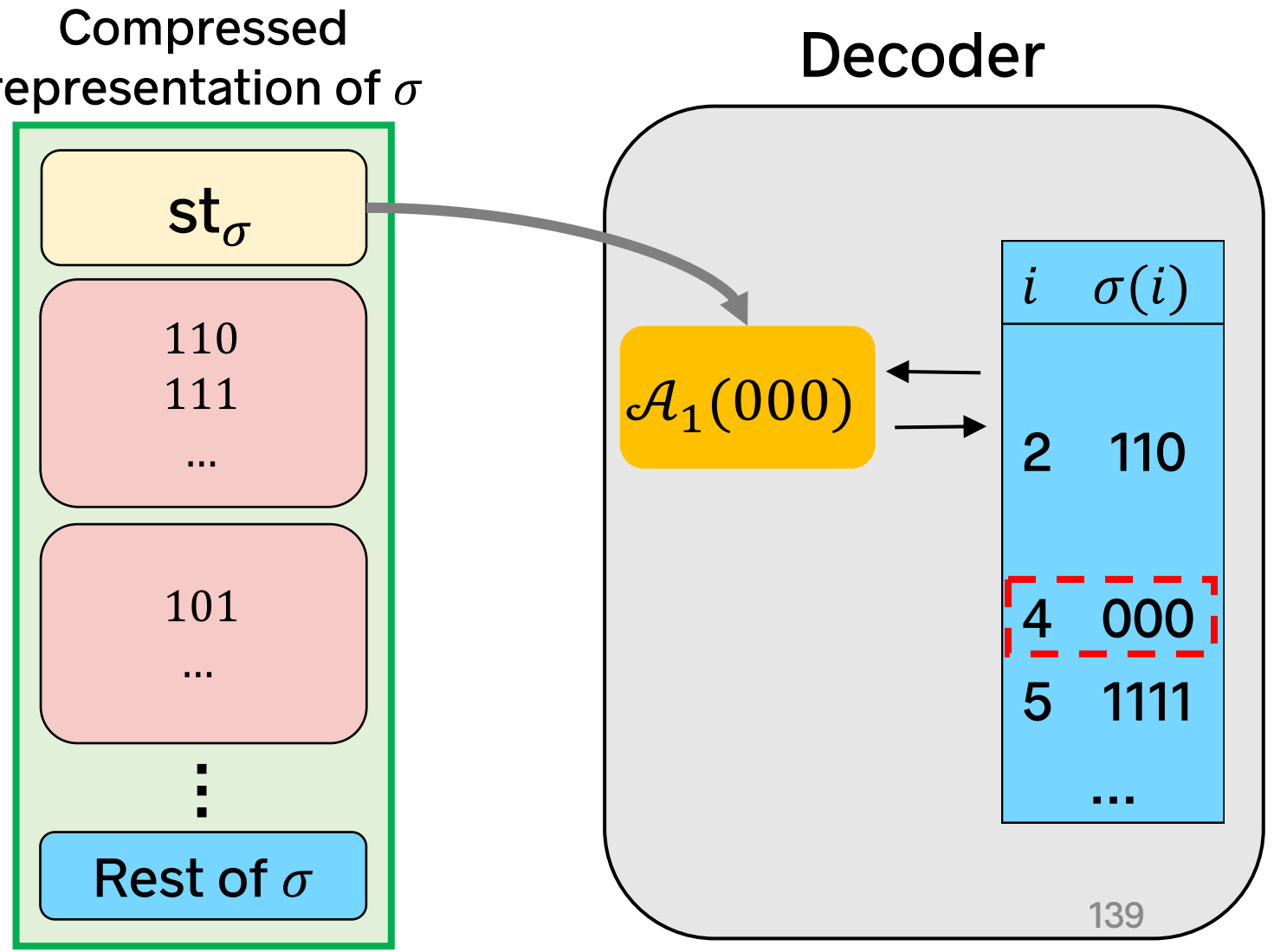

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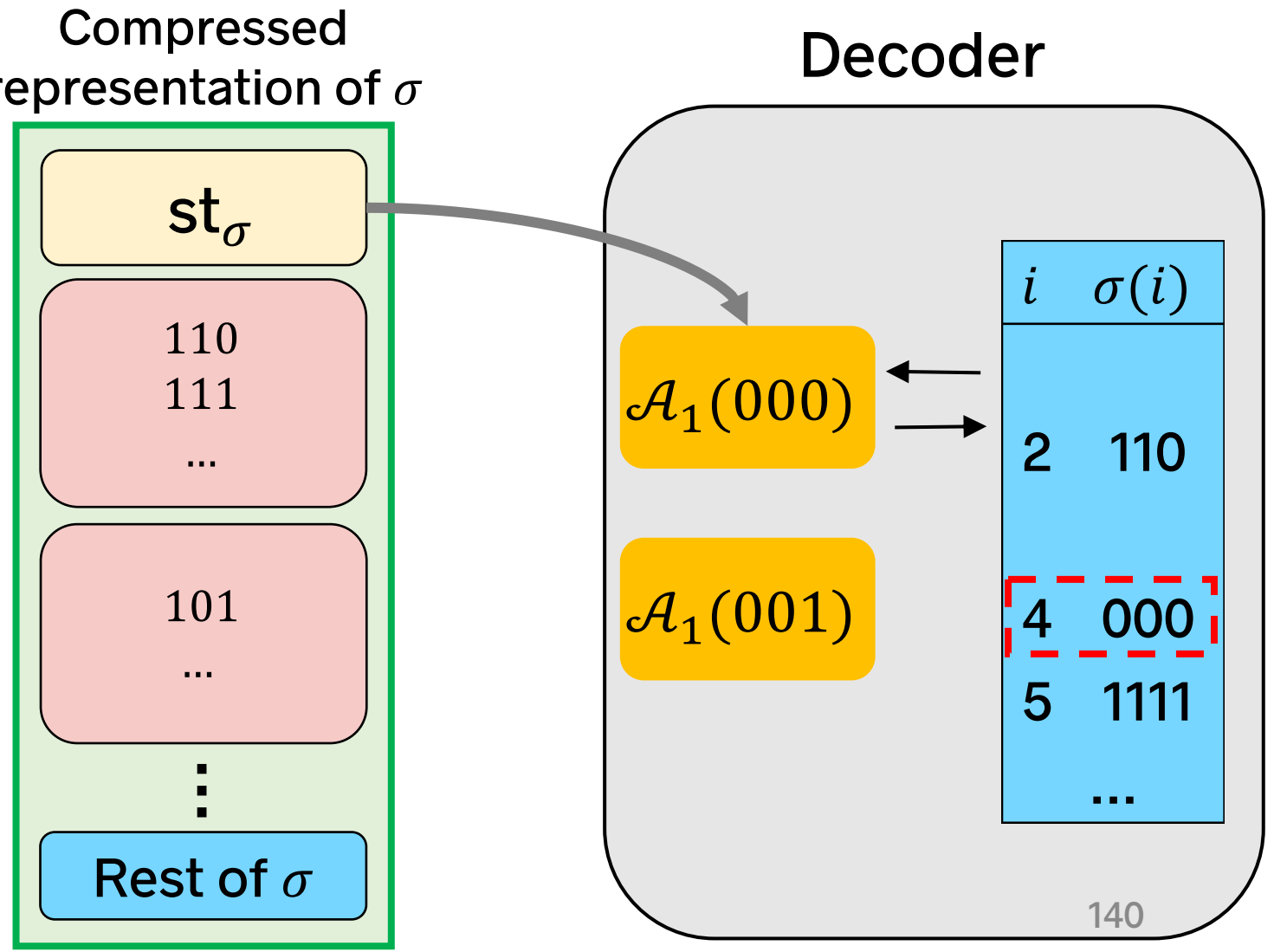

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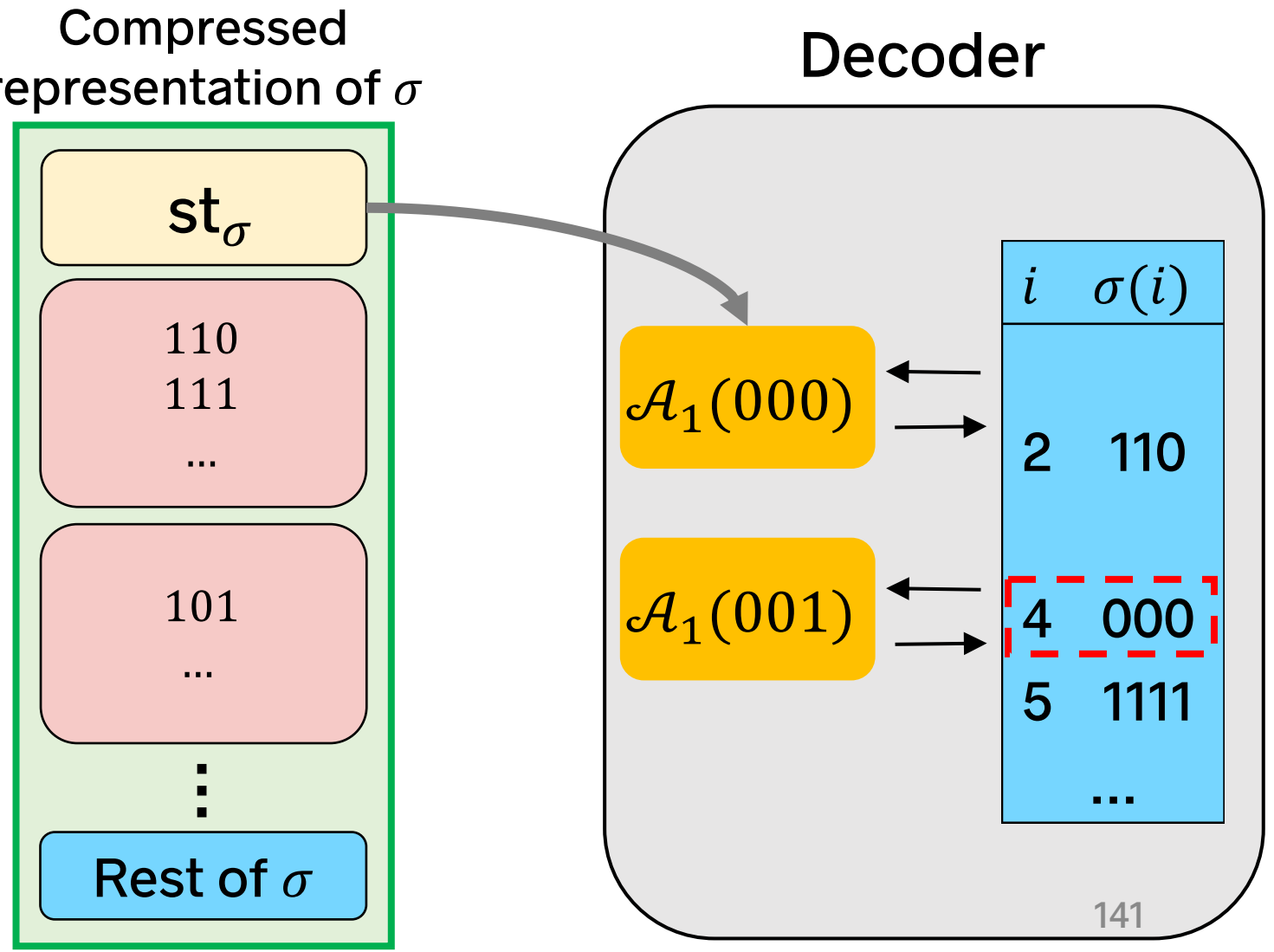

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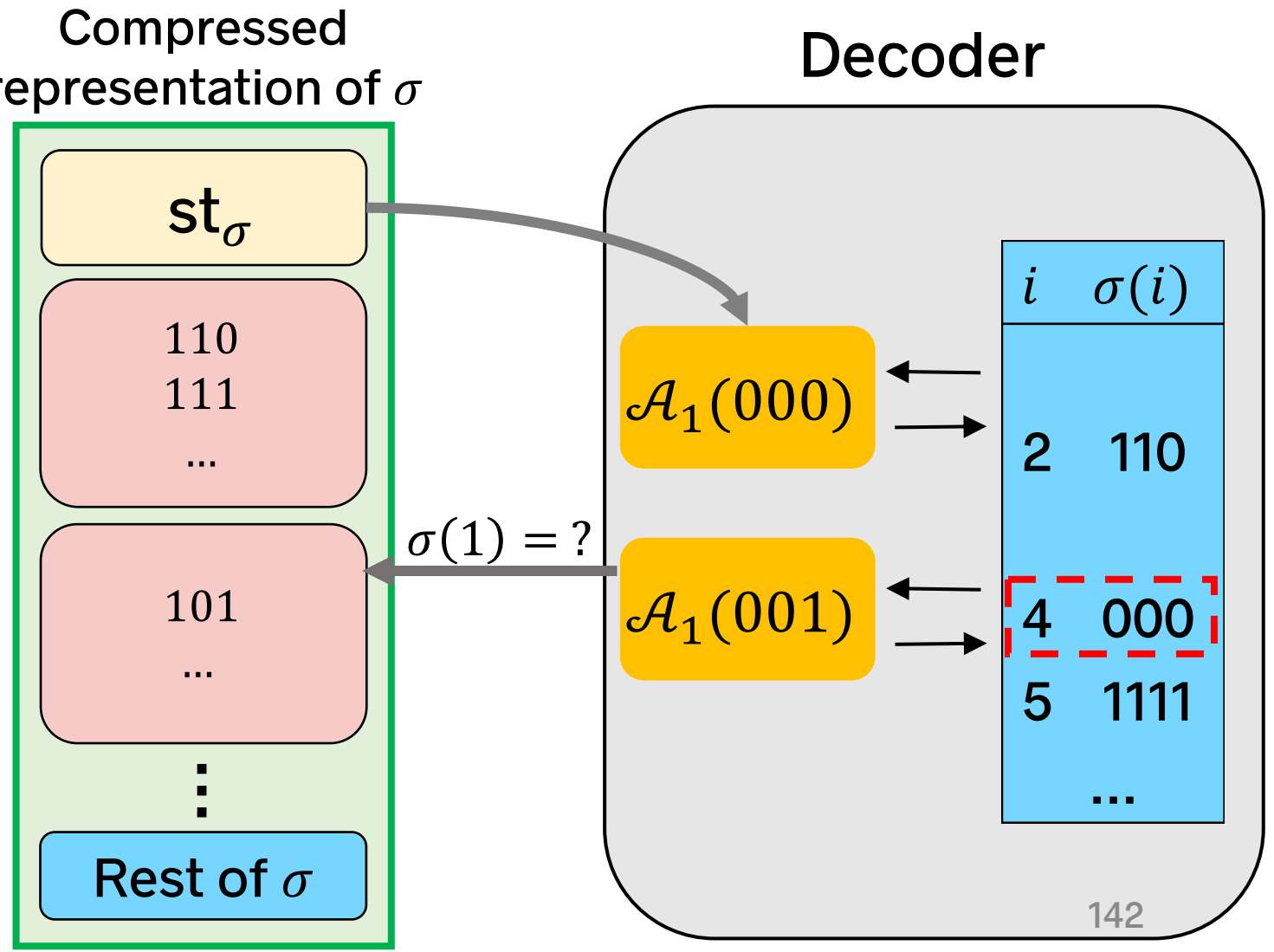

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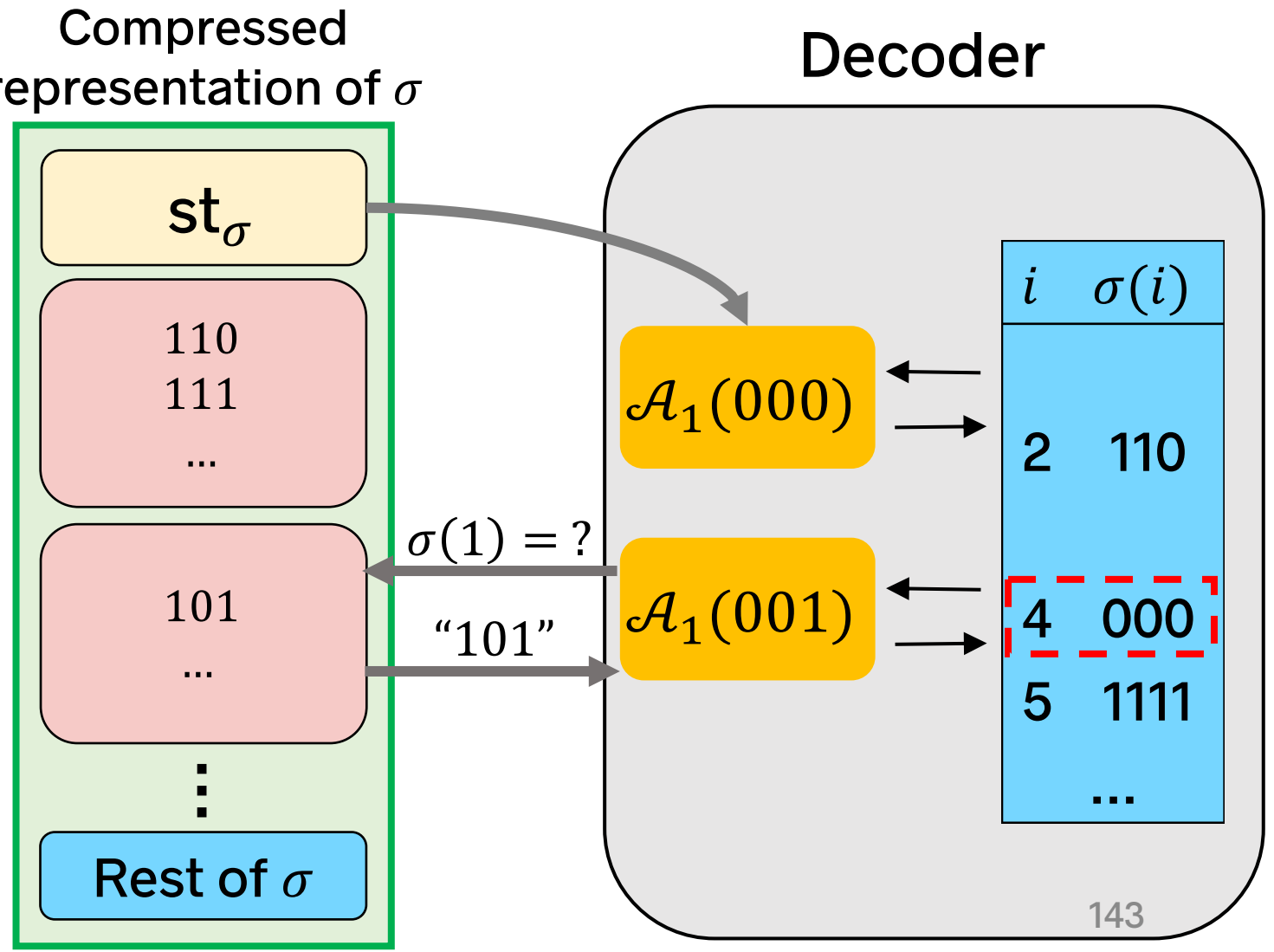

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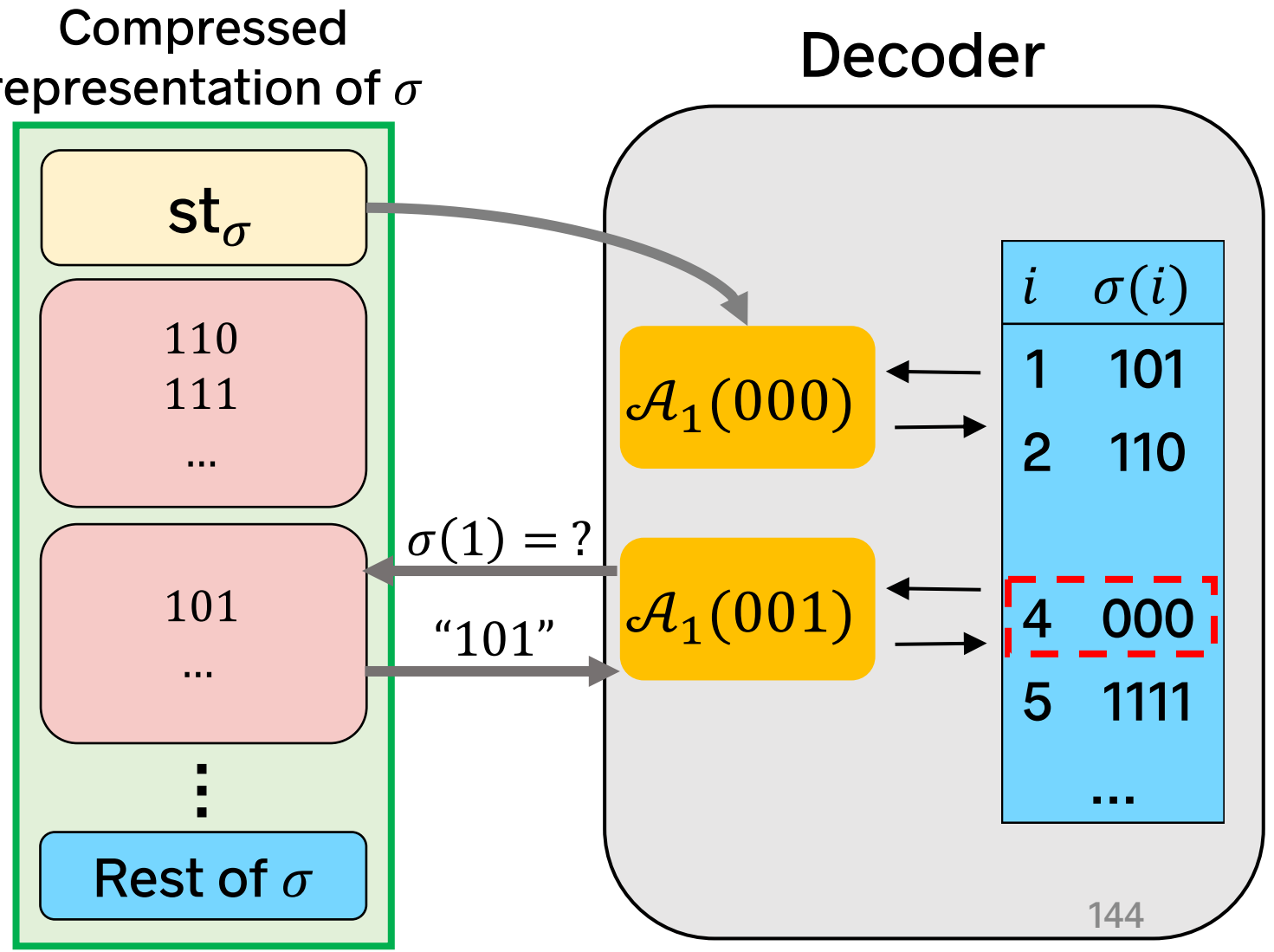

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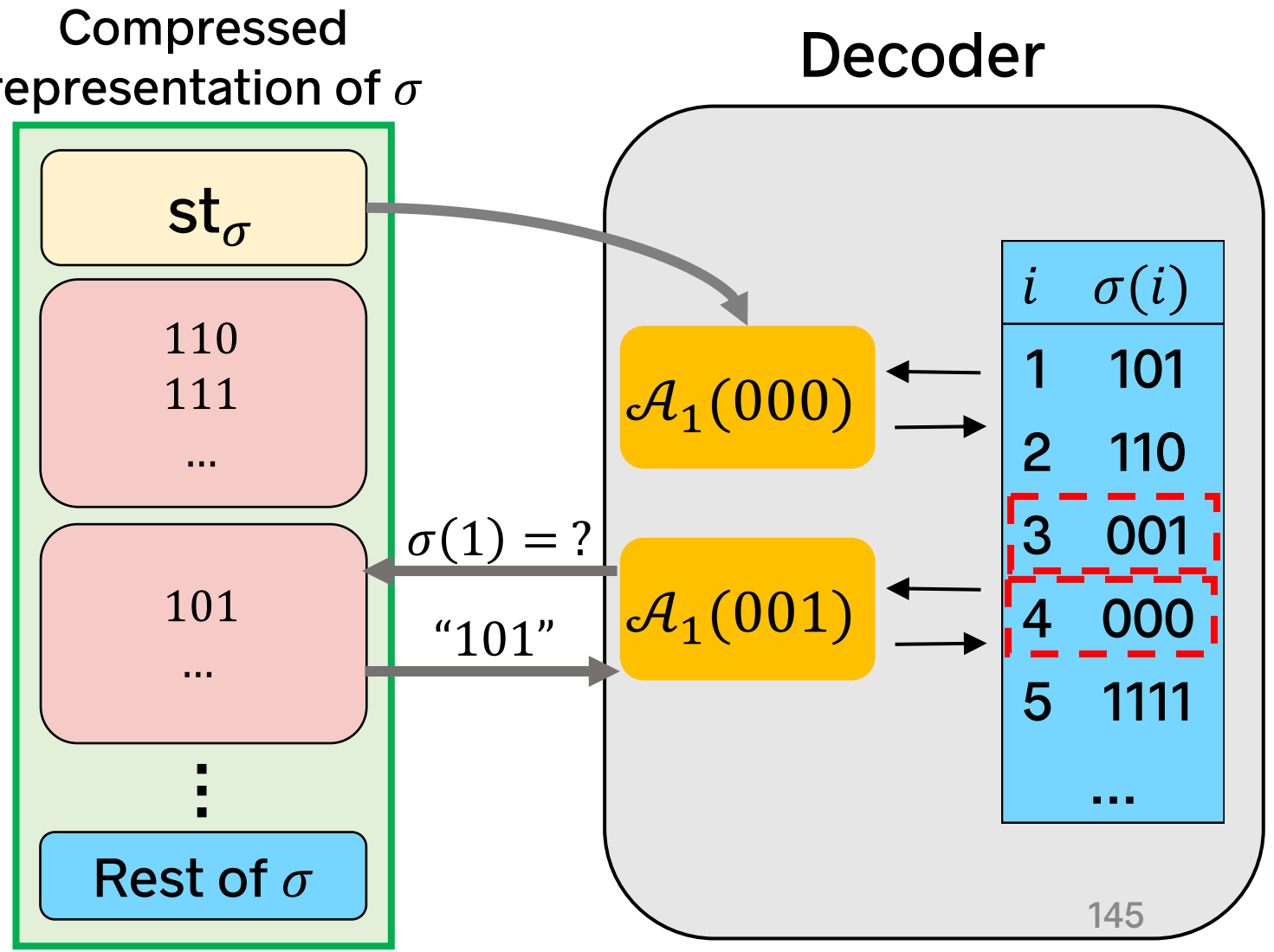

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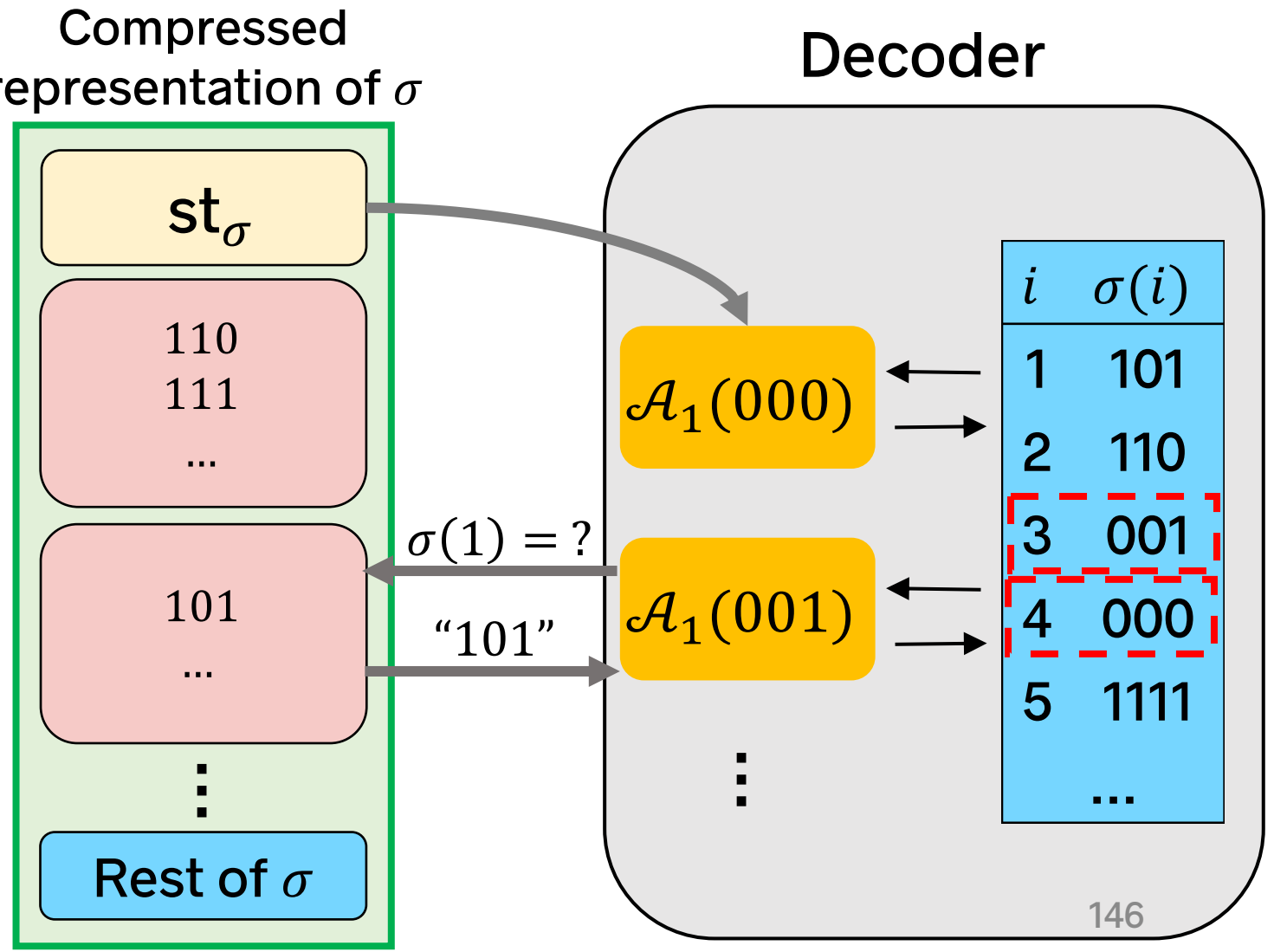

Claim: Each invocation of $\mathcal{A}_1$ allows the encoder to compress σ by at least one bit.

Easy case: The response to all of $\mathcal{A}_1$'s queries are distinct

- $\mathcal{A}_1$ outputs a discrete log "for free" $\Rightarrow$ Compress by $\approx \log N$ bits

Harder case: The response to query t is the same as the response to query $t' < t$.

- A naïve encoding "pays twice" for the same value $\sigma(i) \Rightarrow$ No savings ☹
- Instead, encoder writes a pointer to query t'

  If the encoder runs $\mathcal{A}_1$ on I instances, requires $\underbrace{\log IT}_{\text{Pointer to query } t'} + \underbrace{\log T}_{\text{Index of query } t}$ bits.

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Each execution of $\mathcal{A}_1$ saves at least 1 bit, when:

$$\log IT^2 < \log N, \text{ or } I < N/T^2$$

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Completing the proof

- We run the adversary $\mathcal{A}_1$ on $I = N/T^2$ instances
- Each execution compresses by ≥ 1 bit
- BUT, we have to include the S-bit advice string in the encoding

$$\text{Encoding overhead} = S - \frac{N}{T^2} \geq 0 \qquad \Rightarrow \qquad ST^2 = \Omega(N) \quad \blacksquare$$

Extra complications

- Algorithms that succeed on an ϵ-fraction of group elements
  - Use the random self-reducibility of dlog
  - Hardcode a good set of random coins for $\mathcal{A}_1$ into $\mathbf{Enc}(\sigma)$

- Decisional type problems (DDH, etc.)
  - $\mathcal{A}_1$ only outputs 1 bit—prior argument fails because encoding the runtime in $\log T$ bits is too expensive
  - Run $\mathcal{A}_1$ on batches of inputs [See paper for details]

What about Decision Diffie–Hellman (DDH)?

DDH problem: Distinguish (g, g^x, g^y, g^{xy}) from (g, g^x, g^y, g^z)

	Upper bound	Lower bound	Time T
Discrete log:	$ST^2 = \tilde{O}(\epsilon N)$	$ST^2 = \tilde{\Omega}(\epsilon N)$	$N^{1/4}$
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Better attack?

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Our new results

Definition. The sqDDH problem is to distinguish

$$(g, g^x, g^{(x^2)}) \quad \text{from} \quad (g, g^x, g^y) \qquad \text{for } x, y \leftarrow_R \mathbb{Z}_N.$$

Why it's interesting:

- For generic online-only algs, it's as hard as discrete log
- For generic preprocesssing algs, we show that it's "much easier"

→ A DDH-like problem that is easier than dlog

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- The $\tilde{O}(N^{1/3})$ generic dlog attack is optimal
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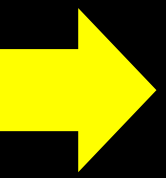

Open questions

Open questions and recent progress

- Tightness of DDH upper/lower bounds?
  - Is it as hard as dlog or as easy as sqDDH?
- Non-generic preprocessing attacks on ECDL?
  - As we have for $\mathbb{Z}_p^*$

Coretti, Dodis, and Guo (2018)

- Elegant proofs of generic-group lower bounds using "presampling" (à la Unruh, 2007)
- Prove hardness of "one-more" dlog, KEA assumptions, …

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- Preexisting $S = T = \tilde{O}(N^{1/3})$ generic attack on discrete log

Our results: Preprocessing lower-bounds and attacks

- The $\tilde{O}(N^{1/3})$ generic dlog attack is optimal
- Any such attack must use lots of preprocessing: $\Omega(N^{2/3})$
- New $\tilde{O}(N^{1/5})$ preprocessing attack on DDH-like problem

Open questions

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https://eprint.iacr.org/2017/1113