Lower Bounds for Tolerant Junta and Unateness Testing via Rejection Sampling of Graphs

Amit Levi Erik Waingarten

The Model

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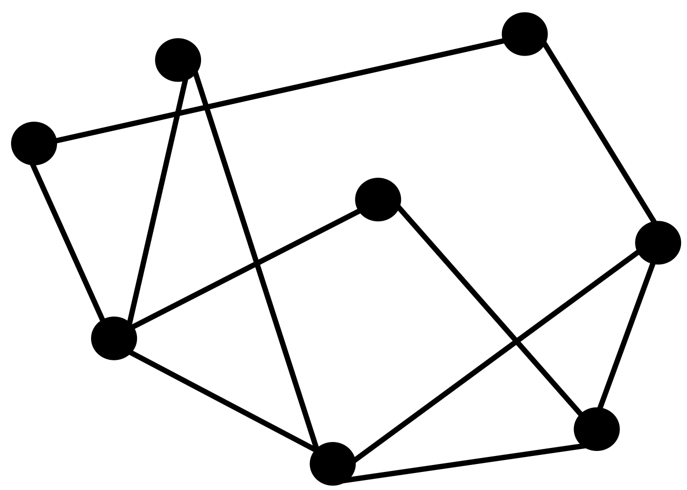

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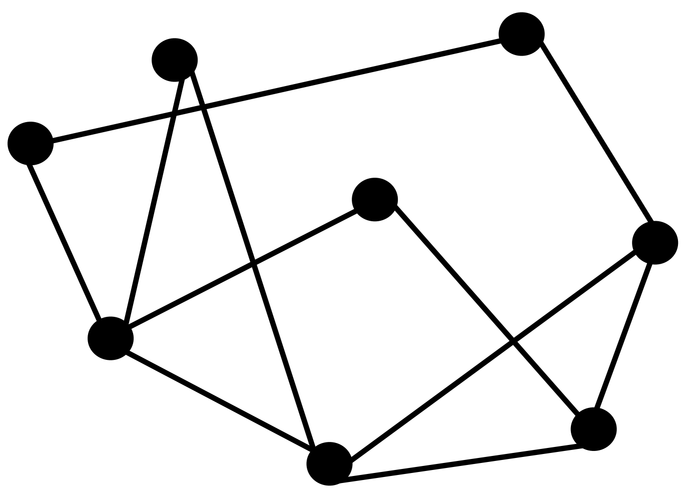

An oracle $\mathcal{O}$ sampling edges u.a.r

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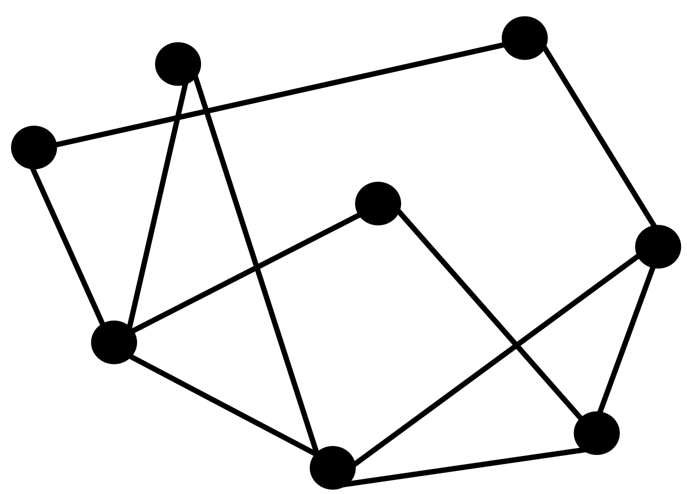

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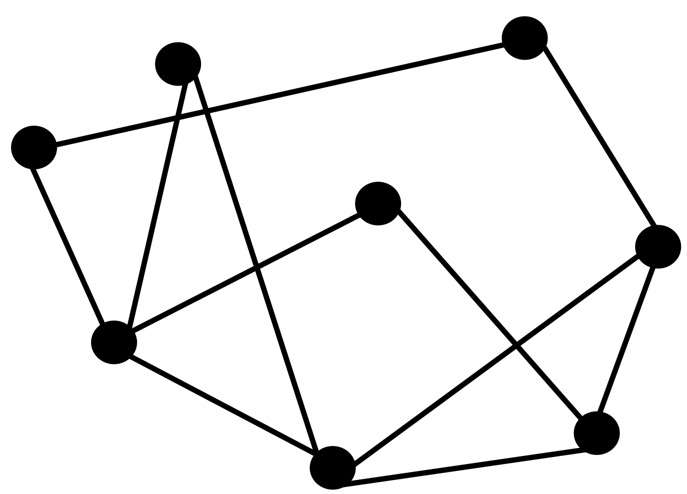

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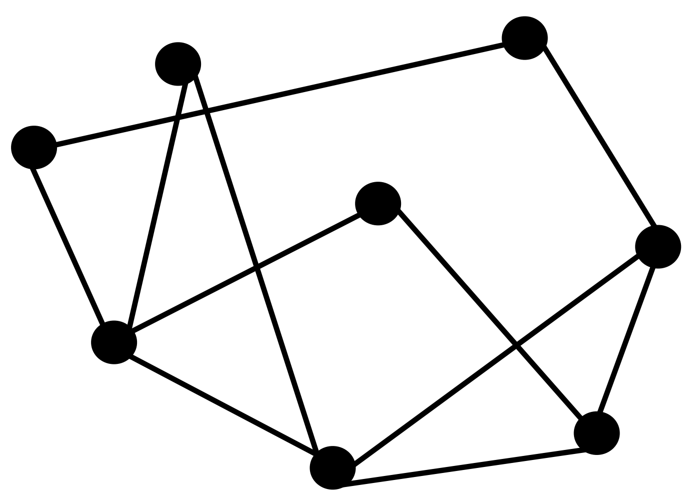

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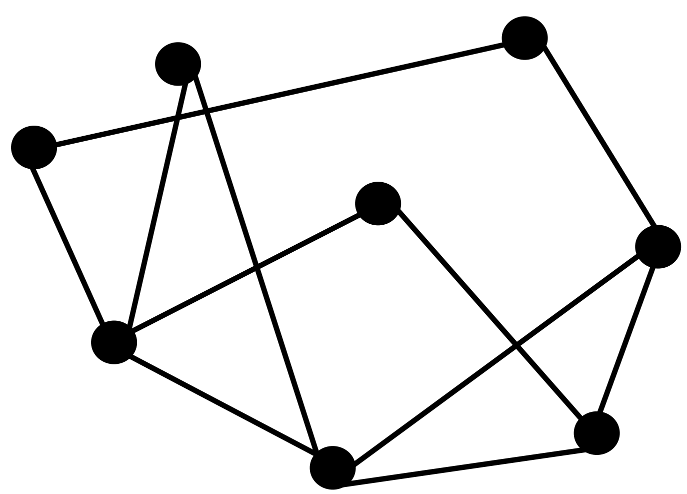

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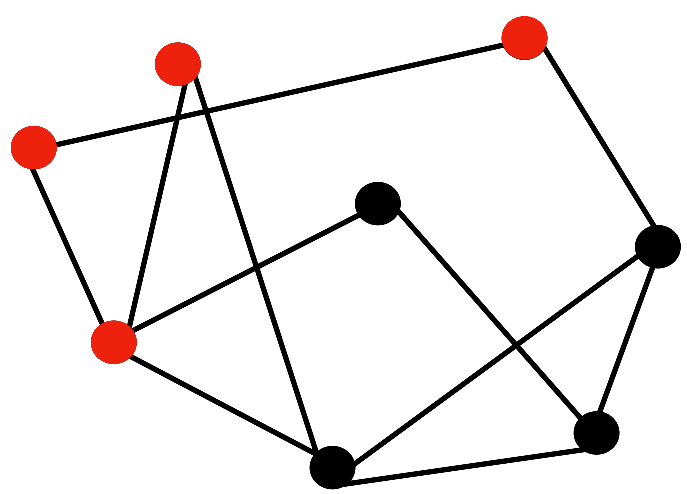

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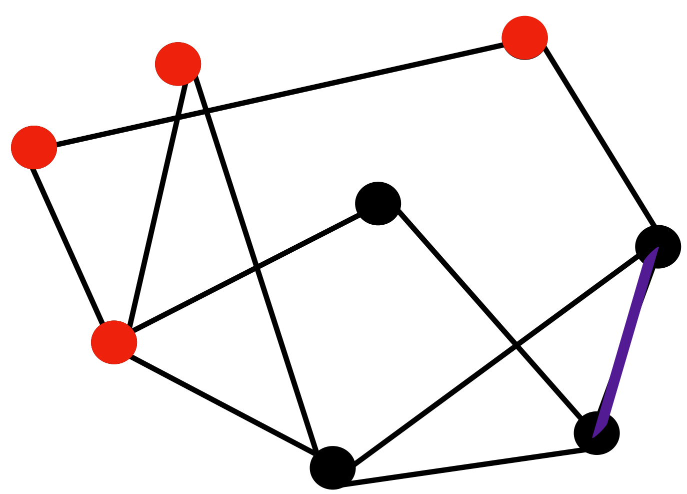

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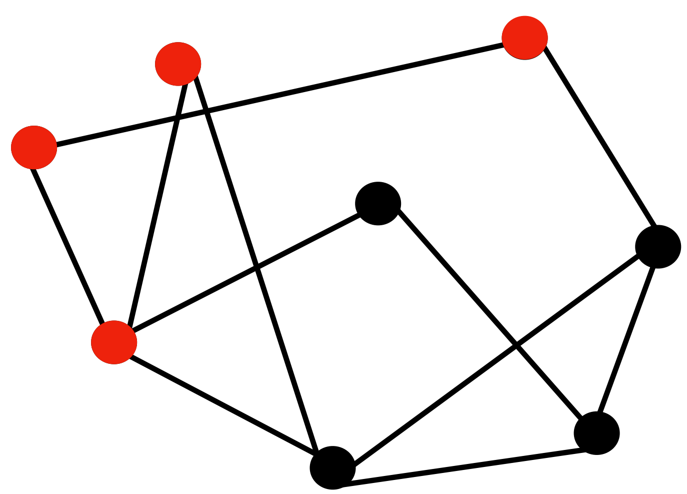

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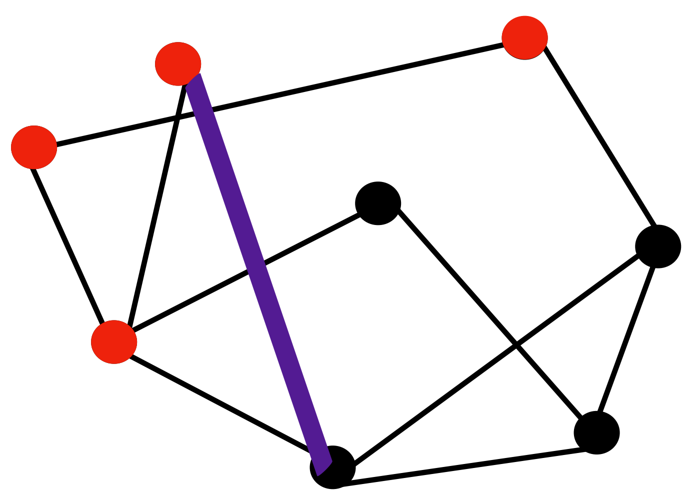

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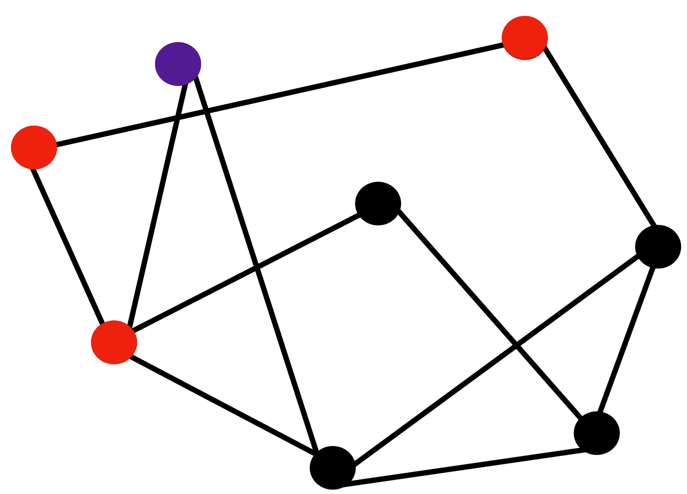

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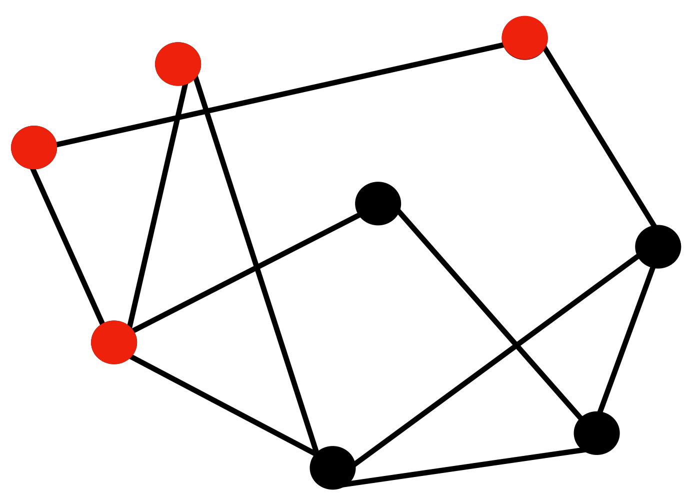

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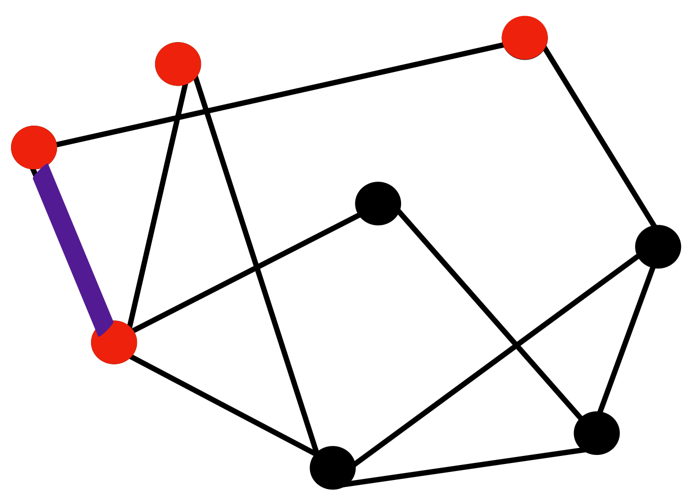

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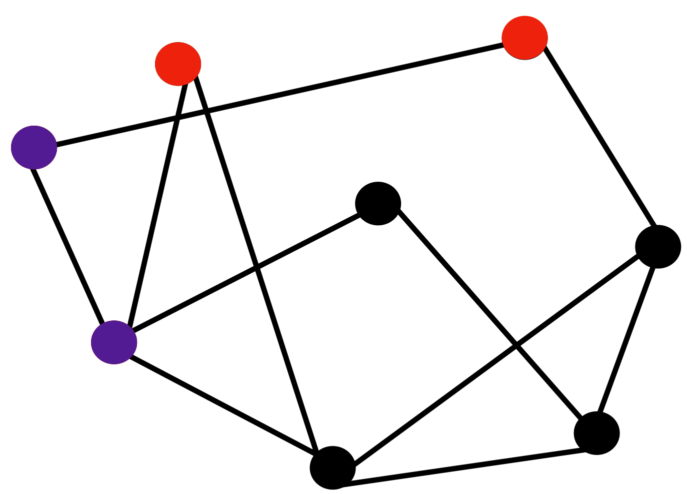

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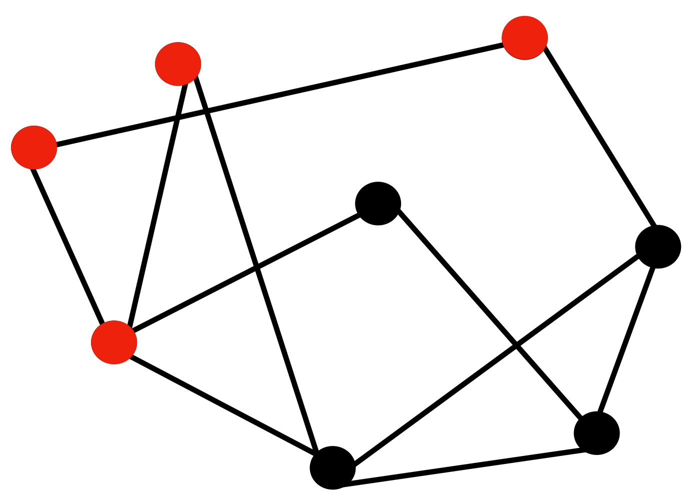

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Thm 2 : There exist $0 < \epsilon_0 < \epsilon_1 < 1$ such that distinguishing a function ϵ_0-close to a k-junta and a function ϵ_1-far from any k-junta requires $\widetilde{\Omega}(k^2)$ non-adaptive queries.

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Given the non-adaptive $\widetilde{O}(k^{3/2})$ tester of Blais [Bla08] (where $\epsilon_0 = 0$), we conclude that non-adaptive tolerant junta testing requires more queries than non-tolerant testing.

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Thm 2 : There exist $0 < \epsilon_0 < \epsilon_1 < 1$ such that distinguishing a function ϵ_0 -close to a unate and a function ϵ_1 -far from any unate function requires $\widetilde{\Omega}(n^{3/2})$ non-adaptive queries.

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Given the non-adaptive $\widetilde{O}(n)$ tester of Baleshzar et al. [BCP+17] (where $\epsilon_0 = 0$), and the adaptive $\widetilde{O}(n^{3/4})$ tester of Chen et al. [CWX17], we conclude that tolerant unateness testing requires more queries than non-tolerant testing in both settings.