ProbGraph: High-Performance and High-Accuracy Graph Mining with Probabilistic Set Representations

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Motivation

- Graph mining is slow
  - Hard to parallelize since there exists little locality and irregularities in some graphs
- Useful to many problems in modern graphs
  - Examples: Triangle Counting, Clique Counting, Vertex Similarity, Graph Clustering
Contributions

- Provides an approximate algorithm trading accuracy for speed
- Helps general class of graph problems requiring set intersections in their routines.
- Approximation is tunable, and claims up to 50x speedups with up to 90% accuracy
Data Review

- Claims appear to lack support in their data, there is high variance, and not a clear link on how they get 98% or 90% accuracy claims.

Fig. 3: Analysis of the accuracy of PG estimators of $|X \cap Y|$. 
Additional Data Review
Overview

- Provide background on triangle counting to use as a motivating example
- Recognize a common subroutine in computation is set intersections
- Delve into Bloom Filters and MinHash approximation algorithms
- Show approximation algorithm given in ProbGraph
Triangle Counting

- Find all unique triples such that each pair of vertices shares an edge.
- Used to analyze real world graphs: cluster coefficient, spam filtering, find structure.
- There is an $n^3$ algorithm: enumerate all triples and check.
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Triangle Counting Faster Approach

- Let $U, V$ be neighboring vertices and $N_x$ be the neighbors of $x$
- Then $N_U \cap N_V \setminus \{U, V\}$ are triangles
Triangle Counting Algorithm

// Derive a vertex order R s.t if R(v) < R(u) then d_v ≤ d_u
for v ∈ V do: N_v^+ = {u ∈ N_v | R(v) < R(u)}

tc = 0
for v ∈ V do:
  for u ∈ N_v^+ do: tc += |N_v^+ ∩ N_u^+|

- Let d be max degree and n be number of nodes
- Initial loop takes O(nd)
- Main loop takes O(nd^2)
// Derive a vertex order $R$ s.t if $R(v) < R(u)$ then $d_v \leq d_u$
for $v \in V$ [in par] do: $N_v^+ = \{u \in N_v \mid R(v) < R(u)\}$

tc = 0
for $v \in V$ [in par] do:
    for $u \in N_v^+$ [in par] do: $tc += |N_v^+ \cap N_v^+|$
Other examples

- Clique Counting
- Vertex Similarity
- Graph Clustering

Listing 2: Reformulated 4-Clique Counting.

```java
1 /* Input: A graph G. Output: Number of 4-cliques ck ∈ N. */
2 / Derive a vertex order R s.t. if R(u) < R(w) then d_u ≤ d_w:
3 for v ∈ V [in par] do: N_v^+ = {u ∈ N_v | R(v) < R(u)}
4 ck = 0;
5 for u ∈ V [in par] do:
6   for v ∈ N_u^+ [in par] do:
7     C_3 = N_v^+ ∩ N_u^+ //Find 3-cliques
8     for w ∈ C_3 do: //For each 3-clique...
9       ck += |N_w^+ ∩ C_3| //Find 4-cliques
```

Listing 4: Jarvis-Patrick clustering.

```java
1 /* Input: A graph G = (V, E). Output: Clustering C ⊆ E */
2 /* of a given prediction scheme. */
3 // Use a similarity S_C(v, u) = |N_v ∩ N_u| (see Listing 3).
4 for e = (v, u) ∈ E [in par] do: //τ is a user-defined threshold
5   if |N_v ∩ N_u| > τ: C ∪= {e}
6 //Other clustering schemes use other similarity measures.
```
Bottleneck

- $|X \cap Y|$ is slow
How to make $|X \cap Y|$ faster?

- Trading some accuracy for speed
- Use of Bloom Filters and MinHash sets to approximate these intersections
Bloom Filter

- Want space efficient/fast answering to membership queries
- False positives
- Bloom filter has L element bit vector
  - Set of hashes, \( \{h_i\} \), computes an integer in \([1, L]\)
- Add Element
  - Compute each hash, set corresponding bit to 1
- Retrieve Element
  - Compute each hash, if all 1, return True

2 hashes, \( L \in [1, 3] \)

Insert a -> \( \{1, 1\} \)
BF = 100
Insert b -> \( \{3\} \)
BF = 101

Now c -> \( \{1, 3\} \) would be “contained” although not inserted
MinHash

- Take $k$ hashes, $h_1, h_2, \ldots, h_k$
- Compute hash for each element
- Keep values that produce the smallest per hash values
- Variant (1-Hash): keep $k$ smallest hash values using 1 hash function

\[
\{\min \{h_1\}, \min \{h_2\}, \ldots, \min \{h_k\}\} \quad \text{or} \quad \{\min \{h\}, \min \{h\} / \min \{h\}, \ldots\} \]
Approximating Intersections

- Two Options:
  - Take bitwise and of Bloom Filter and compute popcount
  - Find intersection of smaller MinHash sets

Size of bloom filter (B), cache word size (W), size of MinHash set (k)
ProbGraph Implementation

- Set a storage limit as a percentage of the graph size
- Now Bloom Filter and MinHash representations exist for the neighborsets of every node with parameters chosen not to exceed this size limit.
- Choose what approximate algorithm you would like to use.
- Very fast to compute as both approximations are much smaller than original neighbor sets.
- Additionally BF is easily vectorized.

```c
1 // Input: Graph G, two vertices u and v
2 // Create a standard CSR graph with G as the input graph
3 CSRGraph g = CSRGraph(G);
4 // Create a ProbGraph representation of G based on Bloom filters
5 ProbGraph pg = ProbGraph(g, BF, 0.25); // Use the 25% storage budget
6
7 // Derive the exact intersection cardinality |N_u \cap N_v|
8 int interEX = pg.int_card(g.N(u), g.N(v));
9 // Derive the estimator |N_u \cap N_v|_{AND}
10 int interBF = pg.int_BF_AND(pg.N(u), pg.N(v));
11
12 // Compute the exact Jaccard coefficient between u and v
13 double jacEX = interEX / (g.N(u).size() + g.N(v).size() - interEX)
14 // Compute the approximate Jaccard coefficient based on BF
15 double jacBF = interBF / (g.N(u).size() + g.N(v).size() - interBF)
```

Listing 5: Obtaining exact and approximate Jaccard (see Listing 3)
Questions?