On Avoiding Spare Aborts in Transactional Memory

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ABSTRACT

This paper takes a step toward developing a theory for understanding aborts in transactional memory systems (TMs). Existing TMs may abort many transactions that could, in fact, commit without violating correctness. We call such unnecessary aborts *spare aborts*. We classify what kinds of spare aborts can be eliminated, and which cannot. We further study what kinds of spare aborts can be avoided efficiently. Specifically, we show that some unnecessary aborts cannot be avoided, and that there is an inherent tradeoff between the overhead of a TM and the extent to which it reduces the number of spare aborts. We also present an efficient example TM algorithm that avoids certain kinds of spare aborts, and analyze its properties and performance.

Categories and Subject Descriptors

D.1.3 [Software]: Concurrent Programming

General Terms

Algorithms, Performance, Theory

Keywords

Transactional memory

1. INTRODUCTION

The emergence of multi-core architectures raises the problem of efficient synchronization in multithreaded programs. Conventional locking introduces a host of well-known problems: coarse grained locks are not scalable, while fine grained locks are error-prone and hard to design. Transactional memory [10, 16] has gained popularity in recent years as a new synchronization abstraction for multithreaded systems, which has the potential to overcome the pitfalls of traditional locking schemes. A *transactional memory* toolkit, or *TM* for short, allows threads to bundle multiple operations on memory objects into one transaction. Similarly to database transactions [17], transactions are executed *atomically*: either all of the transaction's operations appear to take effect

SPAA'09, August 11-13, 2009, Calgary, Alberta, Canada.

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simultaneously (in this case, we say that the transaction *commits*), or none of transaction's operations are seen (in this case, we say that transaction *aborts*). We formally define the model and correctness criterion in Section 3.

A transaction's abort may be initiated by a programmer or may be the result of a TM decision. In the latter case, we say that the transaction is *forcefully aborted* by the TM. For example, when one transaction reads some object A and then writes to some object B, while another transaction reads the old value of B and then attempts to write A, one of the transactions must be aborted in order to ensure atomicity. Most existing TMs perform unnecessary (*spare*) aborts, i.e., aborts of transactions that could have committed without violating correctness; see Section 2. Spare aborts have several drawbacks: work done by the aborted transaction is lost, computer resources are wasted, and the overall throughput decreases. Moreover, after the aborted transactions restart, they may conflict again, leading to livelock and degrading performance even further.

The aim of this paper is to advance the theoretical understanding of TM aborts, by studying what kinds of spare aborts can or cannot be eliminated, and what kinds of spare aborts can or cannot be avoided efficiently. Specifically, we show that some unnecessary aborts cannot be avoided, and that there is an inherent tradeoff between the overhead of a TM and the extent to which it refrains from spare aborts.

Previous works introduced two related notions: commitabort ratio [6] and permissiveness [7]. The latter stipulates that in runs that do not violate correctness, no aborts should happen. However, while shedding insight on the inherent limitations of online TMs, these notions do not provide an interesting yardstick for comparing TMs. This is because under these measures, all online TMs inherently perform poorly for some worst-case workloads, as we show in Section 4.

In Section 5, we then define measures of spare aborts that are appropriate for online TMs. Intuitively, our *strict online permissiveness* property allows a TM to abort some transaction only if not aborting any transaction would violate correctness. Unlike ealier notions, strict online permissiveness does not prevent the TM from taking an action that might lead to an abort in the future. Thus, the information available to the TM at every given moment suffices to implement strict online permissiveness. Clearly, this property depends on the correctness criterion the TM needs to satisfy. In this paper, we consider opacity [8] or slight variants thereof (see Section 3). In this context, strict online permissiveness prohibits aborting a transaction whenever the execution history

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is equivalent to some sequential one. We prove that strict online permissiveness cannot be satisfied efficiently by showing a reduction from the NP-hard view serializability [14] problem. We then define a more relaxed property, online permissiveness, which allows the TM to abort transactions if otherwise it would have to change the serialization order between already committed transactions. We show that online permissiveness also has inherent costs — it cannot be satisfied by a TM using *invisible* reads. Moreover, the information about a read should be exposed in shared memory immediately after the read operation returns.

In Section 6, we show a polynomial time TM protocol satisfying online permissiveness. The protocol maintains a precedence graph of transactions and keeps it acyclic. Unfortunately, we show that the graph must contain some committed transactions. But without removing any committed transactions, detecting cycles in the precedence graph would be impractical as it would induce a high runtime complexity. Hence, we define precise garbage collection rules for removing transactions from the graph. Even so, a naïve traversal of the graph would be costly; we further introduce optimization techniques that decrease the number of nodes traversed during the acyclity check.

Finally, we note that our goal is not to build a better TM, but rather to understand what can and what cannot be achieved, and at what cost. Future work may further explore the practical aspects of the complexity vs. spareaborts tradeoffs; our conclusions appear in Section 7.

2. RELATED WORK

Most existing TM implementations, e.g., [9, 5, 4, 3] abort one transaction whenever two overlapping transactions access the same object and at least one access is a write. While easy to implement, this approach may lead to high abort rates, especially in situations with long-running transactions and contended shared objects. Aydonat and Abdelrahman [2] referred to this problem and proposed a solution based on a conflict serializability graph and multiversioned objects in order to reduce the number of unnecessary aborts. However, their solution still induces spare aborts, and does not characterize exactly when such aborts are avoided. Moreover, they implement a stricter correctness criterion than opacity, which inherently requires more aborts. Riegel et al. [15] looked at the problem of spare aborts from a different angle, and introduced weaker correctness criteria, which allow TMs to reduce the number of aborts.

Napper and Alvisi [13] described a serializable TM, based upon multi-versioned objects, which used cycle detection in the precedence graph when validating the correctness criterion. The focus of the paper was providing a lock-free solution. The authors did not refer to the aspect of spare aborts and, in fact, their TM did lead to spare aborts due to a limitation on write operation, which had to insert the new version after the latest one. In addition, Napper and Alvisi did not refer to the problems of garbage collection and computational complexity of operations.

Gramoli et al. [6] referred to the problem of spare aborts and introduced the notion of *commit-abort ratio*, which is the ratio between the number of committed transactions and the overall number of transactions in the run. Clearly, the commit-abort ratio depends on the choice of the transaction that should be aborted in case of a conflict. This decision is the prerogative of a contention manager [9]. Attiya et al. [1] showed a $\Omega(s)$ lower bound for the competitive ratio for transactions' makespan of any online deterministic contention manager, where s is the number of shared objects. Their proof, however, does not apply to our model, because it is based upon the assumption that whenever multiple transactions need exclusive access to the same shared object, only one of these transactions may continue, while others should be immediately aborted. In contrast, our model allows the TM to postpone the decision regarding which transaction should be aborted till the commit, thus introducing additional knowledge and improving the competitive ratio. In this paper, we show that every TM is $\Omega(L)$ competitive in terms of commit-abort ratio, where L is the number of live transactions in the system. This result suggests that it is not interesting to compare (online) TMs by their commit-abort ratio, as the distance from the optimal result turns out to be an artifact of the workload rather than the algorithm, and every TM has a workload on which it performs poorly by this measure.

Input acceptance is also a notion presented by Gramoli et al. [6] — a TM *accepts* a certain input pattern (sequence of invocations) if it commits all of its transactions. The authors compared different TMs according to their input acceptance patterns. Guerraoui et al. [7] introduced the related notion of π -permissiveness. Informally, a TM satisfies π -permissiveness for a correctness criterion π , if every history that does not violate π is accepted by the TM. Thus, π -permissiveness can be seen as optimal input acceptance. However, Guerraoui et al. focused on a model with singleversion objects, and their correctness criterion was based upon conflict serializability, which is stronger than opacity and thus allows more aborts. They ruled out the idea of ensuring permissiveness deterministically, and instead provide a randomized solution, which is always correct and avoids spare aborts with some positive probability. In contrast, we do not limit the model to include single-version objects only, our correctness criterion is a generalization of opacity [8], and we focus on deterministic guarantees. Although permissiveness does not try to regulate the decisions of the contention manager, we show that no online TM may achieve permissiveness. Intuitively, this results from the freedom of choice for returning the object value during the read operation — returning the wrong value might cause an abort in subsequent operations, which is avoided by a clairvoyant (offline) algorithm.

3. SYSTEM MODEL

Transactions. Our definition of Transactional Memory (TM) is based on [8]. A TM allows threads to run *transac*tions. Transactions perform operations on shared objects. The objects considered in this paper are *read/write regis*ters. The status of a transaction may be either *live*, *aborted*, or *committed*. A transaction can perform operations as long as it is live. Each transaction has a unique identifier (id). Retrying an aborted transaction is interpreted as creating a new transaction with a new id. The maximal possible number of live transactions is L.

The API of the TM includes the following operations. The operation startTransaction() returns the id of a newly created transacton. The status of a newly created transaction is always live. When T_i is live, it can invoke the following operations: $read(T_{i,o})$, which returns the value of register

o, and $write(T_i, o, v)$, which writes value v to register o. For the sake of simplicity, we assume that the values written to the registers are unique. When T_i wishes to terminate, it invokes operation $tryCommit(T_i)$ or $tryAbort(T_i)$. If $tryCommit(T_i)$ returns C_i , the status of T_i changes to committed, while $tryAbort(T_i)$ always returns A_i , indicating that T_i is aborted. The abort value A_i may also be returned as a response to read, write or tryCommit invocations, in which case we say that the TM forcefully aborts transaction T_i . If the TM forcefully aborts transaction T_j as a result of another transaction's operation, then the returned value of the subsequent operation of T_j will be A_j . The read-set and the write-set of T_i are denoted as $read(T_i)$ and $write(T_i)$ respectively, and are not known in advance.

The calls to the TM are blocking — the invoking thread waits for a response before invoking more operations. We assume that TM operations issued by different threads are executed atomically. This allows us to neglect issues related to overlapping operation executions, which are not the focus of this paper; in practice, such atomicity can be implemented using locks or well-known lock-free solutions, e.g., [5]. Note, however, that transactions may overlap.

The TM guarantees that each operation invocation eventually gets a response, even if all other threads are sleeping. This limits the TM's behavior upon operation invocation, so that it may either return an operation response, or abort a transaction, but cannot wait for other transactions to invoke operations.

Transaction histories. A transaction history is the sequence of operations issued by transactions in a given TM execution, ordered by the time at which they are issued (in the rest of the paper we use the notion of run as a synonim to a transaction history). Two histories H_1 and H_2 are equivalent if they contain the same transactions and each transaction T_i issues the same operations in the same order with the same responses in both. A history H is complete if it does not contain live transactions. If history H is not complete, we may build from it a complete history Complete(H) by adding an abort operation for every live transaction. We define committed(H) to be the subsequence of H consisting of all the operations of all the committed transactions in H.

The real-time order on transactions is as follows: if the first event of transaction T_i is issued after the last response of transaction T_j in H, then $T_j \prec_H T_i$. Transactions T_i and T_j are concurrent if neither $T_j \prec_H T_i$, nor $T_i \prec_H T_j$. A history S is sequential if it has no concurrent transactions. A sequential history S is legal if it respects the sequential specification of each object accessed in S. Transaction T_i is legal in S if the largest subsequence S' of S, such that, for every transaction $T_k \in S'$, either (1) k = i, or (2) T_k is committed and $T_k \prec_S T_i$, is a legal history.

Correctness. Our correctness criterion resembles the *opacity* condition of Guerraoui and Kapalka [8]. Let $\Gamma(H)$ be a partial order on transactions. A TM satisfies Γ -*opacity* if for every history H generated by the TM there exists a sequential history S, s.t.:

- S is equivalent to Complete(H).
- Every transaction $T_i \in S$ is legal in S.
- If $(T_i, T_j) \in \Gamma(H)$, then $T_i \prec_S T_j$.

When $\Gamma(H)$ includes all the ordered pairs of non-concurrent transactions in H, the history S should preserve the real-time order of H. On the other hand, when Γ is empty, the correctness criterion is a serializability with consideration of aborted transactions. The use of Γ makes it possible to require transactional ordering that lies between serializability and strict serializability according to any arbitrary rule (e.g., Riegel et al. [15] considered demanding real-time order only from transactions belonging to the same thread). We define a more general criterion in order to broaden the scope of our results. In the rest of this paper, we will assume that $\Gamma(H)$ is a subset of the real-time order on transactions, unless stated otherwise.

We should note that our notion of Γ -opacity is somewhat stronger than the original one defined by Guerraoui and Kapalka [8], in that it is a safety property (i.e., prefix-closed). Since the set of histories of every TM is prefix closed, every TM satisfying the original (weaker) property also satisfies the (stronger) version we define. Moreover, the notion of opacity-permissiveness does not make sense in the context of the former because no TM may generate opaque histories that have non-opaque prefixes. Indeed, the paper that defines permissiveness [7] uses an even stronger notion of opacity, which is in fact prefix-closed.

4. PREVIOUS MEASURES LIMITATIONS

4.1 Commit-Abort Ratio

The commit-abort ratio (τ) [6] is the ratio between the number of committed transactions and the overall number of transactions in the history. Unfortunately, no online TM may guarantee optimal commit-abort ratio. Recall that L is the number of live transactions. We show that every TM is $\Omega(L)$ competitive in terms of its commit-abort ratio.

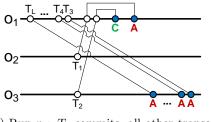
We use the style of [15] to depict transactional runs. Objects are represented as horizontal lines o_1 , o_2 , etc. Transactions are drawn as polylines with circles corresponding to accesses to the objects. Filled circles indicate writes, and empty circles indicate reads. Commit is indicated by the letter **C**, and abort by the letter **A**. If the TM implements the access to the object as if it had appeared in past, the dashed arc indicates the point in time at which the access to the object appears according to the TM serialization.

LEMMA 1. Every TM is $\Omega(L)$ competitive in terms of its commit-abort ratio.

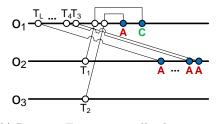
PROOF. Consider the scenarios depicted in Figure 1. The runs are indistinguishable until the time when T_L tries to commit. Transactions T_1 and T_2 cannot both commit because both write o_1 after reading its previous value. In run r_1 (Figure 1(a)), the TM commits T_2 , then T_1 aborts and then the transactions $T_3 \cdots T_L$ try to write to o_3 and must be aborted because they conflict with T_2 , resulting in $\tau = \frac{1}{L}$. In run r_2 (Figure 1(b)), the TM aborts T_2 , T_1 commits and then the transactions $T_3 \cdots T_L$ try to write to o_2 and therefore must be aborted, resulting again in $\tau = \frac{1}{L}$. The optimal offline TM in these cases would abort only one transaction, yielding $\tau = \frac{L-1}{L}$. The online TM, however, cannot distinguish between r_1 and r_2 at the moment it should decide whether to abort T_1 or T_2 , hence the competitive ratio is $\Omega(L)$. \Box

4.2 **Permissiveness**

Since requiring an optimal commit-abort ratio is too restrictive, we consider a weaker notion that limits aborts only



(a) Run r_1 : T_2 commits, all other transactions abort: $\tau = \frac{1}{L}$



(b) Run r_2 : T_1 commits, all other transactions abort: $\tau = \frac{1}{L}$

Figure 1: No online TM may know whether to abort T_1 or T_2 in order to obtain an optimal commit-abort ratio.

in runs where none are necessary: a TM provides *permissiveness* [7] if it accepts every history satisfying Γ -opacity. Gramoli et al. showed that existing TM implementations do not accept all inputs they could have, and hence are not permissive. We show that this is an inherent limitation.

The formal impossibility illustrated in Figure 2 is captured in the following lemma:

LEMMA 2. For any Γ , there is no online TM implementation providing Γ -opacity-permissiveness.

PROOF. Consider the scenario depicted in Figure 2. All the objects have initial values, v_0 . All the transactions start at the same time, t_i , and are therefore not ordered according to the real-time order, thus the third condition of our correctness criterion holds for any Γ .

 T_1 writes values v_1 to o_2 and o_1 . At time t_0 , there is a read operation of T_2 and the TM should decide what value should be returned. In general, the TM has four possibilities: (1) return v_1 , (2) return v_0 , (3) return some value v' different from v_0 and v_1 , and (4) abort T_2 . If the TM chooses to abort, then opacity-permissiveness is violated and we are done. (3) is not possible, for returning such a value would produce a history, for which any equivalent sequential history S would violate the sequential specification of o_1 and thus would not be legal.

Consider case (1): the TM returns v_1 for T_2 at time t_0 . This serializes T_2 after T_1 . Consider run r_1 depicted in Figure 2(a), where T_3 tries to write to o_3 and commit. In this run, the TM has to forcefully abort T_3 , because not doing so would produce a history H with no equivalent sequential history: $T_1 \prec T_2 \prec T_3 \prec T_1$. However, if T_2 would read v_0 in run r_1 , then T_2 , T_1 and T_3 would be legal, and no transaction would have to be forcefully aborted. So Γ -opacitypermissiveness is violated.

In case (2), the TM returns v_0 for transaction T_2 at time t_0 , serializing T_2 before T_1 . Consider run r_2 depicted in Figure 2(b). Transaction T_4 writes to o_2 , and afterwards reads and writes to o_3 . Transaction T_4 has to be serialized after T_1 , because T_1 has read v_0 from o_2 . When T_2 will try to write to o_3 and commit, the TM will have to forcefully abort some transaction, because not doing so would produce a history with no equivalent sequential history: $T_2 \prec T_1 \prec T_4 \prec T_2$. But if T_2 would read v_1 in run r_2 , then no transaction would have to be forcefully aborted. So again, Γ -opacity-permissiveness is violated.

Runs r_1 and r_2 are indistinguishable to the TM at time t_0 . Therefore, no online TM can accept both of the patterns, while an offline TM can accept both of them. \Box

5. ONLINE PERMISSIVENESS — LIMITA-TIONS AND COSTS

5.1 Strict Online Opacity-Permissiveness

We next define a property that prohibits spare aborts, and yet is possible to implement.

DEFINITION 1. A TM satisfies strict online Γ -opacity-permissiveness if the TM forcefully aborts a set S of live transactions only when aborting any subset $S' \subset S$ of transactions violates Γ -opacity for the given Γ .

Note that this property does not define which transaction should be aborted if abort happens, and does not prohibit returning a value that will cause aborts in the future. For example, in the scenarios depicted in Figure 2, at time t_0 , a TM satisfying this property may return either value, even though this might cause an abort in the future.

An algorithm satisfying strict online opacity-permissiveness should be able to detect whether returning a given value creates a history satisfying Γ -opacity. We show that this cannot be detected efficiently. To this end, we recall a wellknown result about checking the serializability of the given history, which was proven by Papadimitriou [14].

Given history H, the *augmented* history H is the history, which is identical to H, except two additional transactions: T_{init} that initializes all variables without reading any, and T_{read} that is the last transaction of H, reading all variables without changing them. The set of *live* transactions in H is defined in the following way: (1) T_{read} is live in H, (2) If for some live transaction T_i , T_i reads a variable from T_i , then T_i is also live in H. Note that aborted transaction cannot be live according to this definition (no transaction may read the values written by the aborted one). Transaction is dead if it is not live. Two histories H and H' are view equivalent if and only if (1) they have the same sets of live transactions and (2) T_i reads from T_i in H if and only if T_i reads from T_i in H'. History H is view serializable, if for every prefix H'of H, complete(H') is view equivalent to some serial history S.

THEOREM 1 (PAPADIMITRIOU). Testing whether the history H is view-serializable is NP-complete in the size of the history, even if H has no dead transactions.

LEMMA 3. For any Γ , detecting whether the history H satisfies Γ -opacity is NP-complete in the size of the history.

PROOF. We will show a reduction from the NP-complete problem of detecting view-seializability of history H without

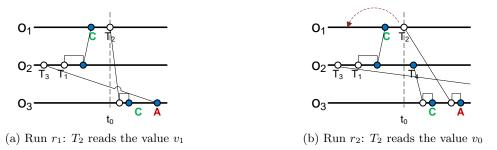


Figure 2: At time t_0 , no online TM knows which value should be returned to T_2 when reading o_1 in order to allow for commit in the future.

dead transactions to the problem of detecting whether some history H' satisfies Γ -opacity. Consider history H with no dead transactions. In the absence of aborted transactions, the definition of view serializability differs from the definition of opacity only in the fact that opacity refers to the partial order Γ , which is a subset of a real-time order. We construct history H', which is identical to history H except the following addition: for each T_i in H, we add $start(T_i)$ at the beginning of H'. We will show that H is view serializable if and only if H' satisfies Γ -opacity.

H is view serializable if and only if there exists a legal sequential history S, which is view equivalent to Complete(H). All the transactions in H' are concurrent $(start(T_i))$ follows before any other event for every T_i), therefore the third condition of Γ -opacity vacuously holds for any Γ . In the absence of aborts in H', H' satisfies Γ -opacity if and only if there exists a legal sequential history S', so that every transaction in H' issues the same invocation events and receives the same response events as in S'. Therefore, H' satisfies Γ -opacity if and only if H' is view-serializable. \Box

5.2 Online Opacity-Permissiveness

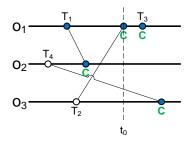


Figure 3: The order of transactions T_1 and T_2 is changed after their commit time.

Intuitively, the problem with strict online opacity-permissiveness lies in the fact that the order of committed transactions may be undefined and may change in the future. Consider, for example, the scenario depicted in Figure 3. Transactions T_1 and T_2 are not ordered according to real-time order, therefore they are not ordered by Γ . At time t_0 , the serialization order is $T_1 \rightarrow T_2$, as o_1 holds the value written by T_2 . When T_3 commits, the serialization order of T_1 and T_2 becomes undefined, since T_3 overwrites o_1 before any transaction reads the value written by T_2 . And when T_4 commits, the serialization order induced by the run cannot change after being defined, the problem becomes much easier. We capture this intuition with the following definition: DEFINITION 2. A TM satisfies λ -persistent ordering for a partial order λ if for every run λ is updated according to the following rules: (1) at the beginning of the run λ is empty; (2) at any point in time λ is a transitive closure over a relation ordering exactly all the pairs of $T_i, T_j, s.t.$ write $(T_i) \cap$ write $(T_j) \neq \emptyset$; and (3) each time λ is updated, its new value preserves the one previously defined.

In other words, if T_i and T_j are committed transactions in H that have written to the same object in a given TM, then they are ordered by λ and their order will persist in every extension of the run.

We now define a more relaxed property, online Γ -opacitypermissiveness, which may be satisfied at a reasonable implementation cost.

DEFINITION 3. A TM satisfies online Γ -opacity-permissiveness for a given Γ if the TM satisfies λ -persistent ordering for some λ consistent with Γ , and the TM forcefully aborts a set S of live transactions only when aborting any subset $S' \subset S$ of transactions violates $(\Gamma \cup \lambda)$ -opacity.

Note that Definition 3 implies that each committing transactions should define its serialization order with regard to all other committed transactions that have written to the same objects. To the best of our knowing, all existing TMs do in fact define the order on two transactions that write to the object by the time the latter commits. We note that this requirement might be limiting for TMs that wish to exploit the benefits of commutative or write-only operations (see [12]), and do not necessarily define the serialization point of the committed transactions. However, this limitation is essential for an effective check of the opacity criterion.

In the following sections we show a polynomial-time TM satisfying online opacity-permissiveness. We now prove that such an implementation, nevertheless, has some inherent costs.

One of the basic decisions that needs to be made during the design of a TM is whether to *expose* the fact that transaction T_i has read the object o, i.e. make a change in shared memory as a result of the read, making the read *visible*. In case we expose the read, there arises another question, regarding whether we can postpone exposing the read until the commit. One of the central problems with exposing the read is that it requires writing metadata in shared memory. One typically tries to avoid writes to shared memory, because writing data that is read by different cores has a high cache penalty. Postponing exposing the read until the commit may save redundant writes in case the transaction eventually aborts. Unfortunately, if a TM satisfying online opacity-permissiveness does not expose a read operation immediately when the read happens, then all read operations through the entire execution must return the respective objects' initial values, rendering such a TM implementation completely useless:

LEMMA 4. For any Γ , if a TM satisfies online Γ -opacitypermissiveness and never exposes read operations when the reads happen, then no transaction T_i can read from a committed transaction T_j if T_j has written to more than one object.

PROOF. Assume by contradiction that there exists a TM, satisfying online Γ -opacity-permissiveness that does not expose read operations when the reads happen, and which allows transactions to read values written by the transactions with more than one object in the write-set. Consider run r_1 depicted in Figure 4. Transaction T_3 reads o_3 without exposing it. Afterwards transaction T_2 writes to o_3 and o_2 . Then T_3 writes to o_1 and tries to commit. We next construct r_2 , where transaction T_1 reads the value written by transaction T_2 , as depicted in Figure 4(b). By our assumption, this value can indeed be read by some transaction. T_1 then continues to read o_1 . Note that T_1 is not aware of transaction T_3 preceding T_2 because T_3 did not expose its read. All the transactions start at the same time and therefore cannot be ordered by real-time order. In run r_1 , T_3 must commit because the run satisfies Γ -opacity. In run r_2 , however, T_3 cannot commit because that would create a precedence cycle $T_1 \to T_3 \to T_2 \to T_1$ and thus violate Γ -opacity. But since we assumed that reads are invisible, runs r_1 and r_2 are indistinguishable to T_3 at time t_0 , a contradiction. \Box

6. THE ABORTS-AVOIDER ALGORITHM

We now present AbortsAvoider, a TM algorithm implementing online opacity-permissiveness. The basic idea behind AbortsAvoider is to maintain a precedence graph of transactions, and keep it acyclic, as explained in Section 6.1. A simplified version of the protocol based on this graph is then presented in Section 6.2. The key challenge AbortsAvoider faces is that completed transactions cannot always be removed from the graph, whereas keeping all transactions forever is clearly impractical. We address this challenge in Section 6.3, presenting a garbage collection mechanism for removing terminated transactions from the graph. In Section 6.4 we present another optimization, which shortens paths in the graph to reduce the number of terminated transactions traversed during the acyclity check. Our complexity analysis appears in the same section.

In this section, we state theorems stipulating the correctness of AbortsAvoider. For space limitations, their formal proofs are deferred to the full version of this paper [11].

6.1 Basic Concept: Precedence Graph

Information bookkeeping. Our protocol maintains *object version lists.* We now explain what such a TM does: (1) each object o is associated with a totally ordered set of versions, (2) a read of o returns the value of one of o's versions, and (3) a write to o adds a new version of o upon commit. For simplicity, at any given moment, we number the versions of the object in increasing order. (Note that the numbering is for analysis purposes only, and the numbers of the versions change during the run as the versions are inserted and removed from the versions list). The object versions of the version of the versions of the versions of the versions list.

sion $o.v_n$ includes the data, $o.v_n.data$, the writer transaction, $o.v_n.writer$, and a set of readers, $o.v_n.readers$. Each transaction has a *readList* and a *writeList*. An entry in a *readList* points to the version that has been read by the transaction. A *writeList* entry points to the object that should be updated after commit, the new data, and the place to insert the new version, (which may be undefined till the commit).

Precedence graph. Transactions may point to one another, forming a directed labelled precedence graph, PG. PG reflects the dependencies among transactions as created during the run. The vertexes of PG are transactions, the edges of PG are as follows (Figure 5):

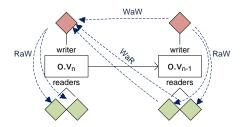


Figure 5: Object versions and the precedence graph.

If $(T_j, T_i) \in \Gamma$, then PG contains (T_j, T_i) labelled L_{Γ} (Γ order). If T_i reads $o.v_n$ and T_j writes $o.v_n$, then PG contains (T_j, T_i) labelled L_{RaW} (Read after Write). If transaction T_i writes $o.v_n$ and T_j writes $o.v_{n-1}$, then PG contains (T_j, T_i) (Write after Write) labelled L_{WaW} . If transaction T_i writes $o.v_n$ and T_j reads $o.v_{n-1}$, then PG contains (T_j, T_i) labelled L_{WaW} . (Write after Read).

Below we present lemmas that link maintaining acyclity in PG and satisfying online-permissiveness. We restrict our discussion to non-local histories, which we now define. We say that a read operation of $T_i \ read_i(o)$ in H is local if it is preceded in $H|T_i$ by a write operation $write_i(o,v)$. A write operation $write_i(o,v)$ is local if it is followed in $H|T_i$ by another write operation $write_i(o,v')$. The non-local history of H is the longest subsequence of H not containing local operations [8]. Note that the precedence graph does not refer to local operations.

LEMMA 5. Consider a TM that maintains object version lists. If PG is acyclic throughout some run, then the nonlocal history H of the run satisfies Γ -opacity.

PROOF. Let H be a history over transactions $\{T_1 \dots T_n\}$. Let $H_C = Complete(H)$, i.e. H with A_i added for every live $T_i \in H$.

Since PG is acyclic, it can be topologically sorted. Let T_{i1}, \ldots, T_{in} be a topological sort of PG, and let S be the sequential history T_{i1}, \ldots, T_{in} . Clearly, S is equivalent to H_C because both of the histories contain the same transactions and each transaction issues the same operations and receives the same responses in both of them.

We now prove that every $T_i \in S$ is legal. Assume by contradiction that there are non-legal transactions in S. Let T_i be the first such transaction. If T_i is non-legal, T_i reads a value of object o that is not the latest value written to o in S by a committed transaction. (Recall that by definition of object version lists, only values written by committed transactions can be read.) S contains only non-local operations, and therefore T_i reads the version $o.v_n$ written by another transaction T_j . Therefore, there is an edge from T_j to T_i in PG. It follows that T_j is committed in S and ordered before

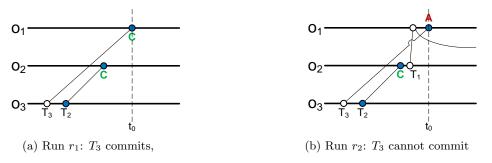


Figure 4: T_3 does not distinguish between r_1 and r_2 at time t_0 . If T_1 does not expose its reads, it is not allowed to read the value written by T_2 , because otherwise the commit of T_3 would violate Γ -opacity.

 T_i according to the topological sort. If the value of $o.v_n$ is not the latest value written in S before T_i , then there exists another committed transaction T'_j that writes to o and is ordered between T_j and T_i in S. If T'_j writes to a version earlier than $o.v_n$, then there is a path from T'_j to T_j in PG, and therefore T'_j is ordered before T_j in S. If T'_j writes to a version later than $o.v_n$, then there is a path from T_i to T'_j in PG, and therefore T'_j is ordered after T_i in S. In any case, T'_j cannot be ordered between T_j and T_i in S, a contradiction.

Finally, for each pair $T_i \prec_{\Gamma} T_j$, PG contains an edge from T_i to T_j . Therefore, according to the topological sort, S preserves the partial order Γ .

Summing up, Complete(H) is equivalent to a legal sequential history S, and S preserves partial order Γ . Therefore H is Γ -opaque.

We define λ_{PG} to be the following binary relation: if PG contains a path from T_i to T_j consisting of L_{WaW} edges, then $T_i \prec_{\lambda_{PG}} T_j$. Note that if PG is acyclic, then λ_{PG} is reflexive, antisymmetric and transitive, and therefore λ_{PG} is a partial order.

LEMMA 6. Every TM that maintains object version lists and keeps PG acyclic satisfies λ_{PG} -persistent ordering.

PROOF. We will show that every TM maintaining object version lists and keeping PG acyclic satisfies the three necessary conditions for λ_{PG} -persistent ordering.

(1) Initially PG does not contain edges and thus λ_{PG} is empty.

(2) Consider an arbitrary point during the run. Let χ be the transitive closure of the partial order containing exactly all the pairs of transactions T_i , T_j committed up to this point, s.t. write $(T_i) \cap \text{write}(T_j) \neq \emptyset$, where $T_i \prec_{\chi} T_j$ iff T_j writes a higher numbered version than T_i . We show that λ_{PG} is equal to χ .

We show first that $\lambda_{PG} \supseteq \chi$. Consider two committed transactions $T_i \prec_{\chi} T_j$ that have a common object o in their write-sets, s.t. T_i has written to the version $o.v_i$ and T_j has written to the version $o.v_j$, where i < j. Then PG contains a path from T_i to T_j consisting of L_{WaW} edges and therefore λ_{PG} contains a pair (T_i, T_j) . λ_{PG} is transitive by definition, and therefore $\lambda_{PG} \supseteq \chi$. It remains to show that $\lambda_{PG} \subseteq \chi$. If $T_i \prec_{\lambda_{PG}} T_j$, then PG contains a path from T_i to T_j consisting of L_{WaW} edges. Every edge on the path defines a pair of committed transactions that have written to the same object and therefore are ordered by χ . By transitivity of χ , we conclude that $T_i \prec_{\chi} T_j$. We have shown that both $\lambda_{PG} \supseteq \chi$ and $\lambda_{PG} \subseteq \chi$. Therefore, $\lambda_{PG} = \chi$. (3) Finally, edges are never removed from PG, therefore each time λ_{PG} is updated, its new value preserves the previously defined order. \Box

LEMMA 7. Consider a TM that maintains object version lists and forcefully aborts a set S of live transactions only when aborting any subset $S' \subset S$ of transactions creates a cycle in PG. Then this TM satisfies online Γ -opacitypermissiveness.

PROOF. Consider the partial order λ_{PG} . As shown in Lemma 6, the TM satisfies λ_{PG} -persistent ordering. We need to show that if there is a cycle in PG, then the run violates $(\Gamma \cup \lambda_{PG})$ -opacity.

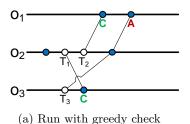
We show first that if there is an edge (T_i, T_j) in PG, then every legal sequential history S preserving $\Gamma \cup \lambda_{PG}$ and equivalent to Complete(H) orders T_i before T_j . Consider two transactions T_i and T_j s.t. there is an edge (T_i, T_j) in PG. If the edge is labelled L_{Γ} , then $(T_i, T_j) \in \Gamma$, and Sorders T_i before T_j . If the edge is labelled L_{RaW} , then T_j reads a value written by T_i and S also orders T_i before T_j . If the edge is labelled L_{WaW} , then $T_i < T_j$ according to λ_{PG} , hence S also orders T_i before T_j . If the edge is labelled L_{WaR} , then T_i reads $o.v_n$ while T_j writes $o.v_{n+1}$. On the one hand, T_j should be ordered after $o.v_n.writer$ in S (there is an edge from $o.v_n.writer$ to T_j labelled L_{WaW}). On the other hand, T_j cannot be ordered between $o.v_n.writer$ and T_i , because T_i must read the value written by $o.v_n.writer$ in S. Therefore, T_j is ordered after T_i in S in this case as well.

Summing up, an edge (T_i, T_j) in the precedence graph induces the order of T_i before T_j in any legal sequential history S preserving $\Gamma \cup \lambda_{PG}$ and equivalent to Complete(H). Therefore, if PG contains a cycle, no such sequential history exists, and the TM cannot satisfy $\Gamma \cup \lambda$ -opacity. \Box

COROLLARY 1. Consider a TM maintaining object version lists that keeps PG acyclic and forcefully aborts a set S of live transactions only when aborting any subset $S' \subset S$ of transactions creates a cycle in PG. Then this TM satisfies Γ -opacity and online Γ -opacity-permissiveness.

6.2 Simplified **r**-AbortsAvoider Algorithm

AbortsAvoider algorithm maintains object version lists as explained above, keeps PG acyclic and forcefully aborts a transaction only if not aborting any transaction would create a cycle in PG. Read and write operations are straightforward, they are depicted in Algorithm 1. A *read* operation (line 4) looks for the latest possible object version to read without creating a cycle in PG. Write operations (line 14) postpone the actual work till the commit.



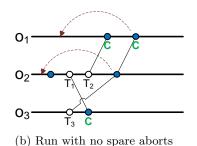


Figure 6: Checking the written objects in a greedy way during the commit may lead to a spare abort.

Algorithm	1 I	Γ -AbortsA	voider	for T_i	- Read/Write.
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1: procedure START()

- $T_{prev} \leftarrow$ the latest transaction preceding T_i by Γ 2:
- $PG.ADDEDGES(\{(T_{prev}, T_i)\})$ 3:
- procedure $READ(Object \ o)$ 4:
- if $o \in T_i$.writeList then return T_i .writeList[o].data 5:
- if $o \in T_i$.readList then return T_i .readList[o].data 6:
- $n \leftarrow$ the latest version that can be read without cre-7:ating a cycle in PG
- if $n = \perp$ then return abort event A_i 8.
- 9: $newEdges \leftarrow \{(o.v_n.writer, T_i), (T_i, o.v_{n+1}.writer)\}$
- PG.ADDEDGES(newEdges)10:
- $o.v_n.readers.ADD(T_i)$ 11:
- $T_i.readList.ADD(\langle o.v_n \rangle)$ 12:
- 13: return $o.v_n.data$
- procedure WRITE(Object o, ObjectData val) 14:

```
15:
        if o \in T_i.writeList then
           T_i.writeList[o].data \leftarrow val; return
16:
17:
        if o \in T_i.readList then
           \triangleright non-blind write, victim version is read version
18:
           writeNode \leftarrow \langle o, readList[o].version, val \rangle
19:
        else
20:
21:
           \triangleright blind write, victim version is not known
           writeNode \leftarrow \langle o, \perp, val \rangle
22:
        T_i.writeList.ADD(writeNode)
23
```

```
The commit operation is more complicated. Intuitively,
for each object written during transaction, the algorithm
should find a place in the object's version list to insert the
new version without creating a cycle. Unfortunately, check-
ing the objects one after another in a greedy way can lead
to spare aborts, as we illustrate in Figure 6(a). Commit-
ting T_3 first seeks for a place to install the new version of
o_1 and decides to install it after the last one (serializing T_3)
after T_2). When T_3 considers o_2, it discovers that the new
version cannot be installed after the last one, because T_3
should precede T_1, but it also cannot be installed before the
last one, because that would make T_3 precede T_2, so T_3 is
aborted. However, installing the new version of o_1 before
the last one would have allowed T_3 to commit, as depicted
in Figure 6(b), that is why aborting T_3 violates online \Gamma-
opacity-permissiveness.
```

Our commit operation (Algorithm 2, line 24) is divided to two phases. We call the object version after which the new version is to be installed a *victim version*. The victim version is known only for the non-blind writes (that is version, which has been read before the write, line 19). In the

first phase the algorithm tries to install the non-blind writes (lines 28–34). In the second phase (lines 36–49) the algorithm tries to find the vicim versions for the blind writes in iterations. Initially, the victim is the object's latest version. In each iteration, the algorithm traverses the objects and for each one searches for the latest possible victim to install the new version without creating a cycle in PG (line 41). When victim $o.v_n$ is found, an edge from T_i to the writer of $o.v_{n+1}$ is added to PG (line 47). We add only the outgoing edges at this point, because changing the victim from $o.v_n$ to $o.v_{n-1}$ may remove some incoming edges to T_i but cannot remove outgoing ones. Meanwhile, incoming edges are kept in in-Edges. After each iteration, there are possibly new outgoing edges added to PG, that would mean that the previously found victim versions might not suit anymore and a new iteration should be run. Once there is an iteration when no new edges are added, the algorithm commits — it installs the new versions after their victims and adds all the edges, including inEdges from the latest iteration, to the PG.

The following theorem shows that AbortsAvoider is correct and avoids unnecessary aborts.

THEOREM 2. Γ -AbortsAvoider satisfies Γ -opacity and online Γ -opacity-permissiveness.

In the rest of the paper we will show the garbage collection rules and optimization techniques for the protocol.

6.3 Garbage Collection

A TM should garbage collect unused metadata. In this section, we describe how terminated transactions may be garbage collected.

Terminated transaction T_i may be garbage collected if it cannot participate in any cycle in PG. Unfortunately, the simple check of whether the terminated transaction has no incoming edges is not enough. Consider, for example, the scenario depicted in Figure 7. At time t_0 , T_1 has no incoming edges, but we are still not allowed to garbage collect it as we now explain. There is a transaction T_2 that read object o_1 with a live preceding transaction T_3 . At the time of T_3 's commit, it discovers that it cannot install the last version of o_1 , and tries to install the preceding version. Had we removed T_1 from PG, this would have caused a consistency violation, because we would miss the cycle between T_1 and T_3 . The example above demonstrates the importance of knowing that from some point onward, T_i may have no new incoming edges. The lemma below shows that some edge additions can be saved:

LEMMA 8. If T_i is a terminated transaction, then no incoming edges need to be added to T_i in PG as long as for

Algorithm 2 Γ -AbortsAvoider for T_i - Commit.

24: procedure COMMIT $newEdges \leftarrow \emptyset$ \triangleright edges added upon commit 25: $blinds \leftarrow \emptyset$ \triangleright the set of blind writes 26 \triangleright Phase I — install the non-blind writes 27: for each n in T_i .writeList do 28:if $n.victim \neq \perp$ then 29: $(v, edgs) \leftarrow VALIDATEWRITE(newEdges, n.victim)$ 30: 31: if v = FALSE then return abort event A_i $newEdges \leftarrow newEdges \cup edgs$ 32: else 33: $blinds \leftarrow blinds \cup \{n\}$ 34: 35: \rhd Phase II — install the blind writes 36: repeat $foundOutEdges \leftarrow False$ 37: 38: $inEdges \leftarrow \emptyset$ for each n in blinds do 39: \triangleright find the latest possible victim 40: $(victim, edges) \leftarrow FINDVICTIM(newEdges, n)$ 41: if *victim* = \perp then return abort event A_i 42: for each e in edges do 43: if e is incoming to T_i then 44: $inEdges \leftarrow inEdges \cup e$ 45:else if $e \notin newEdges$ then 46: $newEdges \leftarrow newEdges \cup \{e\}$ 47: $foundOutEdges \leftarrow TRUE$ $48 \cdot$ until foundOutEdges = FALSE49: 50: \triangleright commit point for each n in T_i .writeList do 51:install the new version right after n.victim52: $PG.ADDEDGES(newEdges \cup inEdges)$ 53: 54: **procedure** FINDVICTIM($List \langle Edge \rangle$ newEdges, WriteNode wn) : (ObjectVersion, $List\langle Edge \rangle$) \triangleright find the latest possible victim 55: if $wn.victim = \perp$ then $vctm \leftarrow wn.latestVersion$ 56: 57: else $vctm \leftarrow wn.victim$ while $vctm \neq \perp do$ 58: \triangleright check installing the new version after *vctm* 59: $(valid, edges) \leftarrow VALIDATEWRITE(newEdges, vctm)$ 60: if valid = TRUE then return (vctm,edges) 61: 62: $vctm \leftarrow vctm.prev$ \triangleright go to the previous version return (\bot, \bot) \triangleright no suitable victim found 63: **procedure** VALIDATEWRITE(*edges*, $o.v_n$) : (boolean. 64: $List\langle Edge \rangle$) $added \leftarrow \{(o.v_n.writer, T_i), (o.v_n.readers, T_i), \}$ 65: $(T_i, o.v_n.next.writer)$ $valid \leftarrow acyclity of PG after adding edges \cup added$ 66: return (valid, added) 67:

each $o.v_n$ written blindly by T_i there is no reader with a live preceding transaction.

Based on Lemma 8 we can optimize the algorithm to add fewer edges.

Garbage collection conditions. We say that a transaction is *stabilized* if no incoming edges may be added to it in the future. At the moment when T_i has no incoming edges and it is stabilized, we know that T_i will not participate in any cycle, and thus may be garbage collected.

THEOREM 3. The terminated transaction T_i is stabilized

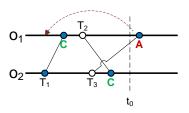


Figure 7: The blind write of transaction T_1 does not allow us to garbage collect it at time t_0 .

at time t_0 if either (1) T_i has not written blindly any object version o.v_n, or (2) all live transactions at time t_0 and all the transactions beginning after t_0 follow T_i according to Γ .

For this, we define that terminated transactions with no incoming edges satisfying one of the conditions of Theorem 3 may be garbage collected. Note that in the runs with no blind writes, every terminated transaction is stabilized and thus the transaction may be garbage collected at the moment it has no incoming edges.

6.4 Path Shortening and Runtime Analysis

AbortsAvoider protocol allows adding new edges to PG only if they do not introduce cycles in PG. The straightforward cycle detection algorithm runs DFS starting from T_i , traversing a set of nodes we refer to as $ingress_i$. We now present an optimization that reduces the number of nodes in $ingress_i$.

Consider stabilized terminated T_i . The idea is to connect the ingress nodes to the egress nodes of T_j directly, thus preventing DFS from traversing T_j . This becomes possible because T_i is stabilized and thus may not have new ingress nodes, hence the egress nodes do not miss the precedence info when they lose their edges from T_j . Once a terminated transaction T_i satisfies the conditions of Lemma 8 and it can no longer have additional incoming edges, (e.g., any transaction with no blind writes), we remove all of its outgoing edges by connecting its ingress nodes directly to its egress nodes as described above, and indicate that T_j is a *sink*, i.e., cannot have outgoing edges in the future. Once a transaction is marked as a sink, any outgoing edge that should be added from it is instead added from its ingress nodes. Note that our path shortening only bypasses stabilized nodes. Had we bypassed also non-stabilized ones, we would have had to later deal with adding new incoming nodes to their egress nodes, which could require a quadratic number of operations in the number of terminated transactions. Hence, we chose not to do that.

Runtime complexity of the operations. Running DFS on $ingress_i$ takes $O(V^2)$, where V is the number of transactions preceding T_i , which have not been garbage collected. In the general case, V = #T + #L, where #T and #L denote the number of terminated and live transactions respectively.

The read operation seeks the proper version to read in the version list. Unfortunately, the number of versions that need to be kept is limited only by the number of terminated transactions. Consider the scenario depicted in Figure 8. Here, the only version of o_2 that may be read by T_1 is the first, all other versions are written by transactions that T_1 precedes. In order to find a latest suitable version, the read operation may use a binary search – $O(\log(\#T))$ versions

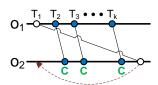


Figure 8: All object versions must be kept, as their writers have a live preceding transaction T_2 .

should be checked. Adding the edges takes O(#L). So altogether, the read complexity is $O(\log(\#T) \cdot \max\{\#L^2, \#T^2\})$, and $O(\log(\#T) \cdot \#L^2)$ when there are no blind writes.

The write operation postpones all the work till the commit. We denote the number of writes of the given transaction as #W. The number of iterations in the commit phase is $O(\#W \cdot \#T)$, and in each iteration O(#W) validate operations should be run. So the overall write cost is $O(\#W^2 \cdot \#T \cdot \max\{\#L^2, \#T^2\})$, and $O(\#L^2)$ when there are no blind writes.

Finally, we would like to emphasize that although in the worst-case, these costs may seem high, transactions without blind writes are garbage collected immediately upon commit. Moreover, the only nodes in *ingress*_i where cycles are checked are transactions that conflict with T_i . Typically, in practice, the number of such conflicts is low, suggesting that our algorithm's common-case complexity is expected to be good. On the other hand, if the number of conflicts is high, then most TMs existing today would abort one of the transactions in each of these cases, which is not necessarily a better alternative.

7. CONCLUSIONS

The paper took a step towards providing a theory for understanding TM aborts, by investigating what kinds of spare aborts can or cannot be eliminated, and what kinds can or cannot be avoided efficiently. We have shown that some unnecessary aborts cannot be avoided, and that there is an inherent tradeoff between the overhead of a TM and the extent to which it reduces the number of spare aborts: while strict online opacity-permissiveness is NP-hard, we presented a polynomial time algorithm AbortsAvoider, satisfying the weaker online opacity-permissiveness property. Understanding the properties of spare aborts is still far from being complete. For example, relaxations of the online opacity-permissiveness property or restrictions of the workload may be amenable to more efficient solutions. Moreover, the implications of the inherent "spare aborts versus time complexity" tradeoff we have shown are yet to be studied.

Acknowledgments

We would like to thank Hagit Attiya, Eshcar Hilel and Alessia Minali, as well as the anonymous reviewers for their helpful comments.

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