Multicore Triangle Computations Without Tuning

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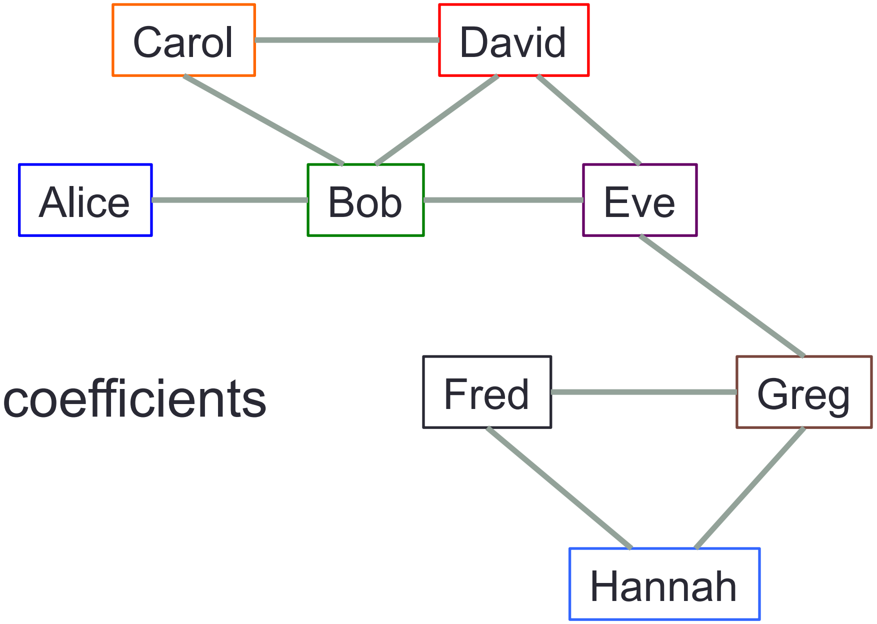
Presentation is based on paper published in International Conference on Data Engineering (ICDE), 2015

Triangle Computations



- Triangle Counting

  Count = 3

- Other variants:
  - Triangle listing
  - Local triangle counting/clustering coefficients
  - Triangle enumeration
  - Approximate counting
  - Analogs on directed graphs

- Numerous applications…
  - Social network analysis, Web structure, spam detection, outlier detection, dense subgraph mining, 3-way database joins, etc.

Need fast triangle computation algorithms!

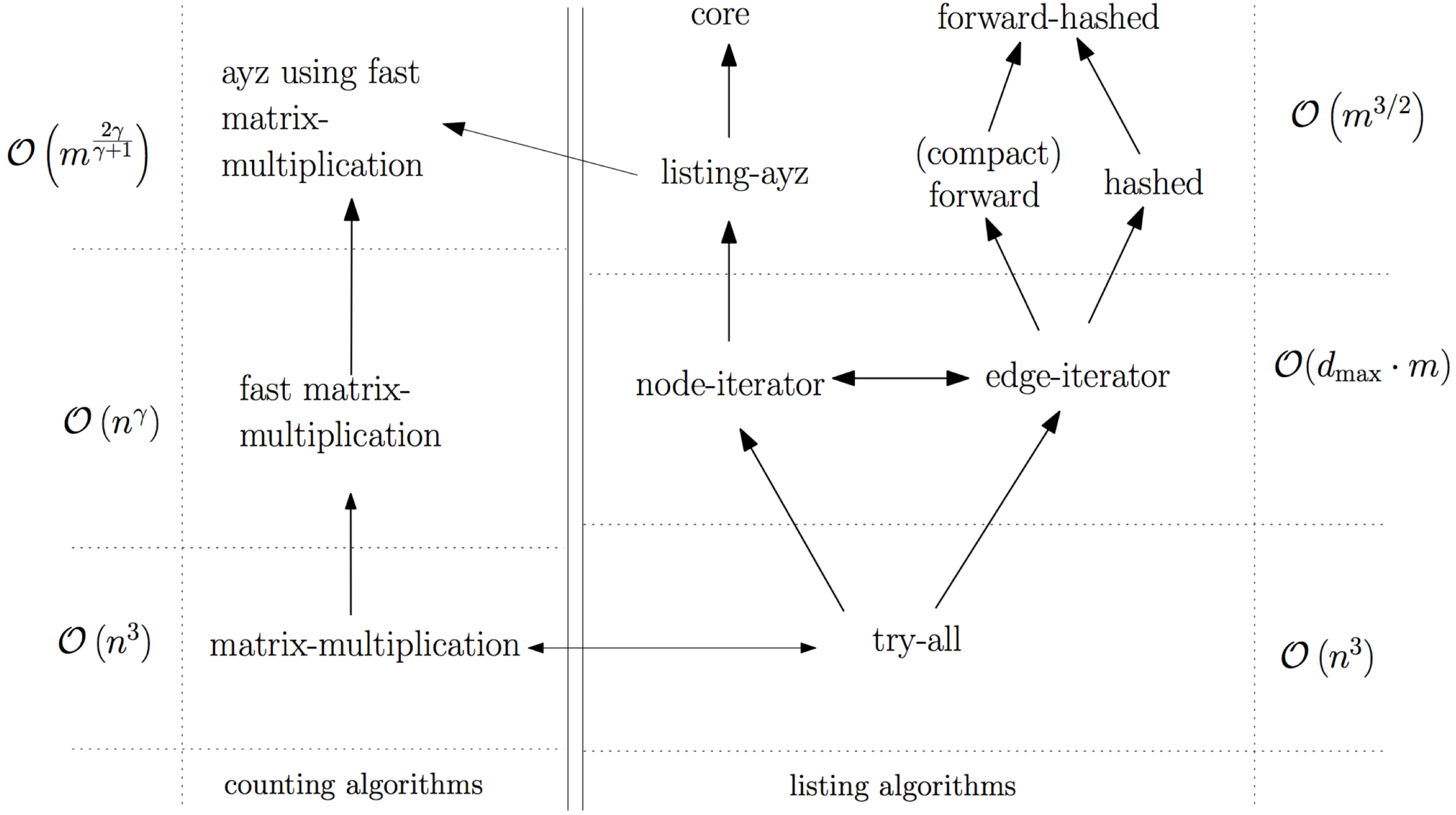
Sequential Triangle Computation Algorithms

V = # vertices E = # edges

- Sequential algorithms for exact counting/listing
  - Naïve algorithm of trying all triplets
    $O(V^3)$ work
  - Node-iterator algorithm [Schank]
    $O(VE)$ work
  - Edge-iterator algorithm [Itai-Rodeh]
    $O(VE)$ work
  - Tree-lister [Itai-Rodeh], forward/compact-forward [Schank-Wagner, Lapaty]
    $O(E^{1.5})$ work
- Sequential algorithms via matrix multiplication
  - $O(V^{2.37})$ work compute A^3, where A is the adjacency matrix
  - $O(E^{1.41})$ work [Alon-Yuster-Zwick]
  - These require superlinear space

Sequential Triangle Computation Algorithms

Source: "Algorithmic Aspects of Triangle-Based Network Analysis", Dissertation by Thomas Schank



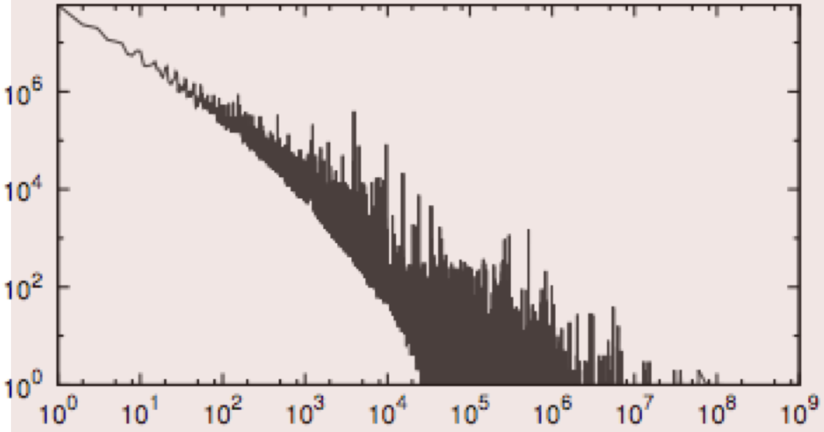
What about parallel algorithms?

Parallel Triangle Computation Algorithms

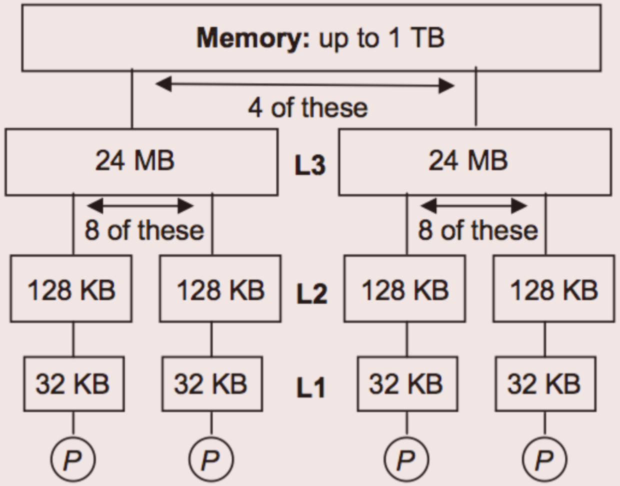
- Most designed for distributed memory
  - MapReduce algorithms [Cohen ’09, Suri-Vassilvitskii ‘11, Park-Chung ‘13, Park et al. ‘14]
  - MPI algorithms [Arifuzzaman et al. ‘13, Graphlab]
- *What about shared-memory multicore?*
  - Multicores are everywhere!
  - Node-iterator algorithm [Green et al. ‘14]
    - O(VE) work in worst case
- *Can we obtain an $O(E^{1.5})$ work shared-memory multicore algorithm?*

Triangle Computation: Challenges for Shared Memory Machines

1 *Irregular computation*



2 *Deep memory hierarchy*


Memory: up to 1 TB

4 of these

24 MB **L3** 24 MB

8 of these 8 of these

128 KB 128 KB **L2** 128 KB 128 KB

32 KB 32 KB **L1** 32 KB 32 KB

P *P* *P* *P*

External-Memory and Cache-Oblivious Triangle Computation

- All previous algorithms are sequential
- External-memory (cache-aware) algorithms

Natural-join	$O(E^3/(M^2 B))$ I/O's
Node-iterator [Dementiev '06]	$O((E^{1.5}/B) \log_{M/B}(E/B))$ I/O's
Compact-forward [Menegola '10]	$O(E + E^{1.5}/B)$ I/O's
[Chu-Cheng '11, Hu et al. '13]	$O(E^2/(MB) + \#triangles/B)$ I/O's

- External-memory and cache-oblivious

[Pagh-Silvestri '14]	$O(E^{1.5}/(M^{0.5} B))$ I/O's or cache misses

- *Parallel cache-oblivious algorithms?*

Our Contributions

1 *Parallel Cache-Oblivious Triangle Counting Algs*

Algorithm	Work	Depth	Cache Complexity
TC-Merge	$O(E^{1.5})$	$O(\log^2 E)$	$O(E + E^{1.5}/B)$
TC-Hash	$O(V \log V + \alpha E)$	$O(\log^2 E)$	$O(\mathrm{sort}(V) + \alpha E)$
Par. Pagh-Silvestri	$O(E^{1.5})$	$O(\log^3 E)$	$O(E^{1.5}/(M^{0.5} B))$

V = # vertices
M = cache size

E = # edges
B = line size

α = arboricity (at most $E^{0.5}$)
$\mathrm{sort}(n) = (n/B) \log_{M/B}(n/B)$

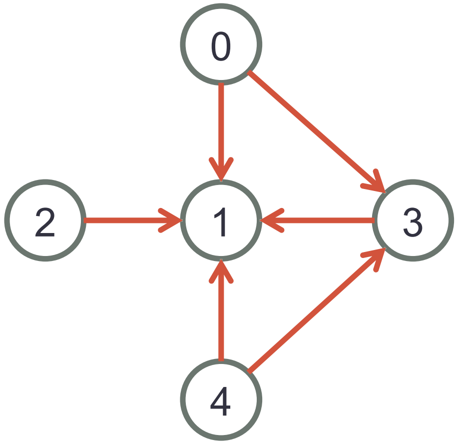
2 *Extensions to Other Triangle Computations:*

Enumeration, Listing, Local Counting/Clustering Coefficients, Approx. Counting, Variants on Directed Graphs

3 *Extensive Experimental Study*

Sequential Triangle Counting (Exact)

(Forward/compact-forward algorithm)



①

Rank vertices by degree (sorting)

Return A[v] for all v storing higher ranked neighbors

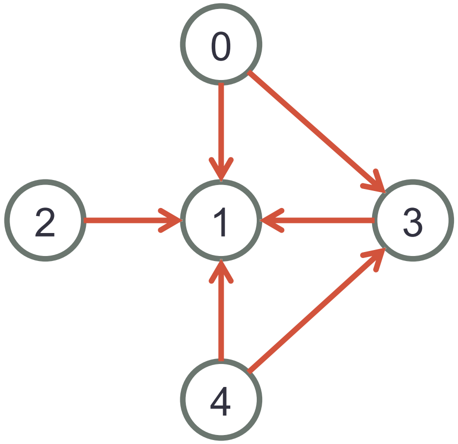
②

```
for each vertex v:
    for each w in A[v]:
        count += intersect(A[v], A[w])
```

Gives all triangles (v, w, x) where rank(v) < rank(w) < rank(x)

Work = $O(E^{1.5})$
[Schank-Wagner ‘05, Latapy ‘08]

Proof of $O(E^{1.5})$ work bound when intersect uses merging



①

Rank vertices by degree (sorting)

Return A[v] for all v storing higher ranked neighbors

②

```
for each vertex v:
    for each w in A[v]:
        count += intersect(A[v], A[w])
```

- Step 1: $O(E+V \log V)$ work
- Step 2:
  - For each edge (v,w), intersect does $O(d^+(v) + d^+(w))$ work
  - For all v, $d^+(v) \leq E^{0.5}$
    - If $d^+(v) > E^{0.5}$, each of its higher degree neighbors also have degree $> E^{0.5}$ and total number of directed edges $> E$, a contradiction
  - Total work = $E * O(E^{0.5}) = O(E^{1.5})$

Parallel Triangle Counting (Exact)

Step 1

Work = O(E+V log V)

Depth = $O(\log^2 V)$

Cache = O(E+sort(V))

Parallel sort and filter

Rank vertices by degree (sorting)

Return A[v] for all v storing higher ranked neighbors

1

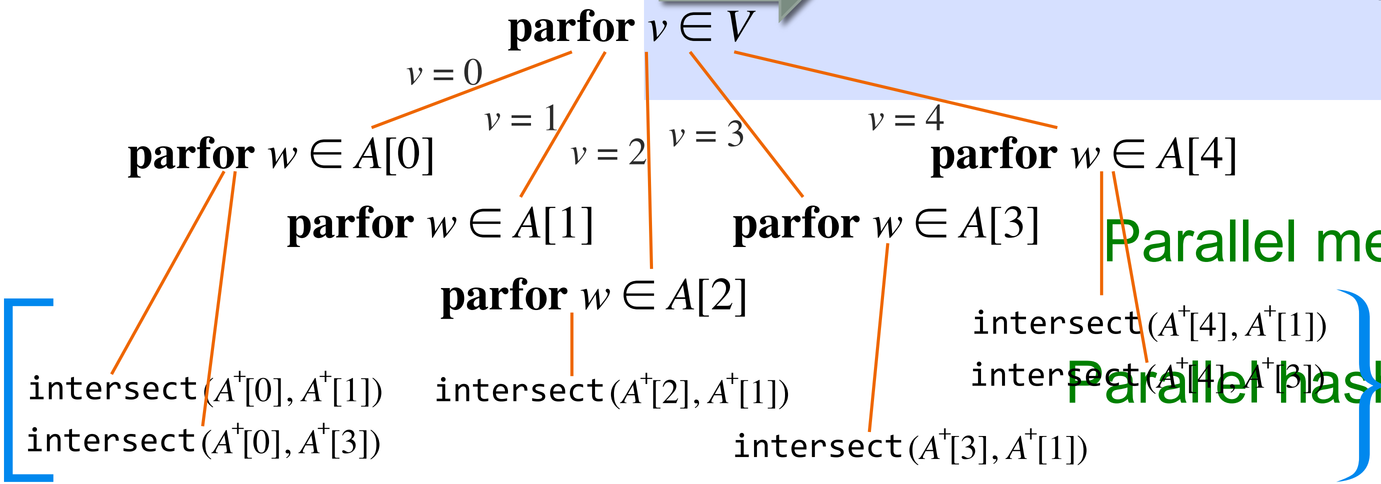
parallel_for each vertex v:

  parallel_for each w in A[v]:

    count += intersect(A[v], A[w])

2

Parallel reduction



$\textbf{parfor } v \in V$

$v = 0$: $\textbf{parfor } w \in A[0]$ — `intersect`$(A^+[0], A^+[1])$, `intersect`$(A^+[0], A^+[3])$

$v = 1$: $\textbf{parfor } w \in A[1]$

$v = 2$: $\textbf{parfor } w \in A[2]$ — `intersect`$(A^+[2], A^+[1])$

$v = 3$: $\textbf{parfor } w \in A[3]$ — `intersect`$(A^+[3], A^+[1])$

$v = 4$: $\textbf{parfor } w \in A[4]$ — `intersect`$(A^+[4], A^+[1])$, `intersect`$(A^+[4], A^+[3])$

safe to run all in parallel

Parallel merge (TC-Merge)

or

Parallel hash table (TC-Hash)

TC-Merge and TC-Hash Details

```
parallel_for each vertex v:
    parallel_for each w in A[v]:
        count += intersect(A[v], A[w])
```

2

Parallel reduction

Parallel merge (TC-Merge)
or
Parallel hash table (TC-Hash)

Step 2: TC-Merge
Work = $O(E^{1.5})$
Depth = $O(\log^2 E)$
Cache = $O(E+E^{1.5}/B)$

Step 2: TC-Hash
Work = $O(\alpha E)$
Depth = $O(\log E)$
Cache = $O(\alpha E)$

(α = arboricity (at most $E^{0.5}$))

- TC-Merge
  - Preprocessing: sort adjacency lists
  - Intersect: use a parallel and cache-oblivious merge based on divide-and-conquer [Blelloch et al. ‘10, Blelloch et al. ‘11]
- TC-Hash
  - Preprocessing: for each vertex, create parallel hash table storing edges [Shun-Blelloch ‘14]
  - Intersect: scan smaller list, querying hash table of larger list in parallel

Comparison of Complexity Bounds

Algorithm	Work	Depth	Cache Complexity
TC-Merge	$O(E^{1.5})$	$O(\log^2 E)$	$O(E + E^{1.5}/B)$ *(oblivious)*
TC-Hash	$O(V \log V + \alpha E)$	$O(\log^2 E)$	$O(\mathrm{sort}(V) + \alpha E)$ *(oblivious)*
Par. Pagh-Silvestri	$O(E^{1.5})$	$O(\log^3 E)$	$O(E^{1.5}/(M^{0.5} B))$ *(oblivious)*
Chu-Cheng ‘11, Hu et al. ‘13	$O(E \log E + E^2/M + \alpha E)$		$O(E^2/(MB) + \#\mathrm{triangles}/B)$ *(aware)*
Pagh-Silvestri ‘14	$O(E^{1.5})$		$O(E^{1.5}/(M^{0.5} B))$ *(oblivious)*
Green et al. ’14	$O(VE)$	$O(\log E)$	

V = # vertices
M = cache size

E = # edges
B = line size

α = arboricity (at most $E^{0.5}$)
$\mathrm{sort}(n) = (n/B) \log_{M/B}(n/B)$

Our Contributions

1 *Parallel Cache-Oblivious Triangle Counting Algs*

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TC-Hash	$O(V \log V + \alpha E)$	$O(\log^2 E)$	$O(sort(V) + \alpha E)$
Par. Pagh-Silvestri	$O(E^{1.5})$	$O(\log^3 E)$	$O(E^{1.5}/(M^{0.5} B))$

V = # vertices

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2 *Extensions to Other Triangle Computations:*

Enumeration, Listing, Local Counting/Clustering Coefficients, Approx. Counting, Variants on Directed Graphs

3 *Extensive Experimental Study*

Extensions of Exact Counting Algorithms

- Triangle enumeration
  - Call **emit** function whenever triangle is found
  - **Listing**: add to hash table to list; return contents at the end
  - **Local counting/clustering coefficients**: atomically increment count of three triangle endpoints
- Directed triangle counting/enumeration
  - Keep separate counts for different types of triangles
- Approximate counting
  - Use colorful triangle sampling scheme to create smaller sub-graph [Pagh-Tsourakakis '12]
  - Run TC-Merge or TC-Hash on sub-graph with pE edges ($0 < p < 1$) and return #triangles/p^2 as estimate

Approximate Counting

Expected # edges = pE

- Colorful triangle counting [Pagh-Tsourakakis ’12]

Sampling rate: $0 < p < 1$

Parallel scan → ① Assign random color in $\{1, \ldots, 1/p\}$ to each vertex

Parallel filter → ② Sampling: Keep edges whose endpoints have the same color

Use TC-Merge or TC-Hash → ③ Run exact triangle counting on sampled graph, return $\Delta_{\text{sampled}}/p^2$

<u>Steps 1 & 2</u>
Work = $O(E)$
Depth = $O(\log E)$
Cache = $O(E/B)$

<u>Step 3: TC-Merge</u>
Work = $O((pE)^{1.5})$
Depth = $O(\log^2 E)$
Cache = $O(pE+(pE)^{1.5}/B)$

<u>Step 3: TC-Hash</u>
Work = $O(V \log V + \alpha pE)$
Depth = $O(\log E)$
Cache = $O(\text{sort}(V)+p\alpha E)$

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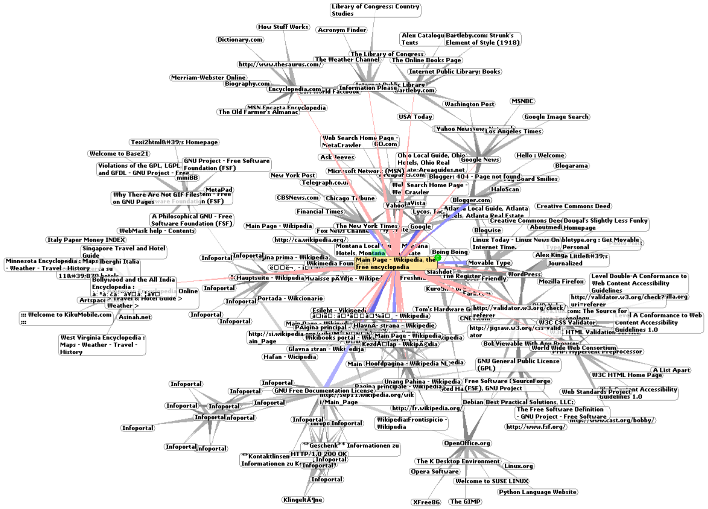
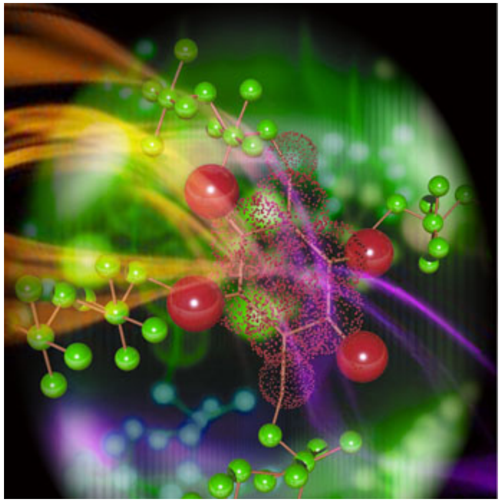
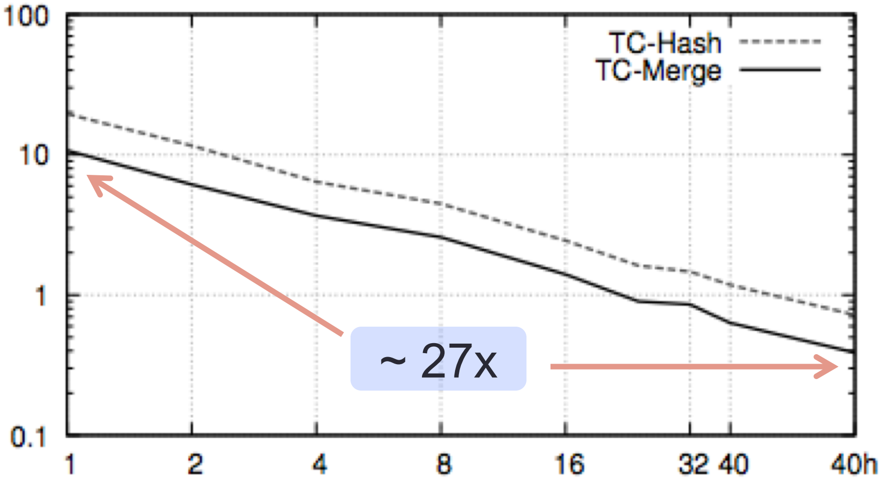
Experimental Setup

- Implementations using Intel Cilk Plus
- 40-core Intel Nehalem machine (with 2-way hyper-threading)
  - 4 sockets, each with 30MB shared L3 cache, 256KB private L2 caches
- Sequential TC-Merge as baseline (faster than existing sequential implementations)
- Other multicore implementations: Green et al. and GraphLab
- Our parallel Pagh-Silvestri algorithm was not competitive
- Variety of real-world and artificial graphs





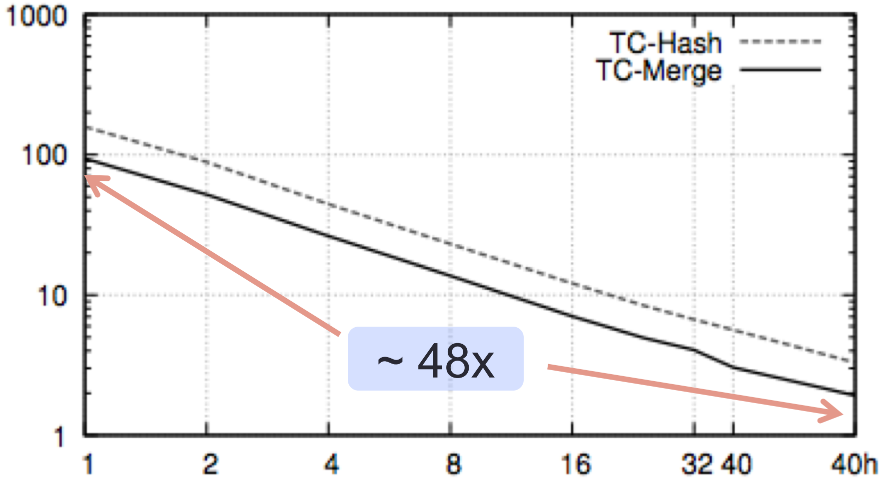
Both TC-Merge and TC-Hash scale well with # of cores:



~ 27x

LiveJournal

4M vtxes, 34.6M edges

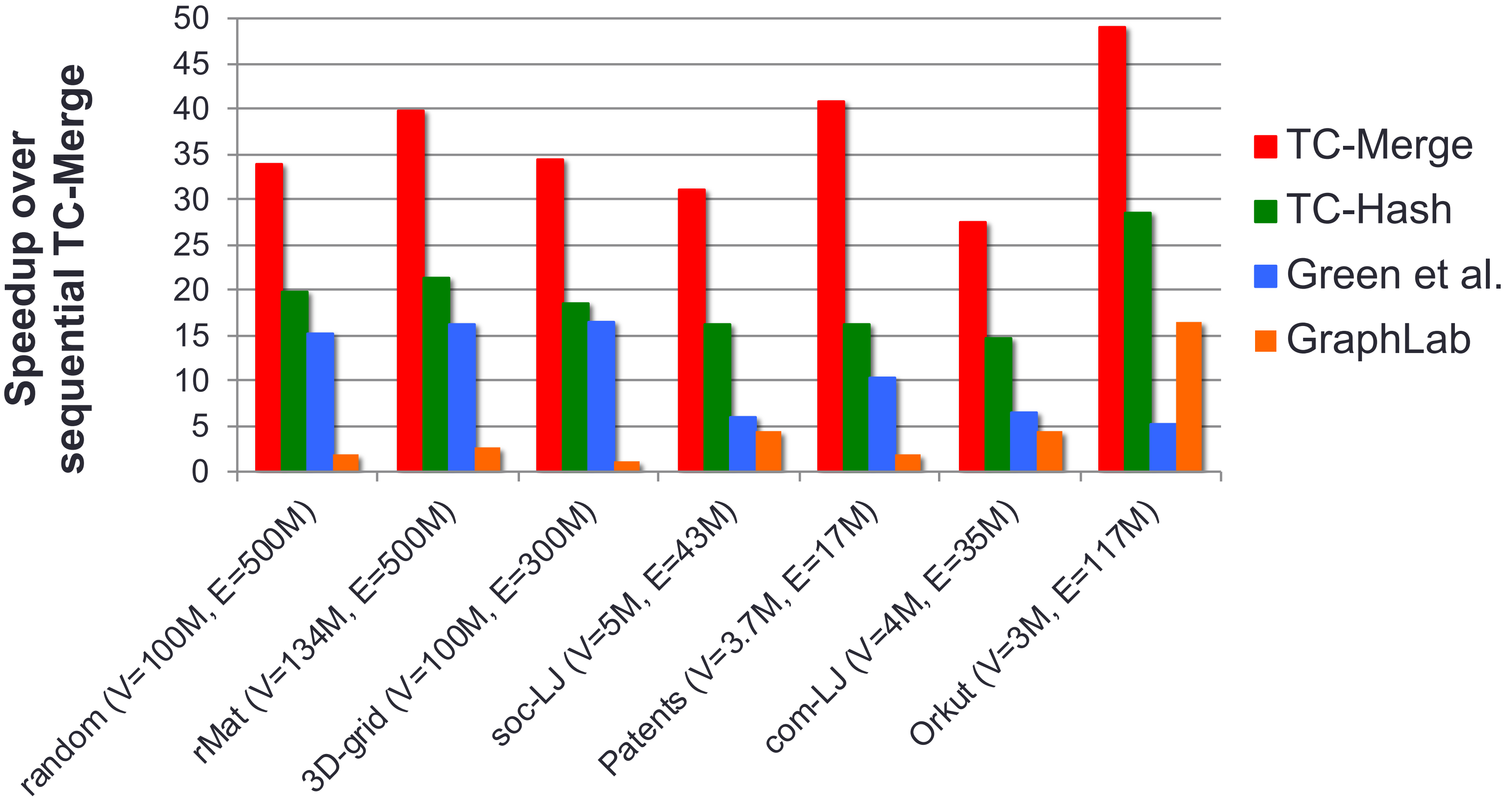


~ 48x

Orkut

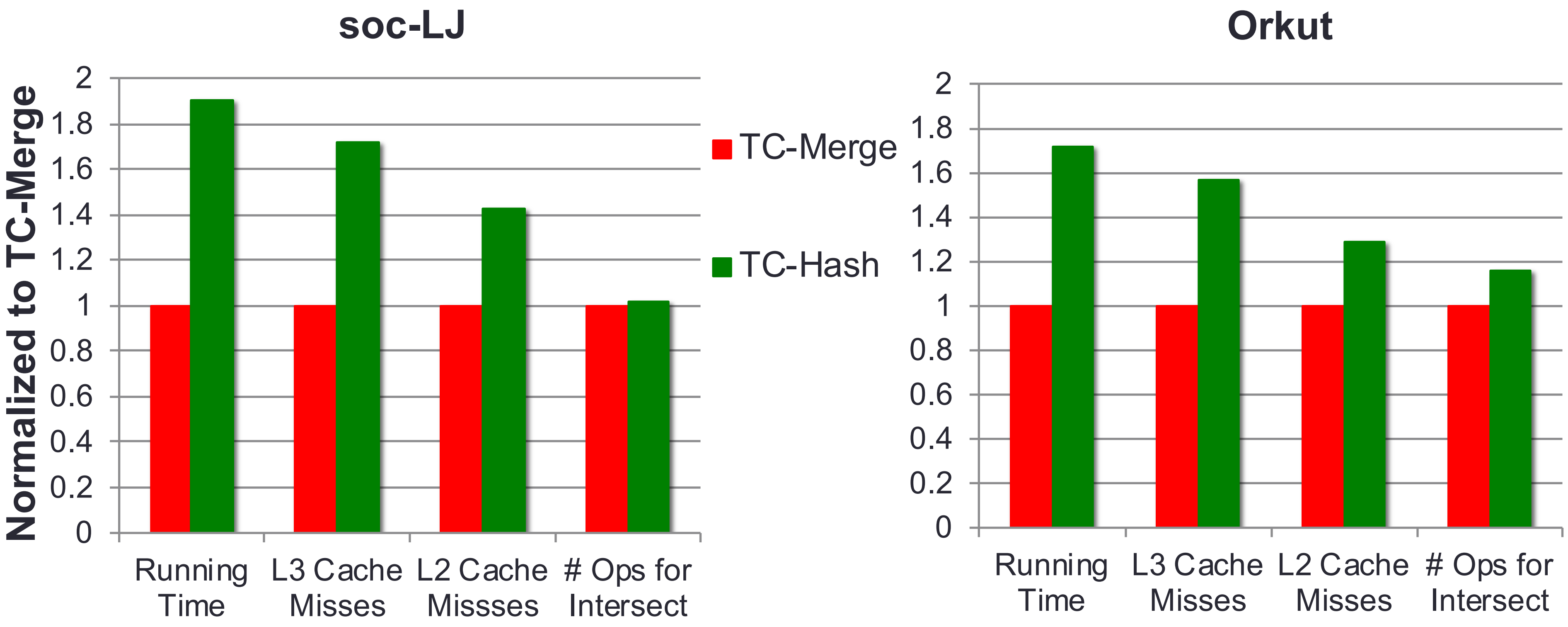
3M vtxes, 117M edges

40-core (with hyper-threading) Performance



- TC-Merge always faster than TC-Hash (by 1.3—2.5x)
- TC-Merge always faster than Green et al. or GraphLab (by 2.1—5.2x)

Why is TC-Merge faster than TC-Hash?



- TC-Hash less cache-efficient than TC-Merge
- Running time more correlated with cache misses than work

Comparison to existing counting algs.

Twitter graph (41M vertices, 1.2B undirected edges, 34.8B triangles)

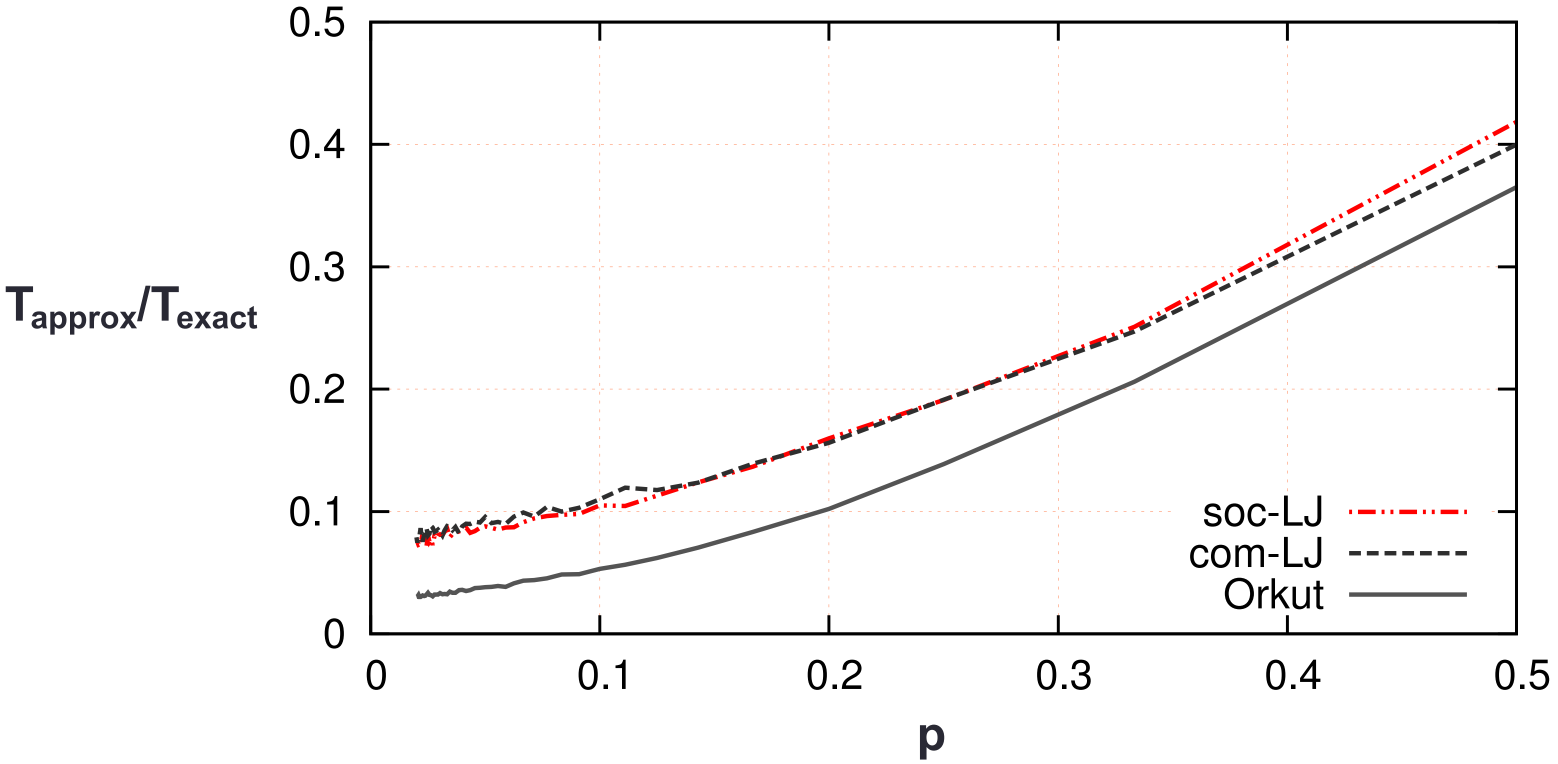
- **Yahoo graph** (1.4B vertices, 6.4B edges, 85.8B triangles) on 40 cores: TC-Merge takes 78 seconds
  - Approximate counting algorithm achieves 99.6% accuracy in 9.1 seconds

Shared vs. distributed memory costs

- Amazon EC2 pricing
  - Captures purchasing costs, maintenance/operating costs, energy costs

Triangle Counting (Twitter)	Our algorithm	GraphLab	GraphLab
Running Time	0.932 min	3 min	1.5 min
Machine	40-core (256 GB memory)	40-core (256 GB memory)	64 x 16-core
Approx. EC2 pricing	< $4/hour	< $4/hour	64 x $0.928/hour
Overall cost	< $0.062	< $0.2	$1.49

Approximate counting



p=1/25	Accuracy	T_{approx}	T_{approx}/T_{exact}
Orkut (V=3M, E=117M)	99.8%	0.067sec	0.035
Twitter (V=41M, E=1.2B)	99.9%	2.4sec	0.043
Yahoo (V=1.4B, E=6.4B)	99.6%	9.1sec	0.117

Conclusion

Algorithm	Work	Depth	Cache Complexity
TC-Merge	$O(E^{1.5})$	$O(\log^2 E)$	$O(E + E^{1.5}/B)$
TC-Hash	$O(V \log V + \alpha E)$	$O(\log^2 E)$	$O(\mathrm{sort}(V) + \alpha E)$
Par. Pagh-Silvestri	$O(E^{1.5})$	$O(\log^3 E)$	$O(E^{1.5}/(M^{0.5} B))$

- Simple multicore algorithms for triangle computations are provably work-efficient, low-depth, and cache-efficient
- Implementations require no load-balancing or tuning for cache
- Experimentally outperforms existing multicore and distributed algorithms

- Future work: Design a practical parallel algorithm achieving $O(E^{1.5}/(M^{0.5} B))$ cache complexity