Convection Measurement Apparatus and Methodology

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Abstract

Presented are the design and operating methodology of an apparatus constructed to make accurate measurements of mixed convection at all horizontal and vertical orientations of an isothermal plate with forced airflow in the plane of the plate.

The measurements from this Convection Machine drove the development and validation of a comprehensive theory of turbulent mixed convection from a rectangular plate having at least one horizontal edge.

Keywords: mixed convection; natural convection; forced convection; wind-tunnel; admissible-roughness

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1. Introduction

Starting in 2015 an apparatus was constructed to measure forced convection in air from heated 305 mm square plates with precisely 3 mm and 1 mm of roughness. Although optimized for downward natural convection mixed with horizontal forced flow, because of the small size of the wind-tunnel chassis (1.3 m × .65 m × .65 m), it also afforded an opportunity to characterize mixed convection at various orientations of plate and flow.

While the total convection could be derived from local convection, measurements of local convection necessarily can’t provide complete coverage of a test surface (particularly the leading edge). Instead, the apparatus was designed to measure total heat flow (and total convection). This decision was fortuitous in that forced convection measurements within a ±2% expected uncertainty were achieved.

The operating methodology was unusual in that it did not wait for temperatures to stabilize before taking measurements. Instead it captured a time sequence of all measured quantities and inferred the convection from that dynamic sequence.

The measurements obtained were the basis for two papers: Skin-Friction and Forced Convection from an Isothermal Rough Plate[1] and Turbulent Mixed Convection from an Isothermal Plate[2].

Section 9 derives a threefold tighter upper bound for the height of admissible-roughness than is given in Schlichting’s book Boundary-layer theory[3].

The Section 12 appendix builds an intricate convection formula for the bi-level plate which models the transition from rough turbulent convection to smooth turbulent convection at increasing Reynolds numbers.
2. **Apparatus**

![Figure 1 Rough surface of plate](image1.png)  
**Figure 1 Rough surface of plate**  
Figure 1 shows the rough surface of the 30.5 cm × 30.5 cm test plate. Machined from MIC-6 aluminum to keep its emissivity low, the reflective surface is composed of (676) 8.28 mm × 8.28 mm × 6 mm posts spaced on 11.7 mm centers. The area of the top of each post is 0.686 cm², half of the 1.37 cm² cell. The arithmetic-mean (also the root-mean-squared) height-of-roughness is thus 3 mm.

The back surface of the plate has 9 precision electronic resistors embedded as heating elements and a temperature sensor. There is 2.54 cm thick thermal foam insulation between the back of the plate and a 0.32 mm thick sheet of aluminum. The aluminum sheet also has a temperature sensor at its center.

Figure 2 is a cross-section drawing of the plate assembly.

![Figure 2 Cross-section of plate](image2.png)

![Figure 3 3 mm roughness plate in wind-tunnel](image3.png)  
**Figure 3 3 mm roughness plate in wind-tunnel**  
The Convection Machine wind-tunnel (Figure 3) has a 61 cm × 35.6 cm cross-section and a 61 cm depth. It allows the plate assembly to be centered with 15 cm of space on all sides. The fan pulling air through the test chamber produces a maximum wind-speed of 4.5 m/s (Re = 9 × 10⁴ for the 0.305 m square plate). Its minimum nonzero wind-speed is 0.12 m/s (Re = 2300).

The 99% boundary-layer depths [4] for the wind-tunnel versus the distance x from its front edge are (laminar and turbulent):
The boundary layer depths from the plate are similar, although the Reynolds numbers are lower with its shorter characteristic-length. Figure 4 shows that 15 cm clearance between the plate and wind-tunnel walls keeps the boundary layers from interacting at airspeeds within its range.

\[
\delta_{\text{lam}} = 4.92 \sqrt{\frac{xV}{V}} \quad \delta_{\text{tur}} = 0.37 x^{4/5} \left( \frac{\mu}{V} \right)^{1/5}
\]

Figure 4  Wind-tunnel boundary layer thickness; \(D_H = 0.449 \text{ m}\)

The rotation rate of the fan is sensed by a photo-transistor receiving the light from a LED being interrupted by the passing fan blades. Software controls a solid-state relay (supplying power to the fan) to maintain the fan rotation rate dialed into switches. At rotation rates below 400 r/min, the software pulses power to the fan according to a phase comparator. Above 400 r/min it servos the duty cycle of a 10 Hz square-wave supplying power. It operates over a range of 32 to 1500 r/min.

The correspondence between fan rotations per second and wind-speed is made using an anemometer. The ABM-200 Airflow & Environmental Meter specifies an accuracy of ±0.5% of reading from 0.22 m/s to 62.5 m/s. At speeds below 5 m/s, 0.5% is much better than can be achieved with hot-wire anemometers, which specify accuracy as a percentage of 10 m/s full-scale.

Figure 5 shows the calibration curve for the empty wind-tunnel. These measurements make clear that the 0.22 m/s lower limit must be a misprint; the smallest nonzero reading my unit was able to report was 0.66 m/s. Even at 1.25 m/s, the slope is on a trajectory to reach 0 when the fan is still moving air.

For any fan, there must be some rotation rate and wind-speed \(V_s\) above which airflow does not increase. That flattening will appear gradually from slower rotation rates. Modeling the stall speed as \(V_s = 7.5 \text{ m/s}\), equation (1) is the solid line in Figure 5. From 500 r/min to 1500 r/min, it matches measurements with a standard deviation of 0.0075 m/s.

\[
V = \left[ (rS)^{-2} + V_s^{-2} \right]^{-1/2} \hfill \\
S = \frac{V_{1500}}{1500 \text{ r/min} \sqrt{1 - (V_{1500}/V_s)^2}} \quad V_{1500} = 4.17195 \text{ m/s}
\]
The plate assembly is suspended from six lengths of 0.38 mm-diameter steel piano wire terminated at twelve zither tuning pins in wooden blocks above and around the test chamber. The wire is sheathed by 0.95 mm Teflon tubing where it would otherwise contact the plate.

With the plate assembly in the wind-tunnel, the airspeed is increased in proportion to the reduction of wind-tunnel aperture by the plate’s cross-sectional area:

\[ V_p = V \frac{A_{wt}}{A_{wt} - C_p} \]  

The increase is 7.7\% for the 3 mm roughness plate and 7.3\% for the 1 mm roughness plate.

The ambient sensor board can be seen at the lower edge of the tunnel in Figure 3. This small board measures the pressure, humidity, and temperature of the air at the wind-tunnel intake. The LM35 temperature sensor projects into the wind-tunnel test chamber; it is wrapped in aluminum tape so that radiative heat transfer is minimized. The temperature sensor is powered only while it is being read so that self-heating doesn’t affect it; self-heating was a problem before it was switched.

Logging of the measurements is performed by a STMicroelectronics STM32F3 Discovery 32-Bit ARM M4 72MHz development board. The custom electronics board which the STM32F3 plugs into, contains power supplies, heater control and drive, and signal conditioning.

A program was written for the microprocessor which takes measurements and writes them to the microprocessor’s non-volatile RAM, controls the heater, servos the fan speed, and (later) uploads captured data to a computer over a USB serial interface. Every second for 108 minutes the program calibrates and reads each on-chip 12 bit analog-to-digital converter 16 times and sums its readings to create a 16 bit value.

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1 It has since been moved to the chamber beneath the wind-tunnel where, although slower to respond to ambient changes, it is less affected by convection from the plate.
3. Measurement Methodology

Previous laboratory measurements (for example [5]) of forced convection have been performed by starting the fluid flow and plate heater, waiting for the system to reach equilibrium (as indicated by a stable plate temperature), then recording the measurements of the physical quantities.

The expectation for this wind-tunnel was that it would support wind-speeds up to 10 m/s. The test plate was made massive in order to maintain uniform temperature at high rates of convection. As a consequence the plate’s temperature settling times range from minutes in the best case to hours at low airflow rates.

Ubiquitous electronic sensors and digital processing make possible an entirely different measurement methodology. Rather than waiting for the plate to reach equilibrium before each measurement, the state of all sensors is captured every second for 108 minutes and compared with simulations of the system, which is rarely in equilibrium.

The physical parameters derived from measurements and specifications of both plate versions are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varepsilon_q)</td>
<td>3 mm</td>
<td>RMS height-of-roughness</td>
</tr>
<tr>
<td>(\varepsilon_a)</td>
<td>3 mm</td>
<td>Arithmetic-mean height-of-roughness</td>
</tr>
<tr>
<td>(A_p)</td>
<td>0.093 m²</td>
<td>Plate Test Area</td>
</tr>
<tr>
<td>(C)</td>
<td>4690 J/K</td>
<td>Plate Thermal Capacity</td>
</tr>
<tr>
<td>(C_b)</td>
<td>146 J/K</td>
<td>Back Thermal Capacity</td>
</tr>
<tr>
<td>(U_I)</td>
<td>0.075 W/K</td>
<td>Insulation Thermal Conductance</td>
</tr>
<tr>
<td>(\varepsilon_p)</td>
<td>0.04</td>
<td>Plate Surface Emissivity</td>
</tr>
<tr>
<td>(\varepsilon_{wt})</td>
<td>0.90</td>
<td>Wind-Tunnel Emissivity</td>
</tr>
</tbody>
</table>

Table 1 Physical parameters

The effective wind-tunnel emissivity \(\varepsilon_{wt}\) may differ from the emissivity of medium-density-fiberboard (0.90 [6]) because the temperatures of the (internal) tunnel surfaces may not be uniform, and the plate exchanges thermal radiation with objects in the room which may have different emissivities and temperatures than the tunnel and air.

With its low emissivity \((\varepsilon_p = 0.07)\), the radiative heat loss from the rough face of the plate is only 35 mW/K, about 3% of the 1.08 W/K expected for \(V = 1\) m/s convection. Even if the \(\varepsilon_{wt}\) value of 0.9 is wrong by 20%, forced convection measurements will be affected by less than 1%.

The measured dynamic physical quantities are:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Pi_H)</td>
<td>W</td>
<td>Heater Power</td>
</tr>
<tr>
<td>(V)</td>
<td>m/s</td>
<td>Fluid Velocity (from fan rotation rate)</td>
</tr>
<tr>
<td>(T_P)</td>
<td>K</td>
<td>Plate Temperature</td>
</tr>
<tr>
<td>(T_F)</td>
<td>K</td>
<td>Fluid (Air) Temperature</td>
</tr>
<tr>
<td>(T_B)</td>
<td>K</td>
<td>Back Surface Temperature</td>
</tr>
</tbody>
</table>

Table 2 Dynamic quantities

The calculated and simulated quantities are:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(U_S(V))</td>
<td>W/K</td>
<td>Side Thermal Conductance</td>
</tr>
<tr>
<td>(h_R)</td>
<td>W/(m²K)</td>
<td>Radiative Surface Conductance</td>
</tr>
<tr>
<td>(h(V))</td>
<td>W/(m²K)</td>
<td>Convective Surface Conductance</td>
</tr>
<tr>
<td>(U_p(t))</td>
<td>W/K</td>
<td>Convective Thermal Conductance</td>
</tr>
<tr>
<td>(T_p(t))</td>
<td>K</td>
<td>Plate Surface Temperature</td>
</tr>
<tr>
<td>(U_{B0})</td>
<td>W/K</td>
<td>Back Correction Conductance</td>
</tr>
</tbody>
</table>

Table 3 Calculated and simulated quantities

The forced convective thermal conductance of the four sides \(U_S(V)\) is computed by integrating the local forced convective surface conductance in series with the insulation thermal conductance over the area of the sides.

The short side is not an isothermal surface; it has a 4 mm metal strip and a wedge of extruded polystyrene (XPS) insulation. Each point’s effective surface conductance will depend on the temperature profile along the side.

A point on the side which is near to metal will have a large conductance. Summing the conductivity divided by the shortest distance to metal at each angle will be roughly proportional to the local conductance (see Figure 6). The conductance through a slab with parallel isothermal faces is well known. The ratio of the slab conductance to the slab sum is 0.637, which is used to normalize the calculated conductivities.
In forced air the four sides have three distinct behaviors. The forced air flows parallel to the long dimension on two sides but flows into the windward side and away from the leeward side.

The forced convective component for the two parallel sides averages the local forced convective surface conductance in series with that point’s insulation conductance as computed above.

For the windward side, a similar calculation is done, but the wind speed grows linearly from 0 m/s at the mid-line to \( V \) along each long edge. For the leeward side the wind speed grows linearly from 0 m/s along each long edge to \( V \) at the mid-line, but with the flow always turbulent. The leeward surface conductance turned out to be within 1% of the windward surface conductance, so the windward value is used for both.

Convection for the sides uses the smooth turbulent correlation \( \text{Nu} = 0.037 \text{Re}^{1/5} \text{Pr}^{1/3} \). The expected convection through the sides is 9% of the convection from the 3 mm rough surface at 4 m/s wind-speed.

Collecting terms which have a factor of \( T \) into \( U \), the heat balance equation for the plate in forced convection is:

\[
U_M(V) = U_S(V) + h(V)A_p + \epsilon_p\varepsilon_w h_R A_p
\]

\[
\Pi_H = U_M(V) [T_P - T_F] + U_I [T_P - T_B] + U_{B0} [T_B - T_F] + C \frac{dT_P}{dt}
\]

To track temperature measurements as functions of time requires characterization of the temperature-time responses of the thermal masses and insulation (a combination of polyisocyanurate (PIR) and XPS foams). Figure 7 shows the temperature-time response of the downward-facing plate to a 57.5 W step in time responses of the thermal masses and insulation (a combination of polyisocyanurate (PIR) and XPS foams). Figure 7 and the heating phase of experiments show that a 15 s delay models the system well.

Including the delays for the insulation and plate thermal masses, and isolating \( T_P(t) \):

\[
T_P(t) = \frac{\Pi_H(t - 14) + U_M(V)T_F(t) + U_I T_B(t - 110) - U_{B0} [T_B(t) - T_F(t)] - C dT_P(t)/dt}{U_M(V) + U_I}
\]

\[
U_{B0} \text{ from equation (8) in Section 5 is the amount of natural convection from a downward-facing back face which reduces the convection from the upward-facing test surface.}
\]

Solving (4) as a finite-difference equation where \( dt = t - t' \):

\[
H(V,t) = \Pi_H(t - 14) + U_M(V)T_F(t) + U_I T_B(t - 110) - U_{B0} [T_B(t) - T_F(t)]
\]

\[
T_P(t) = H(V,t)[t - t'] + C T_P(t')
\]

\[
[U_M(V) + U_I][t - t'] + C
\]

In equation (5), \( T_P(t') \) is the previous simulated value (not the measured value).

Forced convection measurements are usually presented as Nu versus Re. In order to obtain Nu, the heat balance equation is solved for \( U_P = h(V)A_p \):

\[
\eta(V,t) = \Pi_H(t - 14) - U_I [T_P(t) - T_B(t - 110)] - U_{B0} [T_B(t) - T_F(t)]
\]

\[
U_P(t) = \frac{\eta(V,t) - C [T_P(t) - T_P(t')][t - t']}{T_P(t) - T_F(t)} - A_p\varepsilon_p\varepsilon_w h_R - U_S(V)
\]

In the denominator of equation (6), \( T_P(t) \) and \( T_F(t) \) are the 11-element cosine averages of plate and fluid temperature (centered at time \( t \)). Averaging is needed so that the derivative doesn’t correlate with the denominator, causing bias.
Figure 7  57.5 W step response

Figure 8 Upward-facing natural convection run
4. Natural Convection Side Model

In order to measure natural convection from the rough surface, natural convection from the four sides must be deducted from the total heat flow. While the forced (convective) surface conductance of the sides can be modeled by integrating the local forced surface conductance, this is not possible for the natural convective component.

With the plate mounted in the wind-tunnel, the still air convection might be slightly reduced from its true value because the rising plume is obstructed by the ceiling of the wind-tunnel. Low fan speeds move the rising air such that the natural convection is effectively not obstructed. The asymptotic behavior of the low fan speed measurements can be more accurate than the still air measurements.

The natural convection and thermal radiation for the four sides will be modeled using two new parameters. If separate parameters were introduced for upward, downward, and vertical natural convective modes for the sides, then the amount of convection for each mode could be set arbitrarily, proving nothing about the natural convection from the rough plate. So only two parameters are introduced into the model. The natural convection of each side is calculated for a $L_{es} \times L_C$ area instead of its actual $51 \text{ mm} \times L_C$ ($\varepsilon = 3 \text{ mm}$) or $49 \text{ mm} \times L_C$ ($\varepsilon = 1 \text{ mm}$) area. The black-body radiation from each side is calculated for its actual area with an effective emissivity of $\epsilon_{es}$. $L_{es}$ is the sum of $24.5 \text{ mm}$ and the effective height of the plate edge $\sqrt{2} \varepsilon$.

The flow patterns in Fujii and Imura\cite{7} figures 14(e) and 14(f) show plumes rising from the center of an upward-facing plate fed by flow from the plate’s edges. For the test (rough) surface, the upward flow of $0.467 \text{ W/K}$ is more than twice the $0.212 \text{ W/K}$ expected from the back and sides. Convective flow from the upward-facing rough surface will draw in the air heated by the back and sides, reducing heat loss from the test surface. In order to avoid double counting the convected heat from the back and sides, they should not be deducted from the plate heat (the thermal radiation is still deducted). None, rather than some, of this convected heat is deducted because a fraction would introduce another degree-of-freedom into the side model. For the sides, that is probably close to the true situation. But heat from the back is unlikely to entirely displace heat from the rest of the plate. And there should be an expected measurement bias for the displacement of heat flowing from the back. The uncertainty calculations work with symmetric biases; the second law of thermodynamics guarantees that the heat displacement will not be greater than 100%. So the “reuptake” parameter $C_U$ for the back (to upward facing test surface) is set to 75% with an expected bias of $\pm 25\%$.

Not deducting side convection from upward natural convection has a benefit: the upward convection model is not sensitive to $L_{es}$, allowing $\epsilon_{es}$ to be determined from upward convection alone. Figure 9 shows the still-air upward convection measured.

In the vertical case, 1/2 of the heated air from the bottom side flows along the vertical rough surface and would be double counted. And 1/2 of the air drawn by the top side comes from the vertical rough surface and would be double counted.

The reuptake for the vertical orientation is likely to be less efficient than for the upward facing surface, so $C_V$ is set to 62.5% with an expected bias of $\pm 37.5\%$. Only $C_V/2$ of the top and bottom side convection amounts are deducted from the plate total. Figure 10 shows the vertical natural convection measurements.

Figure 11 shows the measurements for downward natural convection.

The 2 years old duct-tape ($\epsilon_{es} = 0.38$) wrapping the plate sides was removed before machining the plate from 3 mm down to 1 mm roughness. When new duct-tape was applied, $\epsilon_{es}$ was inferred (from natural convection of the upward-facing plate) at only 0.30 and 0.21 (for two different brands). With the duct-tape removed, $\epsilon_{es} = 0.23$.

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>3 mm</th>
<th>1 mm</th>
<th>RMS roughness</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{es}$</td>
<td>28.2 mm</td>
<td>25.4 mm</td>
<td>Side effective natural length</td>
</tr>
<tr>
<td>$\epsilon_{es}$</td>
<td>0.38</td>
<td>0.30</td>
<td>Side effective surface emissivity</td>
</tr>
</tbody>
</table>

Table 4 Side physical parameters

Not all of these still-air measurements in Figures 11, 10, and 9 are within their expected measurement uncertainties. Without moving air acting to equalize temperatures in the wind-tunnel, the ambient and plate thermometers tracked poorly, introducing errors not accounted for in the expected uncertainties. Changes in ventilation, and even residual body heat from adjusting the apparatus made the natural convection measurements less repeatable than the forced convection measurements.
For upward-facing, vertical, and downward-facing natural convection Jaffer \cite{8} derives formulas:

\[
\overline{Nu} = \left(0.671 + 0.370 \frac{Ra}{Pr}^{1/6}\right)^2
\]

\[
\overline{Nu}' = 0.682 \left(1 + 0.469 \left[\frac{Ra}{\Xi'(Pr)}\right]^{1/6}\right)^2 \Xi'(Pr) = \left|1.05 \frac{Pr}{Pr}^{1/3}\right|
\]

\[
\overline{Nu}_R = 0.682 \left(1 + 0.806 \left[\frac{Ra}{\Xi_R(Pr)}\right]^{1/5}\right)^2 \Xi_R(Pr) = \left|1.05 \frac{Pr}{Pr}^{1/3}\right|
\]

Figure 9 Natural convection from upward-facing surface

Figure 10 Natural convection from vertical surface

Figure 11 Natural convection from downward-facing surface
5. Mixed Convection Side Model

So far the discussion has dealt with convection as either natural or forced. If the model for natural convection from the sides could be extended to handle mixed convection, then plate measurements could be made at $\text{Re} < 5000$. There is no guarantee that this is possible, but the insulation between the plate and side surfaces reduces the impact of side model inaccuracies on measurement accuracy.

\[ U_{fp}(V) \quad \text{Parallel Side Forced Thermal Conductance} \]
\[ U_{fw}(V) \quad \text{Windward Side Forced Thermal Conductance} \]
\[ U_{mix}(U_F, \theta, L_F, L_V, \psi) \quad \text{Mixed Thermal Conductance} \]
\[ L_C \quad \text{Plate Width = Side Width} = 0.305 \, \text{m} \]

<table>
<thead>
<tr>
<th>Table 5 Thermal conductance functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_F$</td>
</tr>
<tr>
<td>$\theta$</td>
</tr>
<tr>
<td>$L_F$</td>
</tr>
<tr>
<td>$L_V$</td>
</tr>
<tr>
<td>$\psi$</td>
</tr>
</tbody>
</table>

| Table 6 $U_{mix}$ parameters |

$U_S$ is the amount which will be deducted from the plate heat flow. Its components are the thermal radiation from the four sides and convection from the side at top when $\theta = 0^\circ$, the windward and leeward sides (which are always vertical), and the side at bottom when $\theta = 0^\circ$. Each mixed component is paired with the product of the corresponding natural component and a continuous trigonometric function of $\theta$ which goes negative for each side whose natural convection would otherwise be double counted.

In addition to the deductions detailed in Section 4, consider the (vertical) plate as $\theta$ decreases from $0^\circ$. As the bottom side face tilts upward, more (than half of the) heated air will rise toward the rough surface. That heat will reduce the convection from the rough surface. When tilted downward, the heat from the rough surface will reduce the convection from the top side. To handle these cases, equation (7) includes a term $2 \cos \theta \sin \theta$ whose minimum of $-1$ is reached at $\theta = -45^\circ$ and a term $-2 \cos \theta \sin \theta$ whose minimum of $-1$ is reached at $\theta = +45^\circ$.

\[
U_S = 4 A_S \varepsilon_{es} \varepsilon_{wt} h_R \\
+ U_{mix}(U_{fp}(V), \theta - 90^\circ, L_C, L_{es}, 0^\circ, \psi) \\
+ U_{mix}(0, \theta - 90^\circ, L_C, L_{es}, 0^\circ, \psi) \min(0, \sin \theta, -0.5C_C \cos \theta, -2 \cos \theta \sin \theta) \\
+ 2 U_{mix}(U_{fw}(V), 0^\circ, L_{es}, L_C, \theta, \psi) \\
+ 2 U_{mix}(0, 0^\circ, L_{es}, L_C, \theta, \psi) \min(0, \sin \theta) \\
+ U_{mix}(U_{fp}(V), 90^\circ - \theta, L_C, L_{es}, 0^\circ, \psi) \\
+ U_{mix}(0, 90^\circ - \theta, L_C, L_{es}, 0^\circ, \psi) \min(0, \sin \theta, -0.5C_C \cos \theta, 2 \cos \theta \sin \theta) \\
\] (7)

Equation (7) is for horizontal flow $\psi = 90^\circ$. For vertical flow, swap $U_{fp}$ and $U_{fw}$. When $V$ is large, $U_S$ approaches the sum of the forced convection conductances. In still air ($V = 0$) with $\theta$ equal to $-90^\circ$, $0^\circ$, or $+90^\circ$, $U_S$ implements the natural convection calculations described in Section 4.

$U_{B_0}$ (8) is used in equations (3), (5), and (6) to prevent the convection from a downward-facing back from being counted twice. Its min(0, sin $\theta$) term is squared because the heated air from the back must flow around two right-angle edges to affect the rough surface.

\[
U_{B_0} = -U_{mix}(0, 90^\circ, L_C, L_{es}, 0^\circ) C_U \min(0, \sin \theta)^2 \\
\] (8)
6. Airflow Corrections

While formulas worked for vertical-aiding and vertical-opposing measurements of the 3 mm roughness plate, the measurements of the 1 mm roughness plate in both of these orientations showed unexpectedly high convection when \( \text{Re} > 5000 \) in Figure 13.

The wind-tunnel is vertical only for vertical-aiding and vertical-opposing experiments. The distance from its intake to the floor (aiding) or ceiling (opposing) is only 0.5 m. While this crowding might be expected to reduce the airflow; it was an increase in convection that was observed. The crowding might also result in the intake flow being less uniform than when the wind-tunnel is horizontal.

If the intake airflow were turbulent, it would have the effect of lowering the \( \text{Re}_{\ell} \) threshold in the rough-to-smooth turbulence correlation from Jaffer[1] such that the transition between rough turbulence and smooth turbulence is moved closer to the leading edge of the plate.

![Figure 12 Effect of \( \text{Re}_C \) at 1 mm roughness](image)

Figure 12 shows that scaling the threshold \( \text{Re}_{\ell} \) by \( \text{Re}_C = 0.212 \) affects the \( \text{Re} > 50 \times 10^3 \) range, while the \( \ell^2 \)-norm does not. A \( \text{Re}_C \) value of 0.212 brings the 1 mm roughness vertical sets of measurements into agreement with the correlations as shown in Figure 14. The effect from scaling \( \text{Re}_{\ell} \) is subtler for the 3 mm roughness plate; the 3 mm vertical aiding and opposing runs with \( \text{Re}_C = 0.212 \) are in slightly better agreement with the correlations than with \( \text{Re}_C = 1.0 \).

\( \text{Re}_C \) is a parameter of the forced component of convection, not of the mixing; with \( \text{Re}_C = 0.212 \) the mixed correlations work for the vertical wind-tunnel measurements.
Figure 13  Opposing mixed convection vertical plate, 1 mm roughness, \( \text{Re}_C = 1.0 \)

Figure 14  Opposing mixed convection vertical plate, 1 mm roughness, \( \text{Re}_C = 0.212 \)
Figure 15  Upward facing plate, 3 mm roughness, $\Delta T = 10$ K

Figure 16  Upward facing plate with Re correction, 3 mm roughness, $\Delta T = 10$ K
On the non-upward-facing measurements, the plate was suspended so that the (rough) test surface was centered in the wind-tunnel. The upward-facing measurements flipped the plate in its wire suspension, raising the test surface by 30 mm. On both upward-facing plots, the convectons at Re=11000 and Re=22000 were higher than the $\ell^2$-norm expected uncertainty traces as seen in Figure 15. While it was possible to fit the points with a ratio of degree eight polynomials of the forced and natural convection components, the polynomial ratios fitting the 1 mm and 3 mm plates data were not the same.

If these discrepancies were instead due to differences in the airflow when the plate was flipped, the same correction would be expected to work for both roughness plates. Unfortunately, the flow rates of 0.86 m/s and 1.41 m/s are below the range where available small anemometers are accurate enough to directly resolve this question. However, shifting the Reynolds numbers around $Re_b = 15600$ upwards by 20% corrects both roughness plates as seen in Figure 16:

$$Re' = Re \frac{Re^8 + 0.4Re^4 Re_b^4 + Re_b^8}{Re^8 + Re_b^8}$$

7. Laminar Flow

Natural flow velocity profiles are 0 m/s at the plate surface and near zero outside of the boundary layer; forced velocity profiles reach the free stream velocity outside of the boundary layer. Thus opposing flows can cancel each other only over a partial range of depths within the boundary layer. The bi-level test plate interferes with cancellation because its height-of-roughness is comparable to the boundary layer depth on the windward portion of the bi-level surface.

![Figure 17 Opposing mixed convection vertical plate, 1 mm roughness, $\Delta T = 10.4$ K](image)

Figure 17 shows a suite of measurements which resulted when the 1 mm roughness plate was suspended 30 mm off-center (as was done with upward-facing plates) in the vertical wind-tunnel. The five measurements with $2000 < Re < 6000$ show mixed convection lower than the natural convection; they can’t be the result of $\ell^p$-norm ($p > 0$) mixing with the natural convection.

The seven measurements laying on the trace labeled “Tops Smooth Turbulent Asymptote” match convection resulting from smooth turbulent convection from only the post top surfaces. This might be explained
by the opposing flows reducing convection in the vertical channels to relatively low levels, leaving the post-tops to dominate convection.

The lower two measurements laying on the “Laminar Asymptote” trace being less than the natural convection level indicate that turbulent flow was not present at $\text{Re} \leq 4853$. If the effective laminar flow were reduced from its free stream velocity, then these points would have been at higher Reynolds numbers. Their presence on the forced laminar asymptote indicates that the velocity reduction happens in the plate channels.

8. Measurement Uncertainties

The Convection Machine measures heat-flow into the plate from electrical heater dissipation and convective heat flow from the rate of change in plate temperature. For the 3 mm roughness plate, Table 7 computes the sensitivity of heat flow to each parameter and the product of each sensitivity with the estimated parameter bias to yield the component uncertainty. In accordance with ASME Measurement Uncertainty[9], the root-sum-squared (RSS) of these uncorrelated uncertainties is less than 2%. Reducing the roughness $\varepsilon$ to 1 mm ($\varepsilon_{pv} = 2$ mm) raises the combined uncertainty to less than 3% in Table 8.

With a wind-speed of 3 m/s or greater, emissivity and natural convection parameters don’t have a significant effect on the combined uncertainty and aren’t included in Tables 7 and 8.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Nominal</th>
<th>Sensitivity</th>
<th>Bias</th>
<th>Uncertainty Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>19.5°C</td>
<td>-0.151%/°C</td>
<td>1.0°C</td>
<td>0.151% LM35C Temperature Sensor</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>10.5°C</td>
<td>5.814%/°C</td>
<td>0.15°C</td>
<td>0.872% LM35C Differential</td>
</tr>
<tr>
<td>$P$</td>
<td>101kPa</td>
<td>0.001%/Pa</td>
<td>1.5kPa</td>
<td>0.813% MPXH6115A6U Air Pressure</td>
</tr>
<tr>
<td>$V$</td>
<td>3.00m/s</td>
<td>18.306%/m/s</td>
<td>0.20m/s</td>
<td>0.202% Wind-tunnel stall velocity</td>
</tr>
<tr>
<td>$V_b$</td>
<td>7.50m/s</td>
<td>1.011%/m/s</td>
<td>0.20m/s</td>
<td>0.202% Airspeed</td>
</tr>
<tr>
<td>$\text{Re}_C$</td>
<td>1.000</td>
<td>7.808%</td>
<td>0.10</td>
<td>0.781% Rough-Smooth Threshold</td>
</tr>
<tr>
<td>$L_c$</td>
<td>0.305m</td>
<td>13.800%/m</td>
<td>500um</td>
<td>0.007% Characteristic Length</td>
</tr>
<tr>
<td>$L_w$</td>
<td>0.305m</td>
<td>215.555%/m</td>
<td>500um</td>
<td>0.108% Plate Width</td>
</tr>
<tr>
<td>$L_T$</td>
<td>8.29mm</td>
<td>516.239%/m</td>
<td>250um</td>
<td>0.140% Block Length</td>
</tr>
<tr>
<td>$\varepsilon_{pv}$</td>
<td>6.00mm</td>
<td>7851.664%/m</td>
<td>150um</td>
<td>1.178% P2V Height-of-Roughness</td>
</tr>
<tr>
<td>$k_{pir}$</td>
<td>23.0 mW/K/m</td>
<td>-0.052%/mW/K/m</td>
<td>1.2 mW/K/m</td>
<td>0.059% PIR Thermal Conductivity</td>
</tr>
<tr>
<td>$k_{xps}$</td>
<td>34.0 mW/K/m</td>
<td>0.063%/mW/K/m</td>
<td>1.7 mW/K/m</td>
<td>0.108% XPS Thermal Conductivity</td>
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</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Nominal</th>
<th>Sensitivity</th>
<th>Variability</th>
<th>Uncertainty Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>3.00m/s</td>
<td>18.306%/m/s</td>
<td>7.5mm/s</td>
<td>0.137% Airspeed</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.90% RSS Combined Uncertainty</td>
</tr>
</tbody>
</table>

Table 7 Uncertainties for plate with 3mm roughness

Convection measurements are 35 times more sensitive to $\Delta T = T_P - T_F$ than to $T$ (the average of $T_F$ and $T_P$). The offsets in the measurement software for the plate, back, and ambient sensors were matched
to about ±0.15 K. In 300 consecutive readings at constant temperature, the variability for the ambient and plate sensors was 0.03 K, and 0.02 K for the back sensor at \( T = 19^\circ\text{C} \).

The vertical dimensions of the aluminum plate’s rough surface are correct as far as can be determined with spot checks using a caliper depth gauge. The only accuracy specification for the ShopBot PRSstandard CNC (used to machine the rough surface of the plates) is a “Positional Repeatability” of 0.05 mm. Three times this value (150\( \mu \text{m} \)) is used for estimated bias of \( \varepsilon_{pv} \) in Tables 7 and 8.

## 9. Admissible Roughness

“The amount of roughness which is considered “admissible” in engineering applications is that maximum height of individual roughness elements which causes no increase in drag compared with a smooth wall.”

Schlichting[3] develops a formula for \( k_{adm} \) from its Fig. 21.9. (Resistance formula of sand-roughened plate; coefficient of total skin friction):

\[
u k_{adm}/\nu = 100
\]

A footnote states “The estimates performed in this section make no distinction between the equivalent sand height, \( k_S \), and the actual height, \( k \), of a protuberance.” In the Jaffer[1] model of sand-roughness, the protuberance height is \( (1 - \bar{g}/2) k_S \). For Nikuradse’s large grains that is 0.57 \( k_S \); for small grains it is 0.51 \( k_S \). So equation (9) overestimates the admissible-roughness height.

For a given RMS roughness \( \varepsilon \), the equal-area bi-level plate has the smallest possible protuberance height \( k = 2\varepsilon \); while large-grain sand-roughness would have \( k = 0.57 \times 5.333\varepsilon = 3.0\varepsilon \) and small-grain \( k = 0.51 \times 5.333\varepsilon = 2.7\varepsilon \).

The goal is to find the \( Re \) value in the 1 mm trace in Figure 18 below which the slope of measured \( Nu/Pr^{1/3} \) is 4/5. Natural convection obscures the slope below \( Re = 4419 \); at best an upper bound can be found. This upper bound for \( k_{adm} \) in equation (10) is a significant reduction from equation (9):

\[
k_{adm} = 29 \frac{L}{Re}
\]

## 10. Conclusion

The Convection Machine was employed to collect data about forced and mixed convection, capturing 110 datasets with the 3 mm roughness plate and 380 with the 1 mm roughness plate. Each run had a duration of 108 minutes, totaling 880 hours over two and a half years.

The measurements obtained were the basis for two papers: Skin-Friction and Forced Convection from an Isothermal Rough Plate[1] and Turbulent Mixed Convection from an Isothermal Plate[2]. Graphs of the data from the two papers are available at: http://people.csail.mit.edu/jaffer/convect

\( k_{adm} = 29 L/Re \) is an upper bound for admissible-roughness, the maximum protuberance height which causes no increase in drag compared with a smooth plate.

The wind-tunnel will be modified to install a 0.1 N (10 g) load cell at the back of the tunnel. The metal plate will be retired. Plastic plates manufactured in a 3D printer will push on the load cell in order to measure the drag of various roughness profiles.

## Acknowledgments

Thanks to John Cox and Doug Ruuska for machining the bi-level plates. Thanks to Martin Jaffer and Roberta Jaffer for their assistance and problem-solving suggestions.

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2 This definition does not imply that roughness always causes an increase in drag; for example in Figure 18 the 1 mm roughness curve is less than the smooth-turbulent asymptote from 3000 < \( Re < 7000 \).
11. References


Figure 18  Convection and measurement uncertainties from rough plates

The roughness of the bi-level plate is not self-similar; it exhibits different convection behaviors at different scales. The slope of the upper end of both curves in Figure 18 is 4/5, but with a magnitude possible only if operating with a much shorter characteristic-length than the plate’s $L = 0.305 \text{ m}$.

At the leading edge of the plate, the smooth-turbulent boundary-layer depth would be much smaller than the height-of-roughness; so fully rough convection (11) applies.

$$\text{Nu} = \frac{\text{Re} \text{Pr}^{1/3}}{6 \ln^2 (L/\varepsilon)}$$

The transition from fully rough convection to smooth turbulent convection will occur at a location where the boundary-layer thickness is comparable to the height of roughness $\varepsilon$.

From Schlichting[3], the local momentum boundary-layer thickness is $\delta_2 = 0.036 x \text{Re}_x^{-1/5}$. The thickness at the trailing edge of a smooth plate is $\delta_2 = 0.036 L \text{Re}^{-1/5}$. Solving for $\text{Re}$ with $\varepsilon = \delta_2$:

$$\text{Re} = \left( \frac{0.036 L}{\varepsilon} \right)^5$$

Note that $\text{Re}$ depends only on the geometry of the plate. If all lengths of the plate are scaled, $\text{Re}$ does not change. Similarly $\text{Re}_x = x \text{Re}/L$ will be independent of length scaling.

For the bi-level plate the repeat length $L_S = L/26$. The top of the posts are $L_T = 8.3 \text{ mm}$ long in the direction of flow. The length variables involved in plate convection are $L$, $L_T$, $L_S$, $L_S - L_T$, $x$ and $\varepsilon = \delta_2$. From convection measurements it appears that $\text{Re}_x \propto \varepsilon^{9/4}$ with the other lengths held constant.
There are several conditions under which convection should be smooth only; that is accomplished by including dimensionless factors forcing the local $Re_\ell$ threshold to zero when the post length to post spacing ratio is 0 or 1, or if the roughness is 0.

$$Re_\ell = \frac{L_S}{2L_T} \frac{L_S - L_T}{0.036 L_S L_S} \frac{\varepsilon}{0.036 L_S} \left( \frac{\varepsilon}{0.036 L_S} \right)^{5/4} = \frac{L}{0.072 L_T} \left( \frac{\varepsilon}{0.036 L_S} \right)^{9/4} \quad (12)$$

The two bi-level plates tested in the Convection Machine were identical except for their height-of-roughness $\varepsilon = 3 \text{ mm}, 1 \text{ mm}$. So the correctness of the equation (12) dependence on $L_T$ and $L_S$ are less certain than for $\varepsilon$. From the convection measurements it seems that the flow transitions near $Re_x = Re_\ell$ from fully rough convection to a regime consisting of smooth turbulent convection (13) from the channels between posts at scale $L$ and smooth turbulent convection (13) from the post at scale $L_T$.

$$Nu = 0.037 Re^{4/5} Pr^{1/3} \quad (13)$$

The longitudinal (parallel to flow) channels between the posts have convection:

$$Nu_C/Pr^{1/3} = \frac{L_S - L_T}{L_S} \frac{0.037 Re^{4/5}}{ } \quad (14)$$

The top of the posts occupy half ($L_T^2/L_S^2$) of the plate area; their convection is:

$$Nu_T/Pr^{1/3} = \frac{L_T^2}{L_S} \left( \frac{L}{L_T} \right)^{1/5} \frac{0.037 Re^{4/5}}{ } \quad (15)$$

The $(L/L_T)^{1/5}$ factor converts the characteristic-length from $L_T$ to $L$.

Both post sides parallel to the flow also experience smooth turbulent convection at scale $L_T = 8.3 \text{ mm}$, but because the sides are within the channel’s boundary layer, temperature and velocity are not constant with elevation. The channel velocity profile provides velocity as a function of elevation $y$, boundary layer depth $\delta$, and free stream velocity $u$:

$$\frac{u_y}{u} = \begin{cases} (y/\delta)^{1/7} & \text{when } y \leq \delta; \\ 1, & \text{otherwise.} \end{cases}$$

By analogy with the derivation of the thermal profile for laminar flow from Lienhard and Lienhard[4], the turbulent thermal profile would be:

$$\frac{T - T_w}{T_\infty - T_w} = \begin{cases} (y/\delta_t)^{1/7} & \text{when } y \leq \delta_t; \\ 1, & \text{otherwise.} \end{cases} \approx \begin{cases} (y/\left(\delta Pr^{-1/3}\right))^{1/7} & \text{when } y \leq \delta; \\ 1, & \text{otherwise.} \end{cases}$$

The temperature profile will affect side convection by a factor of approximately $Pr^{1/21}(y/\delta)^{1/7}$ when $y \leq \delta$; and 1 otherwise. In air, $Pr^{1/21}$ is a reduction by 1.5% (for the post sides convection only).

![Figure 19 Convection mode regions versus Reynolds number](image-url)
Each vertical slice of Figure 19 represents the regions of convection on the surface of the 1 mm roughness plate at the indicated value of Re. The surface is in rough turbulent convection of correlation (11) below the Re\textsubscript{M} trace where Re\textsubscript{x} < Re\textsubscript{ℓ}. Above the Re\textsubscript{M} trace, the convection is modeled as a combination of channel (14), post-top (15), and post-side (16) convection. When Re is larger than 80000, there is a small region where the channel boundary-layer height is smaller than the post side height \( \varepsilon_{pv} = 2 \) mm. Post side convection from this region was modeled, but its effect on total convection was negligible. For simplicity, the post side model uses formula (16) for all post side convection.

The convection for a vertical slice of the post side is:

\[
\chi(x) = \frac{0.037}{\varepsilon_{pv}} \int_{0}^{\varepsilon_{pv}} \left( \frac{y}{\delta(x) Pr^{1/7}} \right) \left( \frac{Re}{\delta(x)} \right)^{1/7} \frac{4/5}{Pr^{1/21}} \int_{0}^{\varepsilon_{pv}} \left( \frac{y}{0.37 L^{4/5}} \right) \left( \frac{Re}{L} \right)^{1/7} \left( \frac{Re}{L} \right)^{1/7} dy
\]

\[
= \frac{0.037 Pr^{1/21}}{\varepsilon_{pv}} \int_{0}^{\varepsilon_{pv}} \left( \frac{y}{0.37 L^{4/5}} \right) \left( \frac{Re}{L} \right)^{1/7} \left( \frac{Re}{L} \right)^{1/7} dy
\]

\[
= 0.037 \frac{Re^{149/175} Pr^{1/21} L^{113/175}}{L^{158/175}}
\]

The convection from a row of post sides in line with the flow is:

\[
\frac{Nu}{Pr^{1/3}} = \frac{2 \varepsilon_{pv} L_T}{L_S} \left( \frac{L}{L_T} \right)^{1/5} \left[ \frac{L_S - L_T}{L_S} + \frac{L_T^2}{L_S^2} \left( \frac{L}{L_T} \right)^{1/5} \right] 0.0231 Pr^{1/21} \left( \frac{\varepsilon_{pv}}{L} \right)^{9/35} Re^{149/175}
\]

With Re\textsubscript{M} = \text{min}(Re, Re\textsubscript{ℓ}) where Re\textsubscript{ℓ} is from equation (12), integrating the local convections for correlations (11), (14), (15), and (16) over distance:

\[
\frac{Nu}{Pr^{1/3}} = \frac{Re_{\text{mix}}}{6 \ln^2(L/\varepsilon)} + \left[ \frac{L_S - L_T}{L_S} + \frac{L_T^2}{L_S^2} \left( \frac{L}{L_T} \right)^{1/5} \right] 0.037 \left( Re^{4/5} - Re_{\text{M}}^{4/5} \right) + 2 \frac{\varepsilon_{pv} L_T}{L_S} \left( \frac{L}{L_T} \right)^{1/5} 0.0231 Pr^{1/21} \left( \frac{\varepsilon_{pv}}{L} \right)^{9/35} \left( Re_{\text{mix}}^{149/175} - Re_{\text{M}}^{149/175} \right)
\]

Figure 18 shows that this model fits both 3 mm roughness and 1 mm roughness data-sets within their estimated uncertainties below Re = 80000. Figure 20 shows that the percentage discrepancy from the mixed model for 3 mm roughness is within ±2% below Re = 80000.\footnote{While the \( \ell^4 \) Mixed and Rough Turbulent Asymptote traces for 3 mm roughness are close in Figure 18, those traces for 1 mm roughness are not. This is because when \( \varepsilon \) of the plate was reduced from 3 mm to 1 mm, \( L_T \) was not similarly scaled.}
Figure 20  Measurement versus theory of 3.0 mm roughness plate