Improved bounds for MCMC sampling colorings of $G(n, d/n)$

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Joint work with: T. Hayes, D. Štefankovič and E. Vigoda

Workshop on Local Algorithms
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Sampling Problem

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Coloring model μ

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For a graph $G = (V, E)$ and an integer $k > 0$:

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uniform distribution over the proper k-colorings of G

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Input: $G = (V, E)$, k

Output: a k-coloring distributed as in $\mu(\cdot)$

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- *n* vertices

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- unlikely to have efficient algorithm
- focus on efficient approximation algorithms

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it is desirable that the chain "mixes fast"

The local algorithms

The local algorithms

"Glauber dynamics'

- $X_0 = \sigma$
- $X_t \to X_{t+1}$
  - Choose vertex w *uniformly at random* from V
  - Set $X_{t+1}(u) = X_t(u)$, for every vertex $u \neq w$
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Block dynamics

... instead of single vertices, update small the blocks.

The problem

MCMC sampling colorings of $G(n, d/n)$ with Glauber dynamics

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- ergodicity
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Focus

. . . speed of convergence.

How to measure speed . . .

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The number of transitions needed for the chain to reach within *total variation distance* $1/e$ from $\mu(\cdot)$. For worst case X_0.

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Interesting cases

. . . when the mixing time is polynomial in n

. . . we have "rapid mixing"

Rapid Mixing and Maximum Degree Δ

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Maximum Degree Bounds for colorings

Vigoda (1999) $k > \frac{11}{6}\Delta$ for general G

Hayes,Vera,Vigoda (2007) $k = \Omega(\Delta/\log\Delta)$ for planar G

Goldberg, Martin, Paterson (2004) $k \geq (1.763 + \epsilon)\Delta$ for G triangle free and amenable

Dyer, Frieze, Hayes, Vigoda (2004) $k \geq (1.48 + \epsilon)\Delta$ for G of girth $g \geq 7$

Frieze, Vera (2006) $k \geq (1.763 + \epsilon)\Delta$ for G locally sparse.

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the "natural" bound for k is w.r.t. the *expected degree* d

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Conjectured Bound

We have rapid mixing when $k \geq (1 + \epsilon)d$.

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- Efthymiou (2014): $k \geq (11/2)d$

Main Result

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Theorem (Rapid Mixing)

For $\epsilon > 0$ and sufficiently large $d > 0$ the following is true:
For $k \geq (\alpha + \epsilon)d$ and with probability $1 - o(1)$ over $G(n, d/n)$, the Glauber dynamics exhibits

$$T_{\mathrm{mix}} = O\left(n^{2+\frac{1}{\log d}}\right),$$

where $\alpha = 1.763\ldots$ is the solution to the equation $(1/z)e^{(1/z)} = 1$.

The effect of high degrees

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Strategy from Dyer et al. (2005)

"Use block dynamics & hide the high degrees inside the blocks"

The plan

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- define appropriate block partition

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    - use comparison

Block Construction

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Weights [Efthymiou (2014)]

- Each vertex u of degree $\deg(u)$ is assigned weight

$$W(u) = \begin{cases} (1+\gamma)^{-1} & \deg(u) \leq (1+\epsilon)d \\ d^c \cdot \deg(u) & \text{otherwise} \end{cases}$$

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$$\max_{L \in \Gamma(v)} \left\{ \prod_{u \in L} W(u) \right\} \leq 1.$$

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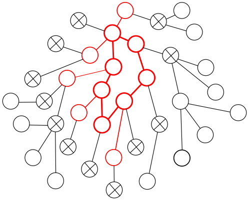

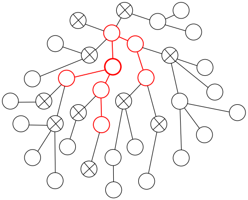

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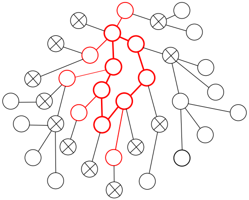

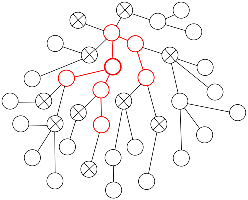

Boundary of the block

Consists only of break points.

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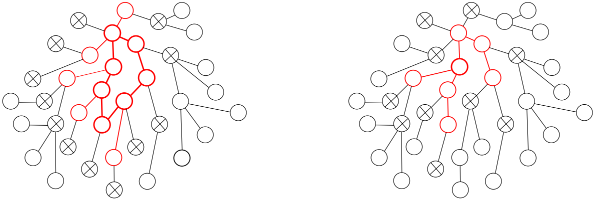

Low degree "buffer"

... between boundary vertices and a high degree vertex

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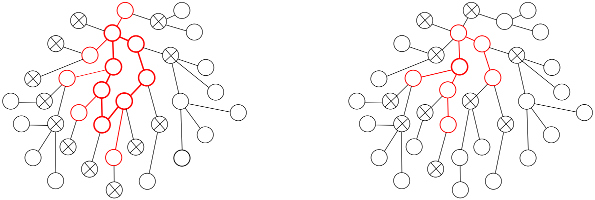

. . . for the analysis

the effect of high degrees disappears

Proving Rapid Mixing

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where

$$\mathrm{dist}(\sigma, \tau) = \sum_{u \in \sigma \oplus \tau} \beta(u)$$

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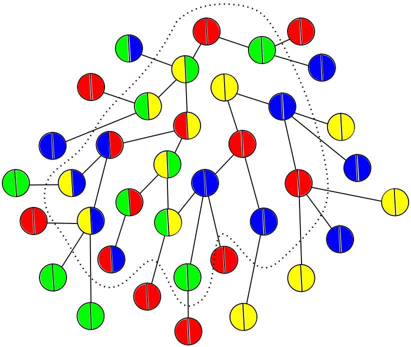

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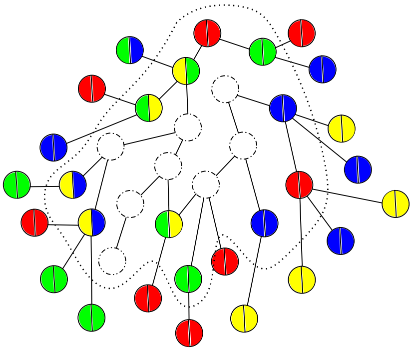
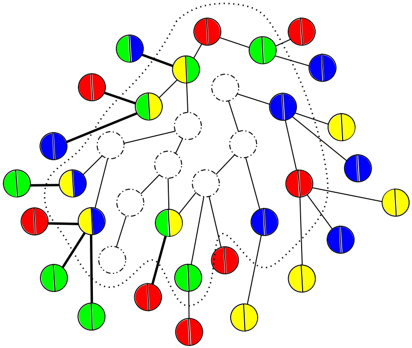


Distance between σ and τ

A distance that counts the disagreeing edges between the blocks



Distance between σ and τ

A new distance metric

Given $G(n, d/n)$ and set of blocks $\mathcal{B}$, for any two σ, τ

$$\mathrm{dist}(\sigma,\tau) = \sum_{v \in \partial\mathcal{B}} \mathbf{1}\{v \in \sigma \oplus \tau\} \deg_{out}(v)$$

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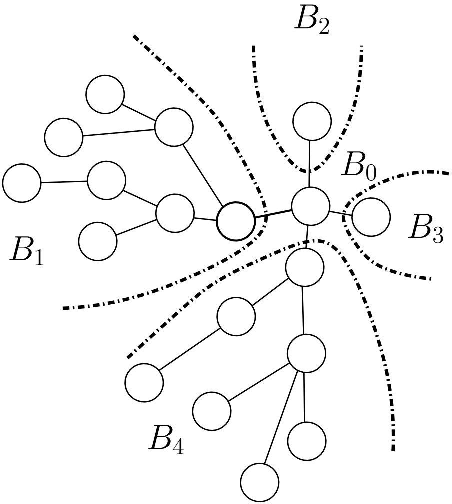

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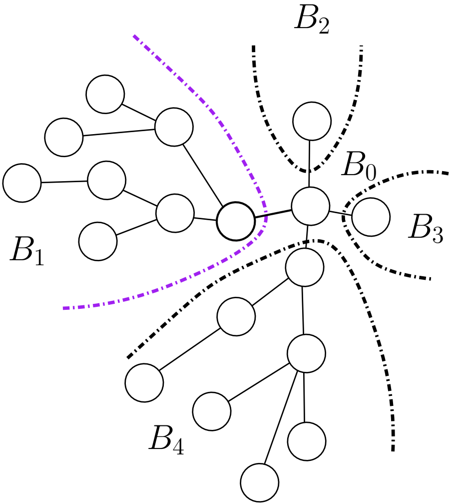

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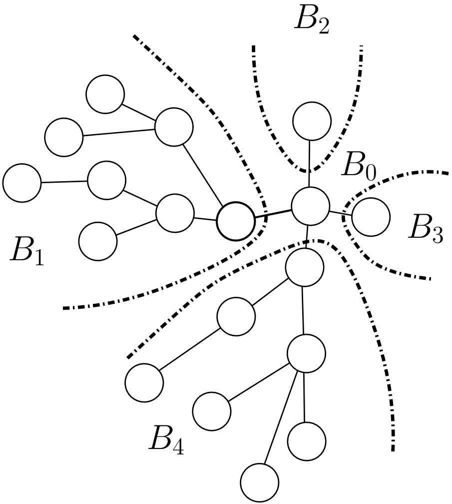

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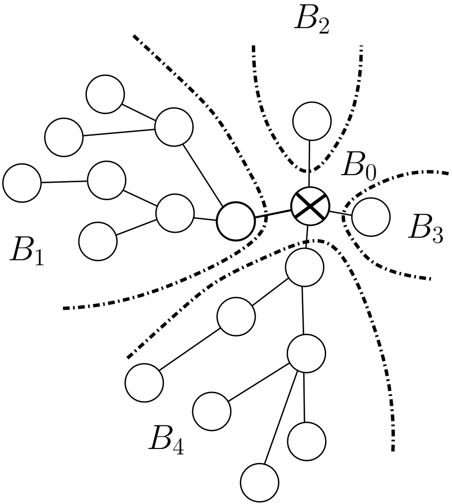

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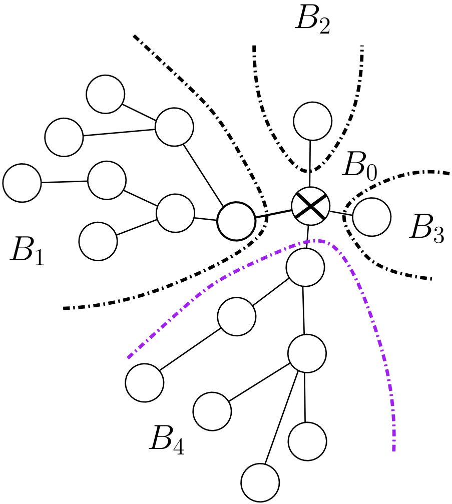
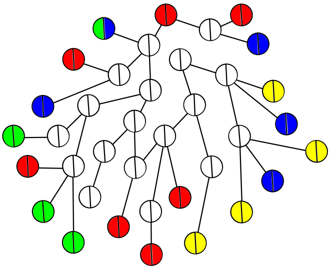


The coupling of $X(B)$ and $Y(B)$



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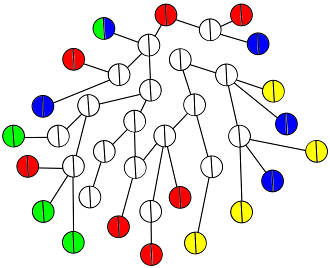

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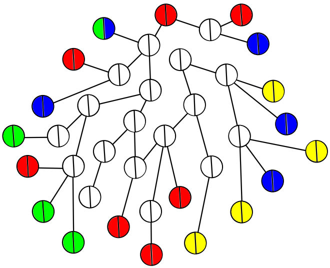

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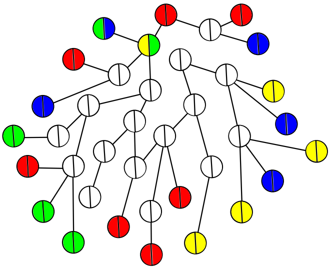

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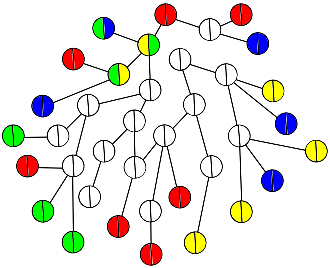

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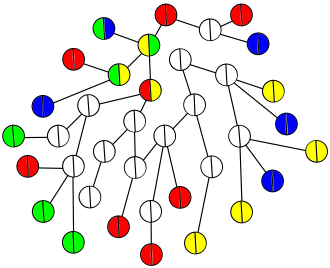

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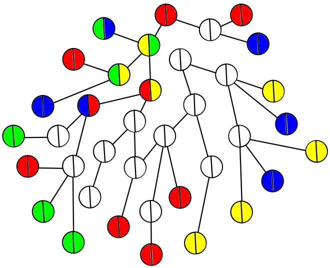

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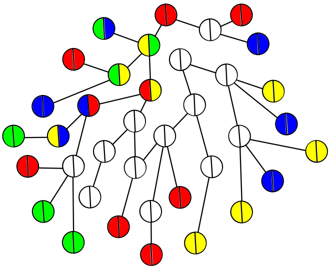

The coupling of $X(B)$ and $Y(B)$

- one vertex at a time
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- disagreement probability

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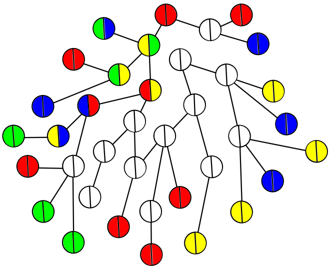

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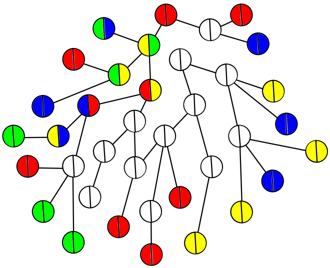

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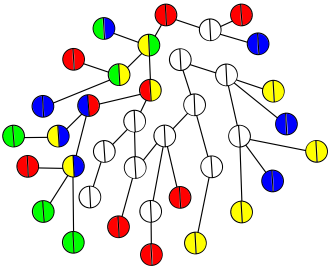

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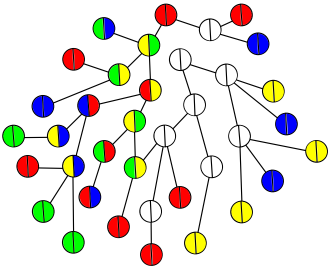

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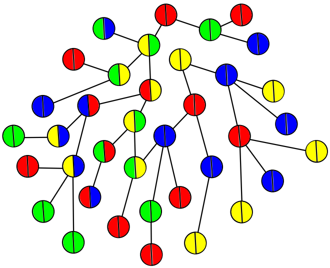

Rapid Mixing for $k > 2d$

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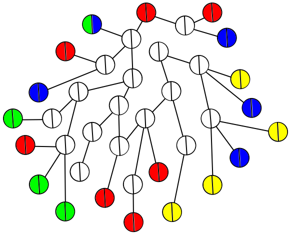

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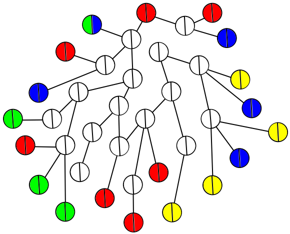
Block partition



Rapid Mixing for $k > 2d$

Probability of Propagation

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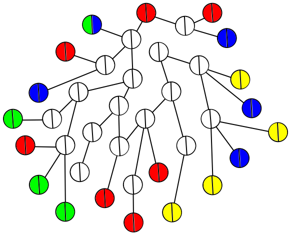

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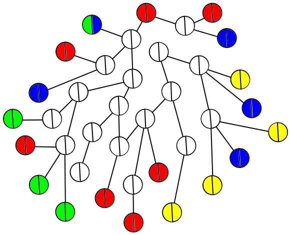

Block partition

Distance metric

Bound for k

Path coupling implies rapid mixing for $k > 2d$.

Better bounds with in-degrees

Goldberg, Martin, Paterson (2004)

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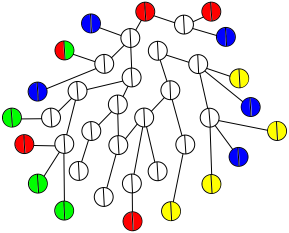


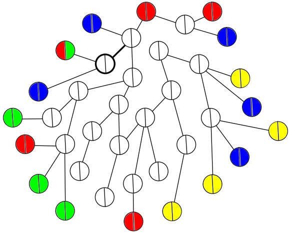
the probability of the most likely color

Better bounds with in-degrees

Goldberg, Martin, Paterson (2004)

Probability of Propagation when $k > \alpha d$

$$\varrho_v = \begin{cases} \dfrac{(1-\epsilon)}{\deg_{\text{in}}(v)} & v \text{ is low degree} \\ 1 & \text{otherwise} \end{cases}$$



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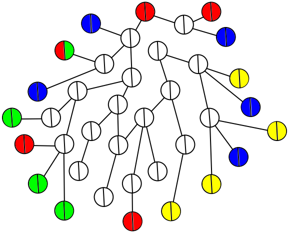

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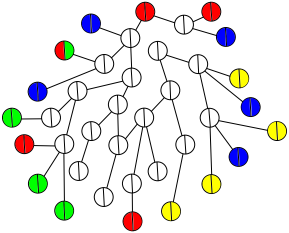

Obstacle for the above

... the coloring at the boundary is “worst case”.

Better bounds with in-degrees

Goldberg, Martin, Paterson (2004)

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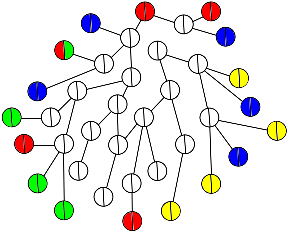

Obstacle for the above

... the neighbors outside use too many different colors!

Local Uniformity

Theorem (Local Uniformity)

With probability $1 - o(1)$ *over* $G(n, d/n)$ *the following is true: For all* $\varepsilon, C_1, C_2 > 0$*, for all* $d > d_0$*, for* $k \geq (\alpha + \varepsilon)d$*, let* $\mathcal{I} = [C_1 N, C_2 N]$*, for a low degree* $v \in V$*,*

$$\Pr\left[\exists t \in \mathcal{I} \text{ s.t. } |\mathrm{Avail}_v(X_t)| \leq \mathbf{1}\{\mathcal{U}_t(v)\}(1 - \varepsilon^2)k \exp\left(-\deg(v)/k\right)\right] \leq \exp\left(-d^{2/3}\right).$$

Rapid Mixing with uniformity

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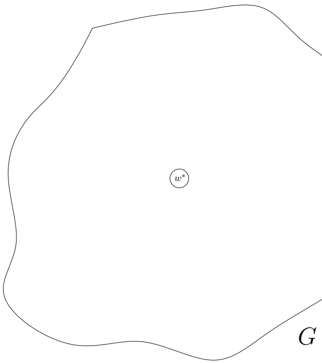
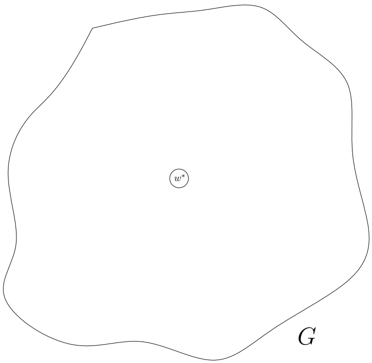


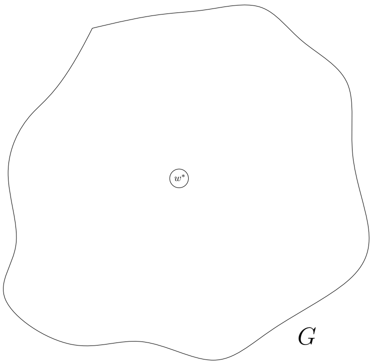
There is a single disagreement at w^*

Rapid Mixing with uniformity



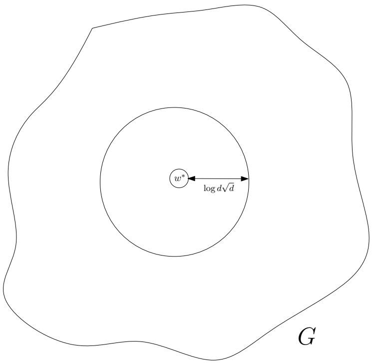
Run the chains for *CN* steps, "burn-in"

Rapid Mixing with uniformity



The disagreements spread in the graph during burn-in

Rapid Mixing with uniformity



Typically the disagreements do not escape the ball

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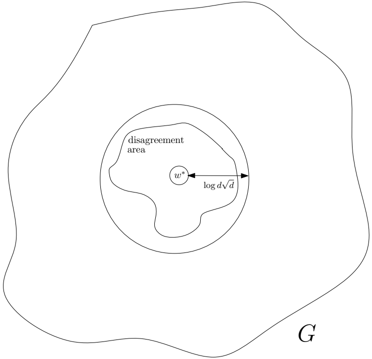

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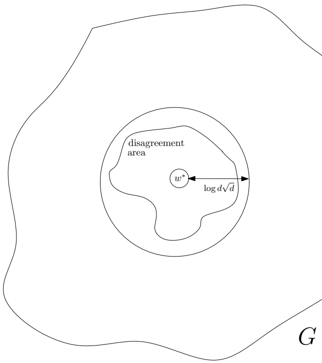
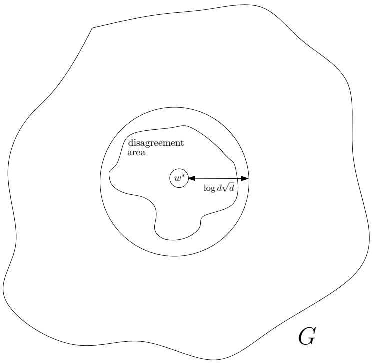


Typically the ball has uniformity.

Rapid Mixing with uniformity



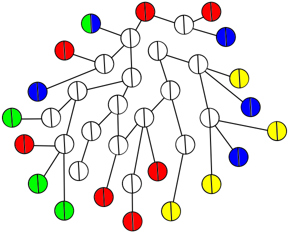
$$\mathbb{E}\left[\mathrm{dist}(X_{CN}, Y_{CN}) \middle| X_0, Y_0\right] \leq (1-\gamma)\mathrm{dist}(X_0, Y_0)$$

Block Update with Uniformity

Block Update with Uniformity

Probability of Propagation for $k > \alpha d$

$$\varrho_v = \begin{cases} \dfrac{1-\epsilon}{\deg_{\text{in}}(v)} & v \text{ is low degree} \\ 1 & \text{otherwise} \end{cases}$$



Block Update with Uniformity

Probability of Propagation for $k > \alpha d$

$v \in \mathrm{Ball}(w^*, (\log d)^2)$

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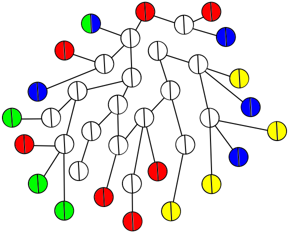

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  - previous bound was $\lambda < 1/(2d)$ [Efthymiou (2014)]

THANK YOU!