

## Distributed Computing Column 37

### *Reconfiguring State Machines ... and the History of Common Knowledge*

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As suggested in the title, this column deals with two distinct issues. First— a debt from last year. In the previous column, (SIGACT News 40(4), December 2009, pp. 64–97), I mentioned that the 2009 Dijkstra Prize was awarded to Joe Halpern and Yoram Moses for their paper “Knowledge and Common Knowledge in a Distributed Environment”. Joe and Yoram received the prize during the DISC 2009 banquet, around 10:30pm, after a seven-course Spanish dinner. This timing did not readily allow for formal speeches. During a coffee break on the following day, some DISC attendees lamented the lack of such a speech, and opined that the Dijkstra Prize ought to be an occasion for the recipients to address the audience, although perhaps at a better timed event. Yoram Moses followed up on this suggestion, but in lieu of delivering a formal speech, wrote down what he might have said in such an address. I am happy to include here his fascinating personal account of the events “behind the scenes” of this paper, and how it came to be written.

Second— our main contribution for this column is a tutorial on reconfiguration in replicated state machines, by Leslie Lamport, Dahlia Malkhi, and Lidong Zhou of Microsoft Research. This is also a bit of a throwback to a column from last year (SIGACT News 40(3), September 2009, pp. 77–85), where Lidong Zhou discussed a number of oft theoretically-neglected issues that arise in practical deployments of large-scale distributed systems. Lidong had written that, in practice, reconfiguration of replicated systems has always been a key focus, though it got little attention in theoretical work. In this column, you will find an instructive (semi-formal) treatment of this vital-but-not-well-understood issue, covering a range of practical approaches to reconfiguration.

Many thanks to Yoram, Leslie, Dahlia and Lidong for their contributions!

**Call for contributions:** I welcome suggestions for material to include in this column, including news, reviews, opinions, open problems, tutorials and surveys, either exposing the community to new and interesting topics, or providing new insight on well-studied topics by organizing them in new ways.

**Behind the Scenes of K&CK:**  
*The Undelivered Speech for the 2009 Dijkstra Prize*

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I was honored to receive the 2009 Dijkstra Prize, joint with Joe Halpern. The circumstances under which the prize was awarded did not allow for more than a brief response. I have since wondered what I might have said in a longer address. I concluded that, rather than discuss the future of the field, I would try to tell the story behind the award-winning paper. This note is a draft of this story, from my very biased and subjective point of view. It is presented here in the style of an added clip in a new DVD of an old movie providing footage of some behind the scenes developments, bloopers etc.

In contrast to my other papers, the K&CK paper does not fit clearly in a previously established line of research; it does not set out to improve, extend, or refine pre-existing work. This is why I think that tracing its origins is of interest. The story of how it came to be touches on some of the issues and topics covered in the paper. Moreover, the published versions of the paper do not properly acknowledge some of the people who have influenced our work on the paper. Perhaps this note can compensate.

As a PhD student at Stanford, I attended a series of lectures on logics of programs given by Joe Halpern, who had just moved west to IBM Almaden Research Center, following his post-doc at MIT. I was looking for an advisor, and Joe was likewise seeking a first graduate student. We agreed to try working together toward my PhD, and decided to start by considering logics of knowledge and belief. We knew, based on the work of Kripke and Hintikka in the late Fifties and early Sixties, that these logics are technically similar to the Temporal and Dynamic logics that Joe had been involved with. One new element in our approach was going to be the study of logics with many “knowers,” whereas most of the classical treatment of knowledge and belief focused on the epistemics of a single agent. In a logic for many knowers one could make (and make sense of) statements about what one agent, say Alice, knows about the world and about what another agent (Bob) may know, or be ignorant of. Indeed, knowledge could be further nested, and statements could involve any number of agents. We started looking at the literature on modal logics of programs trying to see how they may apply to logics of knowledge. We also considered other logical issues such as characterizing ignorance by way of what it might mean for an agent to “know only” some set of facts and nothing more.

I spent the summer of 1983 doing a student internship at IBM Almaden, working with Joe. Danny Dolev was there, and heard that we were working on logics of knowledge. He told us about a talk he had attended by Robert Aumann at the Hebrew University, who described the “cheating wives” puzzle, and explained that *Common Knowledge* played an important role. (Roughly at the same time, Joe’s colleague at IBM Shel

Finkelstein had suggested the cheating wives and the “surprise exam” puzzles to Joe as being related to knowledge about knowledge.) Since “common knowledge” will re-appear in this note, let me digress briefly to describe it. The fact  $A$  is said to be *common knowledge* if everybody knows  $A$ , everybody knows that everybody knows  $A$ , and indeed we can iterate “everybody knows that everybody knows”  $A$  for any finite number of times. This notion appears to have been first studied by Morris Friedell in sociology in 1967 (under the title of “shared awareness”) and termed “common knowledge” by the philosopher David Lewis in 1969. Common knowledge was introduced into game theory by Aumann in a classical 3-page gem of a paper called “Agreeing to Disagree” in 1976. Returning to the cheating wives, Aumann had explained that a public announcement by the king in the town square, regarding the existence of unfaithful wives, makes this fact common knowledge.

Danny observed that when communication is performed via messages, as in a distributed computer network, information evolves differently than it would when announced in the town square. Danny suggested that we consider the cheating wives puzzle under different assumptions about properties of communication channels and about the arrangement of the nodes in the network. This provided a nice set of puzzles, none of which was very hard, that we solved in a few days. What to do with the solved puzzles was another matter: Presenting a variation on a puzzle and its solution can be cute, but presenting eight variants without being repetitive and boring seemed a tough task. The results were only published two years later when the right format was found. The puzzles we considered focused on the interaction between communication and action in a specific setting, but did not deal with knowledge in an explicit fashion. Nevertheless, this exercise had two consequences that seem useful in hindsight: First, it drew our attention to, or at least made us aware of, the notion of common knowledge. Second, and perhaps more importantly, it caused us to explicitly consider how knowledge evolves in a system, and how it depends on the properties of the communication medium. Our focus before that point was on investigating the logics of knowledge, and not on investigating distributed systems using knowledge terms. As a result, Joe and I were led to consider the following question: Suppose that a message  $\mu$  is sent from Alice to Bob over a channel for which there is a small uncertainty, say of  $\epsilon$  time units, regarding the transmission time (the time that the message takes to be delivered along the channel). Moreover, suppose that there is no further communication between the two for a while. When will the fact that Alice sent  $\mu$  to Bob become common knowledge? If  $\epsilon = 0$ , so that the transmission time is known precisely, then this fact would become common knowledge upon arrival. It took us a long while to convince ourselves that even if Alice and Bob can precisely measure time, the fact that Alice sent  $\mu$  will *never* be common knowledge(!) in case  $\epsilon > 0$ . (Indeed, this remains true if both agents share a global clock but  $\mu$  does not specify when it was sent.) In a precise sense, the reason for this is that common knowledge is *public*, in that when a fact  $A$  is common knowledge everyone will know that  $A$  is common knowledge. Moreover, knowledge satisfies the so-called *Knowledge Axiom*, which states that only true facts can be known. Thus, since nobody can know that  $A$  is common knowledge before this is indeed the case, the transition into common knowledge must involve an event that must occur simultaneously at each of the individuals in question. Consequently, if  $A$  is *not* common knowledge at time  $t$  and it *is* common knowledge at  $t+1$ , then there must be some instant  $\tau$  in the range  $t < \tau \leq t+1$  such that  $A$ 's being common knowledge not known to anyone before before  $\tau$ , and it is known to everyone after  $\tau$ . The transition into common knowledge must be simultaneous.

Joe had just completed some work with others on issues of clock synchronization in distributed systems, assuming that upper and lower bounds on the transmission times for each channel are given. In particular, they had shown that there is no way to perfectly synchronize clocks, in general, in such a setting. As a result, perfectly simultaneous events cannot be guaranteed in such a setting. Hence, regardless of the protocol used by Alice and Bob, no further communication could be used to ensure that common knowledge of the sending

of  $\mu$  would be attained. The fact that common knowledge is never attained by Alice and Bob—regardless of how small the bound of  $\epsilon$  may be—is startling, because we may think of  $\epsilon$  as being very small—perhaps a tiny fraction of a second. And if  $\epsilon$  is small enough, how can anyone tell the difference between the case in which a message may arrive within an interval of size  $\epsilon$  and the case in which there is no uncertainty regarding transmission times (and common knowledge is attained)? Moreover, in a setting involving people in the town square (or a very reliable channel), there is always some granularity of our modeling of time at which events can no longer be regarded as simultaneous. Loosely speaking, then, if we stare hard enough at common knowledge, and insist on increasing degrees of precision, it will cease to be! This is sometimes called the *common knowledge paradox*. This issue aside, our result showed that in cases in which the uncertainty is of a measurable magnitude, common knowledge may be impossible to attain. To capture the state of knowledge arising in such situations, we came up with weaker variants of common knowledge that would be attained, chief of which are  $\epsilon$ -common knowledge and *eventual* common knowledge.

The summer break ended, and I was back at Stanford. Interactions with two fellow students had an unexpected influence on our work. One day I met my classmate Tim Mann next to the CS department’s custom-made and state-of-the-art computerized “*Prancing Pony*” vending machine. (You could buy a soda or a sweet and have your computer account automatically charged!) Tim was doing his PhD in computer systems, and asked what I was working on. I tried my best to explain about common knowledge and the “paradox,” and Tim said that this reminded him of Jim Gray’s well-known *Coordinated Attack* problem. This was a problem stated in terms of two generals trying to coordinate a simultaneous attack using an unreliable (potentially lossy) channel. It illustrated the inability to ensure perfect transactions in a distributed database in the presence of links that may lose messages: Any protocol that ensures that either all sites “commit” to a transaction or neither site does, is one in which no transaction can *ever* be committed. I was not familiar with this problem. Significantly, Tim’s intuition was correct, and the proof of the impossibility of attaining common knowledge was generalized to apply to a broad condition on the uncertainty of communication, capturing at once temporal uncertainty (as in the case of the  $\epsilon$  above or asynchronous communication), and the possibility of message loss. In particular, this impossibility result generalized and formalized the coordinated attack problem and its impossibility.

Formalizing the connection to coordinated attack and the impossibility of attaining common knowledge under imperfect communication required the ability to prove statements of the form: “Under particular assumptions, no protocol can give rise to a specific state of knowledge.” To this end, we developed formal machinery by which knowledge can be ascribed and reasoned about in a modular way, for fairly arbitrary systems. Namely, a description of a “system” as the set of all runs of a given protocol of interest (in a given setting that you are interested in studying) directly induces a rigorous definition of what agents running the protocol will know. The paper explains how knowledge and states of knowledge can be readily ascribed to the agents and analyzed in the context of your system.

Another unexpected influence was made following a conversation with Carolyn Foss, who was then doing her PhD in the Psychology department. She told me that Herb Clark, one of the Psychology professors at Stanford, was working on common knowledge, and pointed me to a paper by Clark and Marshall on the subject. They discuss how among people, common knowledge typically arises via *copresence*. In other words, if Alice and Bob are sitting at a cafe for an afternoon tea, then they have common knowledge that they are at the cafe. The situation is inherently one in which both parties know of the situation, and as a result it is common knowledge. The same phenomenon happens when people shake hands to seal an agreement, or when a king announces something in the town square. This is related to what I call the *mafiosi* paradox: A local mafiosi may go to great lengths to make sure that everyone knows that he is a dangerous criminal. This can be good for business, and can provide him with a certain type of respect he may desire.

However, at the same time, he may object to his criminal nature being reported on in the local newspaper's headline: He is interested in ensuring that everyone knows he is a mafiosi, but objects to its being common knowledge! Publishing something in the paper can be viewed as causing it to be common knowledge, in the same manner as copresence does. The Clark and Marshall work taught us how prevalent common knowledge was in interpersonal interaction. Moreover, it pointed out the role of copresence in allowing facts to become common knowledge. Finally, it increased our awareness of the role of common knowledge in everyday agreements among people. The connection between common knowledge and agreement was of special interest to us, since agreement is a fundamental topic in distributed computing. Indeed, at that time the subject of *Byzantine* agreement was gaining quite a bit of attention. The connection between common knowledge and copresence, and its ubiquitous role in everyday interactions among people suggested that common knowledge was not an unreasonable state of knowledge, and understanding it in the context of distributed computing may prove to be of value.

At this stage Joe and I felt that we had enough of a story to write an abstract for a conference, which we did, and submitted it to STOC 1984 – the main CS theory conference at the time. To emphasize the role of common knowledge and the distinction between it and many levels of everybody knows, we used the *muddy children* puzzle, which is a politically correct version of the cheating wives puzzle, due to Jon Barwise. The paper also included the coordinated attack problem and common knowledge paradox, and the discussion on the connection between copresence, common knowledge and agreement. The paper was rejected from STOC. In retrospect, this turned out to be the best outcome (not that this was how we felt about the rejection at the time!). Had it been accepted to STOC, the paper would have had much more limited exposure, and probably a lesser impact. Its presentation at STOC would have been scheduled in the “logics” session, when most people went out to tour the town, and 30 die hards of the 200 participants stayed to discuss the topic the majority cares little for.

Luckily, rejection from STOC came in time for a submission to PODC in 1984, the conference on theoretical principles of distributed computing. For the PODC submission, we emphasized the importance of states of knowledge of groups of agents, and presented a hierarchy of states of knowledge with common knowledge at the top, levels of “everybody knows that everybody knows” in the middle, and at the bottom the notion we now call *distributed knowledge* (then going by *implicit knowledge*), in which the collective knowledge of the group put together implies the “known” something, although none of the agents necessarily knows it. For example, if you know the first four digits of a 7-digit winning number at the lottery, and I know the last three, then (jointly) we have distributed knowledge of the winning number, even though neither of us individually knows it. (Distributed knowledge was informally discussed by Hayek in 1945 and more formally, independently and earlier than us, by Hilpinen in 1977.) We added a discussion of the connection between knowledge and communication, and suggested that communication can often be viewed as the act of changing the system's state of knowledge. Indeed, in some cases it is natural to view communication tasks as moving the system's state of knowledge up the hierarchy. The PODC committee was much more receptive than the STOC committee had been, and the paper was accepted.

There are a couple of additional anecdotes regarding minor hurdles on the paper's way. First, the final proceedings version in those days needed to be mailed in, and to make the most of the time before the submission deadline, that meant sending by courier. The high-quality printer that we used was a room-sized “Xerox Dover laser printer” at the Stanford CS department. Just before sending the paper off, we discovered a nagging typo: A capital “C” (standing for common knowledge) was missing in one of the formulas, so we added it by hand. It then turned out that Danny Dolev was going from Almaden to Stanford to pick up a printout of his paper and mail it by courier. So we generated a version with the typo corrected, and sent it (from IBM) to the Dover printer. We handed Danny the version with the typo, and instructed him to replace

the fourth page (with the typo) by the one from the printer, and add it to the courier package. He duly inserted the newly corrected fourth page, but he removed the third from the stack. . . . And so the PODC proceedings has the fourth page appearing twice, and is missing the third page. Thanks to  $\text{T}_{\text{E}}\text{X}$ '82 typesetting, both the third and fourth pages started in a new paragraph, so the problem was hard to spot with a superficial glance. During the conference Mohamed Gouda, who read the paper, brought the problem to our attention, and we quickly photocopied a single-column version of the missing page and handed it out to the conference participants. I have no idea why, but we did not send an erratum page on this matter for the proceedings of PODC 1985. During the conference, the paper was invited to JACM and promised fast-track treatment. We submitted a journal version in late '84 or early '85. During the first round of reviews, one of the reviewers wrote "*the authors don't define the notions they use, they don't prove anything, and I [the reviewer] have a counterexample to their main theorem [impossibility of attaining CK under uncertainty].*" It turned out that his counterexample did not satisfy the definitions in the paper. . . . The paper was promptly accepted and published by JACM in 1990; so much for the fast track. Needless to say, the final journal version is a much more comprehensive and complete paper than the proceedings version, as our understanding of the subject matter, as well as of its applications and implications, improved over the period in question.

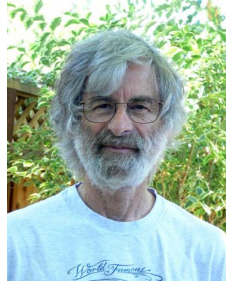
In summary, the roots of the K&CK paper are based on influences from philosophy, economics, psychology, distributed databases, and distributed systems. We set out to perform a study in modal logics of knowledge. We ended up with a framework for defining knowledge in concrete distributed systems and analyzing protocols and systems in terms of knowledge, with a variety of applications that this note has not touched upon. Along the way we discovered the role of common knowledge in agreements, and its interaction with simultaneity and communication.

# Reconfiguring a State Machine

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## Abstract

Reconfiguration means changing the set of processes executing a distributed system. We explain several methods for reconfiguring a system implemented using the state-machine approach, including some new ones. We discuss the relation between these methods and earlier reconfiguration algorithms—especially view changing in group communication.

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# 1 Introduction

A fault-tolerant system may need to change the set of processes that is executing it—a procedure called *reconfiguration*. Reconfiguration can be performed to reduce the vulnerability to further failures after a process has failed [25] or to replace hardware without shutting down the system. Distinguishing process failure from transient communication problems is often a challenging engineering problem. Waiting too long to reconfigure can reduce fault tolerance, but not waiting long enough can lead to thrashing. A reconfigurable system provides an interface for reconfiguring it, separating reconfiguration from the decision of when to reconfigure and what new configuration to use. That decision can be made by any desired algorithm or by a human operator.

A state machine (also called an *object* [12]) accepts commands and produces outputs. The functional behavior of any system can be specified by a state machine [16]. The state-machine approach consists of implementing a fault-tolerant distributed system (or subsystem) by describing it as a state machine and using a general fault-tolerant algorithm to implement that state machine [20, 24]. A state machine’s definition and the correctness of its implementation directly imply the correctness properties of the system.

An important property provided by the state-machine approach is *irrevocability* of output. For a stock exchange, irrevocability means that if a broker receives output from the system confirming that a customer has bought 100 shares of Nocturnal Aviation, then she really owns them. Even if the broker’s computer is destroyed by an asteroid, the customer can go to any other broker and sell those shares. Without irrevocability, it is hard to explain what fault tolerance means for a system.

With the state-machine approach, it is easy to make a system reconfigurable by letting the state machine itself specify the configuration [18, 24]. Although this basic idea was described more than twenty years ago [17] and is implicit in earlier process-control applications [25], it still appears not to be well understood. We therefore review it along with the state-machine approach in Section 2.

Another approach that may seem more intuitive is to reconfigure the system by terminating the state machine, letting it choose the new configuration, then resuming execution with a new state machine that uses the new configuration. This is the way reconfiguration is viewed in group communication [6, 8]. We show in Section 3 that, while it is not as simple as the classic one described in Section 2, this approach does lead to new reconfigurable state-machine implementations. These two approaches to reconfiguration are different paths to obtaining reconfiguration algorithms, but they do not imply any fundamental difference in the resulting algorithms.

The relation between reconfiguration in the state-machine approach, in group communication, and in some particular systems is discussed in Section 4. Although most of our methods for state-machine reconfiguration work in the presence of malicious processes, the body of this paper considers only crash faults. Malicious faults are discussed in the conclusion.

## 2 State Machines

### 2.1 Preliminaries

A state machine is described by a set of states, an initial state, and a function that maps command-state pairs to output-state pairs. If the pair  $\langle c, s \rangle$  is mapped to the pair  $\langle o, s' \rangle$ , then we say that executing command  $c$  in state  $s$  produces output  $o$  and changes the state to  $s'$ . Execution of a state machine consists of executing a sequence of commands in the obvious way, starting with the initial state, to produce a sequence of outputs and new states. A *noop* is a special command that produces a null output and leaves the state unchanged.



A state-machine implementation provides an interface by which clients propose commands to the system and receive outputs from it. Outputs may be sent to clients other than the proposer, but the proposer usually receives output at least informing it that the command was executed. A command can include the identity of the client that proposes it, and the state machine can be specified to treat the command as a *noop* if the client is not authorized to propose it.

The safety requirement for a state-machine implementation is that the outputs received by all clients are generated by a single sequence of *chosen* commands, each of which has been proposed by a client. (A more complete specification also requires the state machine/object to be linearizable [12].) The safety requirement implies irrevocability.

As with any fault-tolerant distributed algorithm, the precise liveness property satisfied by a state-machine implementation depends on the details of how it is implemented. It states approximately that, if enough servers are nonfaulty, and eventually partial synchrony is satisfied, then proposed commands are added to the sequence of chosen commands and their outputs delivered to nonfaulty clients.

There are two (usually non-disjoint) sets of servers called *acceptors* and *learners*. Acceptors choose the commands to be executed; learners maintain the state, learn what commands are chosen, and execute them, generating the outputs. Acceptors essentially provide stable storage and do not care what commands are chosen. Making progress despite the failure of any  $f$  servers requires at least  $2f + 1$  acceptors and  $f + 1$  learners [19].

The classic way of implementing a state machine is with a consensus algorithm for choosing a single command. The implementation runs a sequence of logically separate instances of the consensus algorithm, using instance  $i$  to choose the  $i^{\text{th}}$  command. Most state-machine implementations use a special subset of learners called *leaders*. A client proposes a command  $c$  by sending it to a leader, which assigns it a number  $i$  and proposes  $c$  as the command to be chosen by consensus instance  $i$ . The number  $i$  assigned to command  $c$  can be included in the output generated by executing  $c$ . How client commands are delivered to a leader is an instance of the general problem of how clients of a distributed system locate the servers executing the system. We do not discuss this problem.

A leader does not have to wait until command  $i$  is chosen before proposing another command as number  $i + 1$ . Different commands can be chosen concurrently. In general, command  $i$  cannot be executed and its output determined until all commands numbered less than  $i$  have been chosen. However, there are important cases in which the output of a command can be generated as soon as the command is chosen—for example, if the output reveals only the command’s number and the fact that it has been executed. Because the choice is irrevocable, choosing a command is tantamount to executing it. (Linearizability is satisfied if command numbers are consistent with the order in which the commands are proposed.)

## 2.2 Garbage Collection

In a non-reconfigurable system, we could allow the acceptors to maintain forever their information for each instance of the consensus algorithm. At any time, a learner could then learn the entire sequence of chosen commands by communicating with enough acceptors, where enough generally means a majority. In practice, consensus instances must usually be garbage collected. Periodically, learners checkpoint the state at some point in the execution sequence and instruct the acceptors to forget about consensus instances for earlier commands. Although command indices continue growing monotonically, the memory taken by lower numbered commands may be reclaimed. Exactly how this is done is an engineering detail that does not concern us.

## 2.3 Reconfiguration Made Easy

A configuration is the set of processes (clients, acceptors, etc.) that are executing the system. The consensus algorithm used to implement a state machine assumes a fixed configuration, so we must ensure that each instance of that algorithm is executed by a single configuration. (It is the set of acceptors that is important; adding or removing clients or learners while executing a consensus algorithm is not a problem.) In the *easy* approach, reconfiguration is achieved by using different configurations for different instances. We define the configuration at command number  $i$  to be the one used to execute consensus instance  $i$ .

To prevent chaos, processes must agree on the configuration at command  $i$ . The easy way to obtain a reconfigurable state-machine implementation is to introduce a component  $cfg$  of the state-machine state that specifies the current configuration. In other words, the configuration at command  $i$  is determined by the value of  $cfg$  in the state immediately following execution of command  $i - 1$ , or by its initial value if  $i = 1$ . (We use ordinal numbers for commands, so the first command is number 1.) We add reconfiguration commands of the form  $rcfg(\mathcal{C})$ , which specifies a new configuration  $\mathcal{C}$ .

The obvious way to define the state machine is to let executing  $rcfg(\mathcal{C})$  set  $cfg$  to  $\mathcal{C}$ , so reconfiguration occurs immediately. We call this method  $\mathcal{R}_1$ . The problem with  $\mathcal{R}_1$  is that it prevents concurrent processing of different proposed commands. Since the configuration used to execute instance  $i + 1$  of the consensus algorithm can be changed by executing command  $i$ , the state-machine implementation cannot begin choosing command  $i + 1$  until command  $i$  has been chosen.

To allow concurrent processing of commands, we define the state machine (by introducing additional state) so that executing  $rcfg(\mathcal{C})$  as command number  $i$  causes  $cfg$  to change after executing command  $i + \alpha - 1$ , for some positive integer  $\alpha$ . A reconfiguration command thus takes effect  $\alpha$  commands later, allowing the concurrent processing of up to  $\alpha$  commands. (This was originally described for  $\alpha = 3$  in the mistaken belief that the generalization would be obvious [18]. A practical implementation was later presented in [21]). We call this method  $\mathcal{R}_\alpha$ . As the notation implies,  $\mathcal{R}_1$  is the  $\alpha = 1$  case of  $\mathcal{R}_\alpha$ .

To make the reconfiguration happen right away, a client that proposes a reconfiguration command can propose *noops* as the next  $\alpha - 1$  commands. A sequence of successive commands, with successive command numbers, can be batched so that they are proposed, chosen, and executed as efficiently as (using no more messages than) a single command. Thus,  $\alpha$  can be made arbitrarily large, permitting the concurrent processing of any desired number of commands. To maintain correctness, the implementation must produce the same result as if each instance of the consensus algorithm were executed separately.

With method  $\mathcal{R}_\alpha$ , reconfiguration is performed using the ordinary state-machine interface for proposing commands. Only a subset of the clients might be authorized to propose reconfiguration commands, effectively separating the reconfiguration interface from that used to propose other commands.

## 2.4 Correctness

A simple induction argument shows that  $\mathcal{R}_\alpha$  maintains the safety property of a state-machine implementation. In particular, it satisfies irrevocability. Liveness for a state-machine implementation states that it makes progress if enough servers are nonfaulty. Reconfiguration raises the question, enough of which servers?

The purpose of reconfiguration is to eliminate the dependence on servers that have been reconfigured out of the system. Information maintained by those servers must be transferred to servers in the new configuration. This is done by garbage collecting consensus instances executed under the old configuration and transferring the state to the new configuration's learners. During the transition to the new configuration, progress is guaranteed only if there are enough nonfaulty servers from both the old configuration and the new one. The liveness property satisfied by the resulting algorithm is not easy to state precisely, but its

general nature should be intuitively clear.

### 3 Reconfiguration Made Harder

Algorithm  $\mathcal{R}_\alpha$  of Section 2.3 has the practical drawback that a reconfiguration introduces a sequence of *noop* commands of length about  $\alpha$ . This can be inconvenient for large values of  $\alpha$ —for example,  $\alpha = 2^{64}$ . The bound on concurrency implied by  $\alpha$ , even if not a practical concern, may also be considered inelegant. We now present reconfiguration algorithms that permit any number of commands to be chosen concurrently during normal execution, when no reconfiguration is in progress.

We obtain a reconfigurable state-machine implementation by combining the executions of a sequence of separate non-reconfigurable state-machine implementations. To reconfigure when executing state-machine implementation number  $v$ , we stop that execution, choose the new configuration, and start the execution of state-machine implementation number  $v + 1$  by that configuration. The starting state of state machine  $v + 1$  is the final state of state machine  $v$ .

This approach requires solving three largely orthogonal problems: (i) stopping the current state machine, (ii) choosing the configuration to implement the next state machine, and (iii) combining the sequence of commands chosen for each separate state machine into a single sequence. We consider them separately, starting with three basic methods for implementing a stoppable state machine.

#### 3.1 Stopping a State Machine

##### 3.1.1 The Stop Sign

The Stop-Sign method adds to the state machine a *stop* command that turns every subsequent state-machine command into a *noop*. Unlike algorithm  $\mathcal{R}_1$  of Section 2.3, this method allows multiple commands to be chosen concurrently. Any command chosen after the *stop* command simply has no effect. However, the Stop-Sign method has the following problem. As explained in Section 2.1, an ordinary state-machine implementation allows an output like “this command was executed” to be generated as soon as the command has been chosen. The Stop-Sign method does not allow any command’s output to be generated before every lower-numbered command has been chosen, since one of those commands could turn out to be *stop*.

The *delayed* Stop-Sign method allows command number  $i$  to be executed, if possible, as soon as it and all commands numbered at most  $i - \alpha$  have been chosen. Just as  $\mathcal{R}_\alpha$  generalizes  $\mathcal{R}_1$  to make the reconfiguration command take effect  $\alpha$  commands later, the delayed Stop-Sign method generalizes the *stop* command to take effect  $\alpha$  commands later. In other words, if command number  $i$  is a *stop*, then all commands starting with number  $i + \alpha$  are treated as *noops*. As with  $\mathcal{R}_\alpha$ , the client proposing the *stop* command can at the same time propose  $\alpha - 1$  consecutive *noop* commands.

##### 3.1.2 Padding

We have already observed that we can batch a sequence of commands and process them essentially as easily as a single command. This applies equally well to an infinite sequence of consecutive commands, if all but a finite number of them are *noop* commands. (It is easy to devise a finite encoding of the information needed to execute the infinite number of consensus instances.) In the Padding method, a client stops the state machine by proposing an infinite number of *noop* commands. More precisely, it proposes *noops* for all commands numbered greater than some  $i$ . When command number  $j$  has been chosen for all positive integers  $j$ , the state machine has stopped.

### 3.1.3 The Brick Wall

The Stop-Sign and Padding methods use the underlying state-machine implementation, so their correctness follows immediately from the correctness of that implementation. However, they may seem unintuitive—the Stop-Sign method because it can choose commands that are never executed by the state machine and the Padding method because it fills the sequence with infinitely many *noop* commands. We now describe the *Brick-Wall* method that is conceptually simpler but requires a new state-machine implementation called *Stoppable Paxos*.

Like the Stop-Sign method, Stoppable Paxos assumes a special *stop* command. However, it guarantees that if a *stop* command is chosen as command number  $i$ , then no command can ever be chosen for any command number greater than  $i$ . A precise description and rigorous correctness proof of Stoppable Paxos appears in [14]. Here, we briefly sketch the algorithm, starting with a description of classic Paxos [18].

As described in Section 2.1, Paxos implements a state machine by using logically separate instances of a consensus algorithm. It assumes a method for selecting a leader, guaranteeing progress only when there is a unique leader. (Safety is preserved despite multiple leaders.) The Paxos consensus algorithm uses numbered ballots, each initiated by at most one leader. (Do not confuse ballot numbers with consensus-instance/command numbers.) A newly selected leader chooses a ballot number  $b$  it believes to be greater than any already used, and it begins ballot  $b$  by sending a *phase 1a* message to the acceptors, who respond with *phase 1b* messages. If the leader has chosen  $b$  large enough and it receives phase 1b messages from a majority of the acceptors, then it will learn that either:

- P1. Command  $c$  (and no other command) might have been chosen by a lower-numbered ballot, or
- P2. No command has been chosen by a lower-numbered ballot.

The leader then proposes a command in ballot  $b$  by sending a *phase 2a* message to the acceptors, proposing  $c$  in case P1 and any command in case P2. If no higher-numbered ballot is begun, the proposed command will be chosen when a majority of acceptors receive the phase 2a message. The Paxos state-machine algorithm is efficient because the phase 1a and 1b messages sent by any one process for all consensus instances are bundled into a single physical message.

Stopping Paxos prevents a leader from proposing a command if *stop* could be chosen as a lower-numbered command. To describe how this is done, we first define  $\xi(b, i)$ , for ballot  $b$  of consensus instance  $i$ , to equal the command  $c$  of case P1 above, and to equal  $\perp$  in case P2. Stopping Paxos places the following two additional constraints on what command a leader can propose in ballot  $b$  of instance  $i$ :

- S1. It cannot propose any command if, for some  $j < i$ , it proposed a reconfiguration command in ballot  $b$  of instance  $j$  or  $\xi(b, j)$  is a reconfiguration command.
- S2. It cannot propose a reconfiguration command if, for some  $k > i$ , it has proposed a command in ballot  $b$  of instance  $k$  or  $\xi(b, k) \neq \perp$ .

It is not hard to show that S1 and S2 prevent instance  $i$  from choosing any command if a lower-numbered instance chooses *stop*. However, S1 can prevent progress if a reconfiguration command is proposed but not chosen by a ballot numbered less than  $b$  in instance  $j < i$ . To eliminate this problem, we modify the definition of  $\xi$  approximately as follows: in case P1,  $\xi(b, i)$  equals  $\perp$  if  $c$  is a reconfiguration command proposed in ballot  $b' < b$  and some command was proposed in ballot  $b''$  of instance  $k$  with  $b'' > b'$  and  $k > i$ . The precise definition and the proof that this works are not trivial.

### 3.2 Choosing a New Configuration

When changing from state-machine implementation  $v$  to state-machine implementation  $v + 1$ , there are two basic ways to choose the new configuration that executes implementation  $v + 1$ :

- R1. Let it be chosen by a reconfiguration command executed by state machine  $v$ .
- R2. Use a special instance of the consensus algorithm, executed by the same configuration that executes state machine  $v$ .

Option R2 has the potential advantage of allowing the processes in configuration  $v + 1$  to be determined and commands to be chosen for state machine  $v + 1$  before state machine  $v$  has been stopped. It is not clear if this is ever a good idea in practice. For option R1 to work, we must:

- (a) Make sure that a reconfiguration command is passed before configuration  $v$  is terminated, and
- (b) Decide what to do if multiple reconfiguration commands are passed.

The solution to (a) is obvious for the Stop-Sign and Brick-Wall methods—namely, let the *stop* command specify the new configuration. For the Padding method, a client should make sure that a reconfiguration command has been chosen before it proposes an infinite sequence of *noops*. (If no reconfiguration is chosen, then configuration  $v + 1$  is the same as configuration  $v$ .)

Problem (b) does not exist for the Brick-Wall method. For the other methods, there are two obvious choices: use either the first reconfiguration command that was passed or the last one. If we use the first one, then the system can start choosing commands for configuration  $v + 1$  as soon as all chosen commands through the first reconfiguration command are known.

### 3.3 Combining the Command Sequences

We have shown how to initiate and execute a sequence of state-machine implementations, all but possibly the last one being stopped. State machine number  $v$  is executed with configuration  $v$ .

If a state machine has been stopped, it has a unique “last interesting” command. For the Stop-Sign or Brick-Wall method, that command is the (first) *stop* command. For the delayed Stop-Sign method, it is  $\alpha - 1$  commands after the *stop* command. For the Padding method, it is the last non-*noop* command. Let  $\eta(v)$  be the number of the last interesting command of state machine number  $v$ , and define  $\eta(0)$  to equal 0.

Let us assign the “number”  $\langle v, i \rangle$  to the  $i^{\text{th}}$  command chosen for state machine number  $v$ . With the usual lexicographical ordering of pairs, this provides a linearly ordered numbering of the sequence of all chosen commands. However, it is not a useful numbering scheme because it does not tell a client whether there are any commands between the ones numbered  $\langle 4, 417 \rangle$  and  $\langle 5, 1 \rangle$ . It is better to number commands with consecutive positive integers.

To give commands integer numbers, we observe that there is no reason why a state machine’s command numbers should start with 1. Let us make the starting number a parameter of the state machine, and let the first command of state machine number  $v + 1$  be one greater than that of the last interesting command of state machine  $v$ . In other words the first command of state machine  $v + 1$  has number  $\eta(v) + 1$ . Thus, command  $i$  is the one chosen as command number  $i$  of state machine number  $v$ , where  $\eta(v - 1) < i \leq \eta(v)$ . To implement this, the choosing of commands in state machine  $v + 1$  cannot begin until the value of  $\eta(v)$  is known.

In a naive implementation, each process would maintain a two-dimensional array of data, where element  $\langle v, i \rangle$  contains the data maintained for consensus instance  $i$  of state machine number  $v$ . However, for state

machine  $v$ , there is no consensus instance  $i$  if  $i \leq \eta(v - 1)$ , and it does not matter what value is chosen by instance  $i$  if  $i > \eta(v)$ . In any state-machine implementation that uses a sequence of completely separate consensus instances, a process need maintain information for instance  $i$  only for the largest state machine number  $v$  for which processing of command  $i$  has been initiated. To make this true using a state-machine implementation like Stoppable Paxos, in which the execution of other consensus instances can depend on the state of consensus instance  $i$ , the implementation must be modified so it can forget the state of instance  $i$  when  $i > \eta(v)$ . For Stoppable Paxos, the modification is simple.

### 3.4 The Interface

In the reconfiguration methods based on a sequence of state machines, a state machine is stopped by proposing either a *stop* command or an infinite sequence of *noop* commands. If the new configuration is specified by a state-machine command (option R1), then the entire reconfiguration is performed using the ordinary state-machine interface. If the new configuration is determined by a special consensus instance that is not part of a state-machine execution (option R2), then a separate interface is required to specify the configuration.

In practice, any of these methods would use a separate interface for issuing reconfiguration requests. The state-machine commands needed to perform the reconfiguration would be proposed by a leader, which would also initiate the special consensus instance in option R2.

## 4 Reconfiguration in Group Communication and Other Works

Group communication (GC) is an alternate method for implementing a fault-tolerant distributed system, in which a group of processes called a *view* execute a broadcast mechanism so that all nonfaulty processes in the view receive the same set of messages [6, 5]. Reconfiguration is an integral part of GC, being required to achieve fault tolerance. There are a number of different versions of GC that provide different guarantees. Here, we consider only a few of the versions that produce a consistent total ordering of delivered messages, referring the reader to the survey by Chockler et al. [8] for a more detailed account of previous work.

A GC service provides an interface in which the processes in a view send messages. An ordered sequence of sent messages is *delivered* to the processes. Each delivered message is usually chosen by the view's processes with an unreliable consensus algorithm—an algorithm that guarantees at most one message is chosen, but may be prevented from making progress by the failure of even one process. (Unreliable consensus is usually implemented with a leader whose failure prevents progress.) In the event of failure, the current view is ended and a new one is begun. The view change is performed by using a fault-tolerant consensus algorithm to determine (i) the sequence of messages that were delivered in the old view and (ii) the members of the new view. The interface provided by a GC service typically allows processes to enter and leave a view and to learn what processes belong to the view, but the implementation may spontaneously change the view in response to a failure.

There is an obvious correspondence between GC and the state-machine approach in which a view corresponds to a configuration. We can consider a state-machine implementation to be based on a GC service whose messages are commands, where the delivered messages are the chosen commands. This correspondence is most obvious for the implementations of Section 3 based on a sequence of state machines. Conversely, we can consider a GC service to implement something like a state machine whose commands are of the form *send message m* and whose state is the sequence of delivered messages.



If the GC service does implement a real message-sending state machine with irrevocable message delivery, then using it to implement an arbitrary state machine is straightforward. Whether or not it does depends on exactly how it performs a view change. The property required of the GC view-changing operation was called Virtual Synchrony by Birman and Joseph [6]. As originally defined, Virtual Synchrony requires that the new view contain a majority of the old view’s processes, and that the messages delivered to any process in that majority be declared to have been delivered in the old view. This requirement is not strong enough to ensure irrevocability, since a broker’s computer will not be in the new view if it has been hit by an asteroid, so a message committing a stock sale need not be declared by the view change to have been delivered. (This has been observed before [23, 13].)

It is not hard to implement the stronger requirement that messages delivered by *any* process in the old view are declared to have been delivered. We need only require that before a process delivers a message, it learns that a majority of processes in the view know that the message was sent. A value proposed to the view-changing consensus algorithm as the old view’s sequence of delivered messages must include all messages that some majority of processes knows to have been sent. With the stronger requirement, it is easy to make the GC service implement a state machine. Such stronger services do exist, including Isis’s GBCAST protocol [6] and the “Safe” messages of Transis [9] and Totem [2]. A GBCAST/Safe message is delivered after all members of the view received it. These services have been used to provide consistent totally ordered broadcast by implementing a view-changing decision that forces delivery of any received GBCAST/Safe message [10, 13, 23]. Babaoglu et al. [4] take the direct approach of delaying delivery of a message until a majority of processes acknowledge having received it.

State-machine implementations seem quite different from GC. They use a reliable consensus algorithm to choose each command, while GC implementations use an unreliable consensus algorithm to broadcast a message and invoke a reliable consensus algorithm only on a view change. However, an “asynchronous” consensus algorithm executes a sequence of unreliable consensus algorithms—for example, the individual ballots of Paxos—stopping when a command is chosen. The unreliable consensus algorithm used by GC to broadcast a message can be viewed as just the first one of a reliable consensus algorithm, the rest of the consensus algorithm being performed simultaneously for all messages at the view change. This is effectively what happens in Paxos when a leader fails and a new ballot is begun simultaneously for all instances of the consensus algorithm. A state-machine implementation and a strong GC service perform essentially the same actions; those actions are just viewed differently. The state-machine view makes correctness obvious; the GC view makes the implementation more obvious.

All protocols that provide a strong GC service prevent any messages from being delivered after a view change has begun. They therefore essentially use the Brick-Wall method for stopping a state machine. Totem [2] chooses the number of the last message delivered in the old configuration and the new reconfiguration at the same time, a procedure analogous to option R1 of Section 3.2. CoRel [13] first selects a new configuration and only later determines which messages are delivered in the old configuration, which is analogous to option R2.

In traditional GC, the view-changing consensus algorithm is executed by the processes in the new view, while in  $\mathcal{R}_\alpha$  and in the methods of Section 3, the new configuration is chosen with a consensus algorithm executed by the old configuration. (In  $\mathcal{R}_\alpha$  and in option R1 of Section 3.2, that consensus algorithm is the one that chooses an ordinary state-machine command.)

Maintaining a unique sequence of configurations requires the new configuration to be chosen by the acceptors of the old configuration, but making progress requires it to be learned by the learners of the new configuration. Because the original work on GC did not distinguish between acceptors and learners, it ensured progress by having the processes in the new view execute the consensus algorithm and ensured

uniqueness of views by requiring that the new view contain a majority of the processes from the old view. As our algorithms show, there is no need for any overlap between the old and new views. The processes in the new view just have to learn the new view chosen by the old one.

There are also reconfigurable systems that do not provide the power of a state-machine implementation. RAMBO [7, 11, 22] is a fault-tolerant system that provides simple read and write operations. It is based on the ABD algorithm [3], which plays the role that a consensus algorithm does in a state-machine implementation. In Rambo, reconfiguration is done using a separate state machine to choose a new configuration that can take effect in the middle of executing an individual read or write. This is similar to an alternative approach to reconfiguration in Paxos that we have not discussed [15]. Interestingly, it was shown recently that it is possible to construct reconfigurable atomic read/write objects without relying on a consensus for reconfiguration decisions [1].

## 5 Conclusions

In the state-machine approach, a fault-tolerant distributed system is built on top of a fault-tolerant state-machine implementation. The correctness properties of the system, including irrevocability, follow from the definition of the state machine and correctness of the state-machine implementation. A reconfigurable state-machine implementation provides an interface for choosing the configuration that executes the system. Section 2 reviewed the obviously correct method  $\mathcal{R}_\alpha$  for making a state-machine implementation reconfigurable. Section 3 introduced other methods for doing this. Except for the Brick-Wall method, their correctness follows directly from the correctness of the underlying consensus algorithm.

We have considered only non-malicious faults. The state-machine implementation obtained with any of our algorithms except the Brick-Wall method (Section 3.1.3) tolerates malicious failures if the underlying consensus algorithms does. We must also design the state machine to tolerate malicious clients—including ones that propose reconfigurations. This is done by requiring that a critical operation such as stopping the current configuration or choosing a new one be performed only when requested by enough different clients. In the Stop-Sign method (Section 3.1.1), the *stop* command is the request that triggers stopping. For option R2 in which the new view is chosen by a separate consensus algorithm (Section 3.2), that algorithm is replaced by a fault-tolerant state machine.

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