

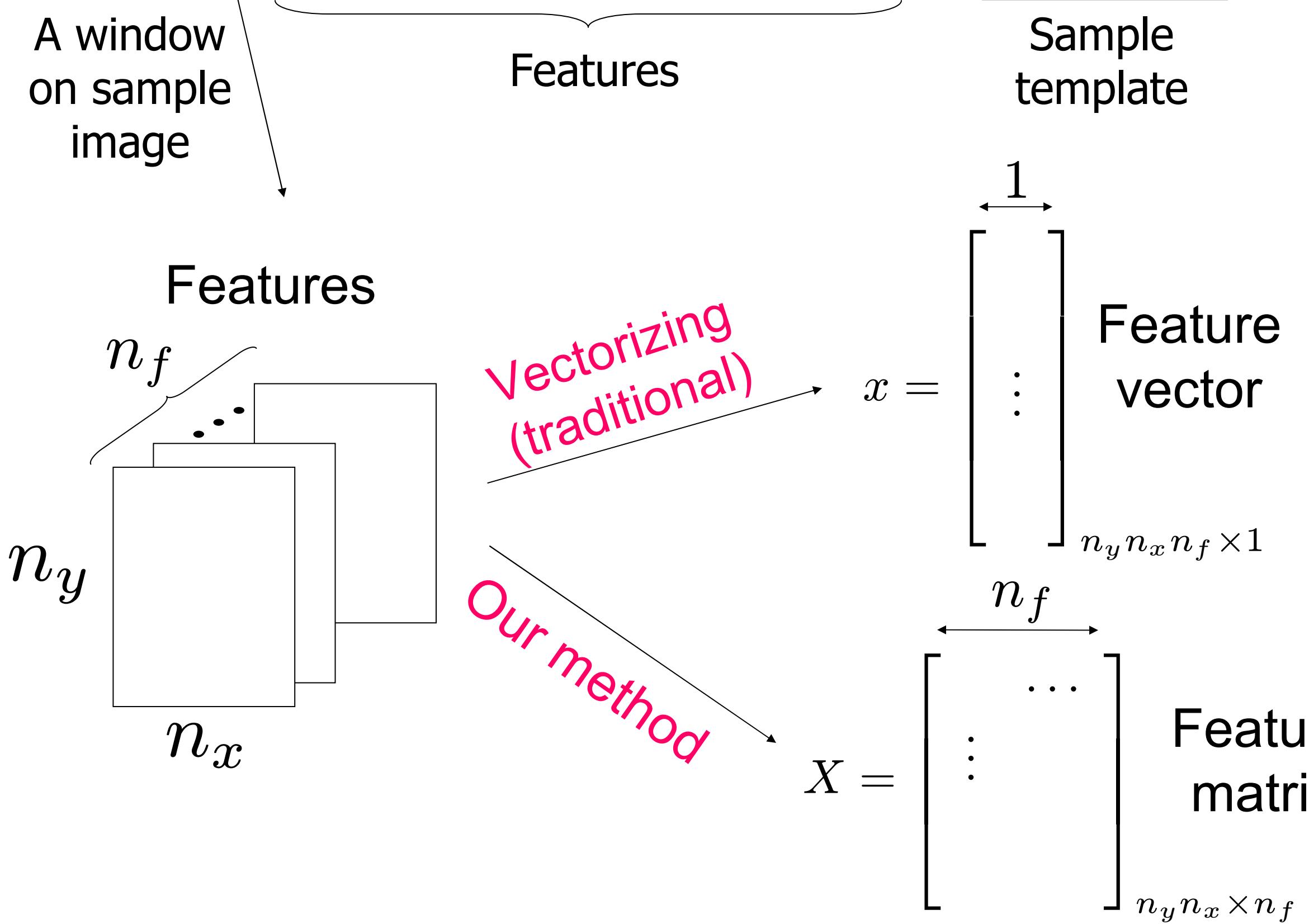
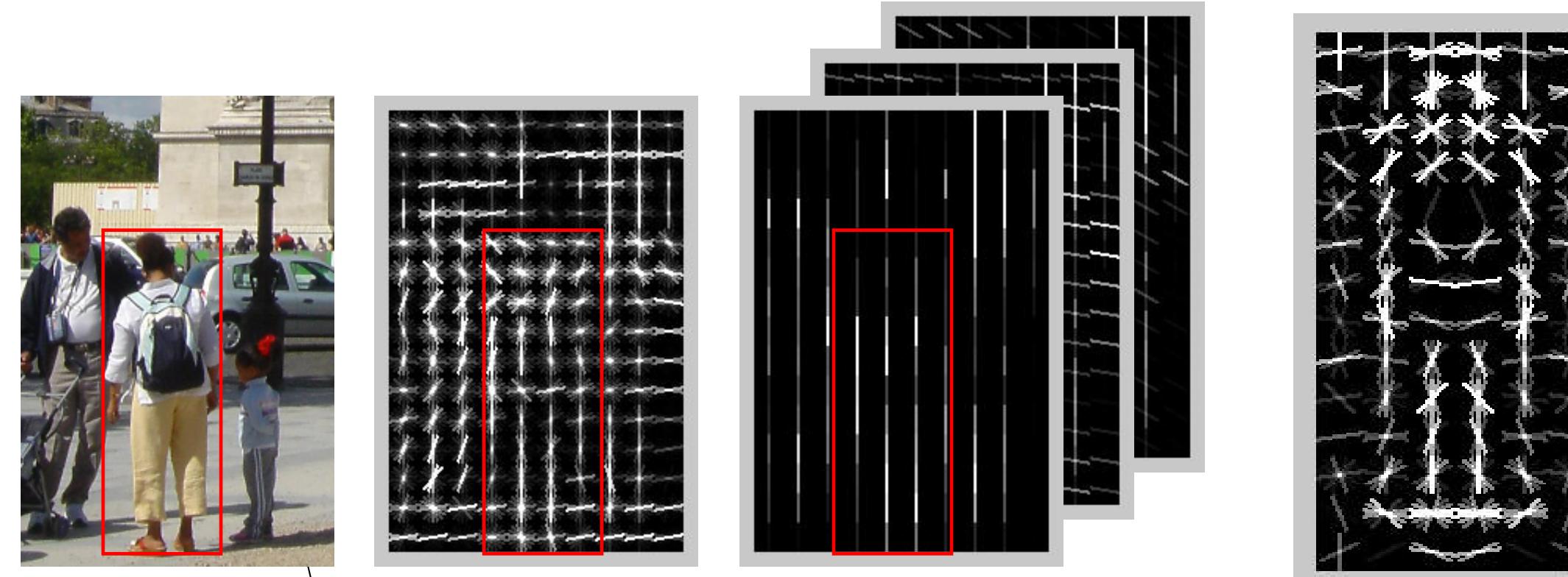
# Bilinear Classifiers for Visual Recognition

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## Main ideas:

- Treat image features and templates as matrices or tensors rather than simply “vectorizing” them
- Learn a low rank template by performing alternating minimization using standard linear SVM solver
- Share a subset of parameters (subspace) across different problems (transfer learning)



**Rank restriction:**  $d \leq \min(n_s, n_f)$  where  $n_s = n_y n_x$

$$W = W_s W_f^T$$

$$\begin{bmatrix} & \cdots \\ \vdots & \end{bmatrix}_{n_s \times n_f} = \begin{bmatrix} d \\ \vdots \end{bmatrix}_{n_s \times d} \times \begin{bmatrix} & \cdots \\ \vdots & \end{bmatrix}_{d \times n_f}$$

Low dim template      Subspace

**Linear SVM:** For a given set of training pairs:  $\{X_n, y_n\}$

$$\min_W L(W) = \underbrace{\frac{1}{2} \text{Tr}(W^T W)}_{\text{Regularizer}} + C \sum_n \max(0, 1 - y_n \text{Tr}(W^T X_n)) \underbrace{\text{Tr}(W^T X_n)}_{\text{Constraints}}$$

**Detection function:** Was Linear:  $f(X) = \text{Tr}(W^T X) > 0$

Now it is Bilinear:  $f(X) = \text{Tr}(W_s^T X W_f) > 0$

**Learning:** It is **biconvex** so we use coordinate descent with off-the-shelf linear SVM solver in the loop

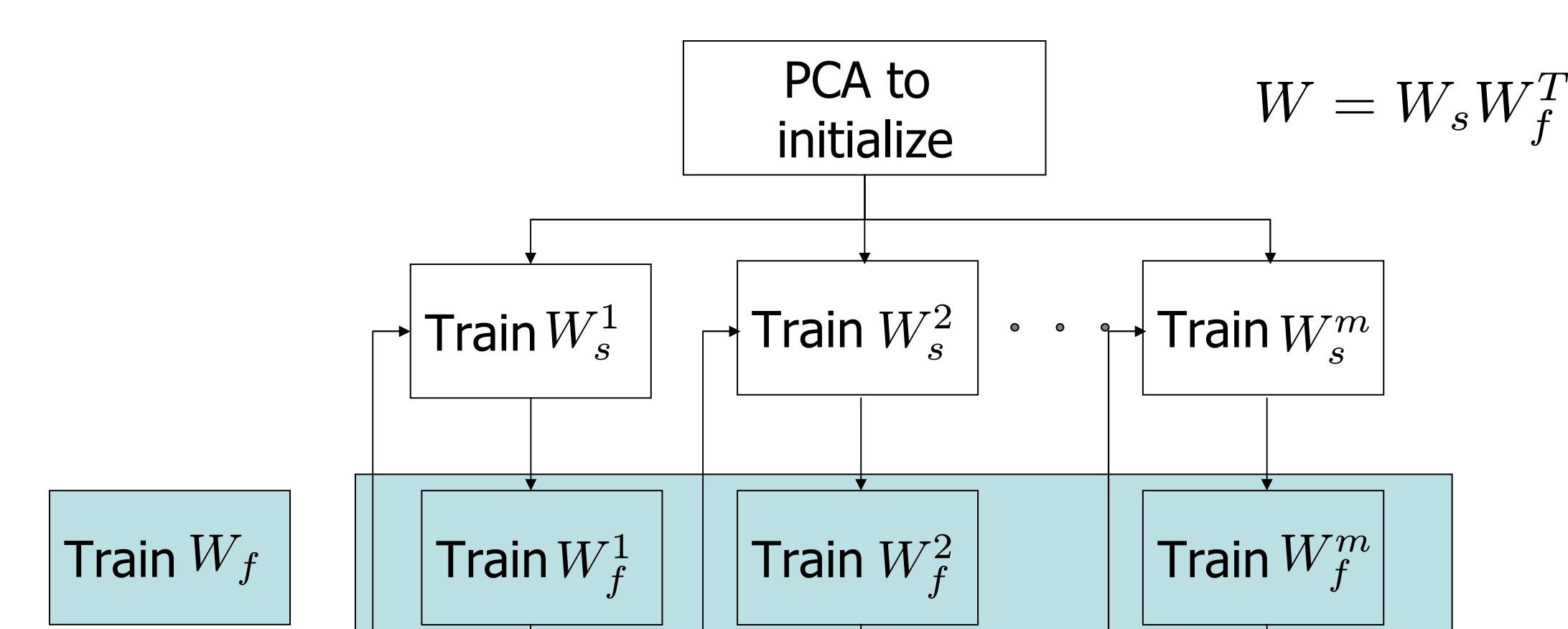
Change of basis:  $\tilde{W}_f = A^{\frac{1}{2}} W_f^T, \tilde{X}_n = A^{-\frac{1}{2}} W_s^T X_n, A = W_s^T W_s$

$$\min_{\tilde{W}_f} L(\tilde{W}_f, W_s) = \frac{1}{2} \text{Tr}(\tilde{W}_f^T \tilde{W}_f) + C \sum_n \max(0, 1 - y_n \text{Tr}(\tilde{W}_f^T \tilde{X}_n))$$

**Advantages :**

- Rank constraint provides natural regularization.
- 10X runtime speedup with no loss in performance.  
(Fewer number of convolutions in detection)
- Learn shared subspaces across multiple classes or training sets to allow for transfer learning.  
(e.g., share subspace between “human detector” and “cat detector”)

Joint minimization:  $\min_{\tilde{W}_f} \sum_{i=1}^m L(\tilde{W}_f, W_s^i)$

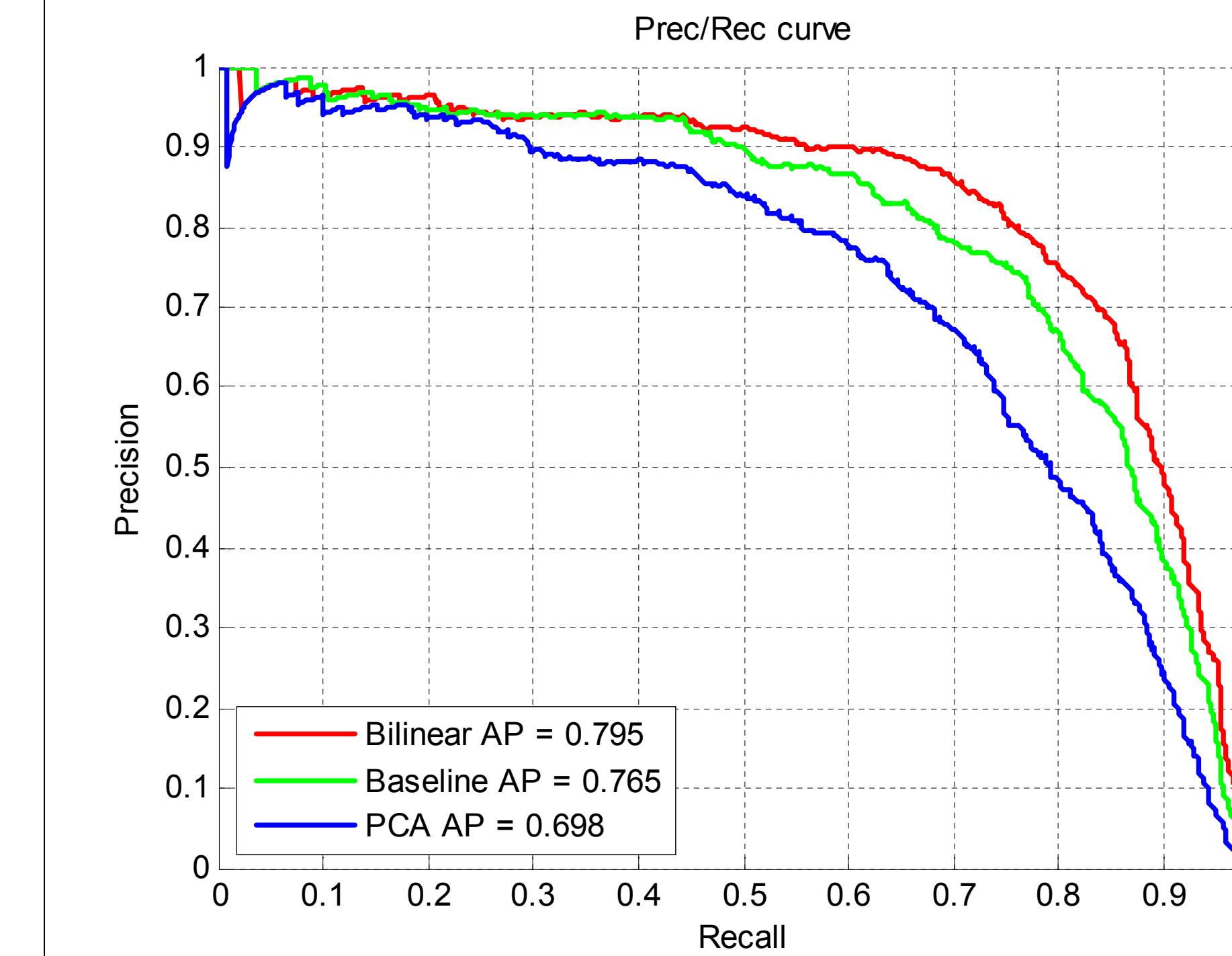


**Related work :**

- Tenenbaum & Freeman's *bilinear model* (Neural Computation'00)
- Wolf et al's *low-rank SVM* paper (CVPR'07)
- Ando and Zhang's *transfer learning* paper (JMLR'05)

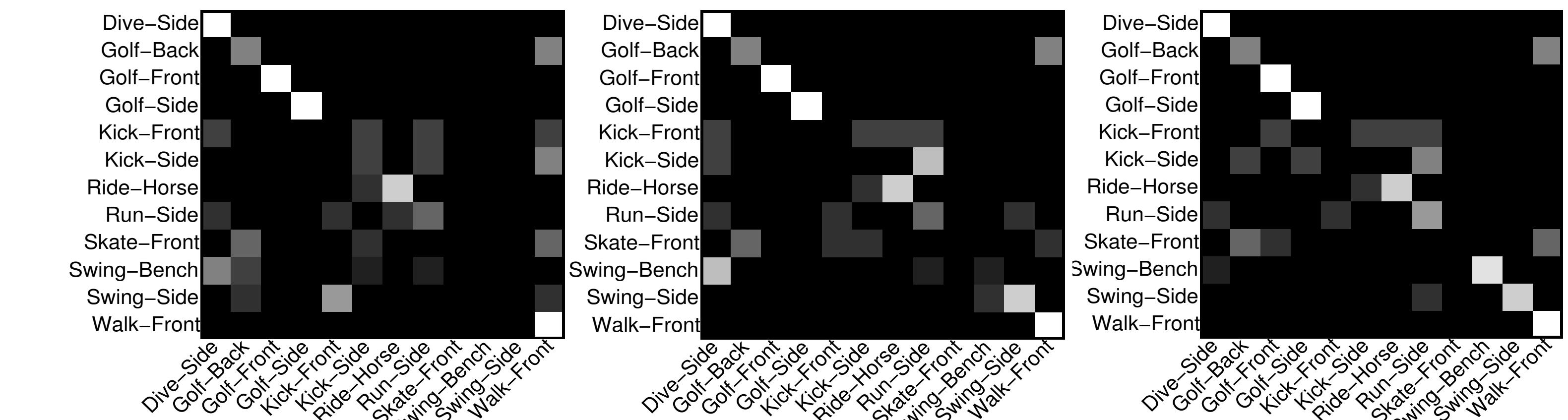
**Experiment 1:** Pedestrian detection on INRIA motion dataset

$$n_s = 14 \times 6, n_f = 84, d = 5$$



**Experiment 2:** Human action recognition on UCF sports dataset

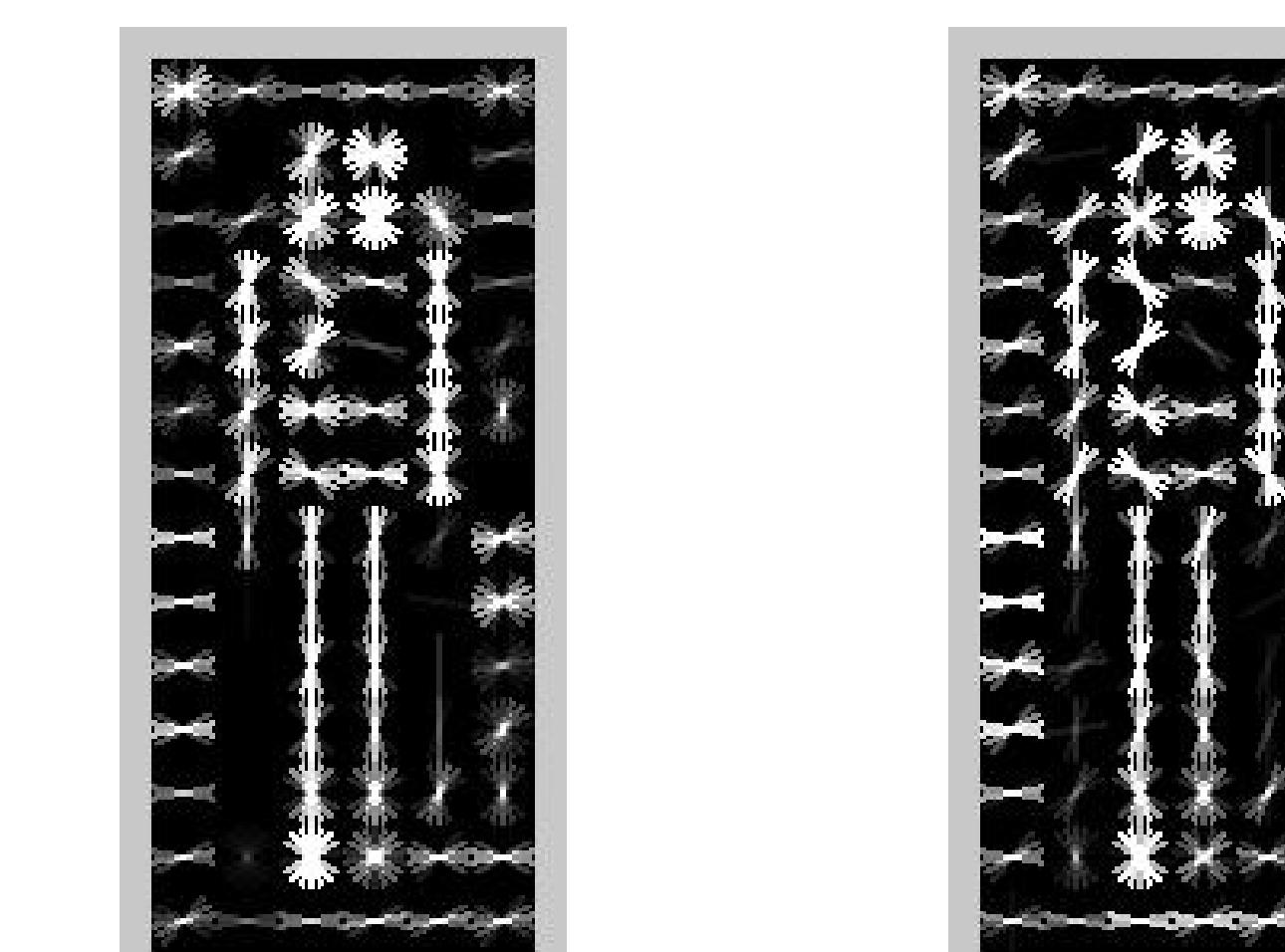
Linear (0.518)  
PCA on features (0.444)  
(Not always feasible)



**Experiment 3:** Sharing subspace on UCF sports dataset

(Using only two training samples for each class)

Iteration 1      Refined at Iteration 2



Average classification rate

	Coordinate decent iteration 1	Coordinate decent iteration 2
Independent bilinear (C=0.01)	0.222	0.289
Joint bilinear (C=0.1)	0.269	0.356