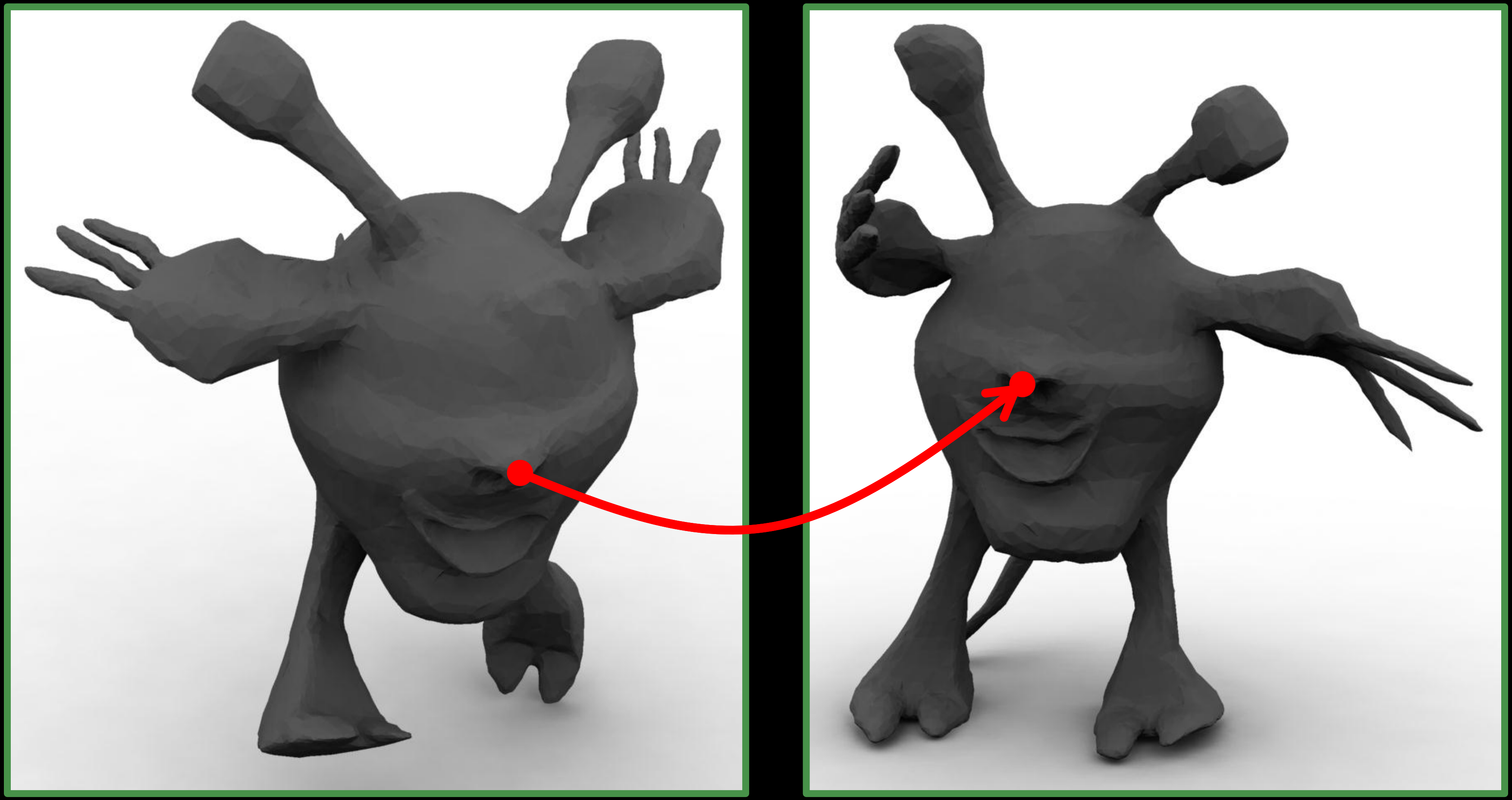


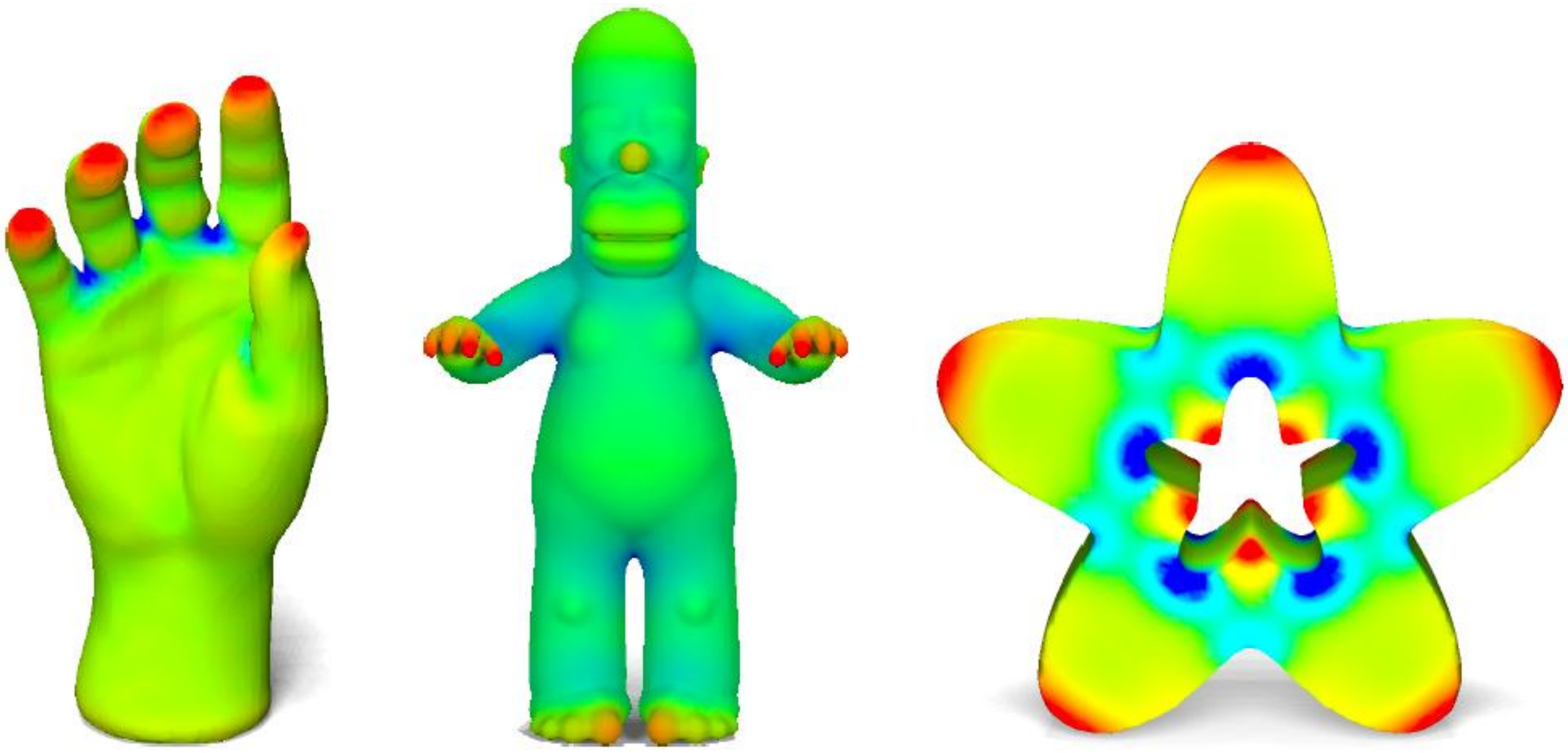
Shape Analysis and Correspondence

Justin Solomon

Geometric Computing Group

Stanford University

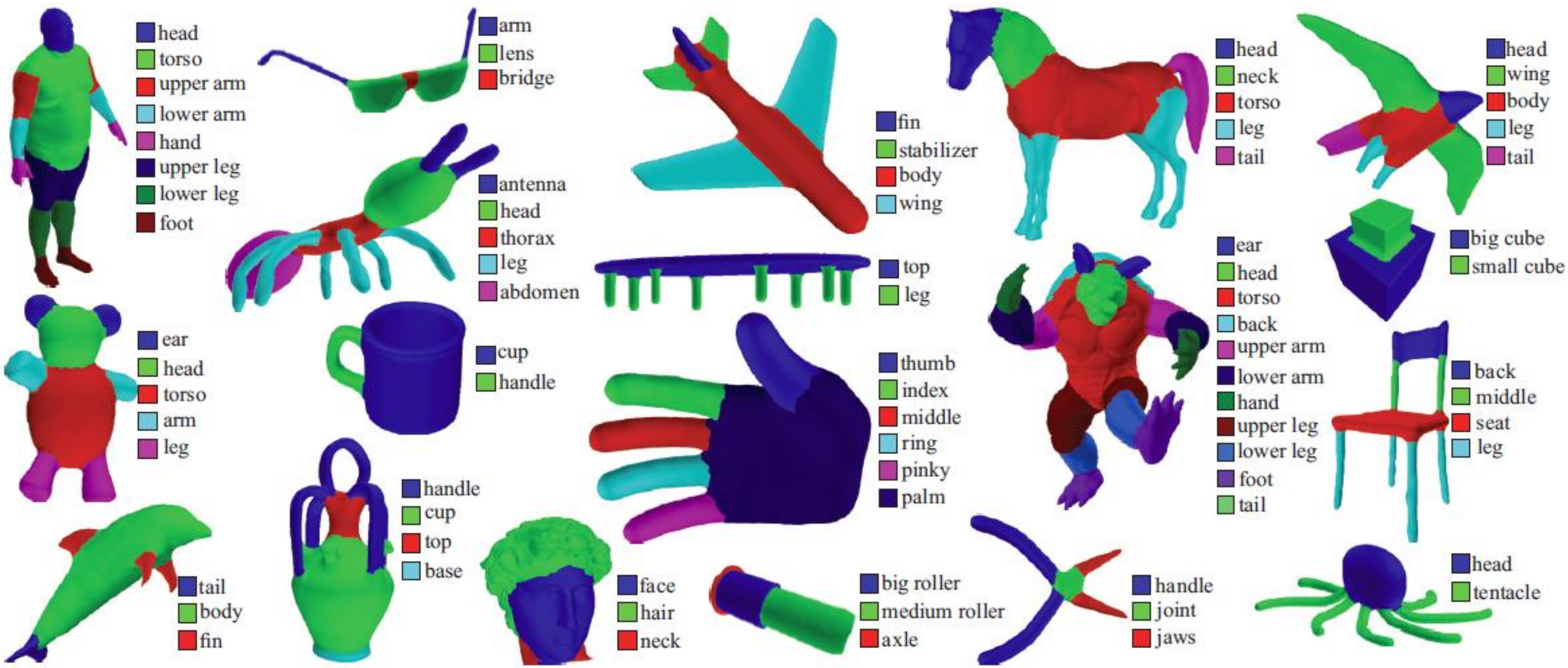
Basic Goals



http://graphics.stanford.edu/projects/lgl/papers/sog-hks-09/sog-hks-09.pdf

Compute shape descriptors

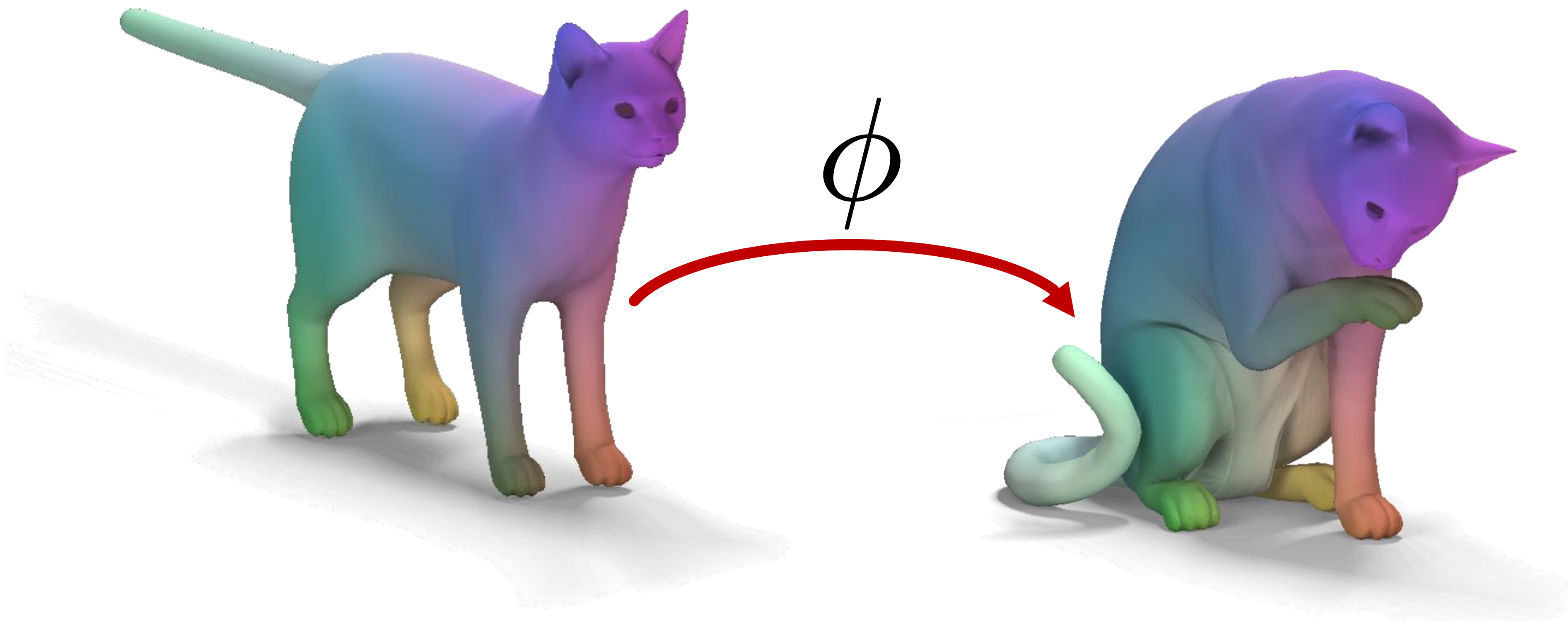
Basic Goals



http://people.cs.umass.edu/~kalo/papers/LabelMeshes/LabelMeshes.pdf

Extract important features

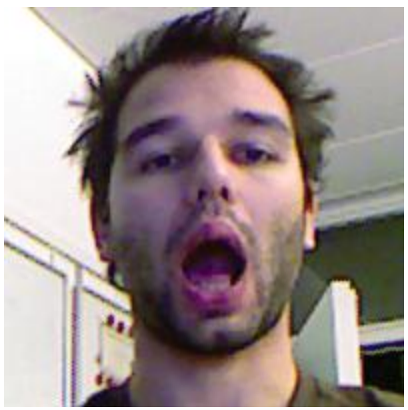
Basic Goals



http://www.stanford.edu/~justso1/assets/fmaps.pdf

Map shapes to one another

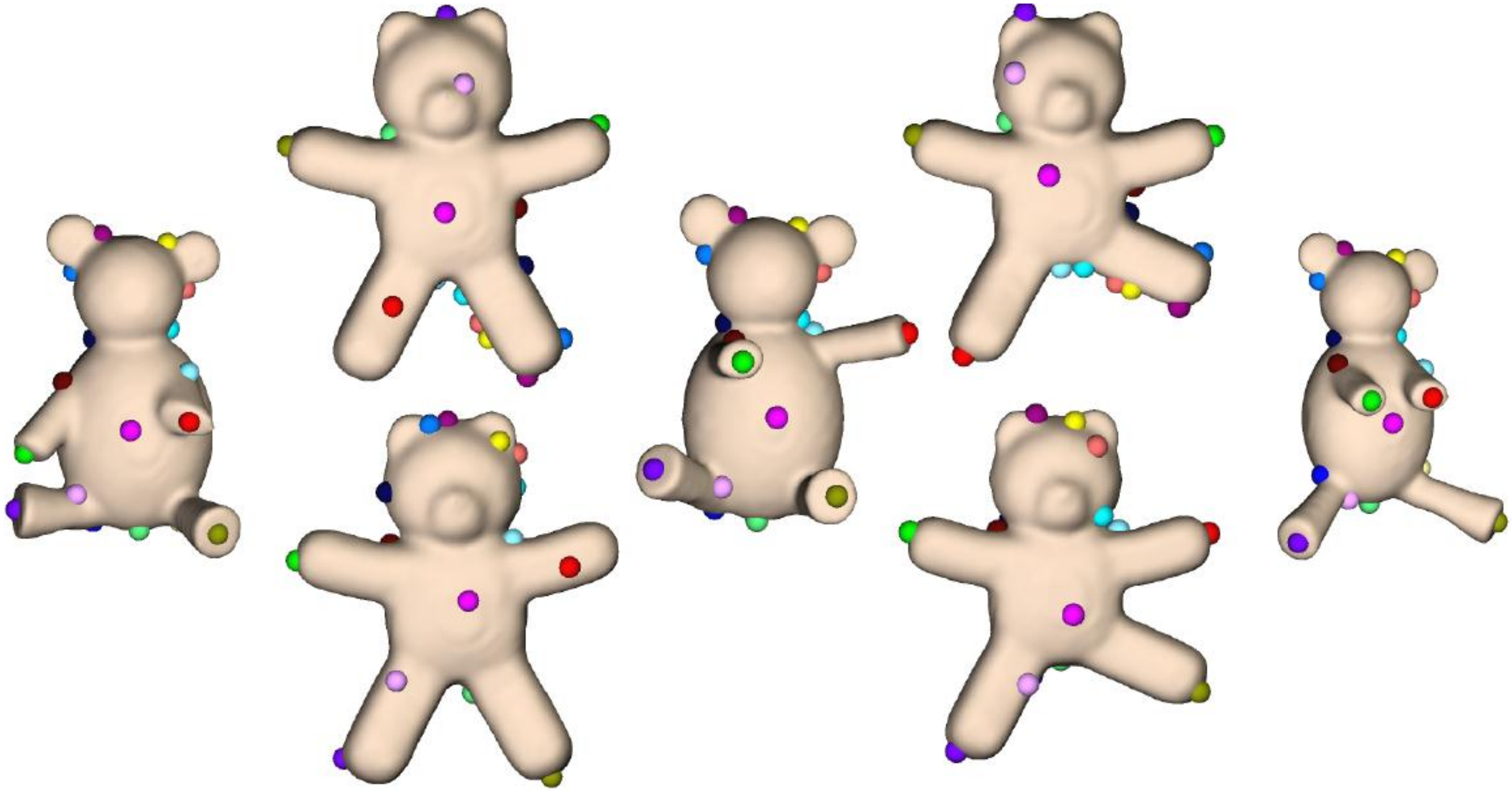
Basic Goals



http://www.hao-li.com/publications/papers/siggraph2011RPBFA.pdf

Relate new scans to known models

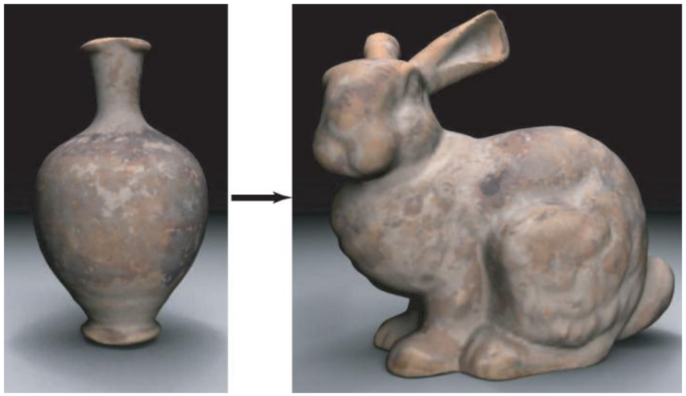
Basic Goals



http://graphics.stanford.edu/projects/lgl/papers/nbwyg-oaicsm-11/nbwyg-oaicsm-11.pdf

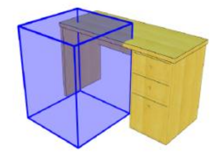
Understand collections of shapes

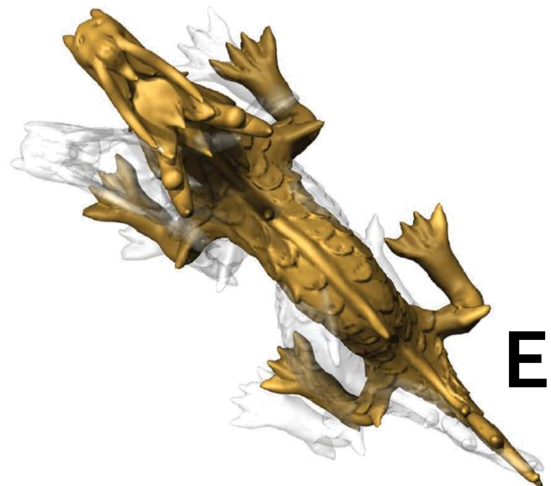
Example Applications



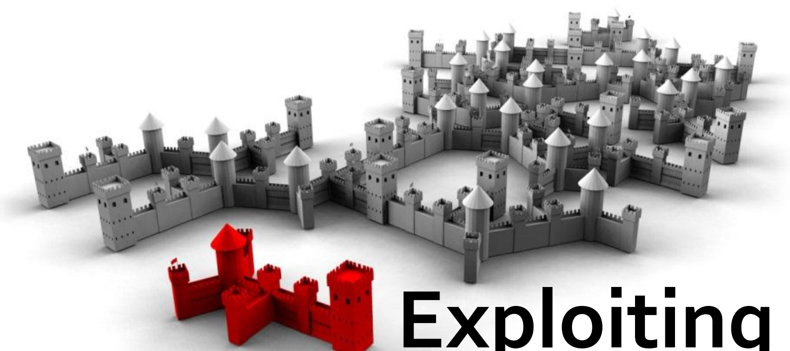
Transfer

Retrieval





Editing

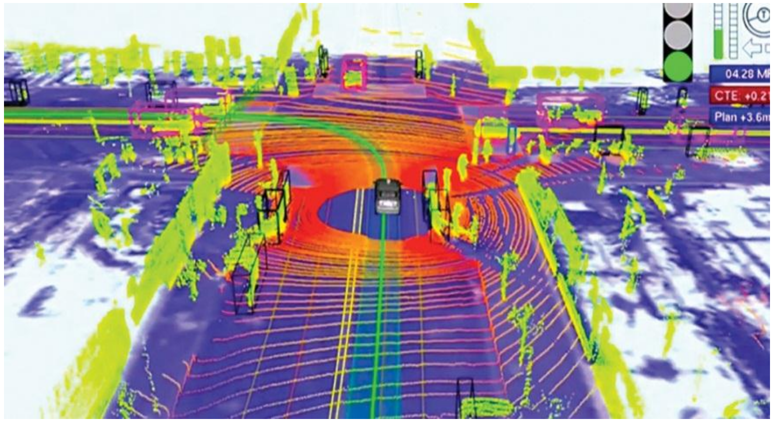


Exploiting patterns

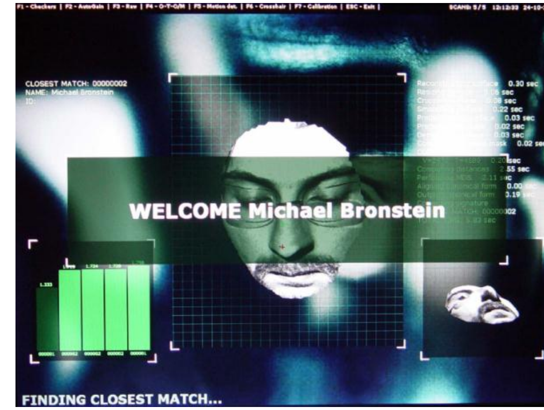
http://people.csail.mit.edu/tmertens/papers/textransfer_electronic.pdf
http://graphics.stanford.edu/~mdfisher/Data/Context.pdf
http://graphics.stanford.edu/~niloy/research/symmetrization/paper_docs/symmetrization_sig_07.pdf
http://www.mpi-inf.mpg.de/~mbokeloh/project_dockingSites.html

Graphics

Example Applications

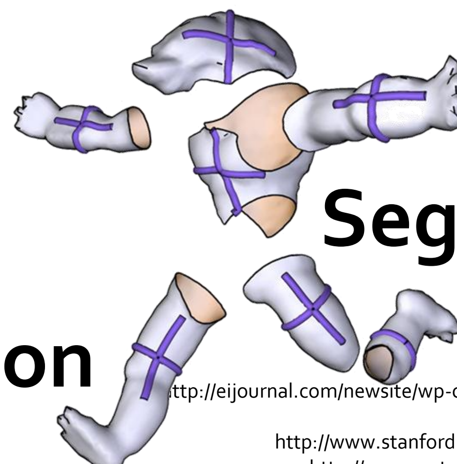


Recognition



Navigation

Reconstruction

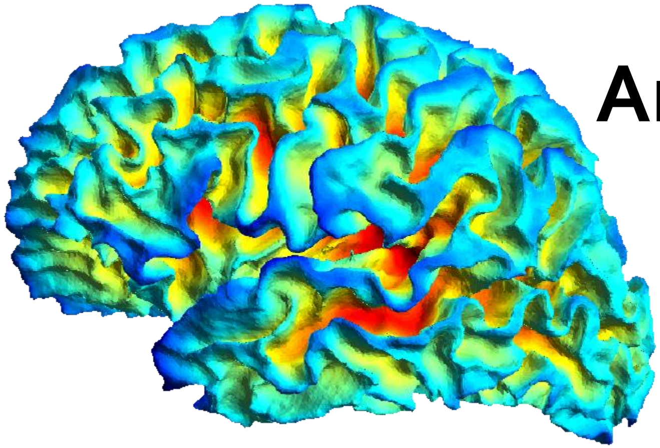


Segmentation

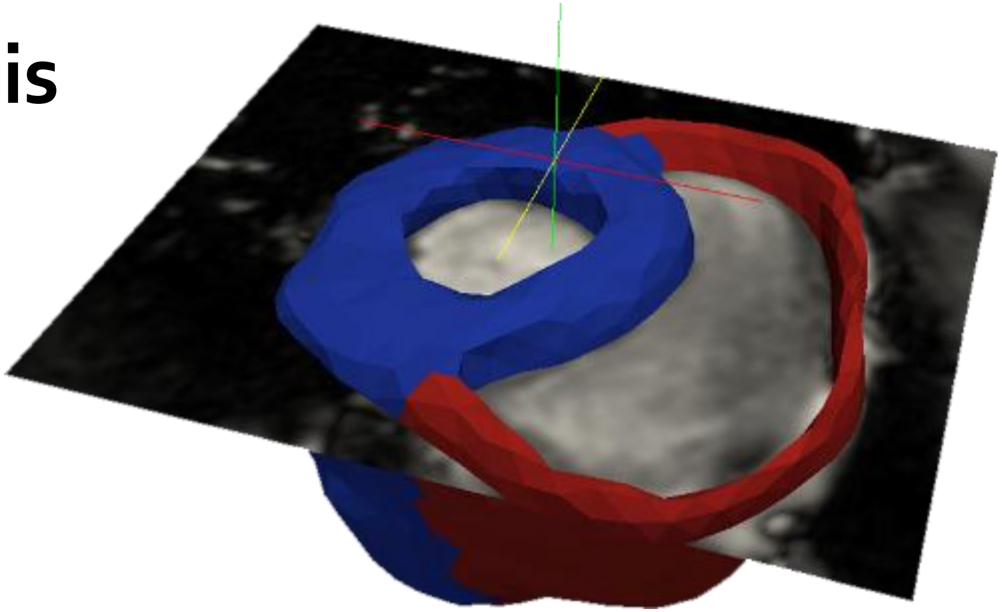
ttp://eijournal.com/newsite/wp-content/uploads/2012/01/VELODYNE-IMAGE.jpg
http://www.stanford.edu/~jinhae/iccv09/
http://www.stanford.edu/~justso1/assets/intrinsic_part_discovery.pdf
http://www.cs.technion.ac.il/~ron/PAPERS/BroBroKimIJCV05.pdf

Vision

Example Applications



Analysis



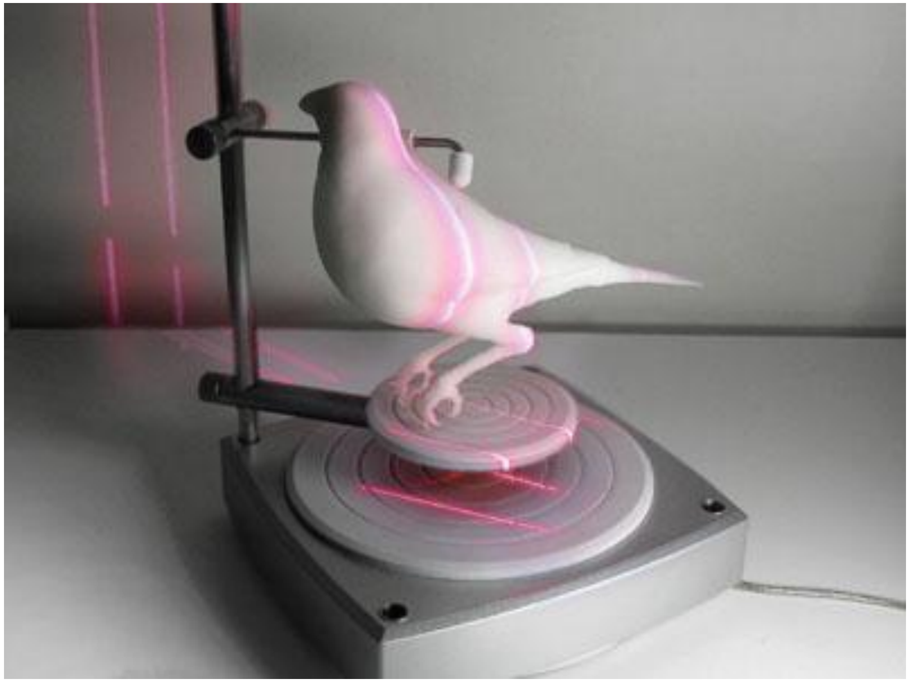
Segmentation

Registration

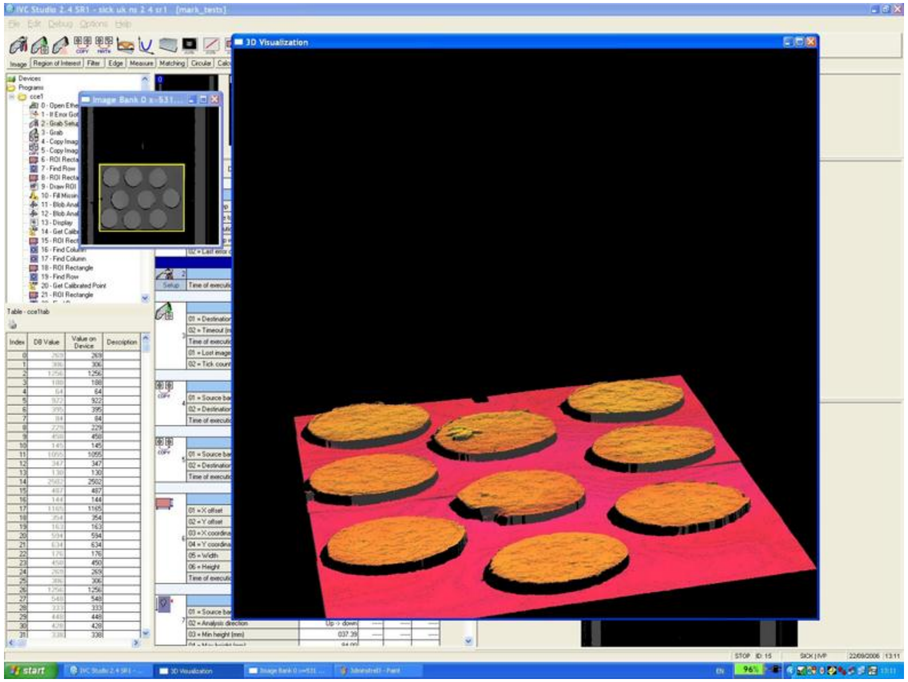
http://dmfr.birjournals.org/content/33/4/226/F3.large.jpg
http://www-sop.inria.fr/asclepios/software/inriaviz4d/SphericalImTransp.png
http://www.creatis.insa-lyon.fr/site/sites/default/files/segm2.png

Medical Imaging

Example Applications



Scanning



Defect detection

http://www.conduitprojects.com/php/images/scan.jpg
http://www.emeraldinsight.com/content_images/fig/0330290204005.png

Manufacturing

Obvious Observation

Analysis and correspondence form a **large and diverse** field.

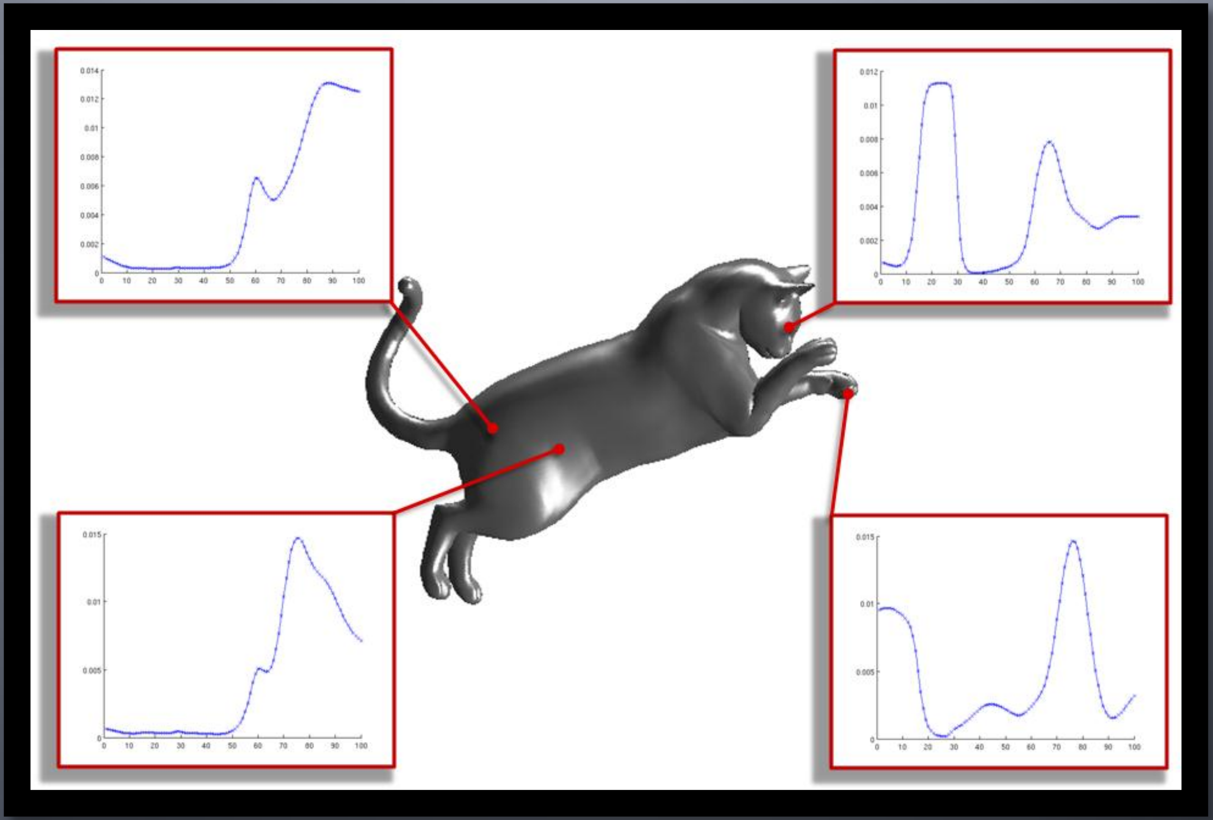
Plan for today

Summarize approaches to

- Local descriptors
- Shape understanding
- Correspondence
- Shape collections

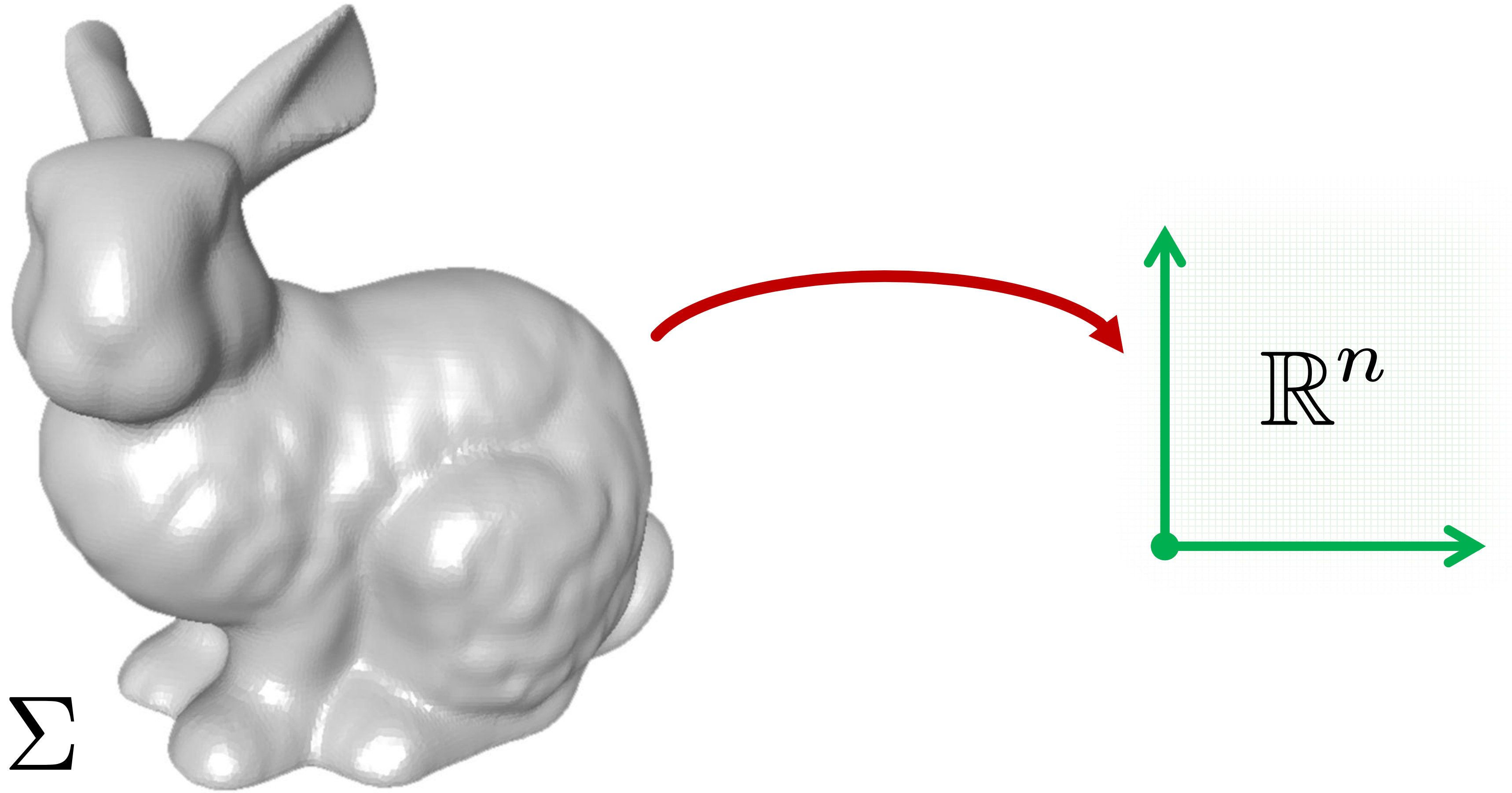
Part I:

Local Descriptors



vision.in.tum.de/_media/spezial/bib/aubry-et-al-4dmod11.pdf

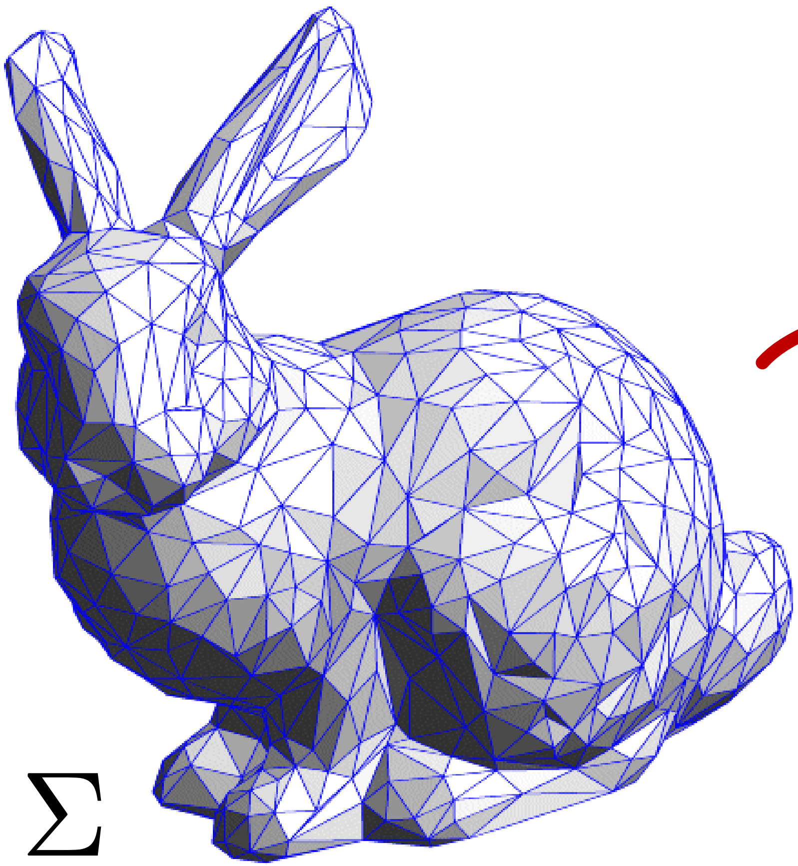
Shape Descriptors



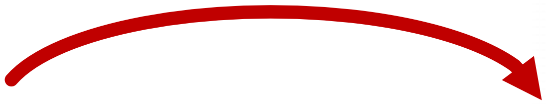
http://liris.cnrs.fr/meshbenchmark/images/fig_attacks.jpg

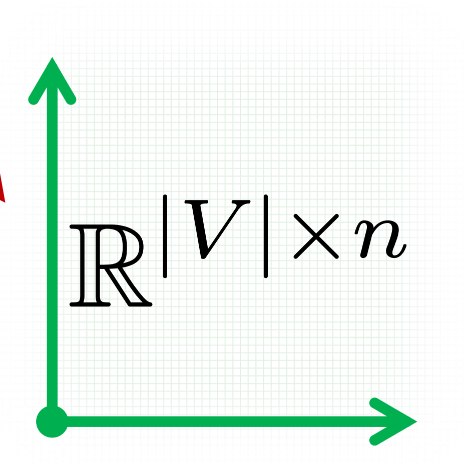
Pointwise quantity

Discrete Representation



http://isg.cs.tcd.ie/spheretree/pics/bunny.gif





Pointwise quantity

Desirable Properties

- **Distinguishing**

  Provides useful information about a point

- **Stable**

  Numerically and geometrically

- **Intrinsic**

  No dependence on embedding

Desirable Properties

- **Distinguishing**
  Provides useful information about a point
- **Stable**
  Numerically and geometrically
- **Intrinsic**
  No dependence on embedding

Controversial!

Isometry

[ahy-**som**-i-tree]**:**

Bending without stretching.

Intrinsic Descriptors

http://www.revedreams.com/crochet/yarncrochet/nonorientable-crochet/

Isometry invariant

Isometry Invariance: Hope

http://www.flickr.com/photos/melvinvoskuijl/galleries/72157624236168459

Isometry Invariance: Reality

http://www.4tnz.com/content/got-toilet-paper

Few shapes *can* deform isometrically

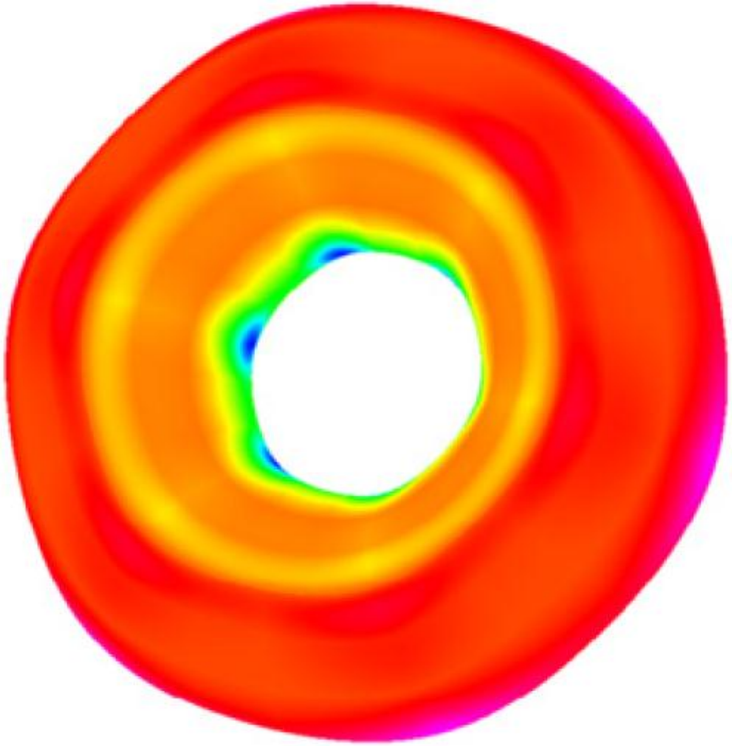
Isometry Invariance: Reality

Behavior for approximate isometry is important, too!

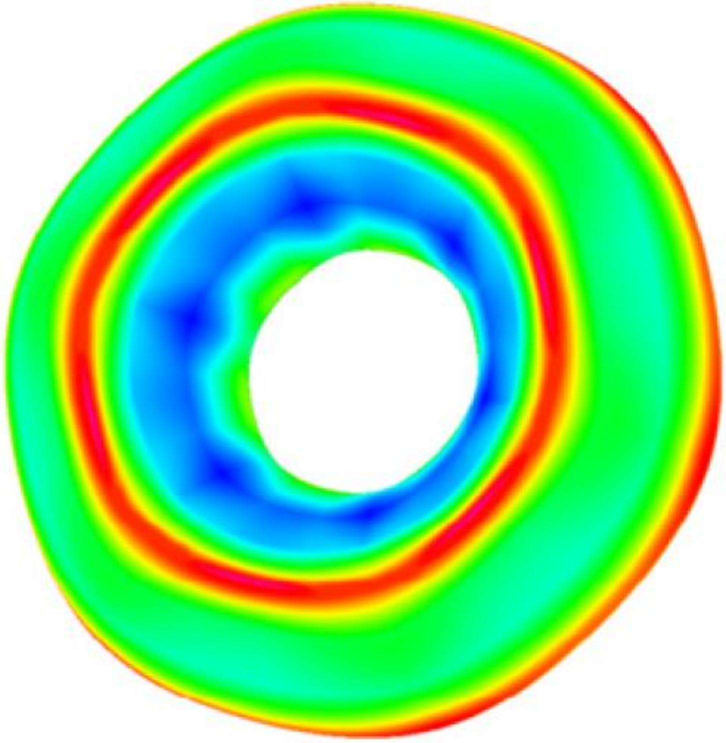
http://www.4tnz.com/content/got-toilet-paper

Few shapes *can* deform isometrically

Descriptors We've Seen Before



$$K = \kappa_1 \kappa_2$$

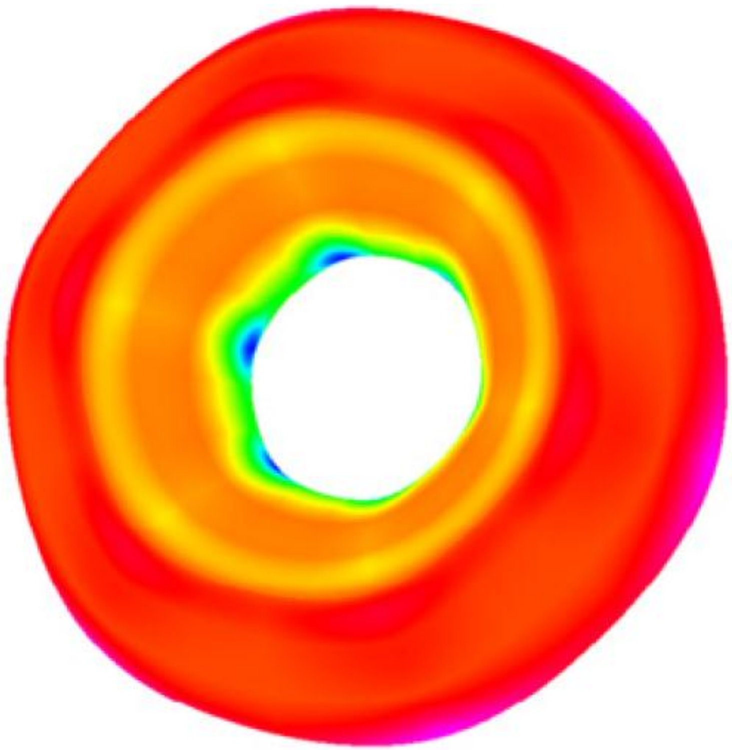


$$H = \tfrac{1}{2}(\kappa_1 + \kappa_2)$$

http://www.sciencedirect.com/science/article/pii/S0010448510001983

Gaussian and mean curvature

Descriptors We've Seen Before



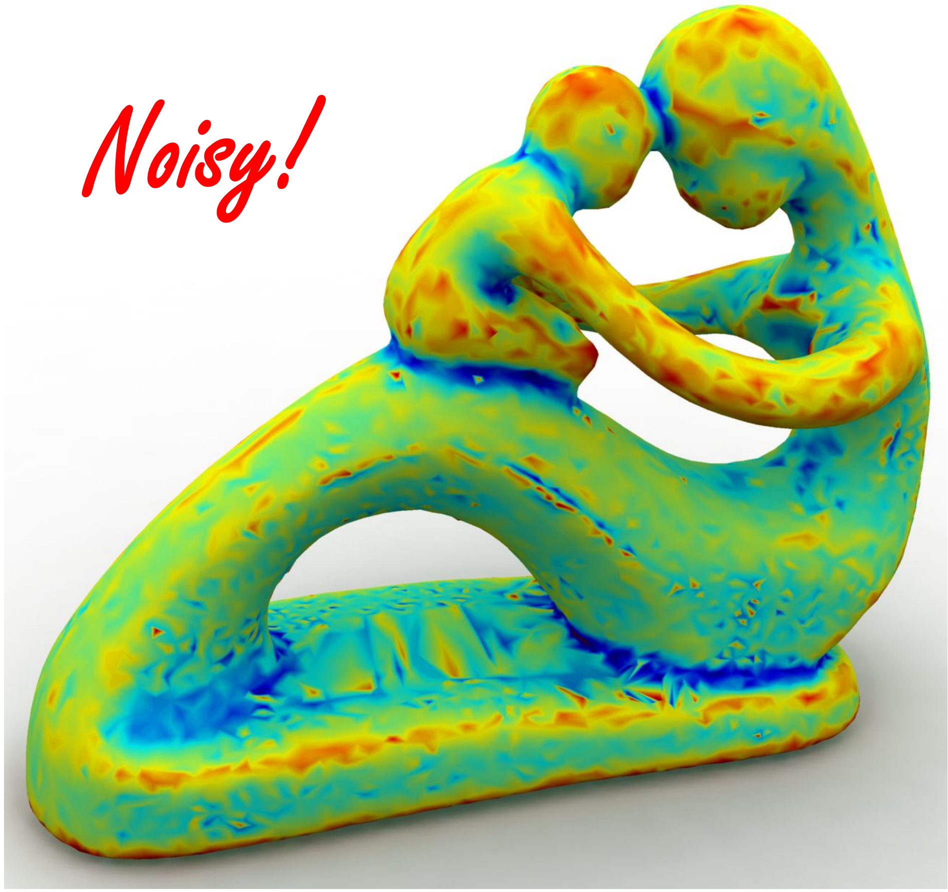
$$K = \kappa_1 \kappa_2$$

Theorema Egregium
("Remarkable Theorem"):
Gaussian curvature is intrinsic.

http://www.sciencedirect.com/science/article/pii/S0010448510001983

Gaussian and mean curvature

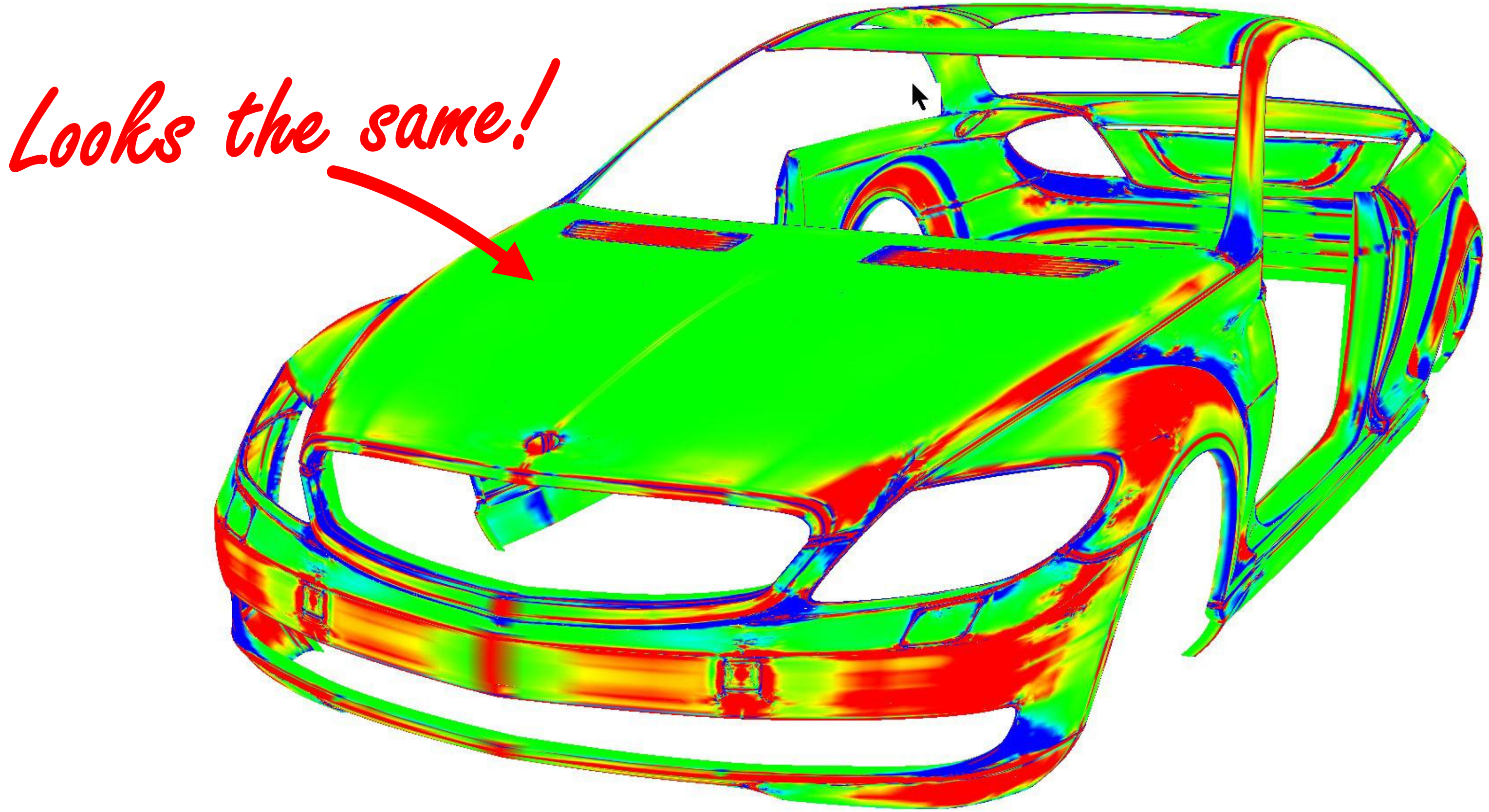
Problems



$$K = \kappa_1 \kappa_2$$

Localized differential descriptors

Problems



http://www.integrityware.com/images/MerceedesGaussianCurvature.jpg

Nonunique

Functions of Curvature

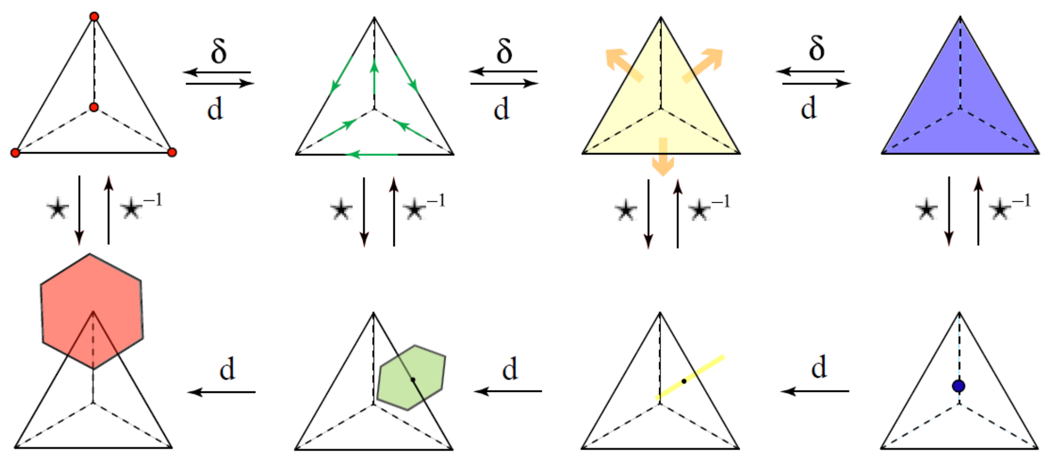
- Principal curvatures κ_1, κ_2
- Shape index $\frac{2}{\pi}\arctan\left(\frac{\kappa_1+\kappa_2}{\kappa_1-\kappa_2}\right)$
- Curvedness $\sqrt{\frac{1}{2}(\kappa_1^2+\kappa_2^2)}$

Goal

Incorporate **neighborhood information** in an intrinsic fashion.

Goal

Incorporate **neighborhood information** in an intrinsic fashion.



http://ddg.cs.columbia.edu/SIGGRAPH06/DDGCourse2006.pdf

Recall: The Laplacian

$$\boxed{\Delta = d \star d \star + \star d \star d}$$

An *intrinsic* operator

Recall: The Laplacian

$$\boxed{\Delta = d \star d \star + \star d \star d}$$

$\Delta\phi_1 = \lambda_1\phi_1$ $\Delta\phi_2 = \lambda_2\phi_2$ $\Delta\phi_3 = \lambda_3\phi_3$ $\Delta\phi_4 = \lambda_4\phi_4$ $\Delta\phi_5 = \lambda_5\phi_5$ $\cdots$

$(\Delta\phi_0 = 0)$

An *intrinsic* operator

Global Point Signature (GPS)

$$GPS(p) = \left(\frac{1}{\sqrt{\lambda_1}}\phi_1(p), \frac{1}{\sqrt{\lambda_2}}\phi_2(p), \frac{1}{\sqrt{\lambda_3}}\phi_3(p), \cdots \right)$$

Good properties:

- Isometry-invariant
- Unique to each point
- Complete description of intrinsic geometry
- Dot products, distances meaningful

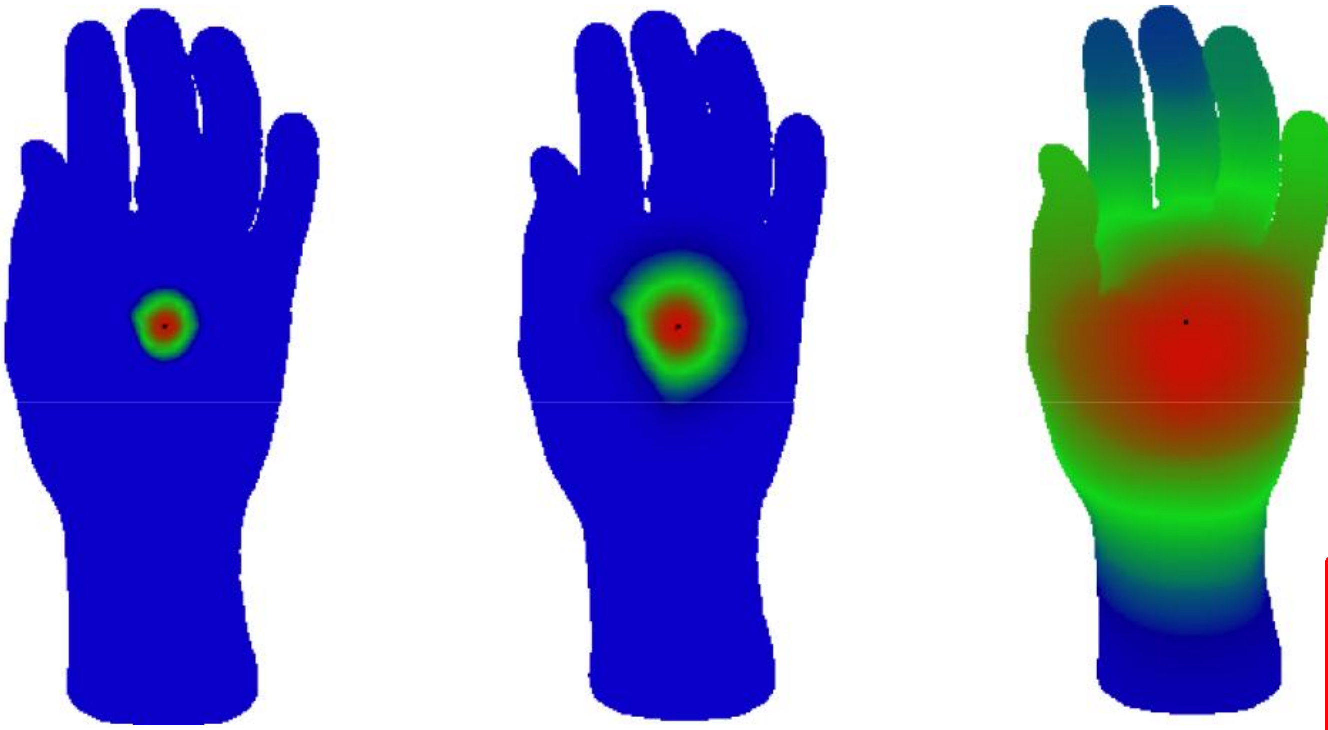
Global Point Signature (GPS)

$$GPS(p) = \left(\frac{1}{\sqrt{\lambda_1}}\phi_1(p), \frac{1}{\sqrt{\lambda_2}}\phi_2(p), \frac{1}{\sqrt{\lambda_3}}\phi_3(p), \cdots \right)$$

Bad properties:

- **Assumes unique λ's**
- **Potential for eigenfunction "switching" upon deformation**
- **Nonlocal feature**

PDE Applications of the Laplacian

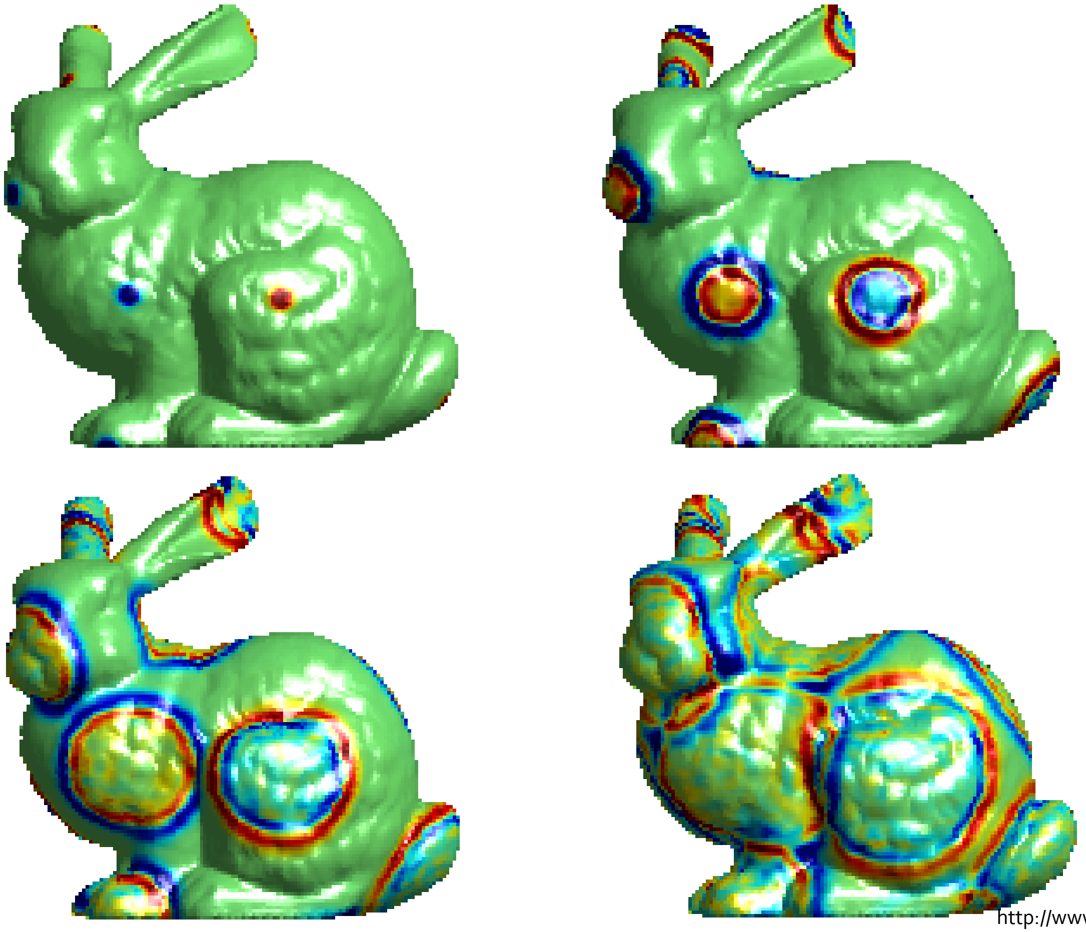


$$\boxed{\frac{\partial u}{\partial t} = -\Delta u}$$

http://graphics.stanford.edu/courses/cs468-10-fall/LectureSlides/11_shape_matching.pdf

Heat equation

PDE Applications of the Laplacian



$$\boxed{\frac{\partial^2 u}{\partial t^2} = -i\Delta u}$$

http://www.ceremade.dauphine.fr/~peyre/numerical-tour/tours/meshproc_5_pde/index_06.png

Wave equation

PDE Applications of the Laplacian

Use this behavior to characterize shape.

$$\boxed{\frac{\partial^2 u}{\partial t^2} = -i\Delta u}$$

http://www.ceremade.dauphine.fr/~peyre/numerical-tour/tours/meshproc_5_pde/index_06.png

Wave equation

Solutions in the LB Basis

$$\boxed{\frac{\partial u}{\partial t} = -\Delta u}$$

Heat equation

$$u = \sum_{n=0}^{\infty} a_n e^{-\lambda_n t} \phi_n(x)$$

$$\left(a_n = \int_{\Sigma} u_0 \cdot \phi_n \; dA \right)$$

Heat Kernel Signature (HKS)

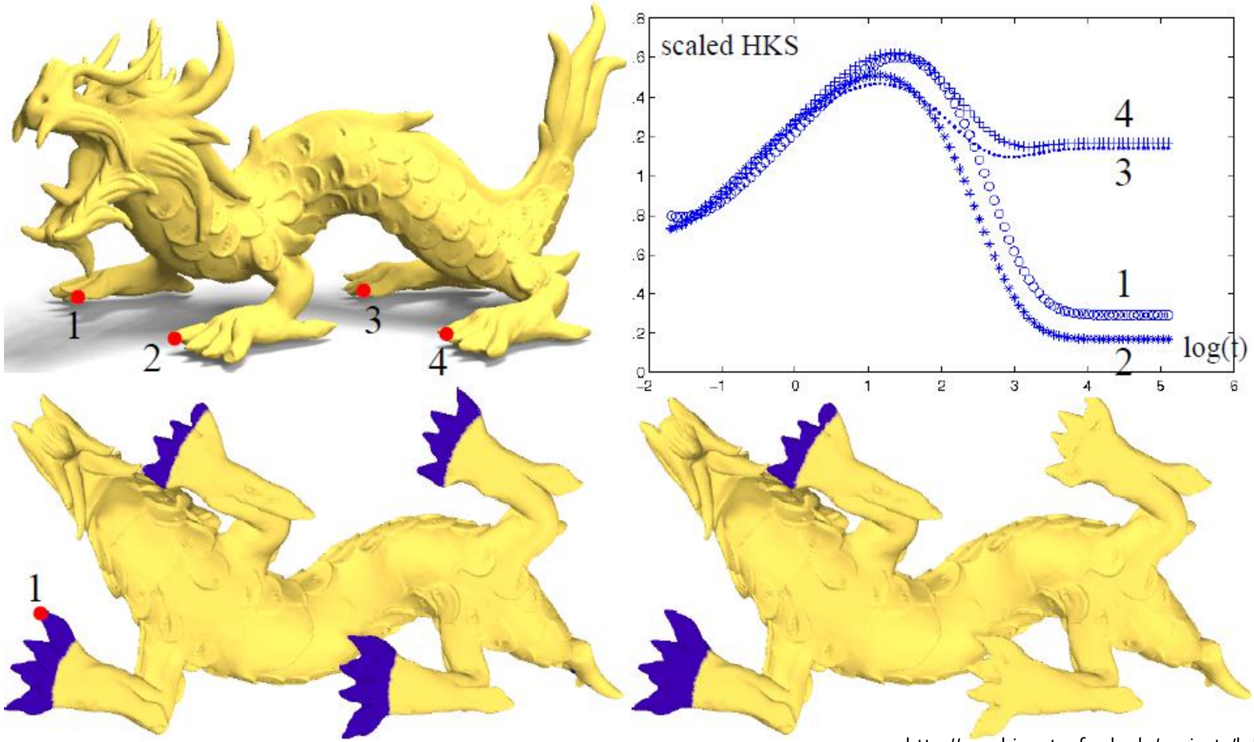
$$\boxed{k_t(x,x) = \sum_{n=0}^{\infty} e^{-\lambda_i t} \phi_n(x)^2}$$

Continuous function on [0,∞)

How much heat diffuses from *x* to itself in time *t*?

Heat Kernel Signature (HKS)

$$k_t(x,x) = \sum_{n=0}^{\infty} e^{-\lambda_i t} \phi_n(x)^2$$



http://graphics.stanford.edu/projects/lgl/papers/sog-hks-09/sog-hks-09.pdf

Heat Kernel Signature (HKS)

$$\boxed{k_t(x,x) = \sum_{n=0}^{\infty} e^{-\lambda_i t} \phi_n(x)^2}$$

Good properties:

- Isometry-invariant
- Multiscale
- Not subject to switching
- Easy to compute
- Related to curvature at small scales

Heat Kernel Signature (HKS)

$$\boxed{k_t(x,x) = \sum_{n=0}^{\infty} e^{-\lambda_i t} \phi_n(x)^2}$$

Bad properties:

- Issues remain with repeated eigenvalues
- Theoretical guarantees require (near-)isometry

HKS Extensions

http://www.cs.technion.ac.il/~mbron/publications_conference.html

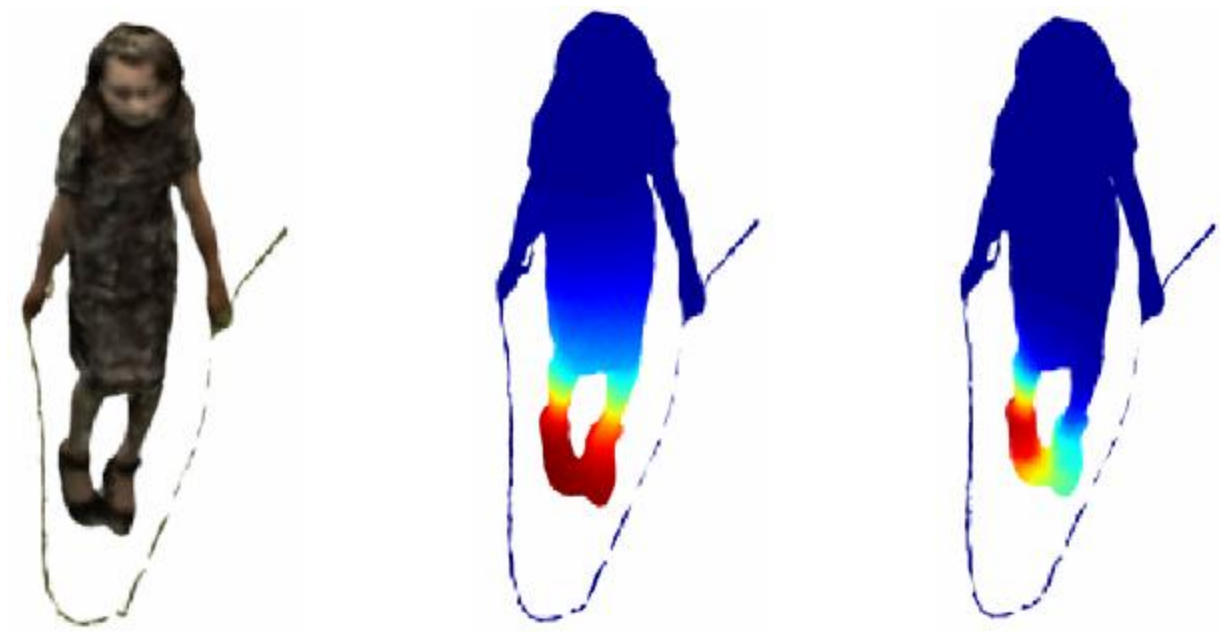
Scale-Invariant HKS (SI-HKS)

HKS Extensions

http://www.cs.technion.ac.il/~darav/RavBroBroKimAffine10TR.pdf

Affine-Invariant HKS

HKS Extensions



http://www.cs.technion.ac.il/~mbron/publications_conference.html

Photometric HKS

HKS Extensions

http://www.cs.technion.ac.il/~mbron/publications_conference.html

Volumetric HKS

Wave Kernel Signature (WKS)

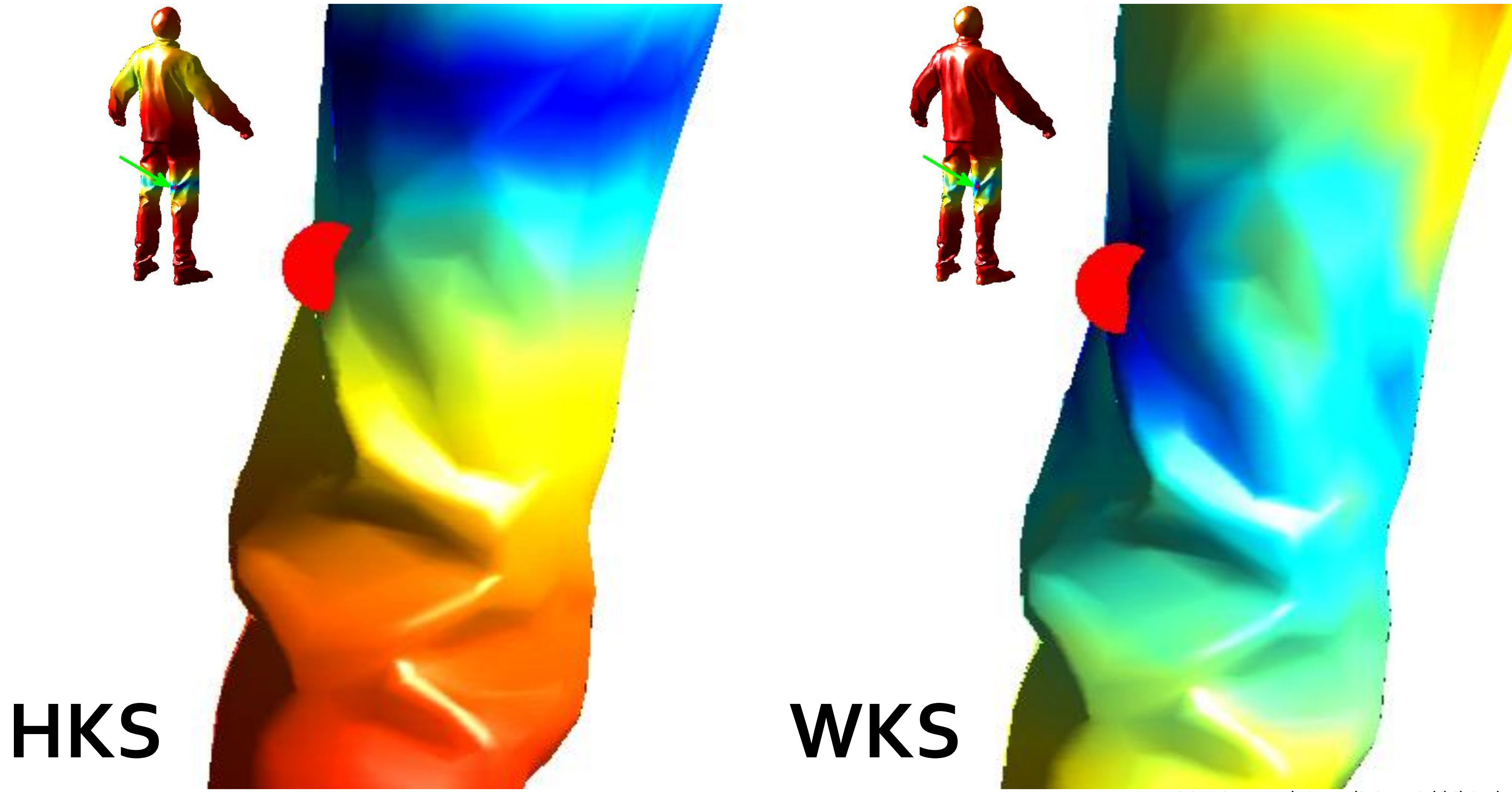
$$WKS(E,x) = \lim_{T\to\infty} \frac{1}{T}\int_0^T |\psi_E(x,t)|^2\, dt = \sum_{n=0}^{\infty} \phi_n(x)^2 f_E(\lambda_k)^2$$

Initial energy distribution

Average probability over time that particle is at x.

Wave Kernel Signature (WKS)

$$WKS(E,x) = \lim_{T\to\infty} \frac{1}{T} \int_0^T |\psi_E(x,t)|^2 \, dt = \sum_{n=0}^{\infty} \phi_n(x)^2 f_E(\lambda_k)^2$$



vision.in.tum.de/_media/spezial/bib/aubry-et-al-4dmod11.pdf

Wave Kernel Signature (WKS)

$$WKS(E,x) = \lim_{T\to\infty} \frac{1}{T} \int_0^T \left|\psi_E(x,t)\right|^2 \, dt = \sum_{n=0}^{\infty} \phi_n(x)^2 f_E(\lambda_k)^2$$

Good properties:

- [Similar to HKS]
- Localized in frequency
- Stable under some non-isometric deformation
- Some multi-scale properties

Wave Kernel Signature (WKS)

$$WKS(E,x) = \lim_{T\to\infty} \frac{1}{T} \int_0^T |\psi_E(x,t)|^2 \, dt = \sum_{n=0}^{\infty} \phi_n(x)^2 f_E(\lambda_k)^2$$

Bad properties:

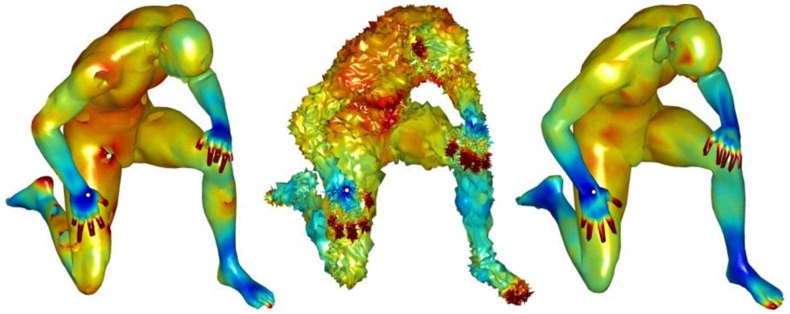
- [Similar to HKS]
- Can filter out *large*-scale features

Spectral Descriptors

$$\boxed{\sum_{n=0}^{\infty} f(\lambda_n)\phi_n(x)^2}$$

Considerations:

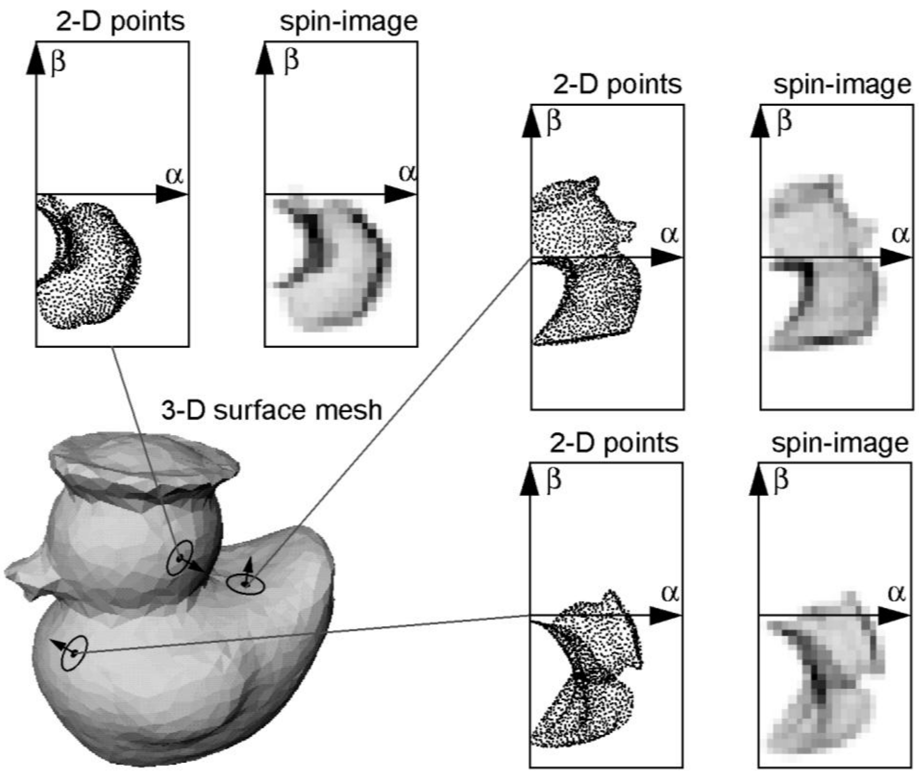
- Collection of shapes
- Potential transformations/noise



http://arxiv.org/pdf/1110.5015.pdf

Can you *learn* the function *f*?

Other Descriptors



Bin points using:

$$\alpha = \text{distance to normal line}$$

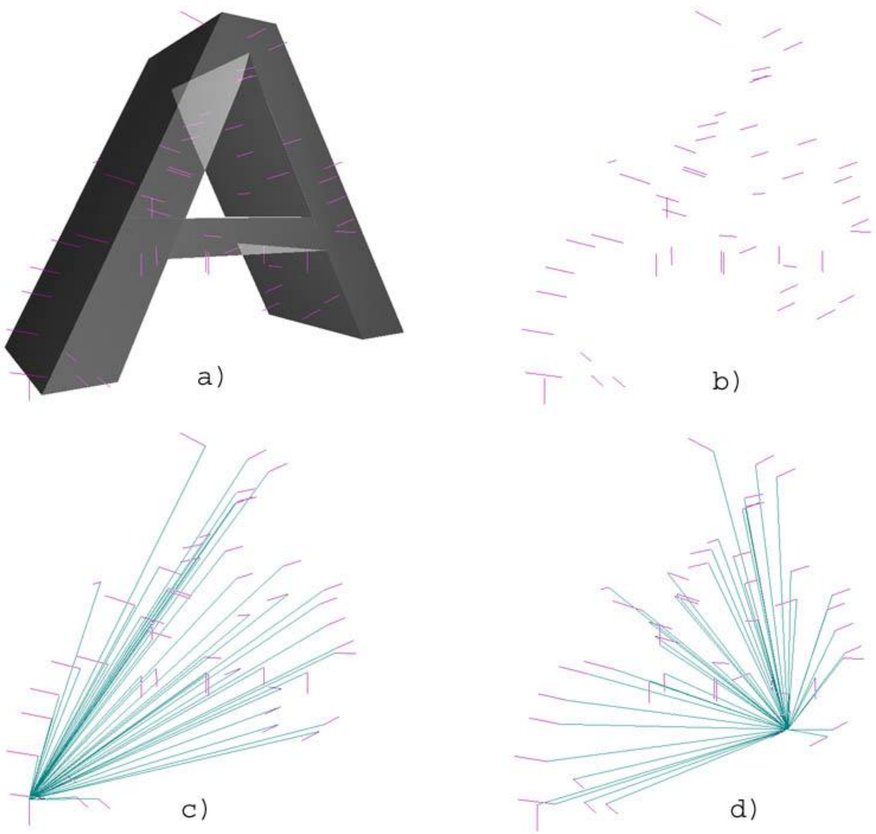
$$\beta = \text{distance to tangent plane}$$

Can use low-rank approximation!

http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=00765655

Spin images

Other Descriptors

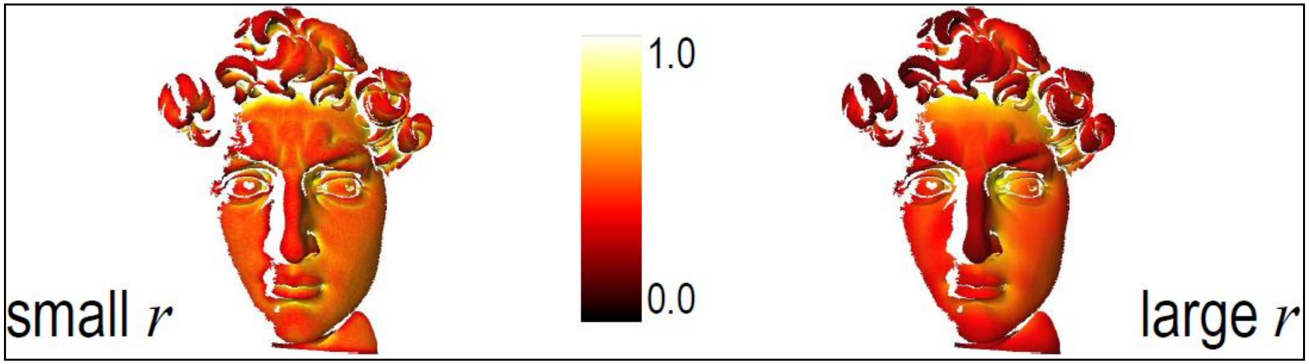


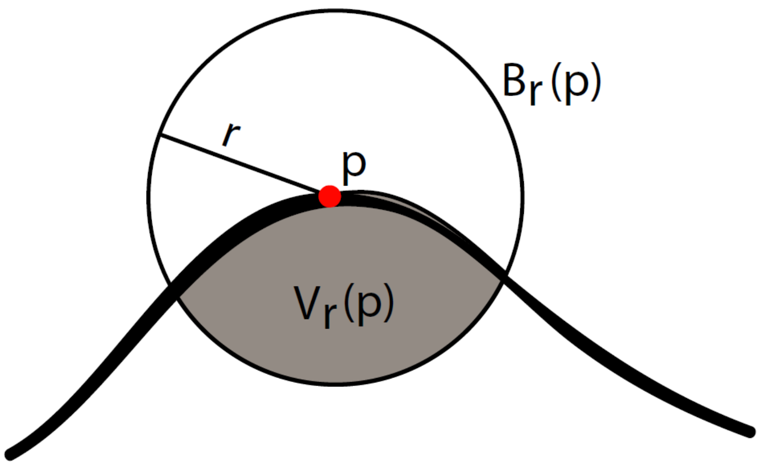
Bin directions *y*-*x* for each *x*

http://cg.tuwien.ac.at/hostings/cescg/CESCG-2003/MKoertgen/paper.pdf

Shape context

Other Descriptors





$$V_r(p) = \int_{B_r(p) \cap S} dV$$

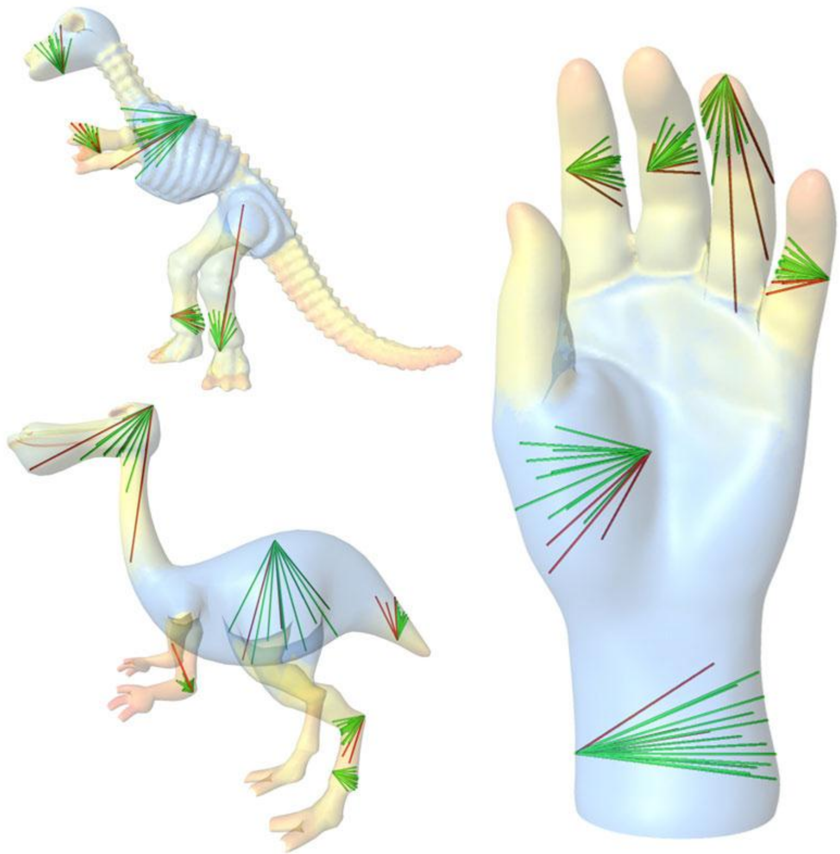
Related to mean curvature

Integral invariants → robust

http://graphics.stanford.edu/~niloy/research/global_registration/paper_docs/global_registration_sgp_05_poster.pdf

Integral volume

Other Descriptors



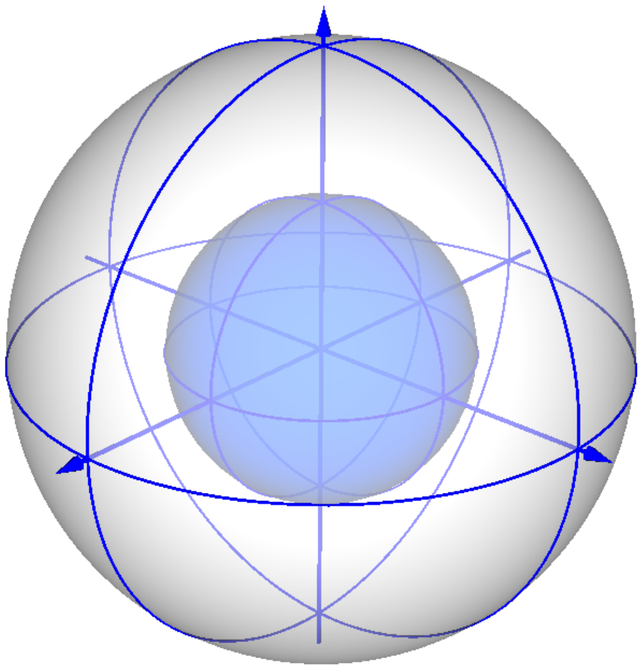
Weighted average distance along the surface

Lightweight version of medial axis distance

http://www.cs.jhu.edu/~misha/ReadingSeminar/Papers/Shapira08.pdf

Shape Diameter Function

Other Descriptors



Bin nearby normals in a canonical orientation

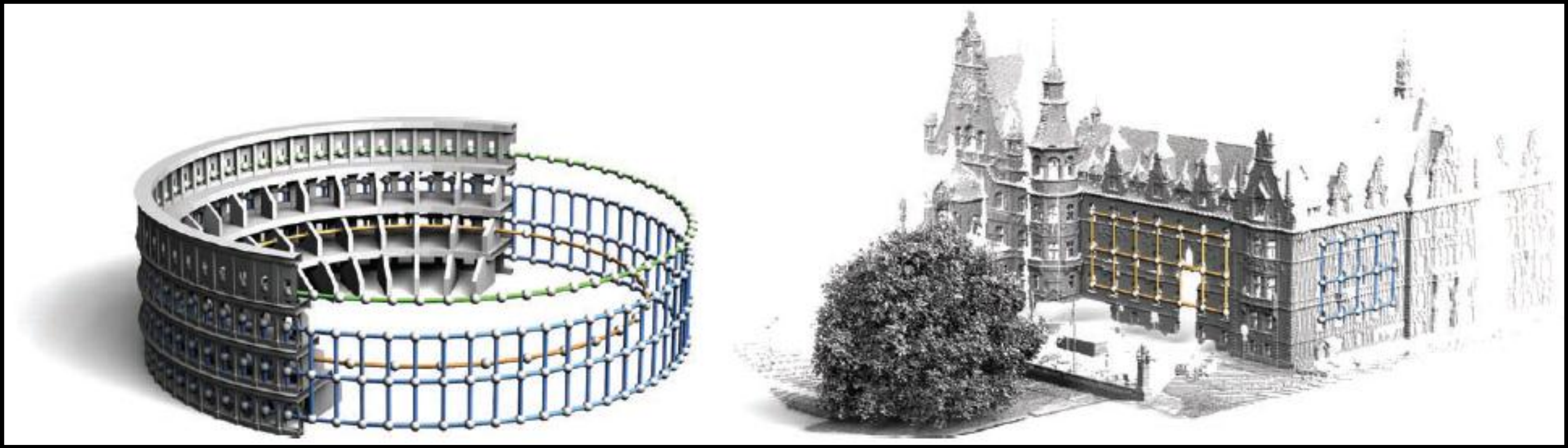
http://www.vision.deis.unibo.it/fede/papers/eccv10.pdf

Signature of Histograms of OrienTations

Many Others

- Structural indexing
- Point signatures
- Point fingerprints
- Intrinsic shape signature
- Multi-scale surface descriptors
- Slippage
- Spherical harmonics
- RIFT
- HMM
- …

Part II:
Shape Understanding



http://graphics.stanford.edu/~niloy/research/structure/paper_docs/structure_sig_08.pdf

Many Potential Tasks

- Segmentation
- Symmetry detection
- Global shape description
- Retrieval
- Recognition
- Feature extraction
- Alignment
- …

Many Potential Tasks

- Segmentation
- Symmetry detection
- Global shape description
- Retrieval
- Recognition
- Feature extraction
- Alignment
- …

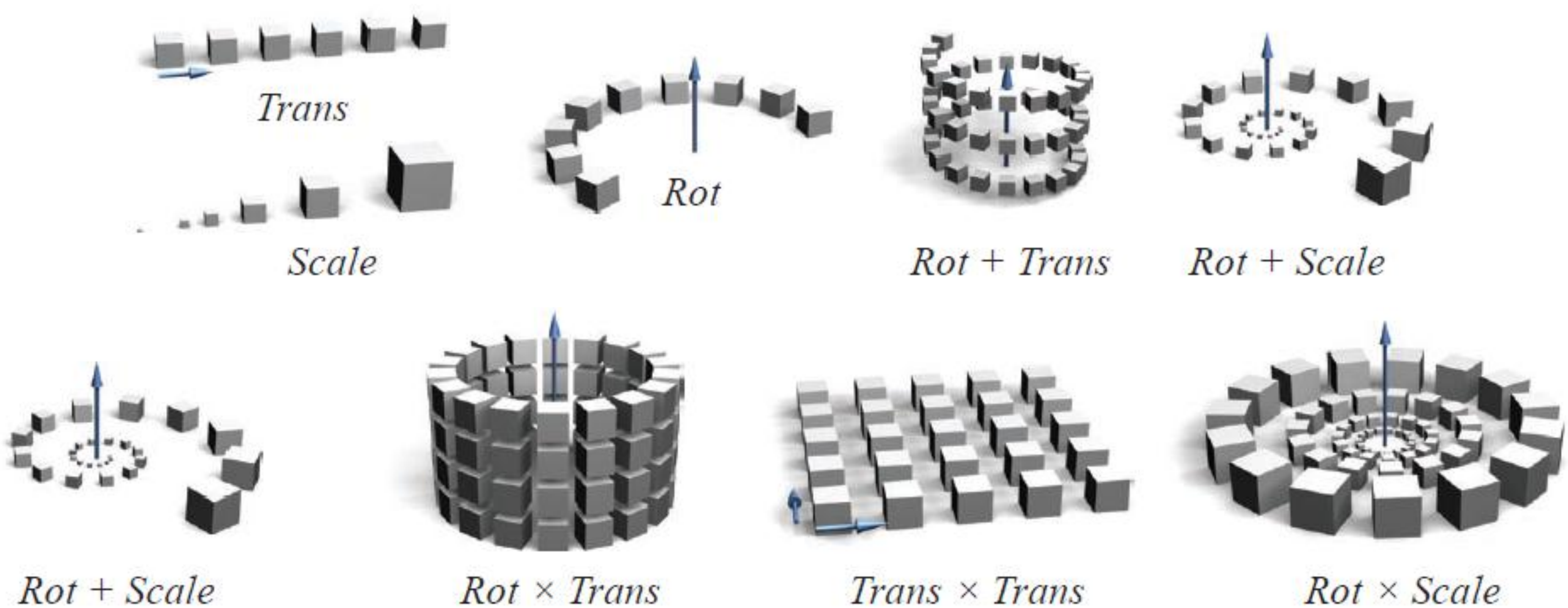
We'll sample a few!

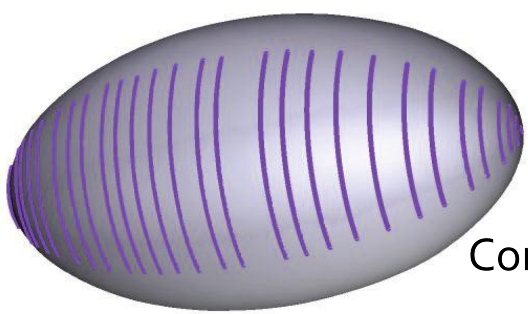
Symmetry Detection

- Compression
- Reconstruction
- Classification
- Analysis
- Alignment
- Matching

http://wwwo.cs.ucl.ac.uk/staff/n.mitra/research/symmetry_survey/symmetrySurvey_12.html
http://www.cs.princeton.edu/courses/archive/fall03/cs597D/lectures/symmetry.pdf

Types of Symmetries

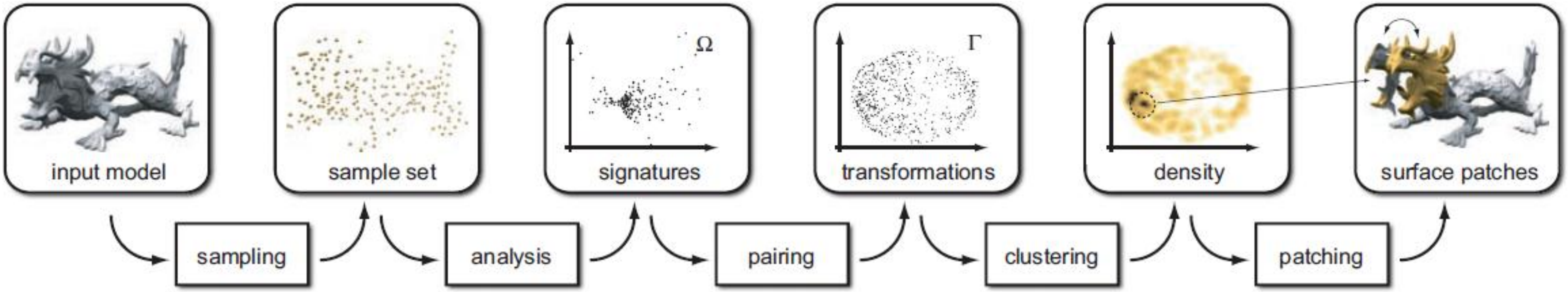




Continuous

http://graphics.stanford.edu/~niloy/research/structure/paper_docs/structure_sig_08.pdf

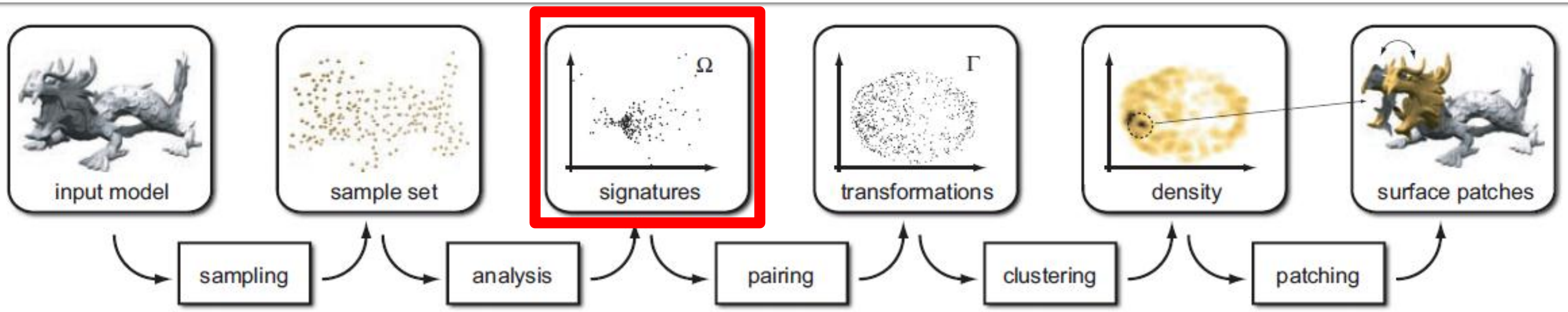
Symmetry Detection Methods



Partial and Approximate Symmetry Detection for 3D Geometry
Mitra, Guibas, Pauly 2006

Ex. 1: Discrete extrinsic symmetries

Symmetry Detection Methods

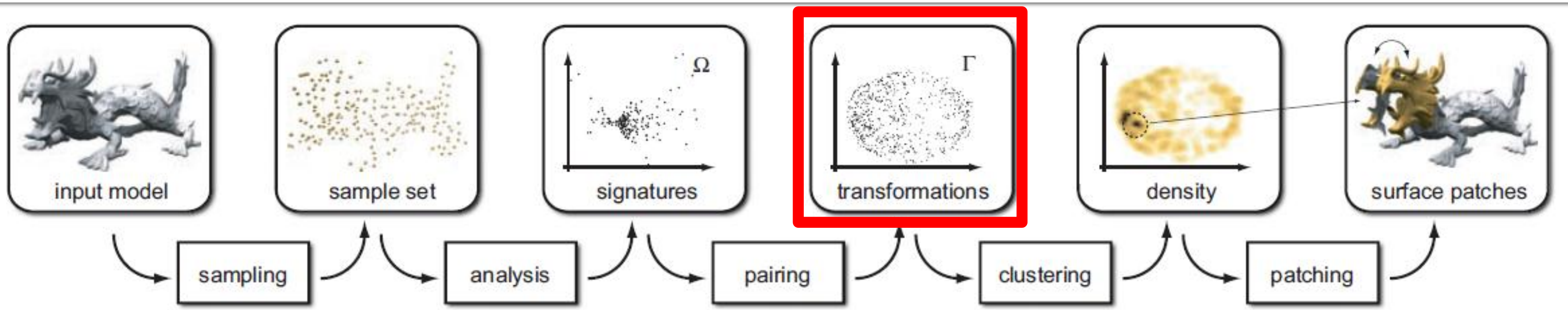


Compute simple curvature features to help pair similar points.

Partial and Approximate Symmetry Detection for 3D Geometry
Mitra, Guibas, Pauly 2006

Ex. 1: Discrete extrinsic symmetries

Symmetry Detection Methods

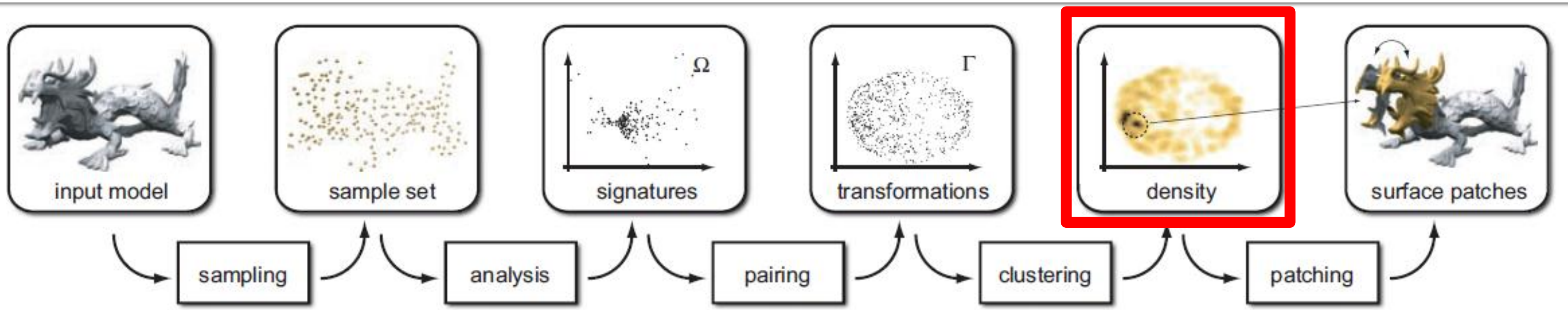


Pairs of points with similar signatures vote for different transformations.

Partial and Approximate Symmetry Detection for 3D Geometry
Mitra, Guibas, Pauly 2006

Ex. 1: Discrete extrinsic symmetries

Symmetry Detection Methods

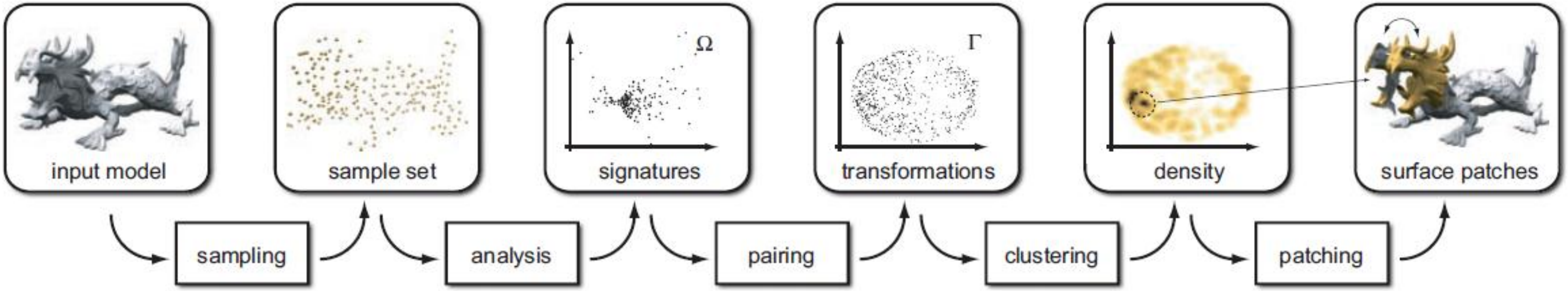


Use mean shift clustering to find prominent transformations.

Partial and Approximate Symmetry Detection for 3D Geometry
Mitra, Guibas, Pauly 2006

Ex. 1: Discrete extrinsic symmetries

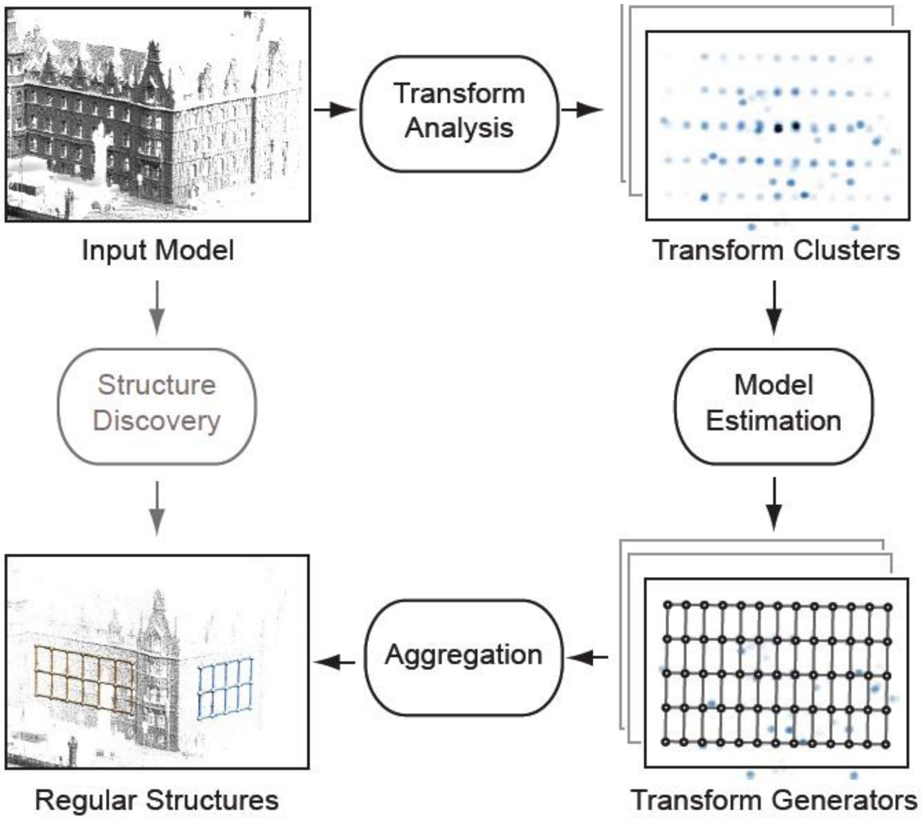
Symmetry Detection Methods



Partial and Approximate Symmetry Detection for 3D Geometry
Mitra, Guibas, Pauly 2006

Ex. 1: Discrete extrinsic symmetries

Symmetry Detection Methods

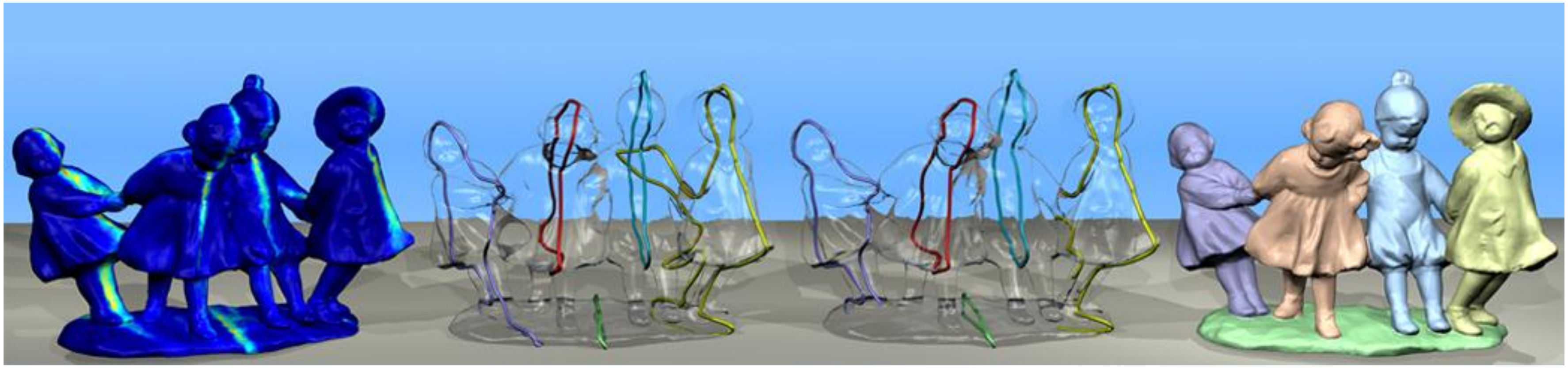


Discovering Structural Regularity in 3D Geometry

Pauly et al. 2008

Ex. 1: Discrete extrinsic symmetries

Symmetry Detection Methods

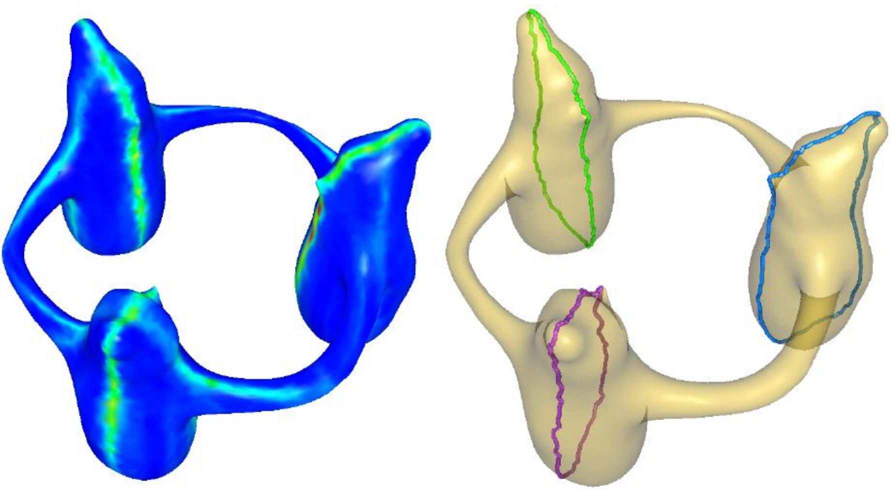


Partial Intrinsic Reflectional Symmetry of 3D Shapes
Xu et al. 2009

Ex. 2: Discrete intrinsic symmetries

Symmetry Detection Methods

IRSA Transform
("Intrinsic Reflectional Symmetry Axis")



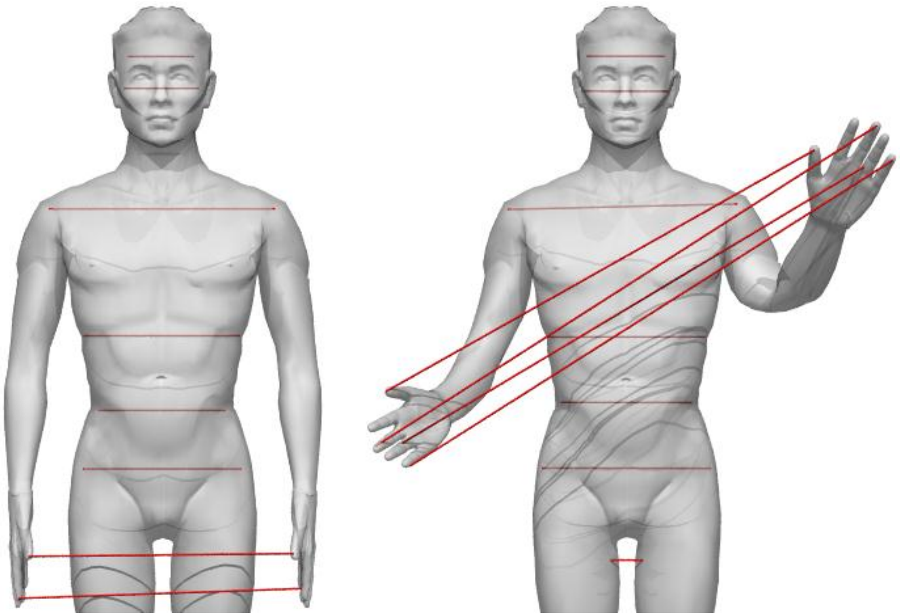
Want T: M→M (or parts thereof) preserving geodesic distances; fixed points are symmetry axis

Sample potential axes; voting scheme for IRSA transform

Partial Intrinsic Reflectional Symmetry of 3D Shapes
Xu et al. 2009

Ex. 2: Discrete intrinsic symmetries

Symmetry Detection Methods

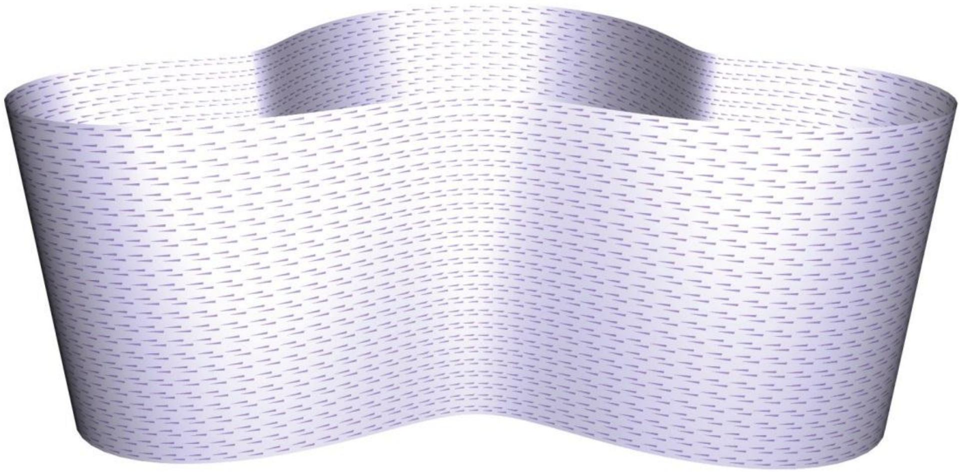


Intrinsic symmetries become extrinsic in GPS space!

Global Intrinsic Symmetries of Shapes
Ovsjanikov, Sun, and Guibas 2008

Ex. 2: Discrete intrinsic symmetries

Symmetry Detection Methods



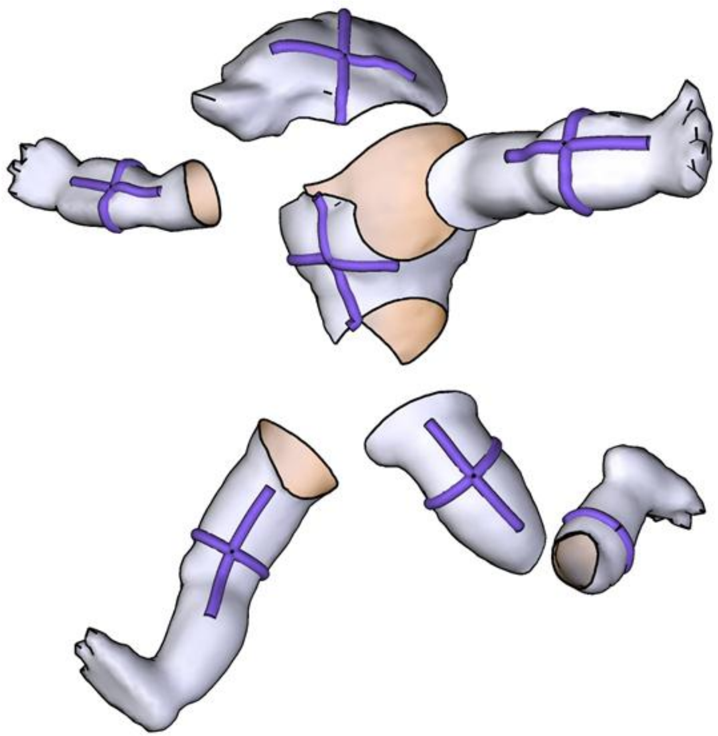
Flows of Killing vector fields (KVFs) generate isometries

DEC framework for finding approximate KVFs

On Discrete Killing Fields and Patterns on Surfaces
Ben Chen et al. 2010

Ex. 3: Continuous intrinsic symmetries

Symmetry Detection Methods



Approximate KVFs can be used to find nearly symmetric pieces

Discovery of Intrinsic Primitives on Triangle Meshes
Solomon et al. 2011

Ex. 3: Continuous intrinsic symmetries

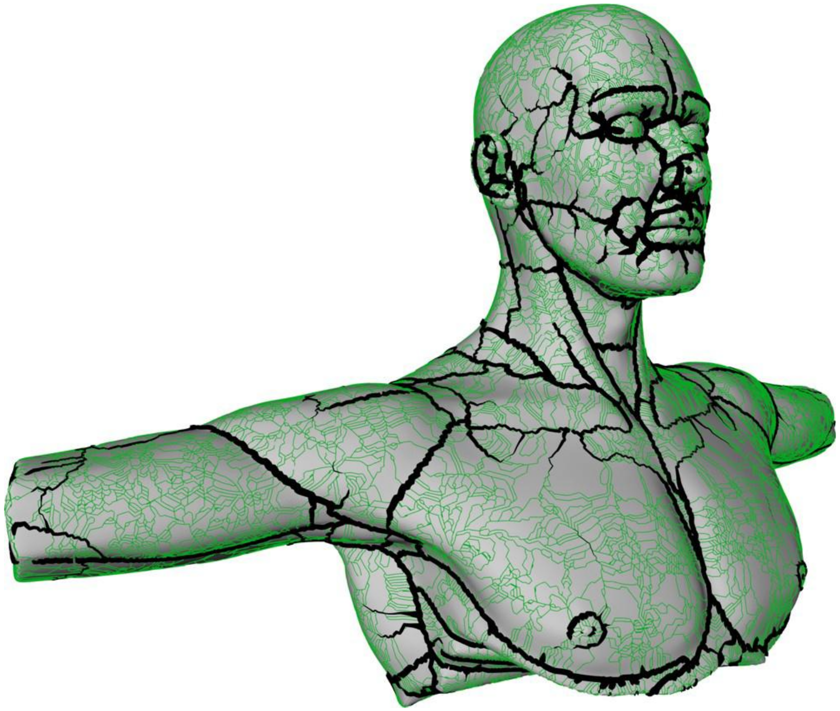
Feature Extraction Methods

Maxima of $k_t(x,x)$ for large t.

A Concise and Provably Informative Multi-Scale Signature Based on Heat Diffusion
Sun, Ovsjanikov, and Guibas 2009

Feature points

Feature Extraction Methods

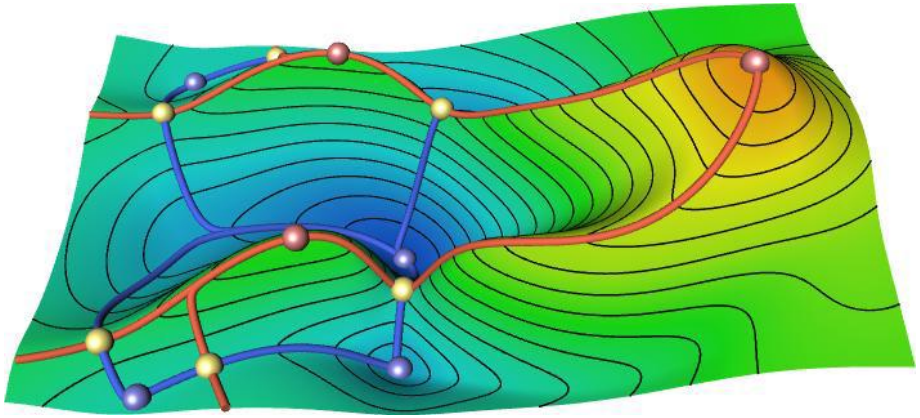


Filter out extraneous feature curves

Separatrix Persistence: Extraction of Salient Edges on Surfaces Using Topological Methods
Weinkauf and Gunther 2009

Feature curves

Feature Extraction Methods



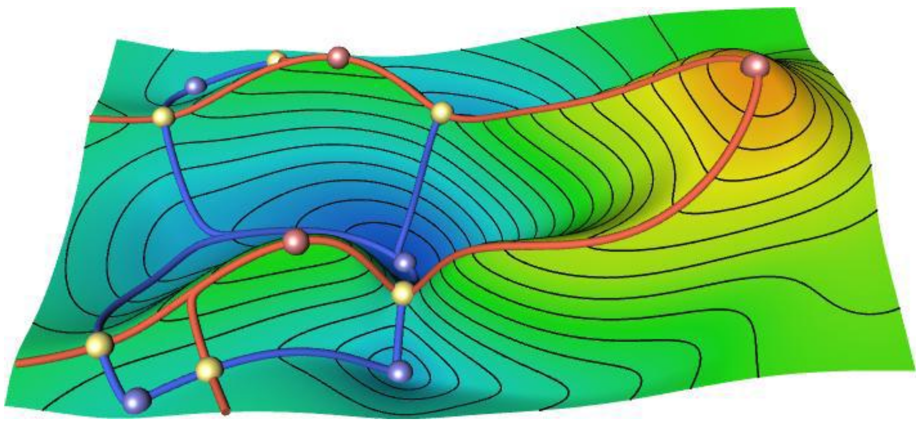
Morse-Smale Complex: Topological skeleton of critical points and separatricies

x* is in the descending manifold of critical point *p* if there exists a gradient flow curve connecting *p* to *x

Separatrix Persistence: Extraction of Salient Edges on Surfaces Using Topological Methods
Weinkauf and Gunther 2009

Feature curves

Feature Extraction Methods



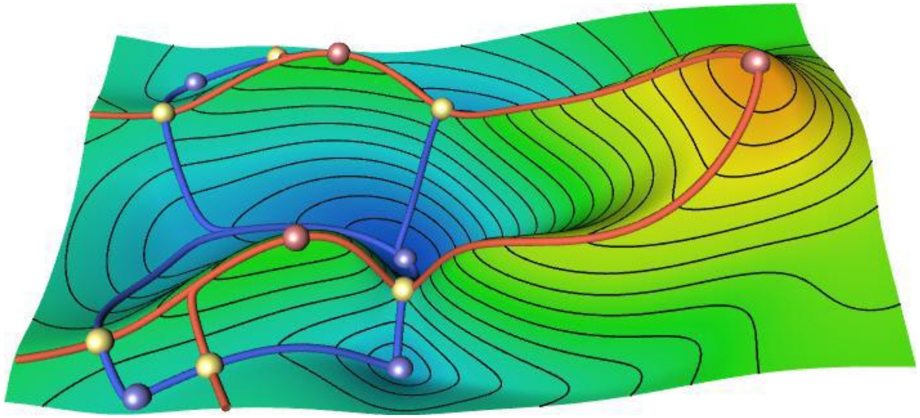
Morse-Smale Complex:
Topological skeleton of critical points and separatricies

x **is in the ascending manifold of critical point** ***p*** **if there exists a gradient flow curve connecting** ***x*** **to** ***p***

Separatrix Persistence: Extraction of Salient Edges on Surfaces Using Topological Methods
Weinkauf and Gunther 2009

Feature curves

Feature Extraction Methods



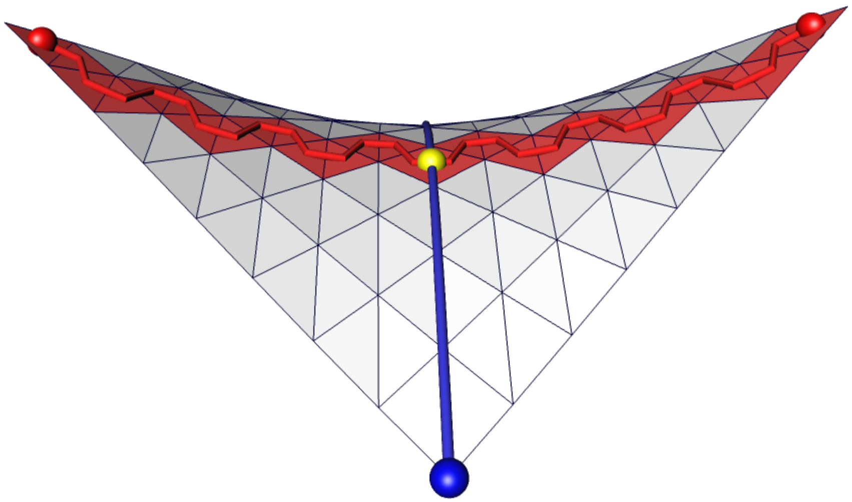
Morse-Smale Complex: Topological skeleton of critical points and separatricies

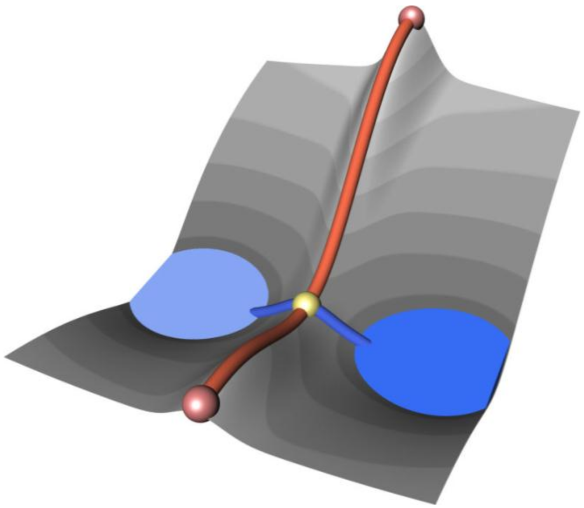
Separatrix: intersection of one ascending and one descending manifold

Separatrix Persistence: Extraction of Salient Edges on Surfaces Using Topological Methods
Weinkauf and Gunther 2009

Feature curves

Feature Extraction Methods





1. Build combinatorial Morse-Smale complex.

2. Apply persistence to simplify.

Separatrix Persistence: Extraction of Salient Edges on Surfaces Using Topological Methods
Weinkauf and Gunther 2009

Feature curves

Feature Extraction Methods

Use curvature to choose better contour lines

Suggestive contour generator: Points with zero/increasing curvature in view direction

http://www.cs.rutgers.edu/~decarlo/pubs/sg03.pdf

Feature curves

Feature Extraction Methods

A Benchmark for 3D Mesh Segmentation

Xiaobai Chen, Aleksey Golovinskiy, Thomas Funkhouser
Princeton University

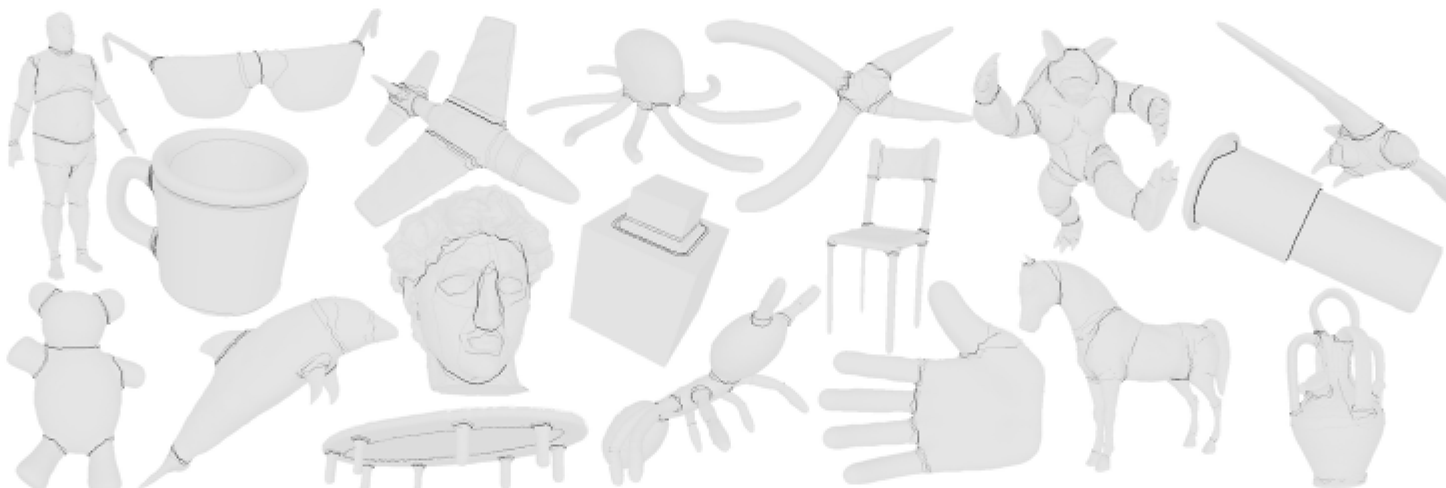


Figure 1: *Composite images of segment boundaries selected by different people (the darker the seam the more people have chosen a cut along that edge). One example is shown for each of the 19 object categories considered in this study.*

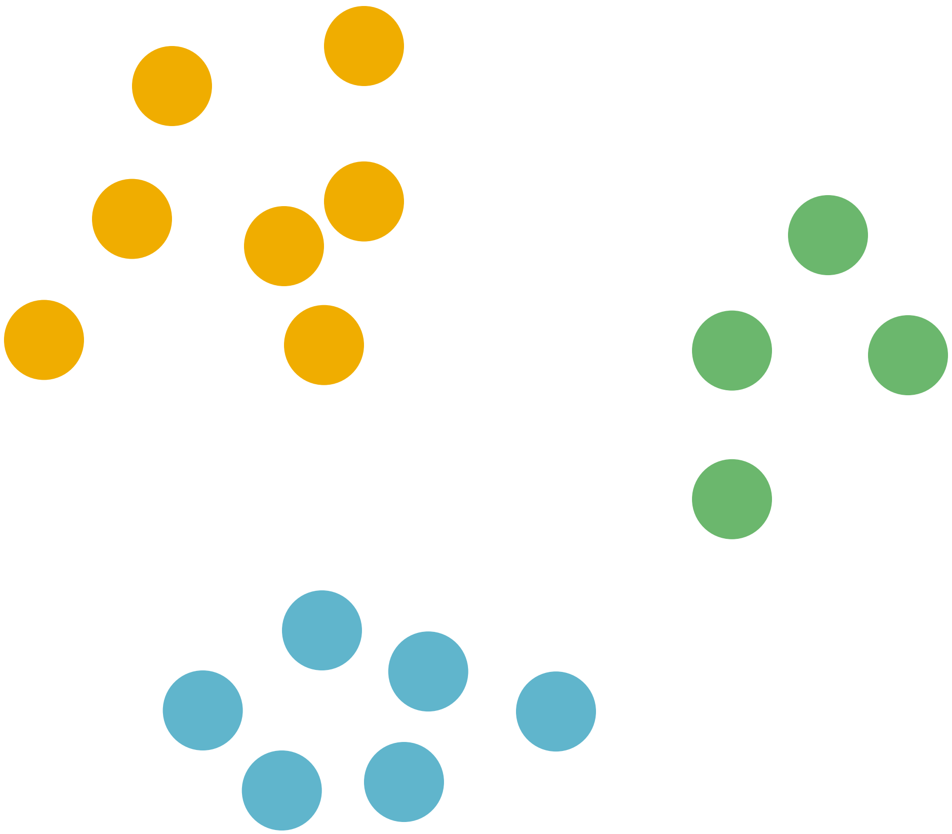
Abstract

This paper describes a benchmark for evaluation of 3D mesh seg-

processing algorithms, including skeleton extraction [Biasotti et al. 2003; Katz and Tal 2003], modeling [Funkhouser et al. 2004], morphing [Zöckler et al. 2000; Gregory et al. 1999], shape-based

"Feature patches" → Segmentation

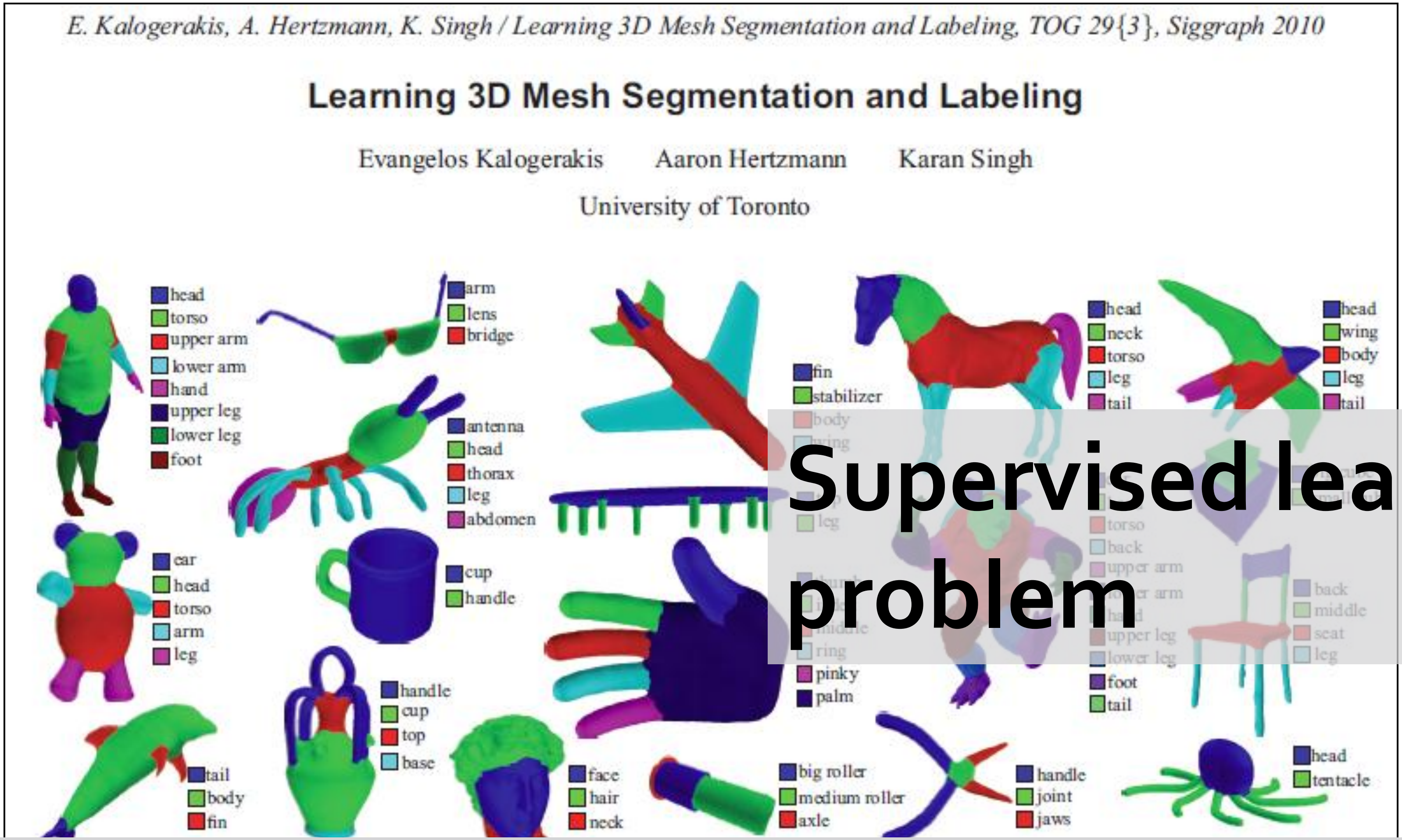
Feature Extraction Methods



Simplest strategy:
Cluster feature points (*k*-means, mean shift, etc.); use standard vision techniques for continuous regions

"Feature patches" → Segmentation

Feature Extraction Methods



Supervised learning problem

“Feature patches” → Segmentation

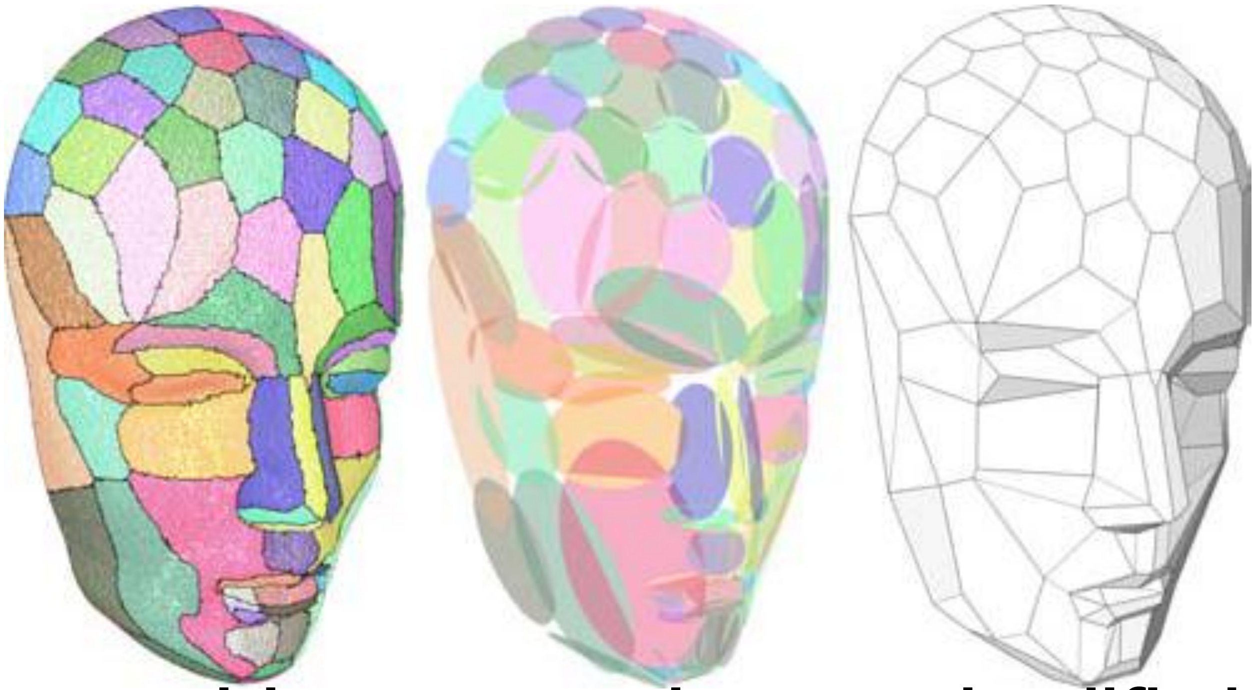
Feature Extraction Methods

Partition Proxies Simplified

Variational Shape Approximation
Cohen-Steiner, Alliez, and Desbrun 2004

"Feature patches" → Segmentation

Feature Extraction Methods



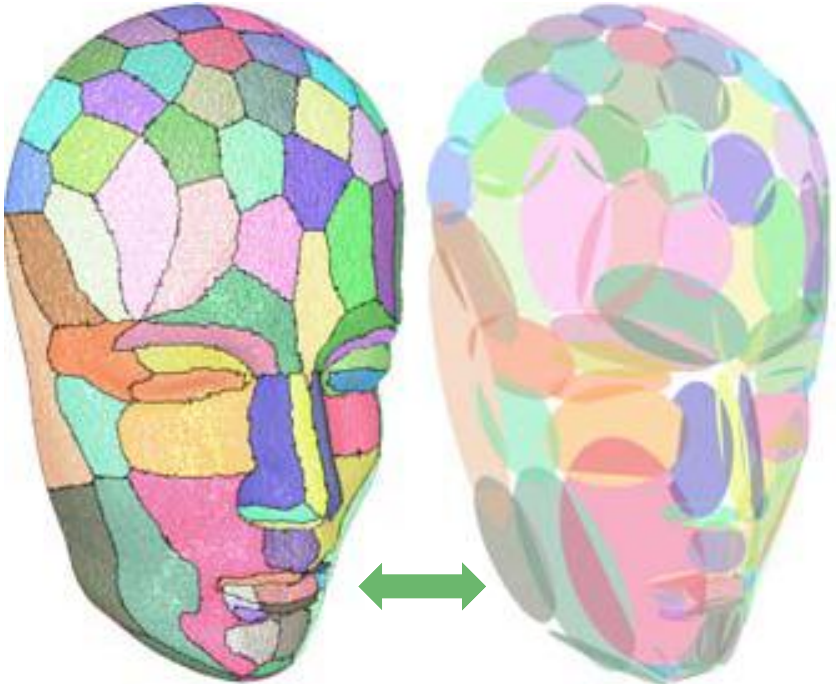
Partition Proxies Simplified

Variational Shape Approximation
Cohen-Steiner, Alliez, and Desbrun 2004

"Feature patches" → Segmentation

Feature Extraction Methods

Flood using priority queue



(X_i, N_i) minimizing fixed functional

Partition Proxies

Variational Shape Approximation
Cohen-Steiner, Alliez, and Desbrun 2004

"Feature patches" → Segmentation

Distances Between Surfaces

$$d_H(X,Y) = \max\left\{\sup_{x\in X}\inf_{y\in Y} d(x,y), \sup_{y\in Y}\inf_{x\in X} d(x,y)\right\}$$

Easy to compute

$$d_{GH}(X,Y) = \min_{f,g \text{ isometries}} d_H(f(x), g(y))$$

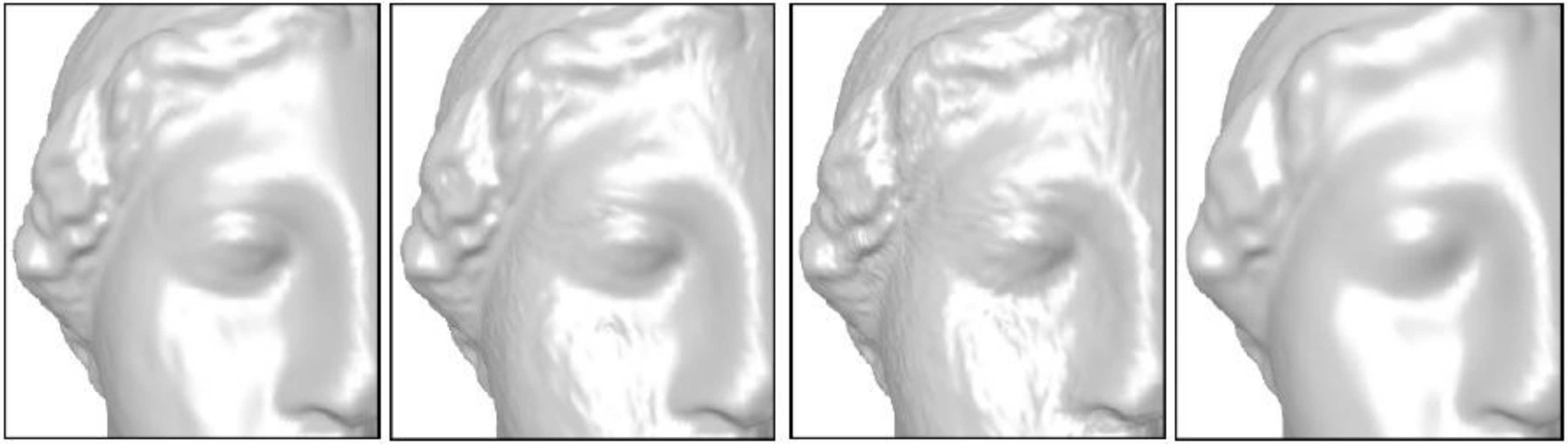
Hard to compute

Less hard to approximate

Related to Gromov-Wasserstein distance

(Gromov-)Hausdorff distance

Distances Between Surfaces



(a) Original model (b) High noise on smooth areas (c) High noise uniform (d) High smoothing uniform

Combine simple metrics (curvature, edge length distortion, etc.) with user studies.

http://liris.cnrs.fr/guillaume.lavoue/travaux/conference/SPIE-2006.pdf

Perceptual distance

Many Potential Tasks

- Segmentation
- Symmetry detection
- Global shape description
- Retrieval
- Recognition
- Feature extraction
- Alignment
- …

Intermission

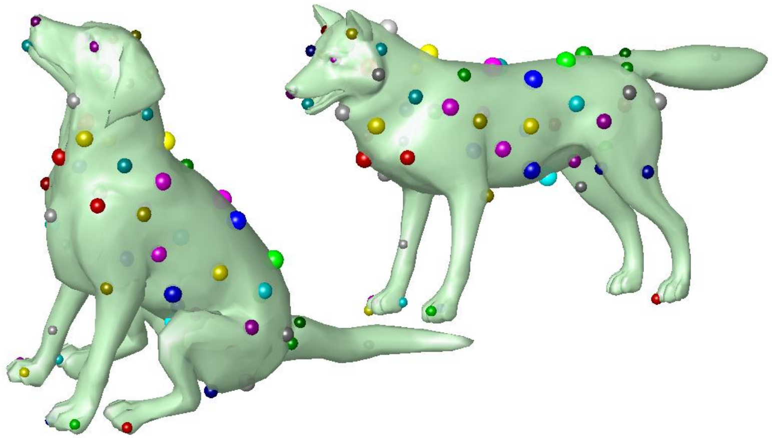
http://en.wikipedia.org/wiki/Let%27s_All_Go_to_the_Lobby

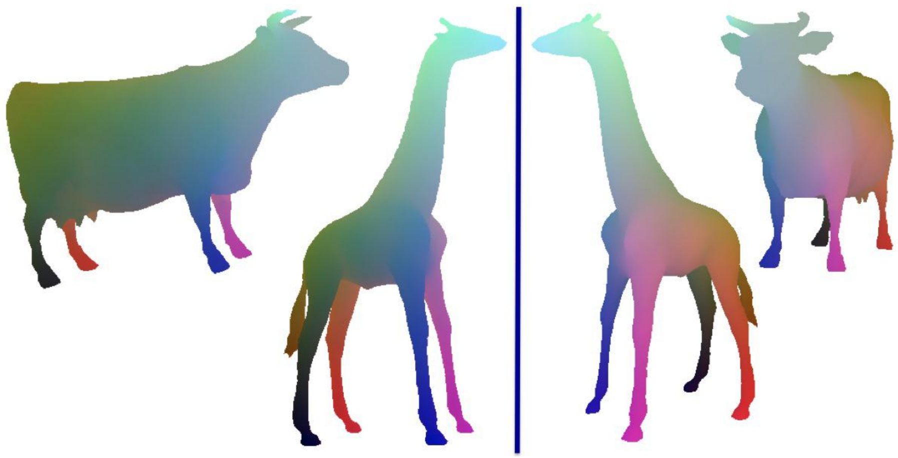
Part III:

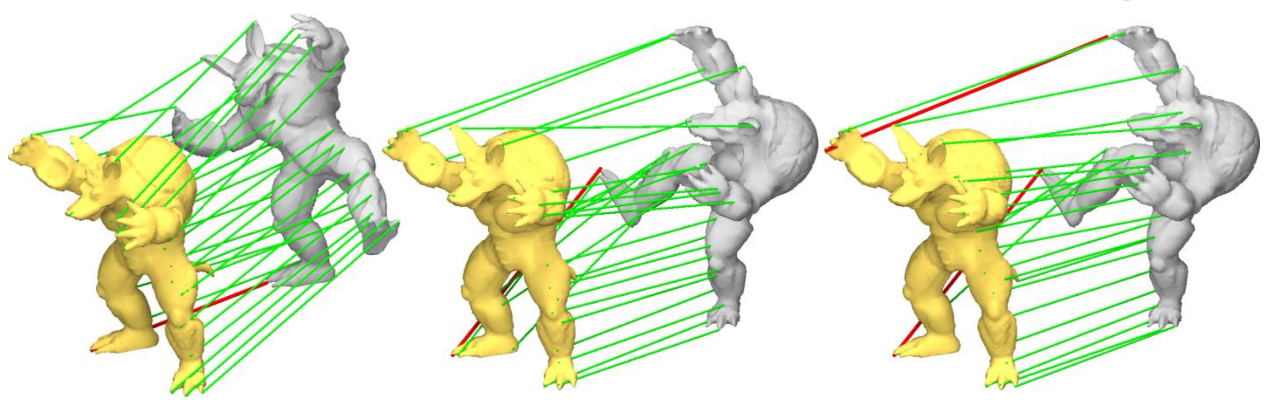
Correspondence

http://www.cs.princeton.edu/~funk/sig11.pdf

Goal







http://graphics.stanford.edu/projects/lgl/papers/ommg-opimhk-10/ommg-opimhk-10.pdf
http://www.cs.princeton.edu/~funk/sig11.pdf
http://gfx.cs.princeton.edu/pubs/Lipman_2009_MVF/mobius.pdf

Which points map to which?

Maks Ovsjanikov and Mirela Ben-Chen, CS 468

Taxonomy

- **Local vs. global**

  Refinement or alignment?

- **Rigid vs. deformable**

  Rotation/translation or stretching?

- **Pair vs. collection**

  Two shapes or many shapes?

http://graphics.stanford.edu/courses/cs468-10-fall/LectureSlides/11_shape_matching.pdf

(Only?) Solved Case

- **Local vs. global**
  Refinement or alignment?

- **Rigid vs. deformable**
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- **Pair vs. collection**
  Two shapes or many shapes?

http://graphics.stanford.edu/courses/cs468-10-fall/LectureSlides/11_shape_matching.pdf

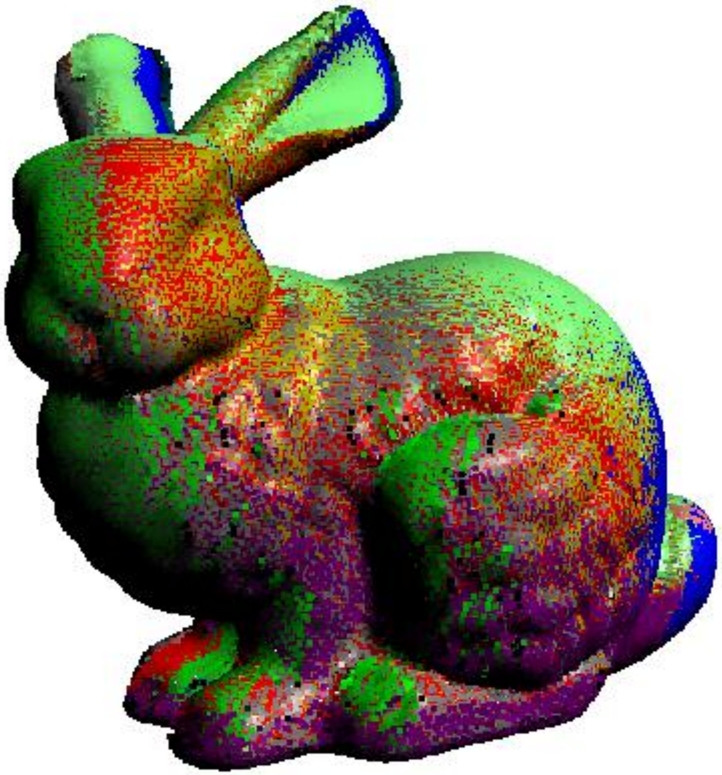
(Only?) Solved Case

- **Local vs. global**
  Refinement or alignment?
- **Rigid vs. deformable**
  Rotation/translation or stretching?
- **Pair vs. collection**
  Two shapes or many shapes?

Lots of room for research!

http://graphics.stanford.edu/courses/cs468-10-fall/LectureSlides/11_shape_matching.pdf

Local/Rigid/Pairwise Mapping



Repeat:

1. For each x_i in X, find closest y_i in Y.
2. Find rigid deformation (R,T) minimizing

$$\sum_i \|(Rx_i + T) - y_i\|$$

http://graphics.stanford.edu/courses/cs468-10-fall/LectureSlides/11_shape_matching.pdf
http://www.gris.uni-tuebingen.de/people/staff/bokeloh/gallery/bunny_res1.png

Iterative Closest Point (ICP)

Local/Rigid/Pairwise Mapping

Repeat:

1. For each x_i in X, find closest y_i in Y.
2. Find rigid deformation (R,T) minimizing

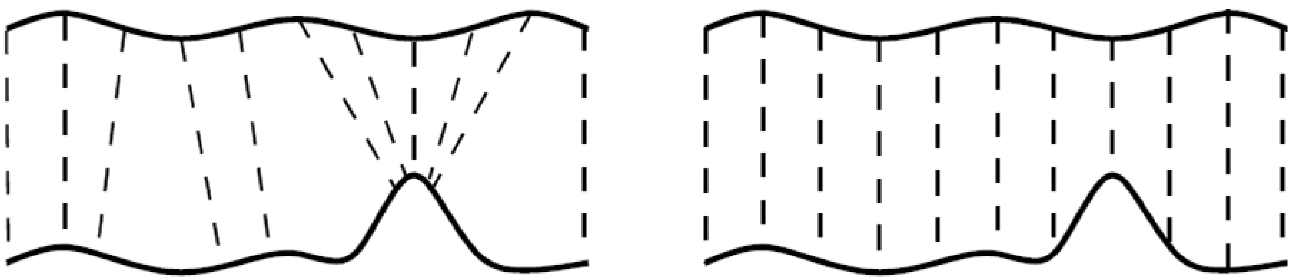
$$\sum_i \|(Rx_i + T) - y_i\|$$

Converges to local minimum.

http://graphics.stanford.edu/courses/cs468-10-fall/LectureSlides/11_shape_matching.pdf
http://www.gris.uni-tuebingen.de/people/staff/bokeloh/gallery/bunny_res1.png

Iterative Closest Point (ICP)

ICP Variations



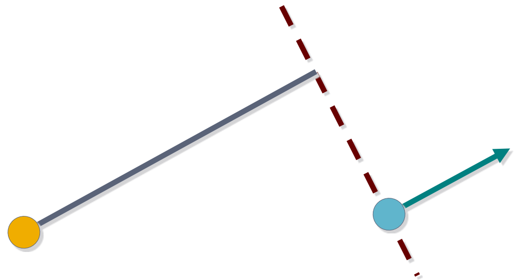
- **Selection of sample points**
  One or both surfaces? How many?

- **Matching points on the surfaces**
  Closest? Approximate nearest? Normal lines? Compatible normal/curvature/color?

- **Weighting correspondences**
  Distance? Compatibility? Scanner certainty?

www.math.tau.ac.il/~dcor/Graphics/adv-slides/ICP.ppt

ICP Variations



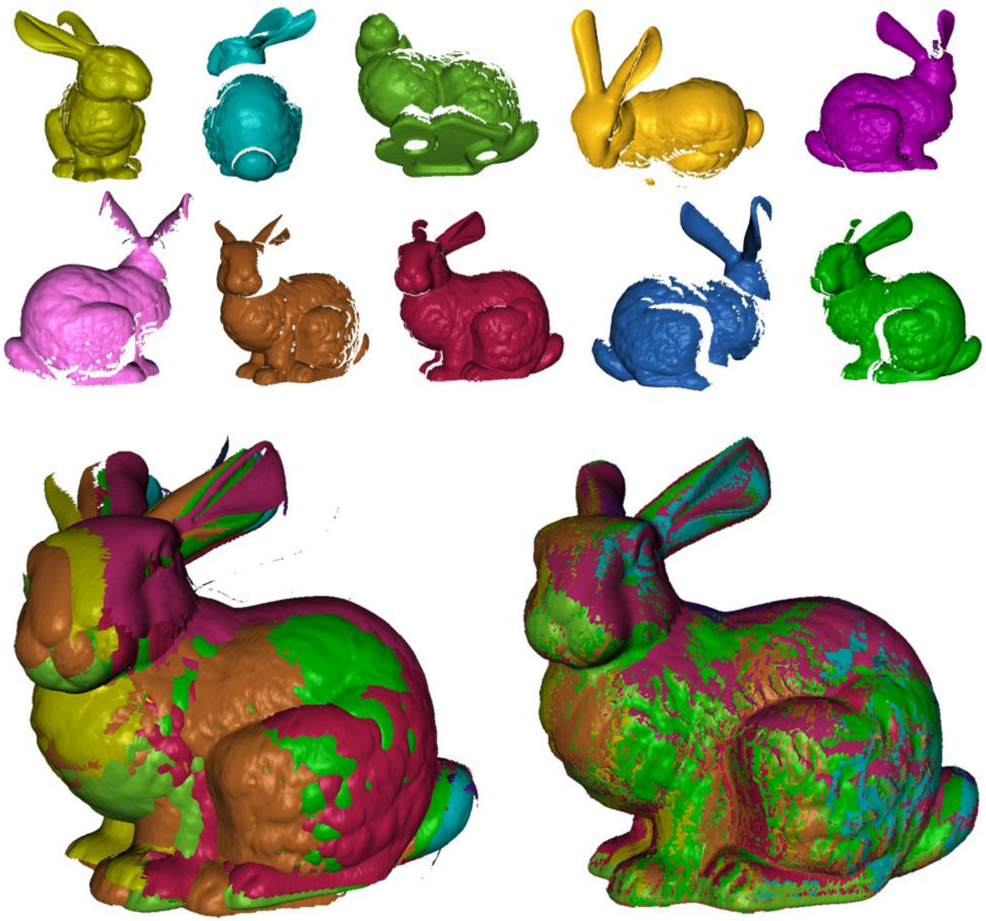
- **Reject outlier pairs**
  Too far? Inconsistent with neighbors? Incompatible descriptors?

- **Modified error metric**
  Allow affine transformations? Nonrigid motion?

- **Optimization technique**
  Avoid local minima?

www.math.tau.ac.il/~dcor/Graphics/adv-slides/ICP.ppt

Global Matching



Align shapes in arbitrary positions

Starting point for ICP

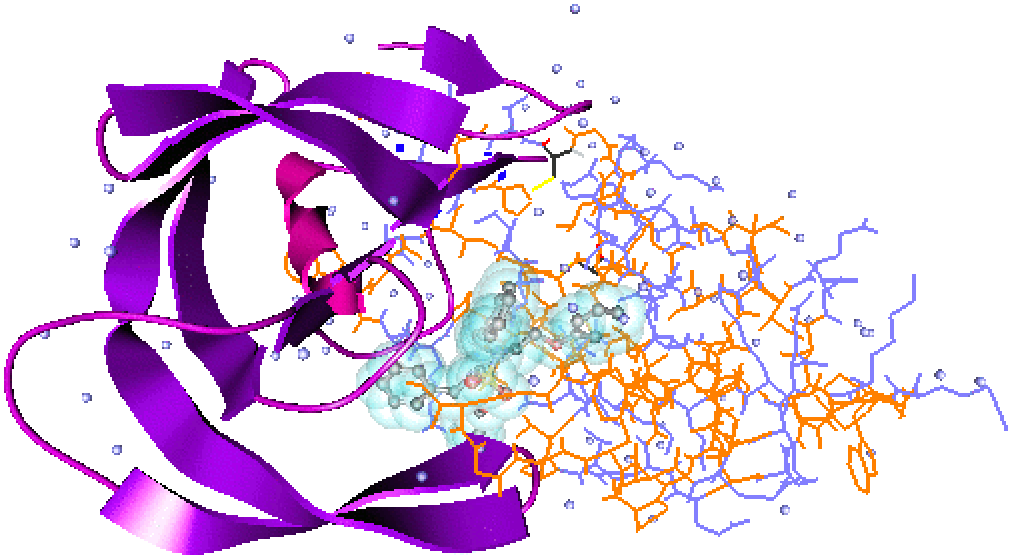
http://gmsv.kaust.edu.sa/people/faculty/pottmann/pottmann_pdf/registration.pdf

Global Matching Strategies

- Exhaustive search
- Normalization
- Random sampling
- Invariance

http://graphics.stanford.edu/courses/cs468-10-fall/LectureSlides/11_shape_matching.pdf

Global Matching Strategies



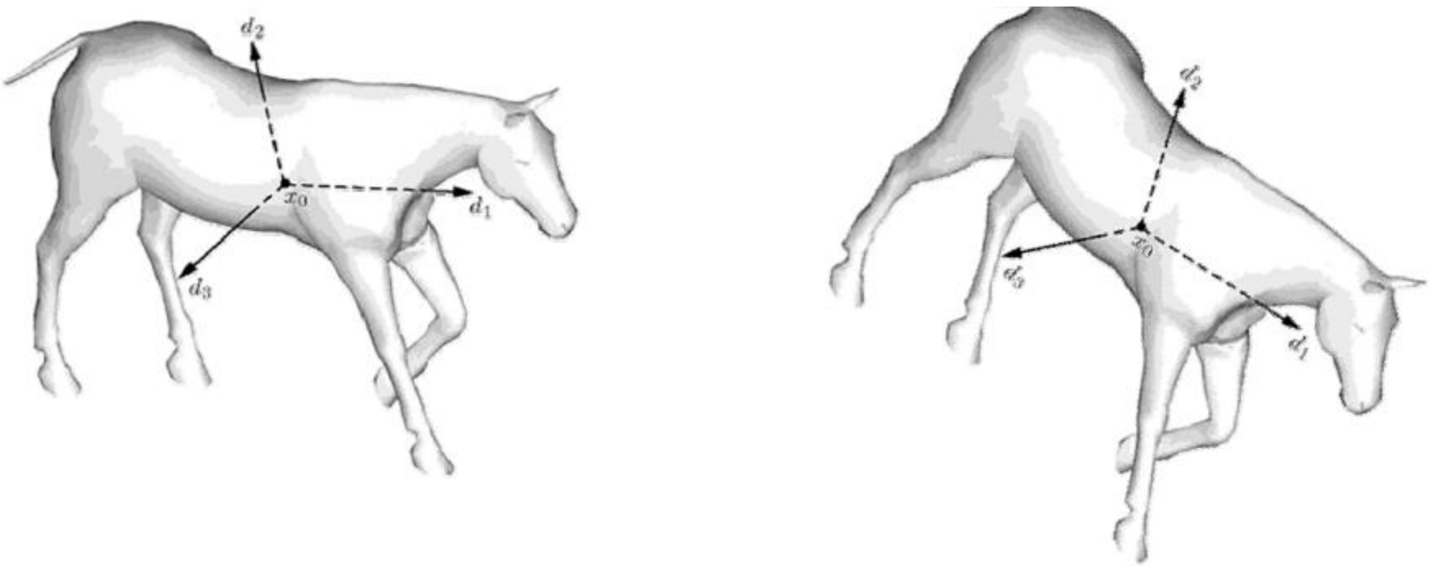
Sample possible alignments

Keep best post-ICP
(Slow, only for rigid!)

http://graphics.stanford.edu/courses/cs468-10-fall/LectureSlides/11_shape_matching.pdf
http://vis.lbl.gov/~scrivelli/DShop_research.html

Exhaustive search

Global Matching Strategies



Find canonical alignment

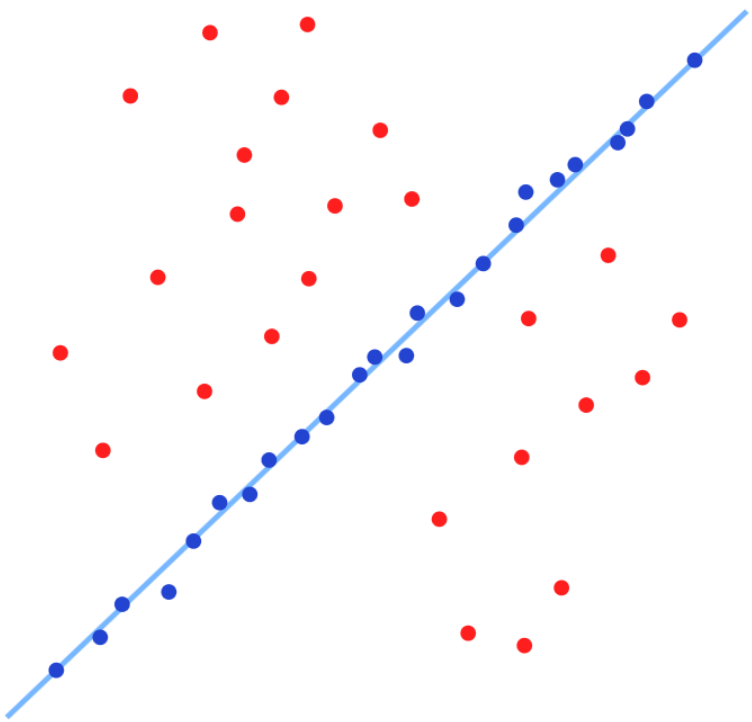
e.g. using PCA; reduces number of starting points

http://graphics.stanford.edu/courses/cs468-10-fall/LectureSlides/11_shape_matching.pdf

Normalization

Global Matching Strategies

RANSAC: Random Sample Consensus



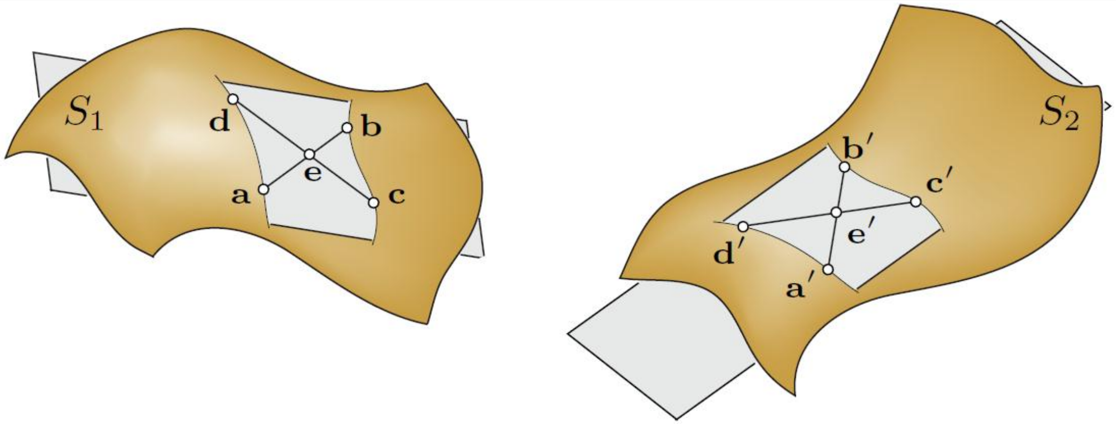
http://upload.wikimedia.org/wikipedia/commons/d/de/Fitted_line.svg

Repeat:

1. Guess minimum number of points to determine parameters
2. Check if model works for other points

Random sampling

Global Matching Strategies



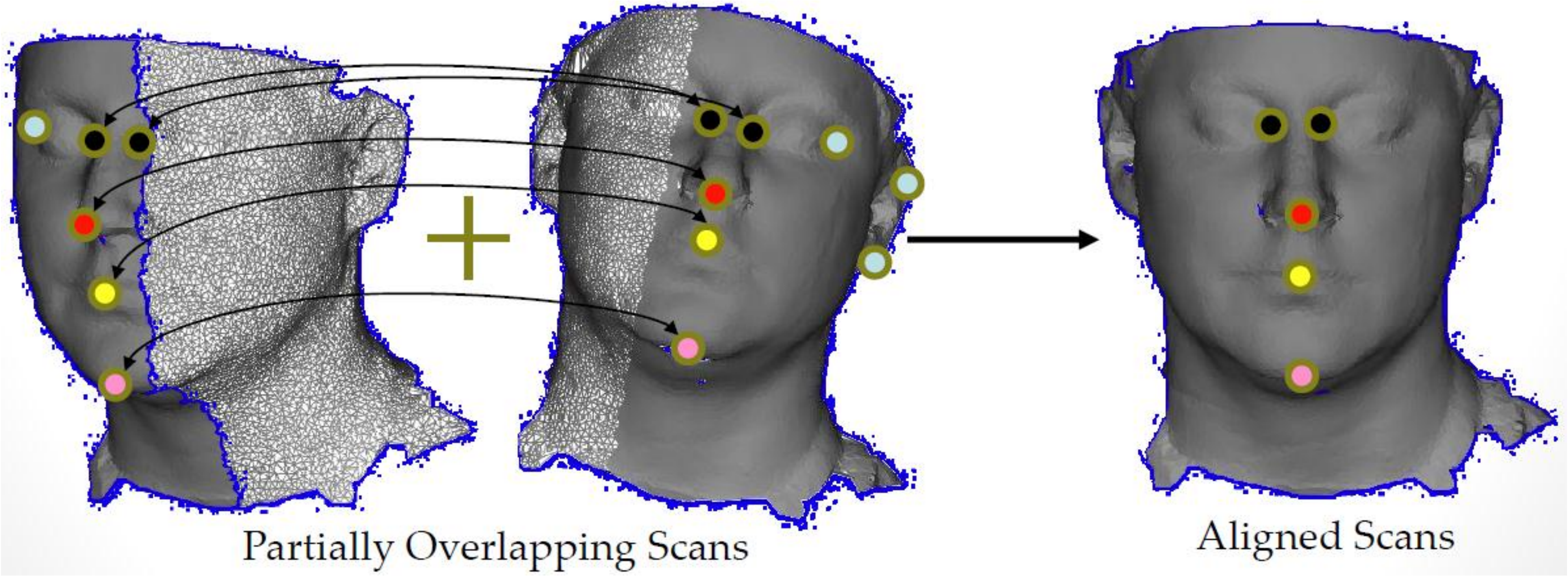
RANSAC with sets of four near-coplanar points. Affine maps preserve ||c-e||/||c-d||, so sample points *e′* with these ratios (n^2 time), then match those.

4-Points Congruent Sets for Robust Pairwise Surface Registration
Aiger, Mitra, and Cohen-Or 2008

Random sampling

Global Matching Strategies

1. Find interesting **points**.
2. Match **feature vectors** on those points.
3. Compute the **aligning transformation**.

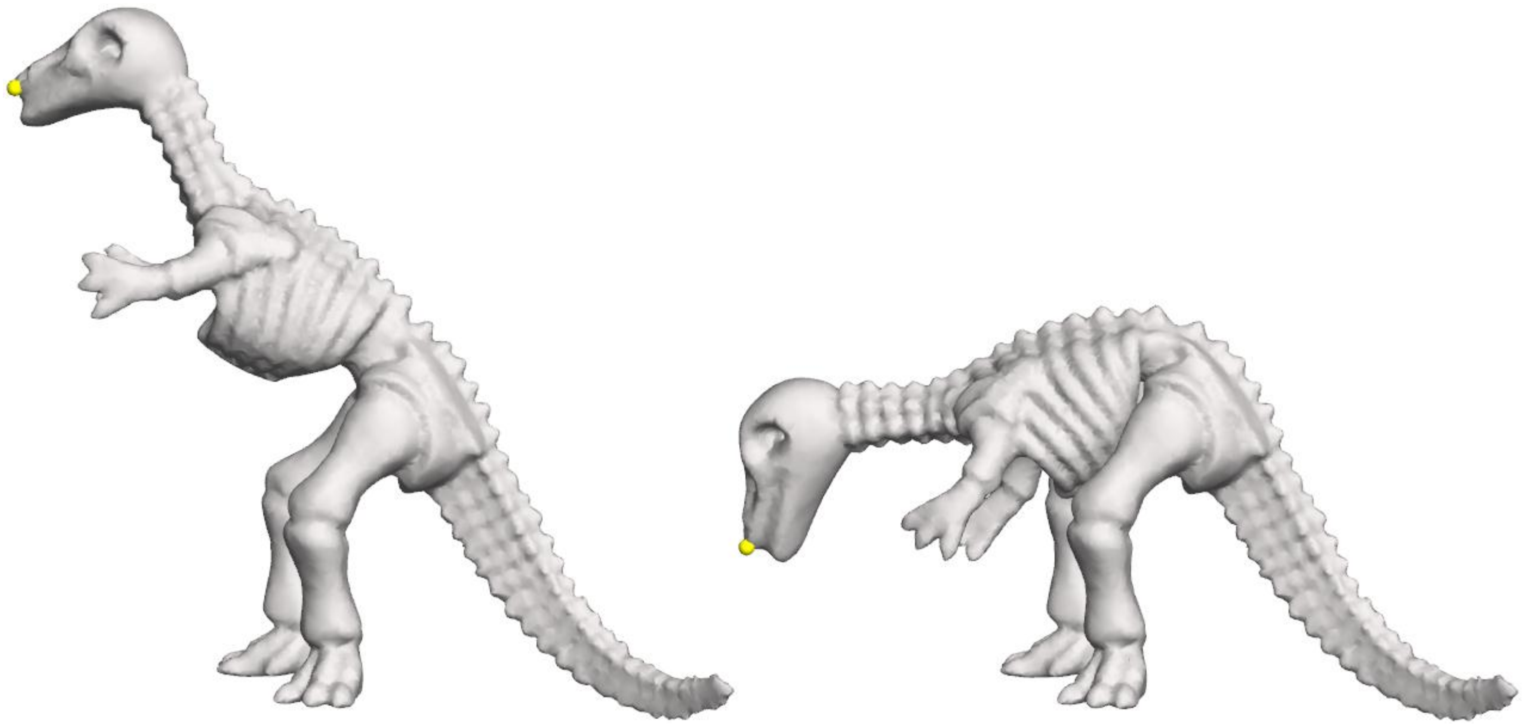


http://graphics.stanford.edu/courses/cs468-10-fall/LectureSlides/11_shape_matching.pdf

Invariance: Already done!

Deformable Shape Matching

Elastic, thin shell, volumetric, ARAP, bending, …



igl.ethz.ch/projects/ARAP/

Needs a deformation model

Deformable Shape Matching

Elastic, thin shell, volumetric, ARAP, bending, …

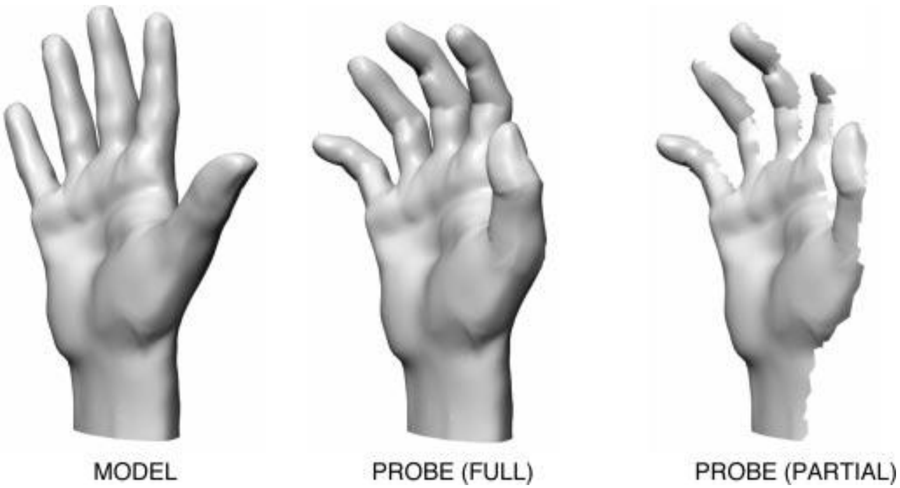
igl.ethz.ch/projects/ARAP/

Needs a deformation model

Local Deformable Matching

http://www.hao-li.com/publications/papers/sgp2008GCO.pdf

Global Deformable Matching



Embed samples of one surface directly over another by minimizing a "generalized stress" involving geodesics.

Generalized Multidimensional Scaling
Bronstein, Bronstein, Kimmel 2006

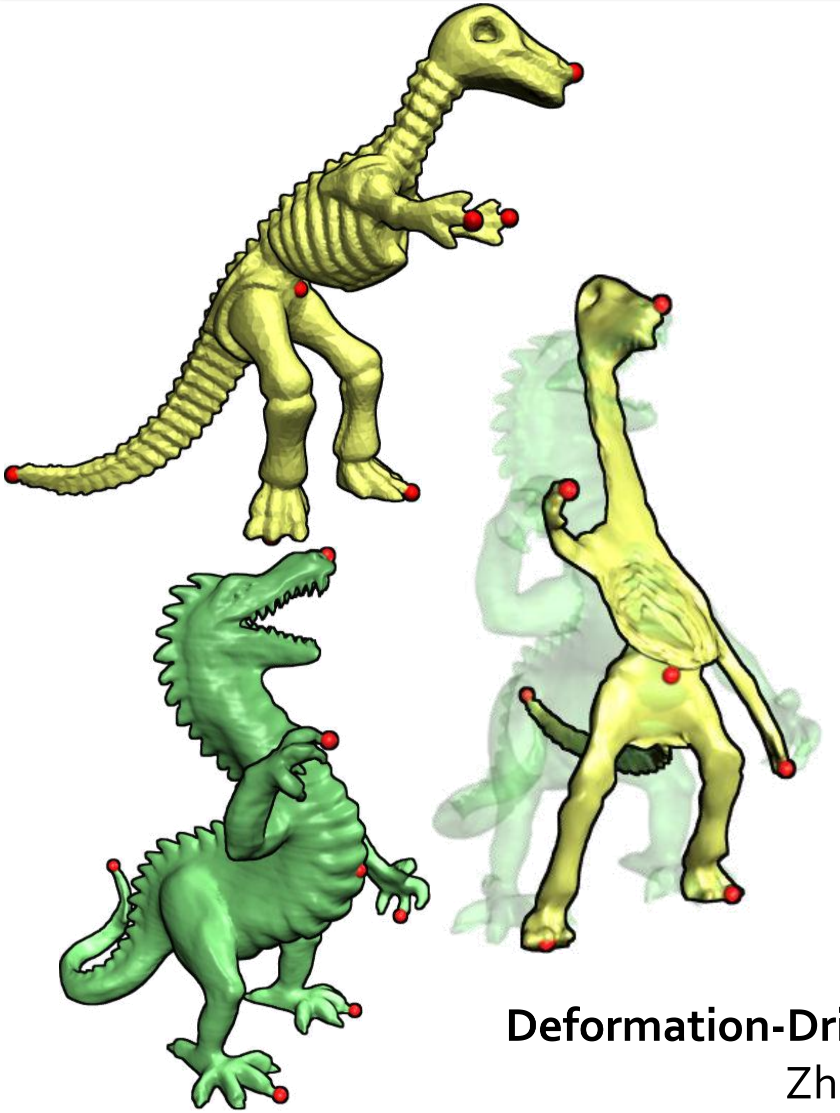
Generalized Multi-Dimensional Scaling (GMDS)

Global Deformable Matching

Alternate between **matching feature points** using descriptors and moving other points in **rigid clusters**; isometry assumption helps prune bad matches.

Non-Rigid Registration Under Isometric Deformations
Huang et al. 2008

Global Deformable Matching

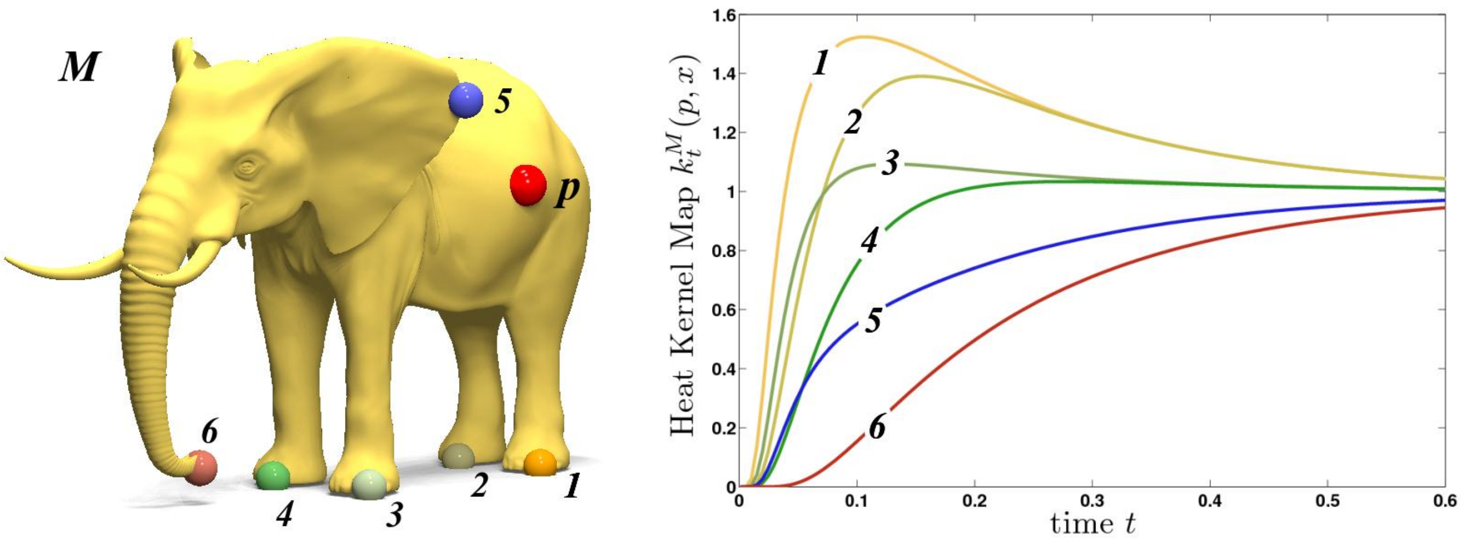


1. Extract feature points (part extrema)
2. Combinatorial search matching features; evaluate by deforming the mesh

Slow!

Deformation-Driven Shape Correspondence
Zheng et al. 2008

Global Deformable Matching

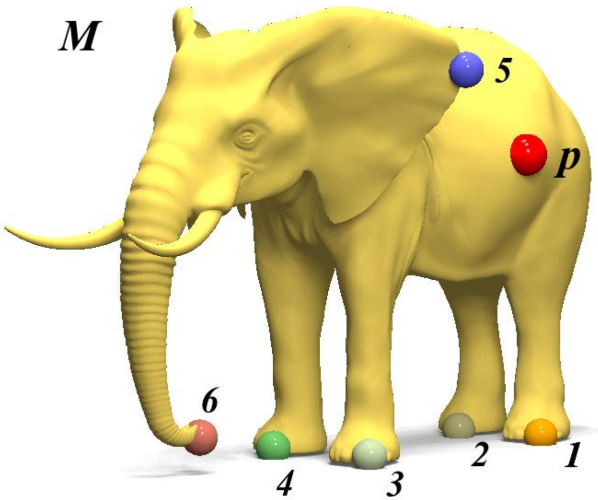


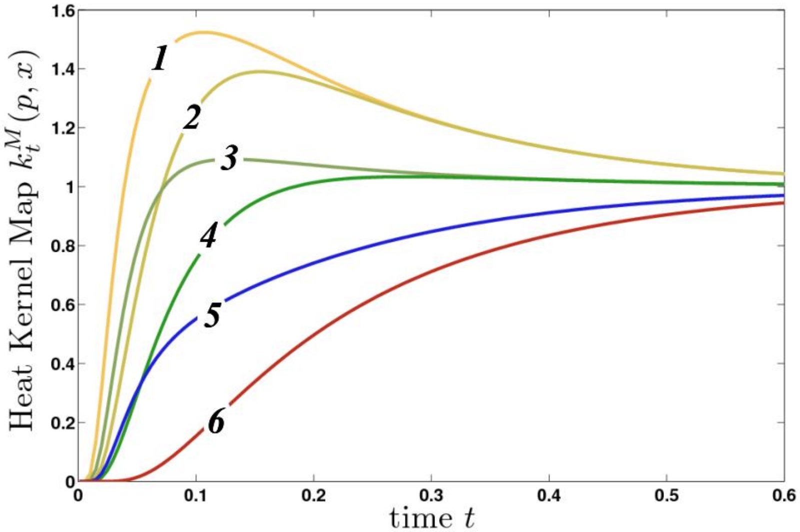
$$\boxed{HKM_p(x,t) = k_t(p,x)}$$

How much heat diffuses from *p* to *x* in time *t*?

One Point Isometric Matching with the Heat Kernel
Ovsjanikov et al. 2010

Global Deformable Matching





$$\boxed{HKM_p(x,t) = k_t(p,x)}$$

Theorem: Only have to match one point!

KNN

One Point Isometric Matching with the Heat Kernel

Ovsjanikov et al. 2010

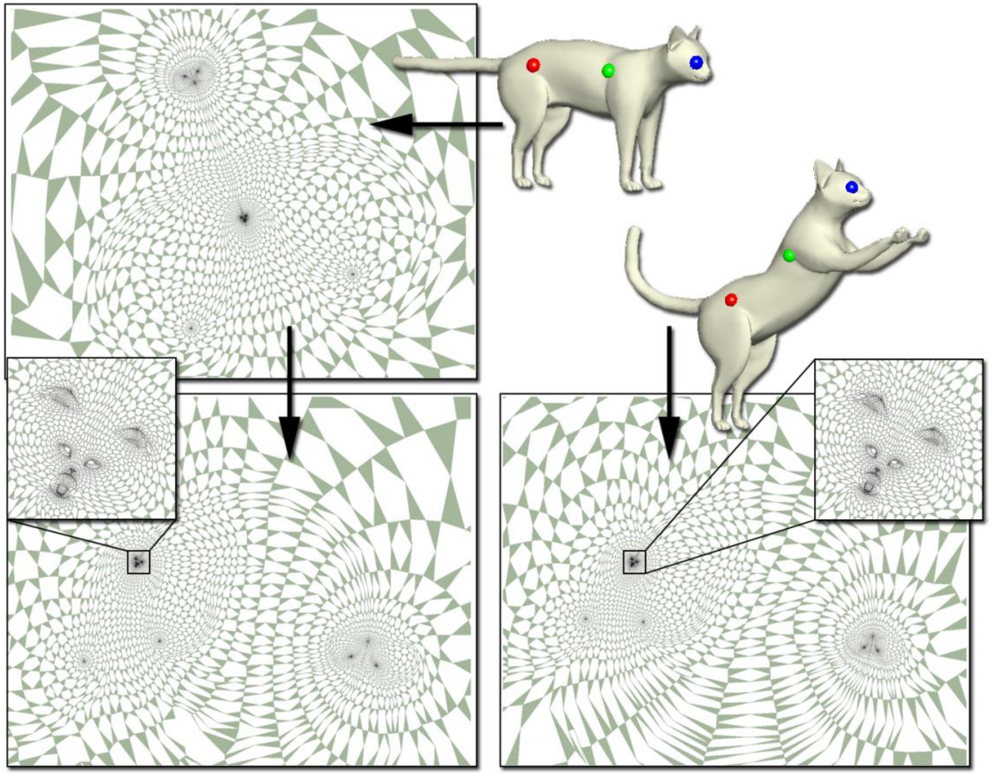
Global Deformable Matching

$$\text{isometries} \subseteq \text{conformal maps}$$

Hard! Easier

Möbius Voting for Surface Correspondence
Lipman and Funkhouser 2009

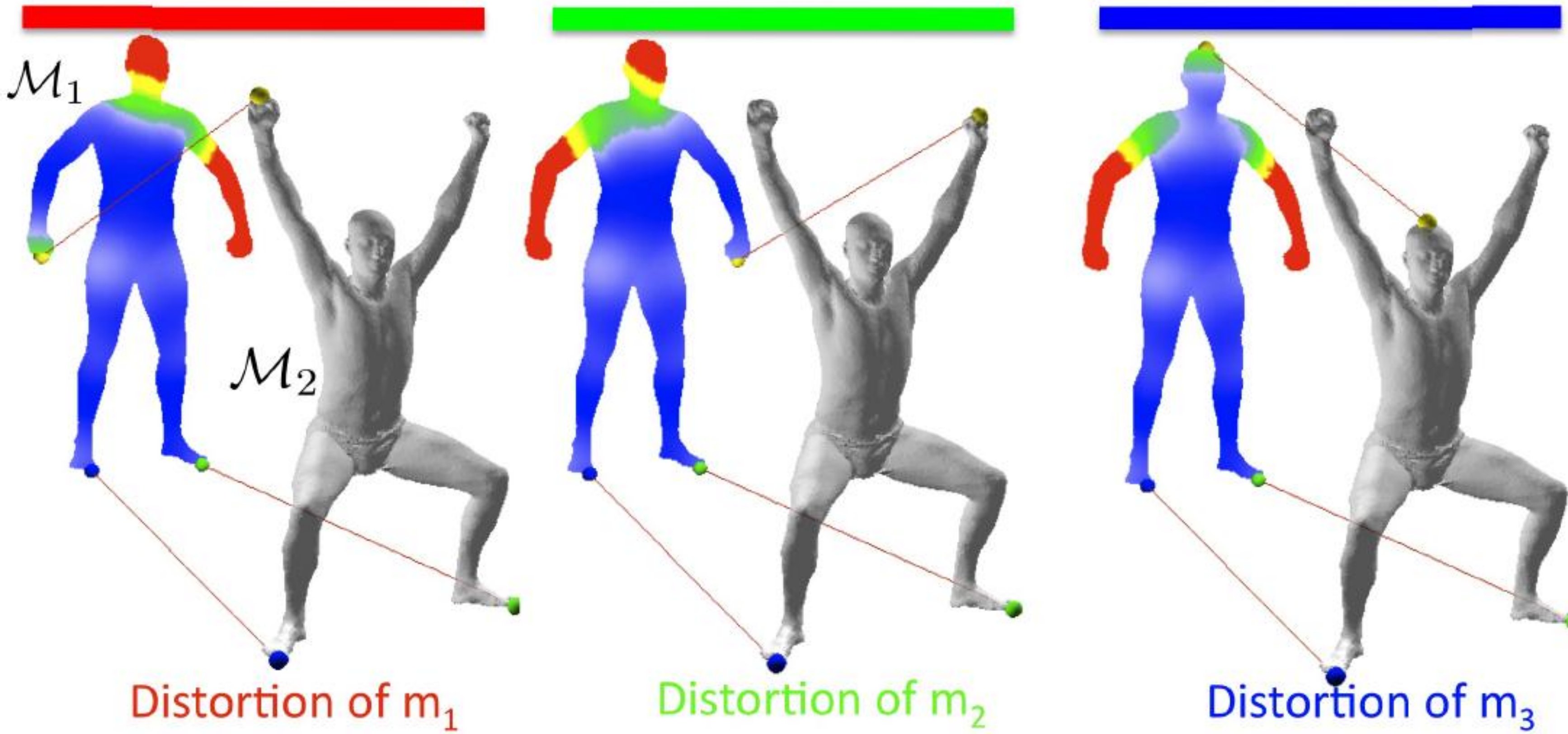
Global Deformable Matching



1. Map surfaces to complex plane
2. Select three points
3. Map plane to itself matching these points
4. Vote for pairings using distortion metric to weight
5. Return to 2

Möbius Voting for Surface Correspondence
Lipman and Funkhouser 2009

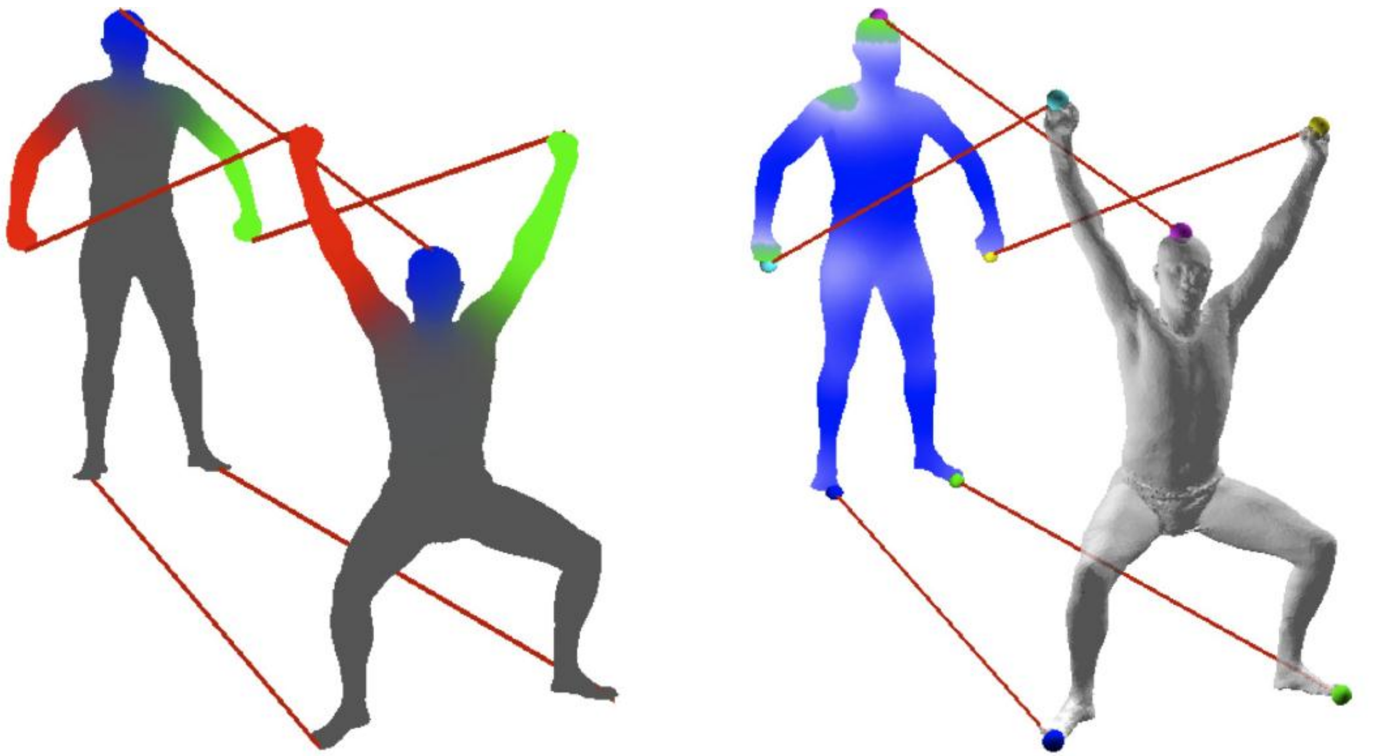
Global Deformable Matching



Different simple maps might be good in different places.

Blended Intrinsic Maps
Kim, Lipman, and Funkhouser 2011

Global Deformable Matching

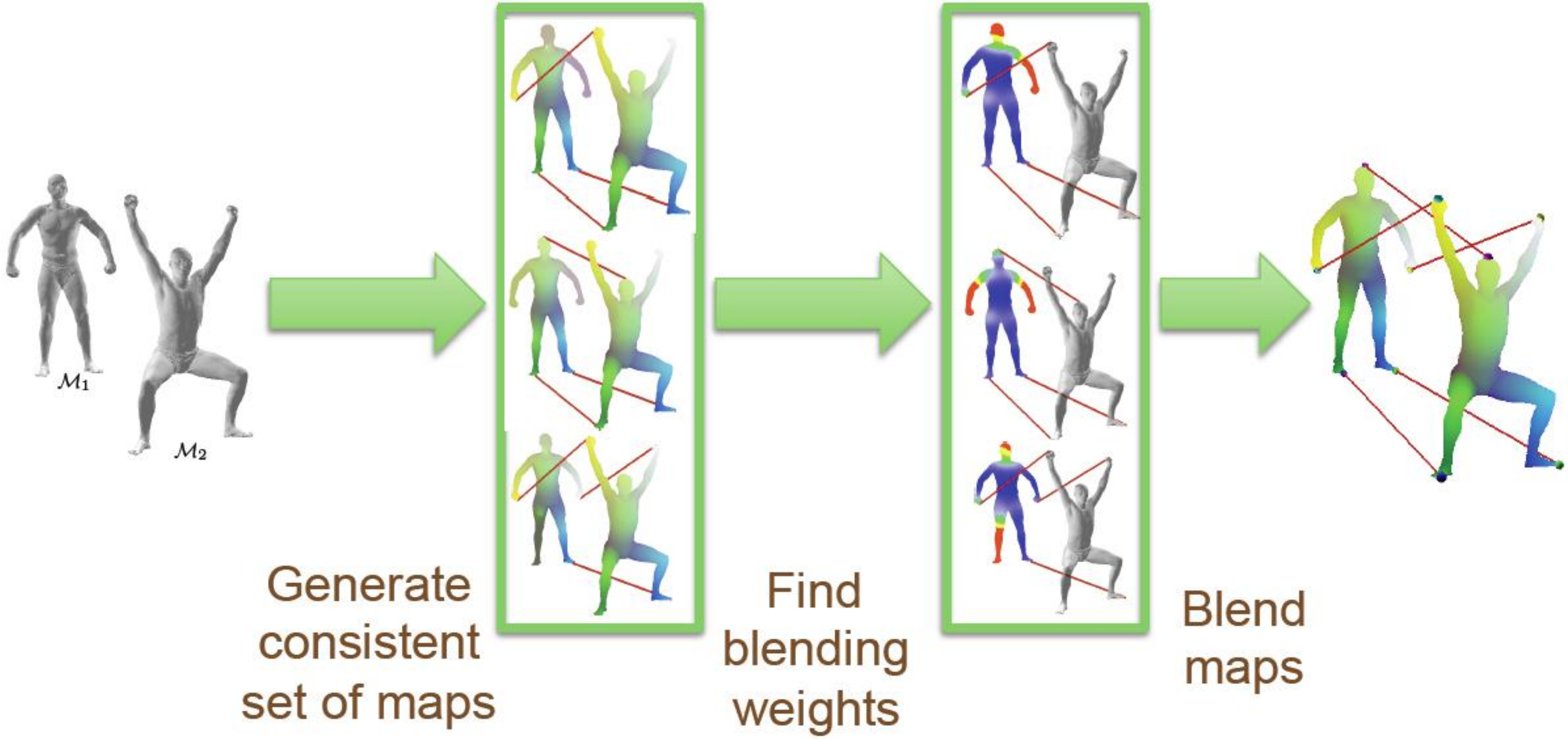


Blending Weights for m_1, m_2, and m_3 | Distortion of the Blended Map

Combine good parts of different maps!

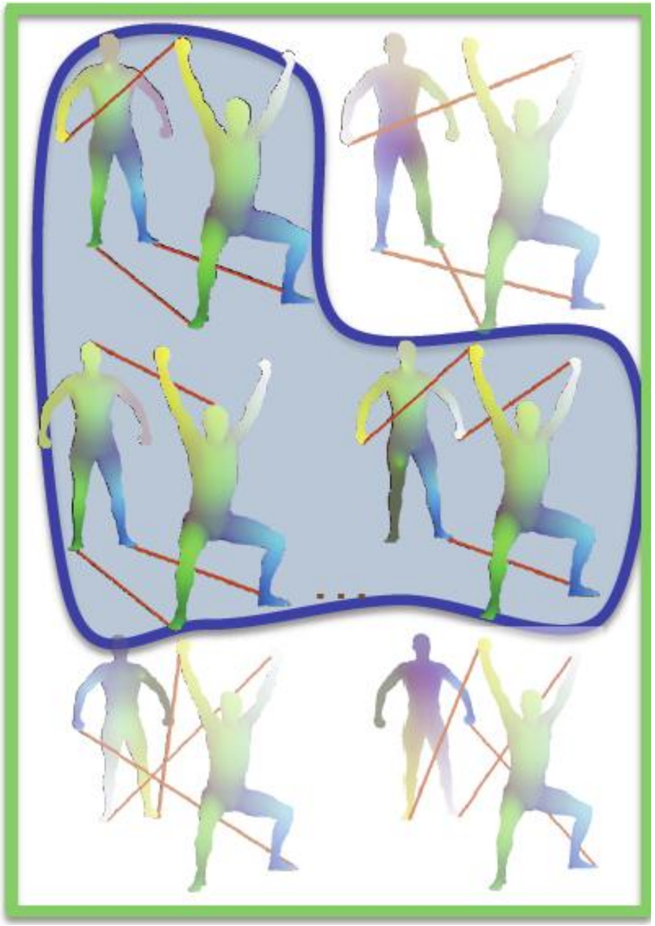
Blended Intrinsic Maps
Kim, Lipman, and Funkhouser 2011

Global Deformable Matching



Blended Intrinsic Maps
Kim, Lipman, and Funkhouser 2011

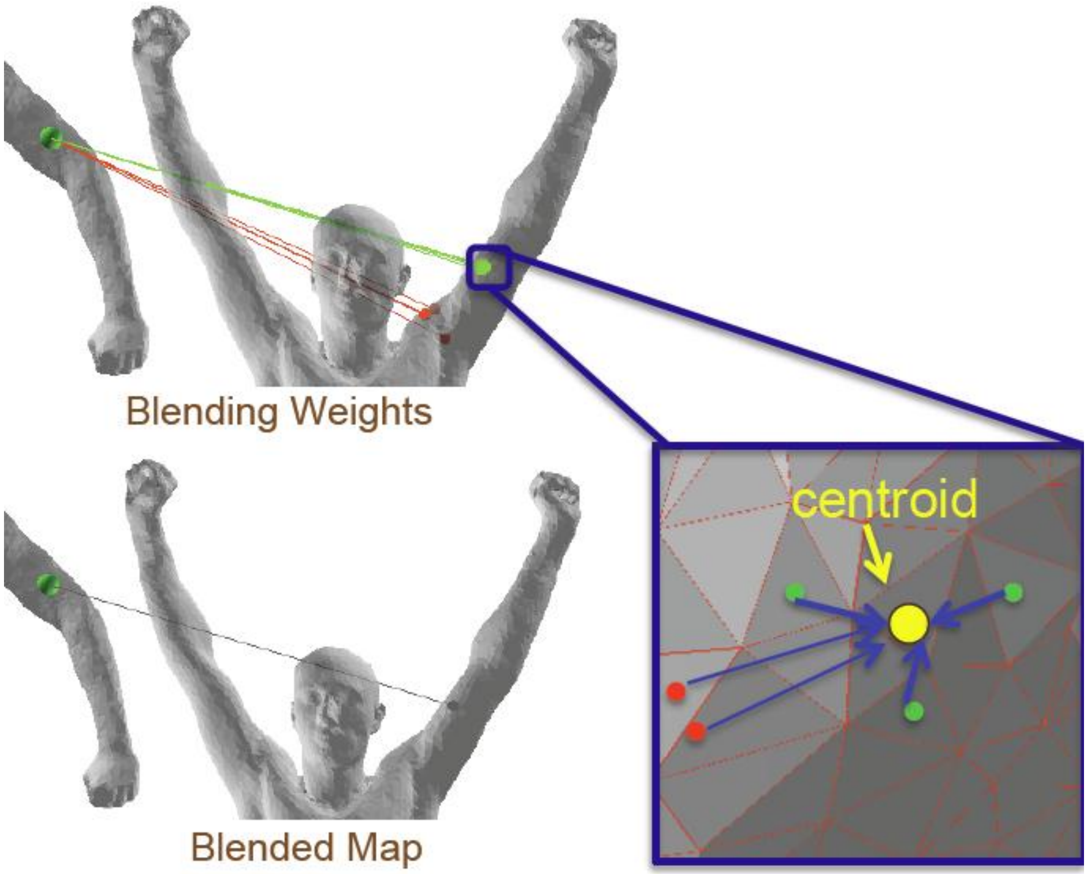
Global Deformable Matching



Find groups of consistent/similar maps by clustering in a similarity matrix.

Blended Intrinsic Maps
Kim, Lipman, and Funkhouser 2011

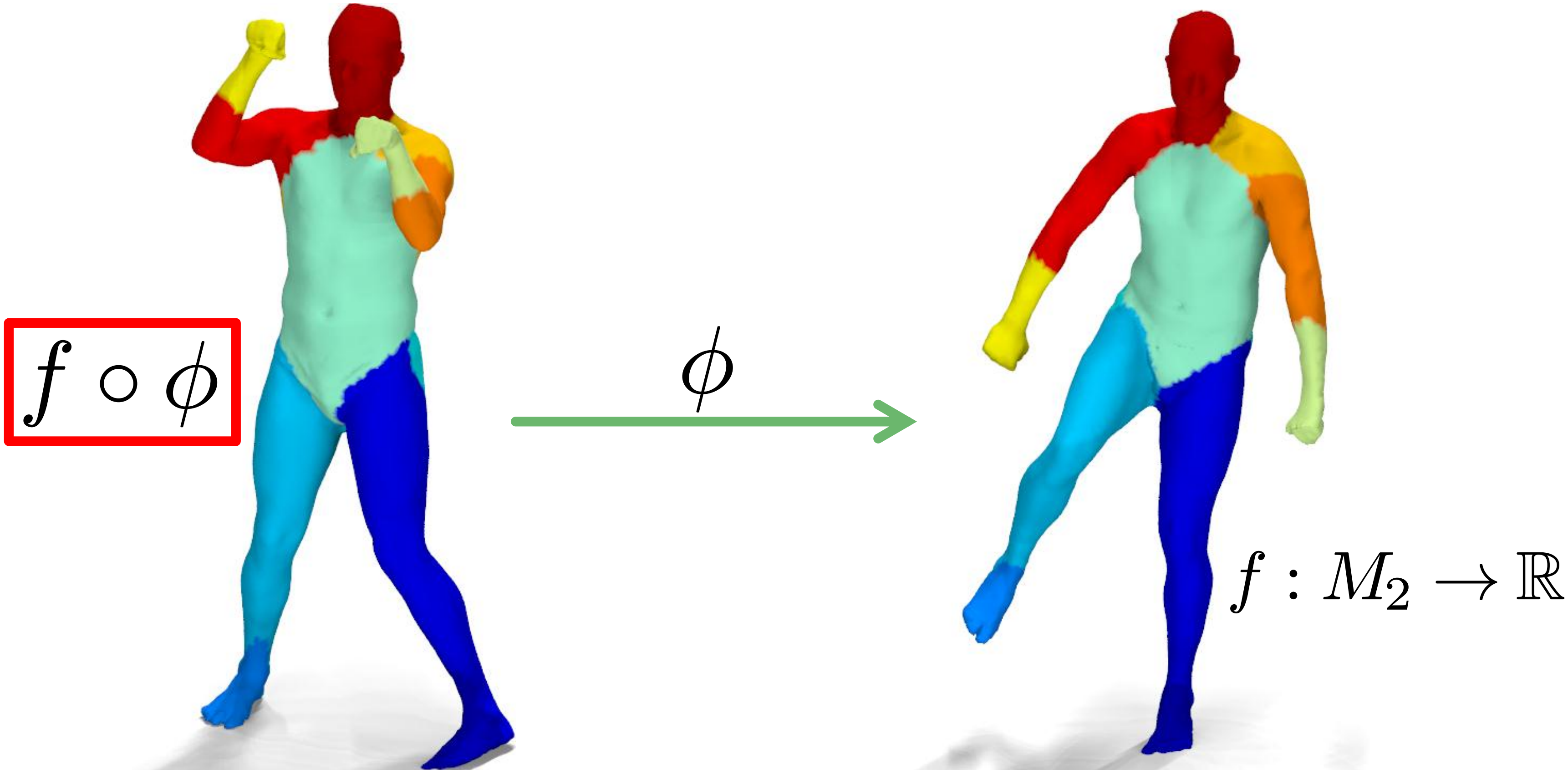
Global Deformable Matching



Weight maps at each vertex based on deviation from isometry. Output **weighted geodesic centroid**.

Blended Intrinsic Maps
Kim, Lipman, and Funkhouser 2011

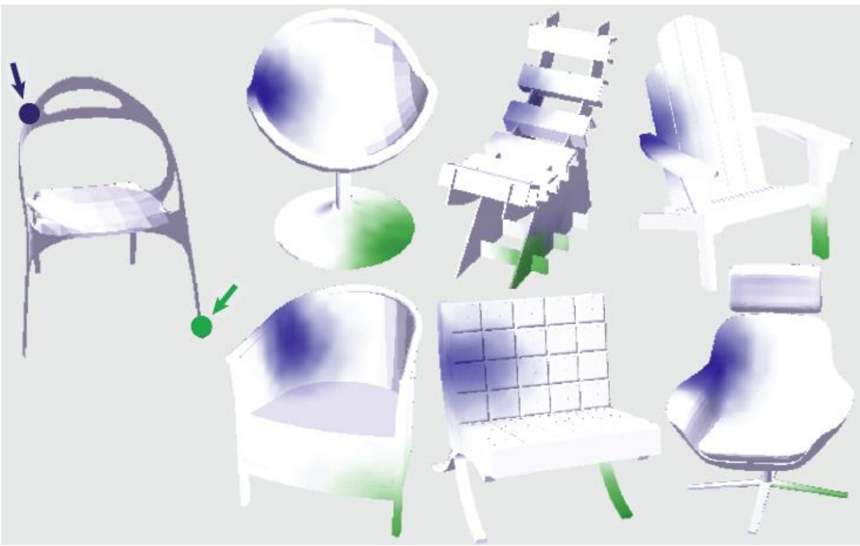
New Frontier in Mapping



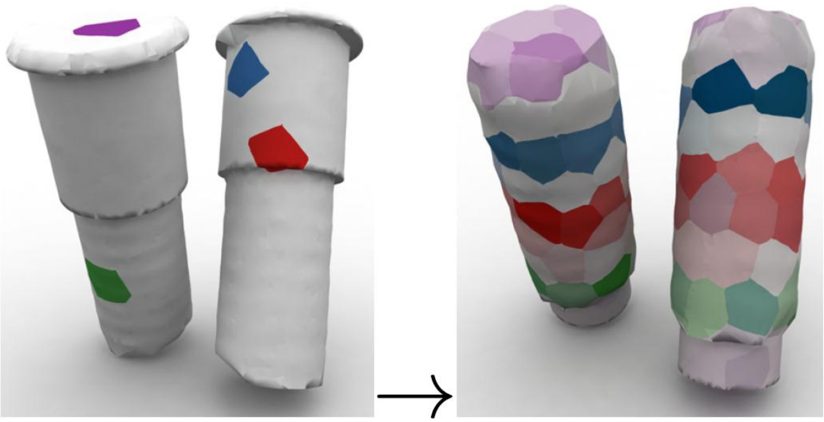
Functional Maps: A Flexible Representation of Maps Between Shapes
Ovsjanikov et al. 2012 (to appear)

Map representations

New Frontier in Mapping



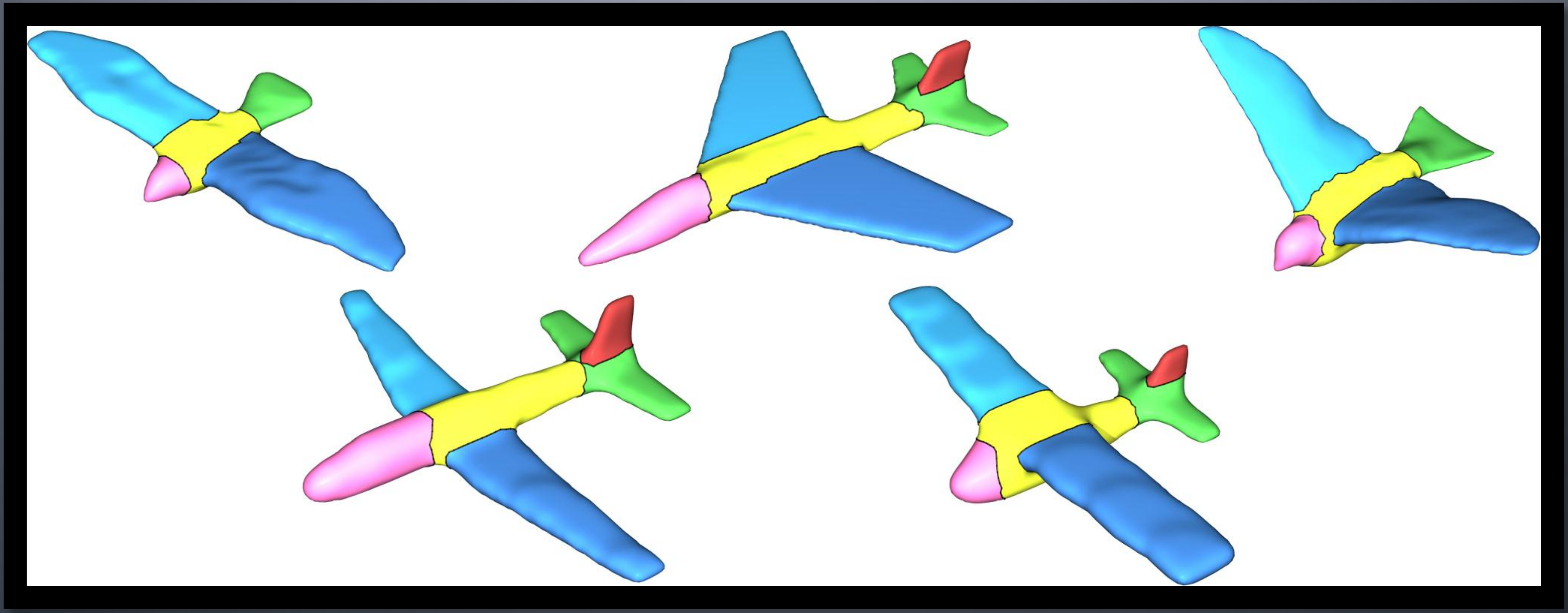
Exploring Collections of 3D Models using Fuzzy Correspondences
Kim et al. 2012 (to appear)



Soft Maps Between Surfaces
Solomon et al. 2012 (to appear … shortly!)

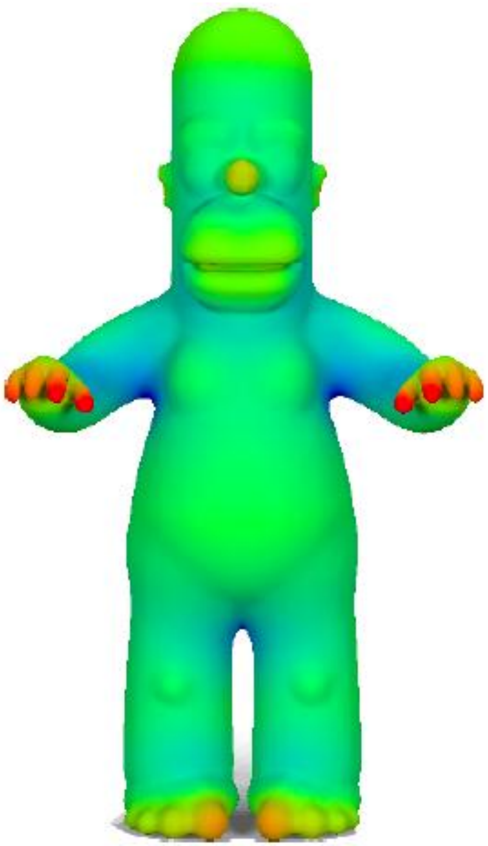
Map representations

Part IV:
Shape Collections

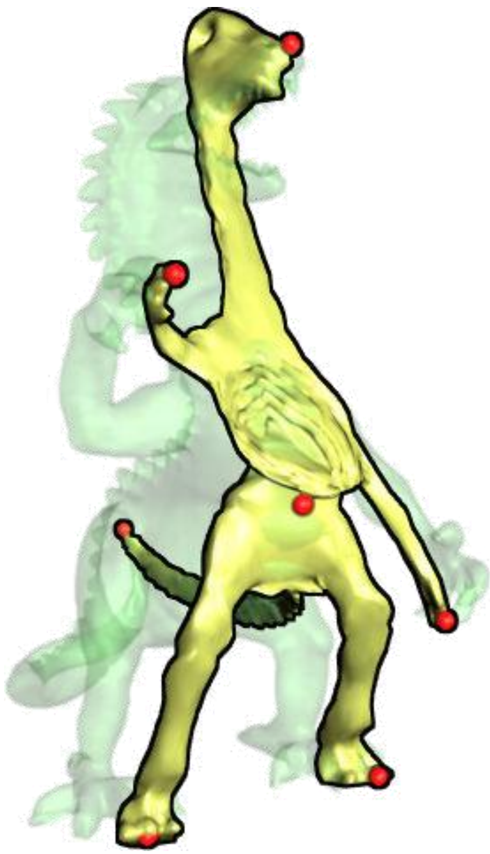


http://www.stanford.edu/~huangqx/siga11_jointseg.pdf

Our Story So Far



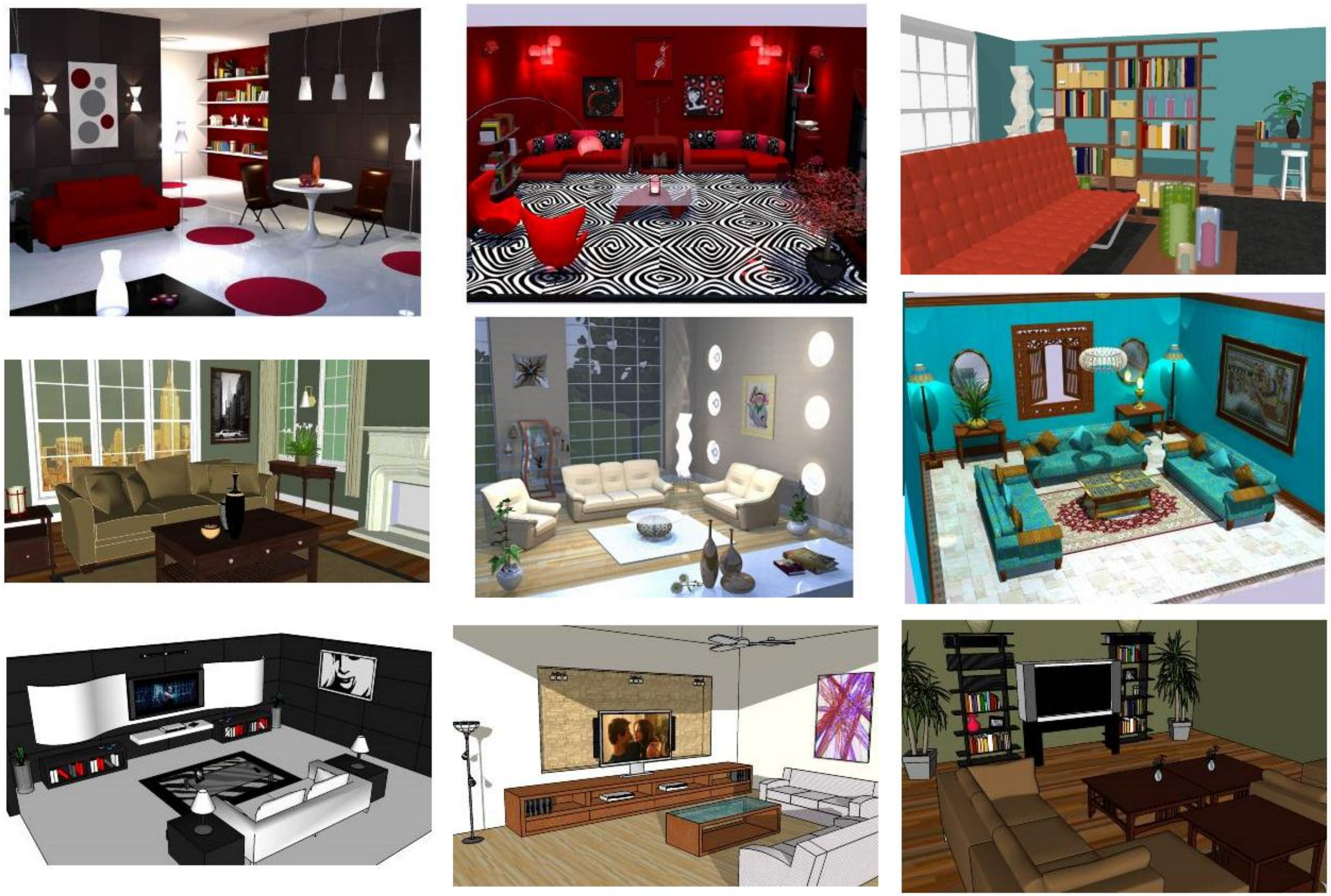
One surface



Two surfaces

...

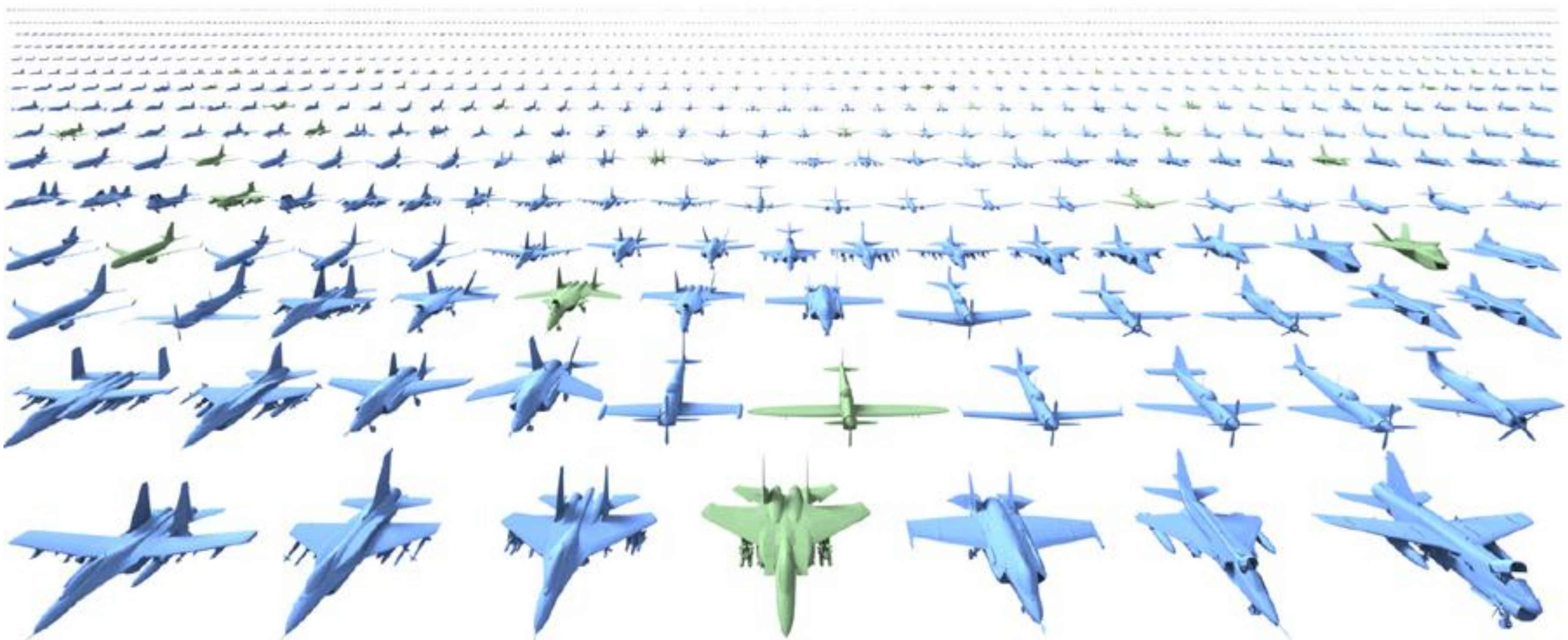
Shape Rarely Exist in a Vacuum



http://graphics.stanford.edu/~mdfisher/Data/GraphKernel.pdf

Scenes

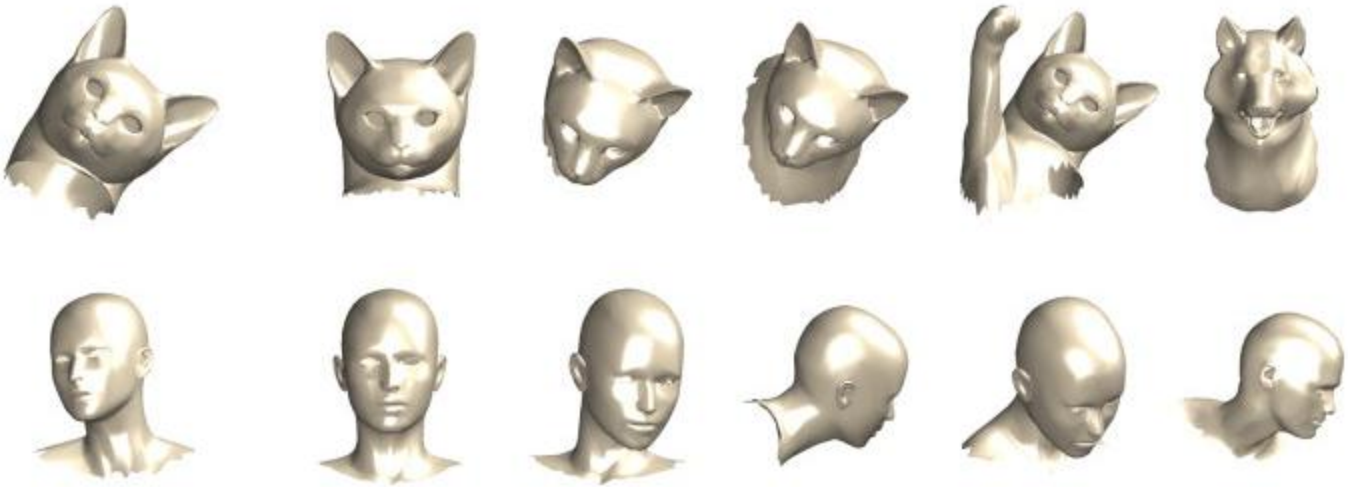
Shape Rarely Exist in a Vacuum



http://people.cs.umass.edu/~kalo/papers/ShapeSynthesis/index.html

Databases

Shape Rarely Exist in a Vacuum



http://ars.sciencedirect.com/content/image/1-s2.0-S0097849311000501-gr9.jpg

Motions of one object

Motivation

You can learn about one shape using its relationship to other shapes.

Examples

- Function
- Key features
- Deformation model
- Usability
- Structure
- Symmetries
- Missing information
- …

Shape Space

"There are manifoldnesses in which the determination of position requires not a finite number, but . . . a continuous manifoldness of determinations of quantity. Such manifoldnesses are, for example, the possible determinations of a function for a given region, the possible shapes of a solid figure, and so on."

- Riemann (via Clifford)

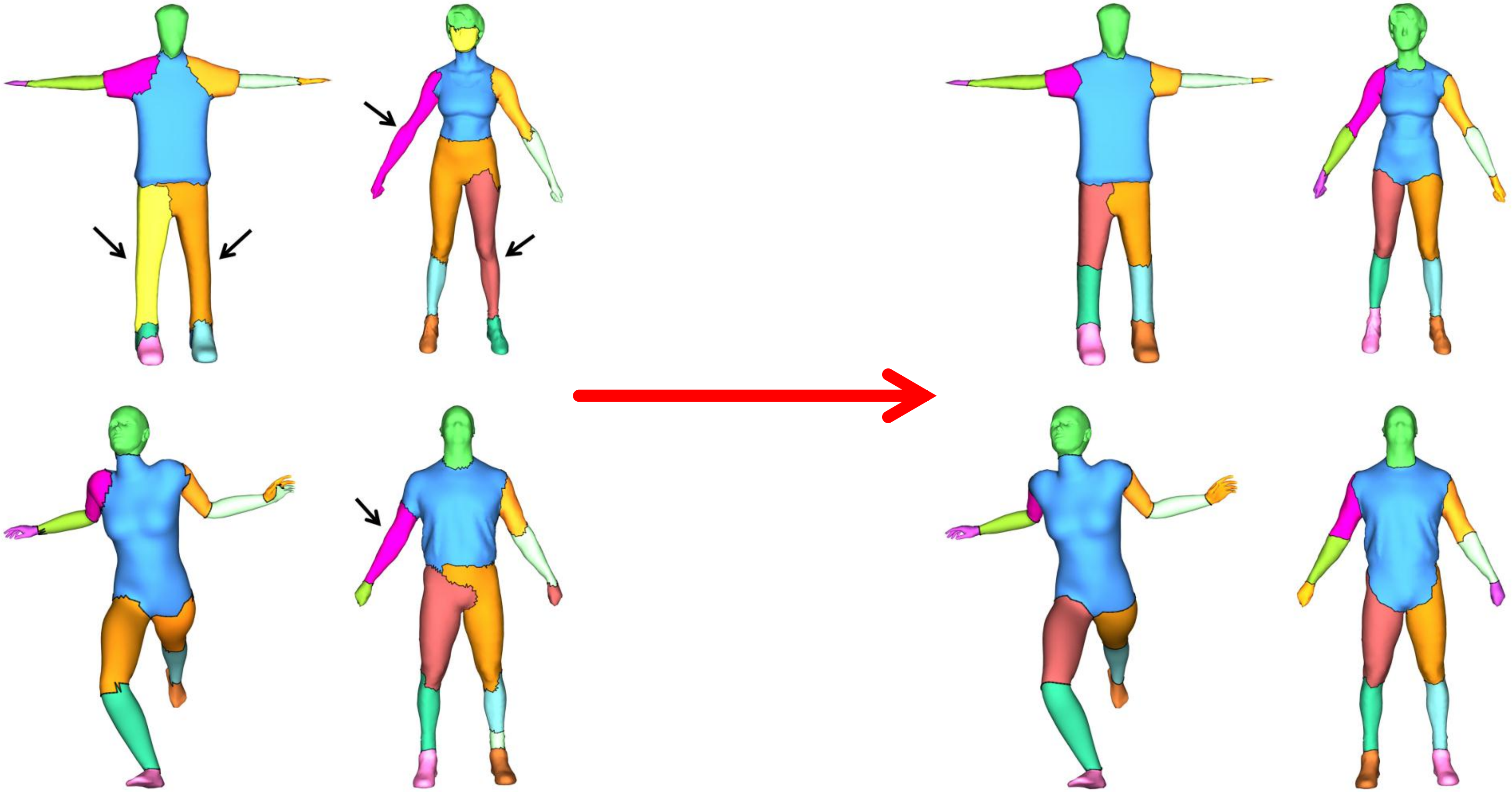
http://www.ics.uci.edu/~fowlkes/papers/f-ma240.pdf

Machine Learning Philosophy

http://graphics.ethz.ch/Downloads/Publications/Papers/2011/Mar11/Mar11.pdf

Learn shape space from examples

Segmentation



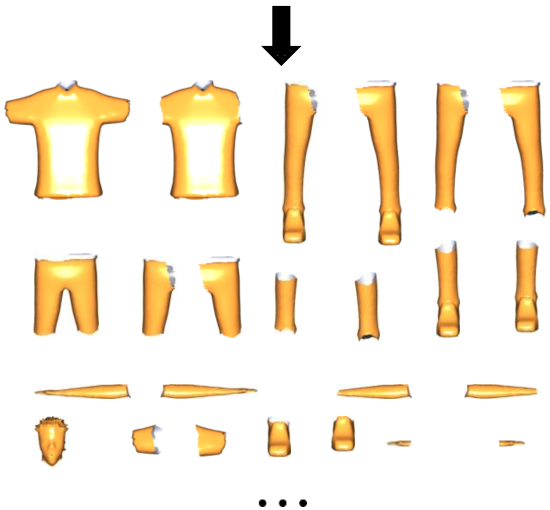
Individual **Joint**

Joint Shape Segmentation with Linear Programming

Huang, Koltun, and Guibas 2011

Segmentation

$$\max_{\text{segmentations } S_1, S_2} [\text{score}(S_1) + \text{score}(S_2) + \text{consistency}(S_1, S_2)]$$



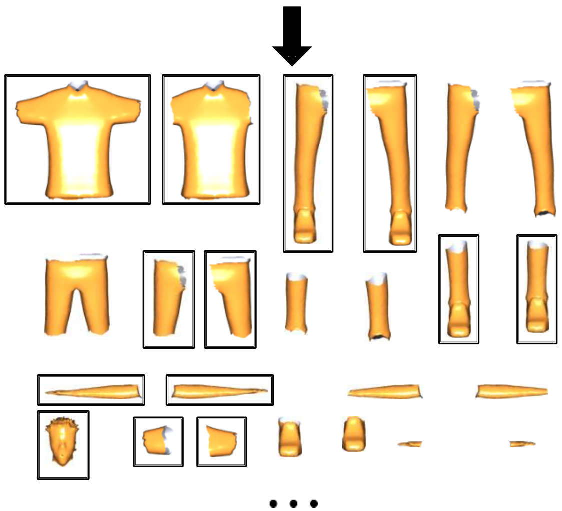
Initial Segments

Create small discrete pieces by cutting surface in different ways.

Joint Shape Segmentation with Linear Programming
Huang, Koltun, and Guibas 2011

Segmentation

$$\max_{\text{segmentations } S_1, S_2} [\text{score}(S_1) + \text{score}(S_2) + \text{consistency}(S_1, S_2)]$$



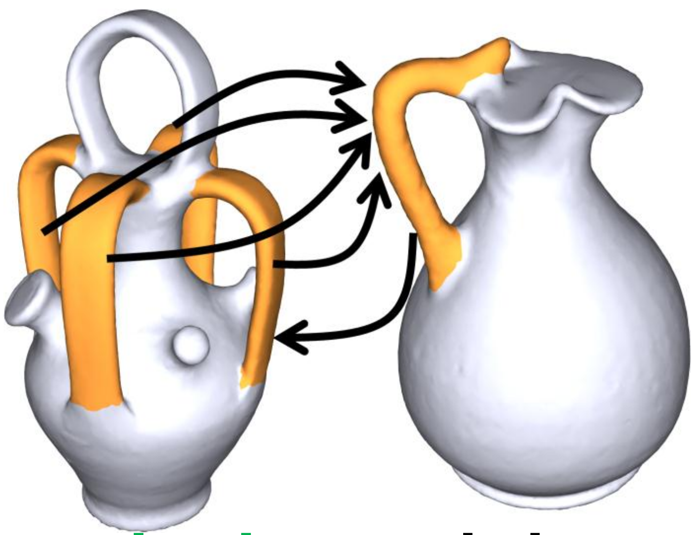
Initial Segments

A segmentation consists of decisions about whether to include each piece, where each point is covered once.

Joint Shape Segmentation with Linear Programming
Huang, Koltun, and Guibas 2011

Segmentation

$$\max_{\text{segmentations } S_1, S_2} [\text{score}(S_1) + \text{score}(S_2) + \text{consistency}(S_1, S_2)]$$



Pairwise and then **joint**

Unary segmentation scores measure **segment quality** with area.

Binary consistency scores match segments (many-to-one) using **geometric similarity** and **adjacency**.

Joint Shape Segmentation with Linear Programming

Huang, Koltun, and Guibas 2011

Segmentation

$$\max_{\text{segmentations } S_1, S_2} \left[\text{score}(S_1) + \text{score}(S_2) + \text{consistency}(S_1, S_2)\right]$$

Pairwise and then joint

Unary segmentation scores measure segment quality with area.

Binary consistency scores match segments (many-to-one) using geometric similarity and adjacency.

Linear program relaxation

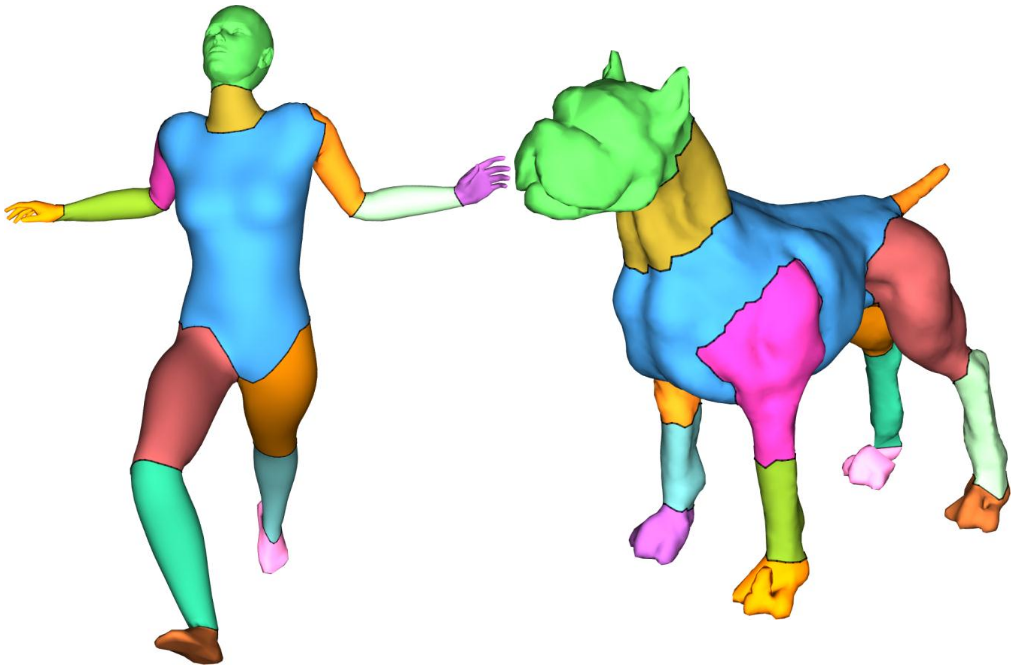
Joint Shape Segmentation with Linear Programming

Huang, Koltun, and Guibas 2011

Segmentation

	SD	RC	Supervised	**Joint**	**JointAll**	Human
Average	17.2	15.3	10.7	**10.5**	**10.1**	10.3

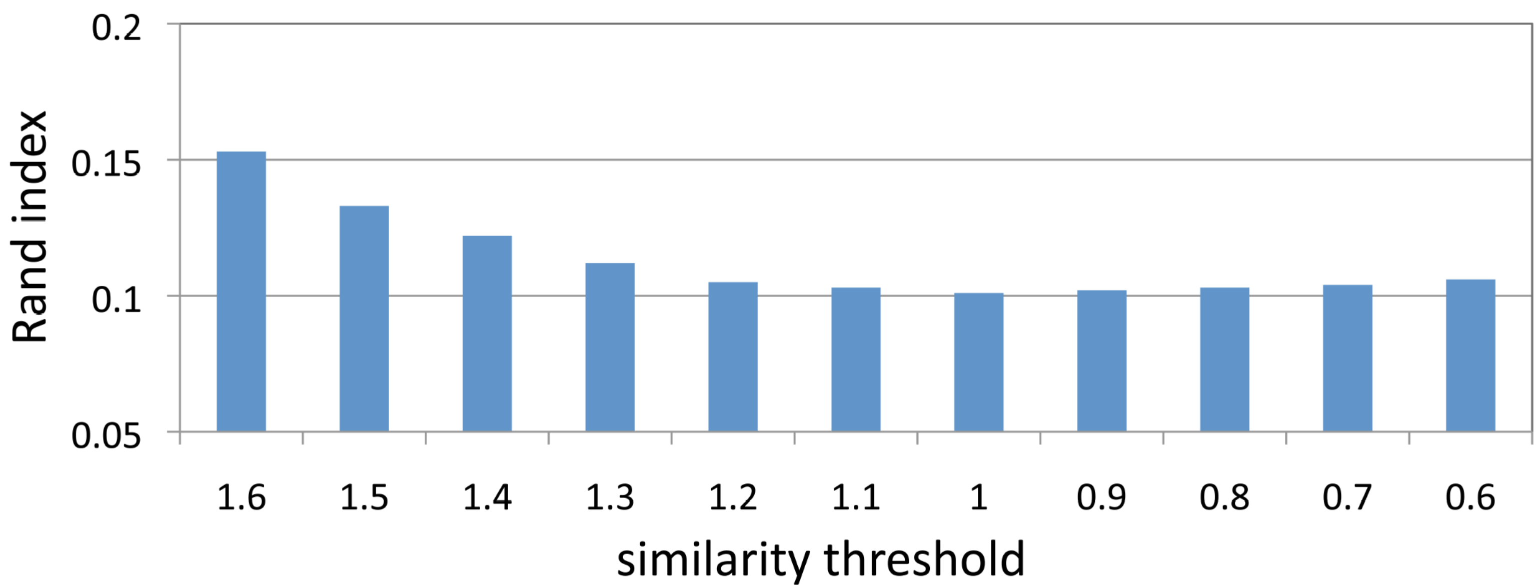
Rand index (smaller is better)



JointAll uses the dog's neck to help segment the geometry of the human.

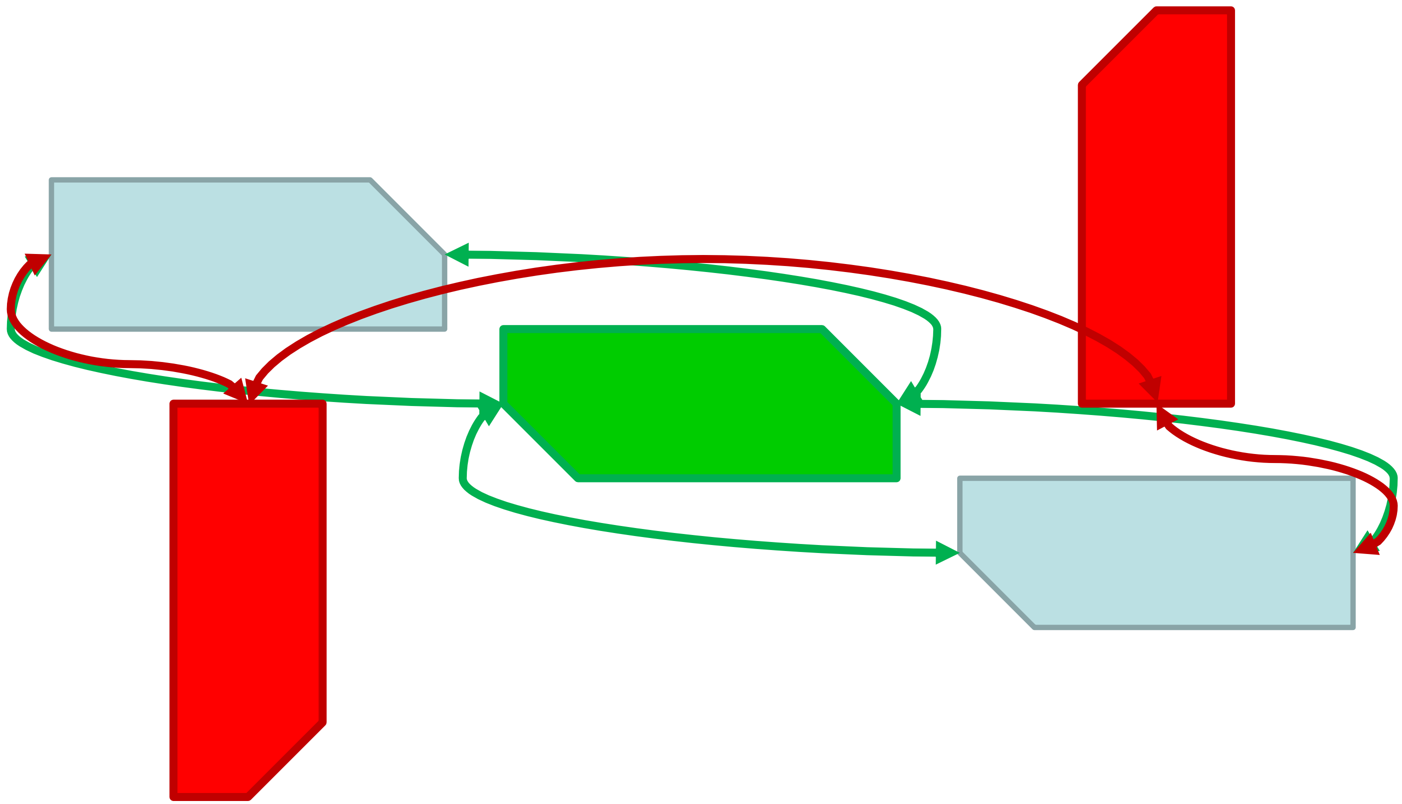
Joint Shape Segmentation with Linear Programming
Huang, Koltun, and Guibas 2011

Segmentation



Joint Shape Segmentation with Linear Programming
Huang, Koltun, and Guibas 2011

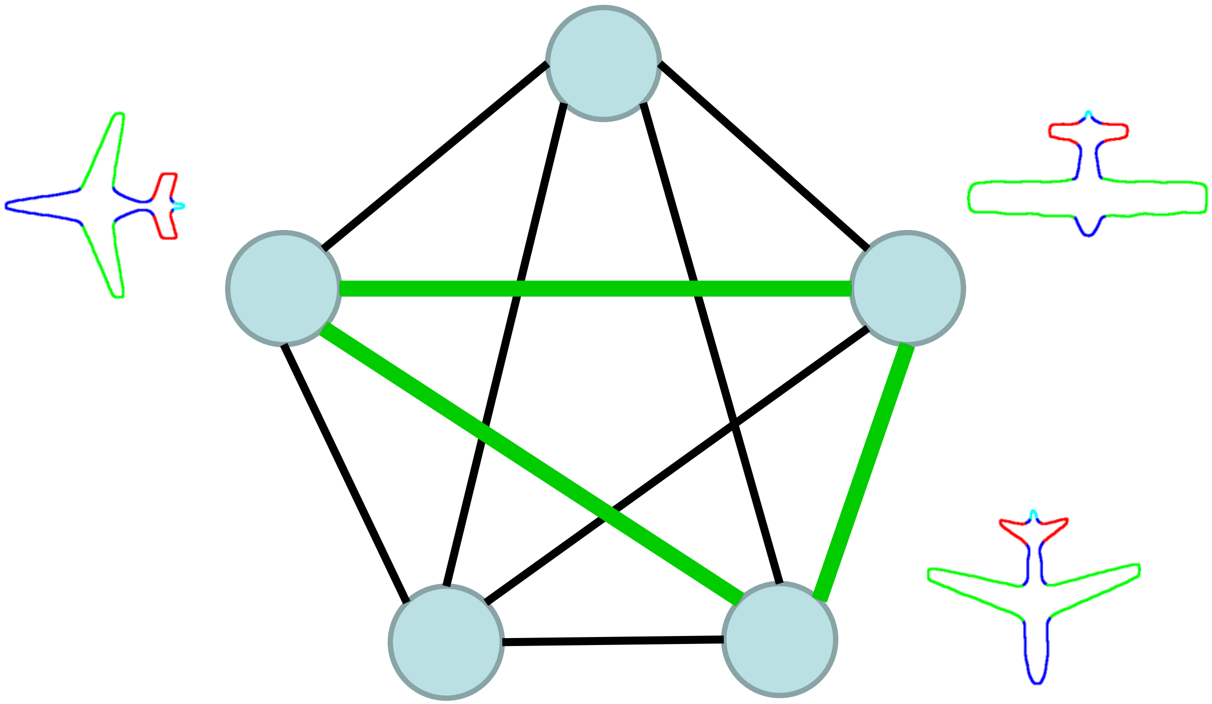
Mapping



Shape collections indicate which maps make sense.

An Optimization Approach to Improving Collections of Shape Maps
Nguyen et al. 2011

Mapping

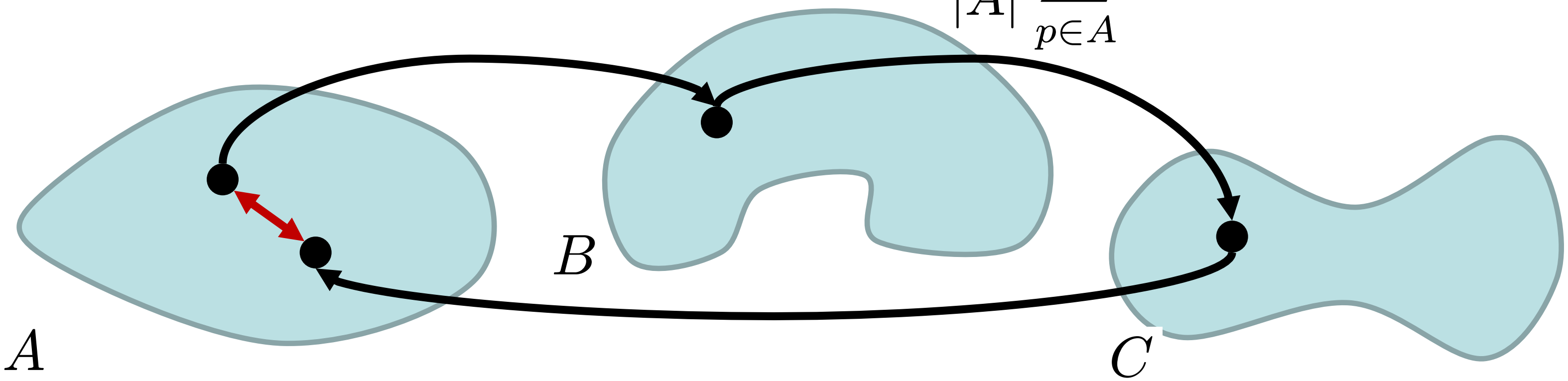


Maps are edges in a graph of shapes.
Cycles are self maps after composition.

An Optimization Approach to Improving Collections of Shape Maps
Nguyen et al. 2011

Mapping

$$E_{cons}(\gamma) = \frac{1}{|A|} \sum_{p \in A} d_A(p, m_\gamma(p))$$

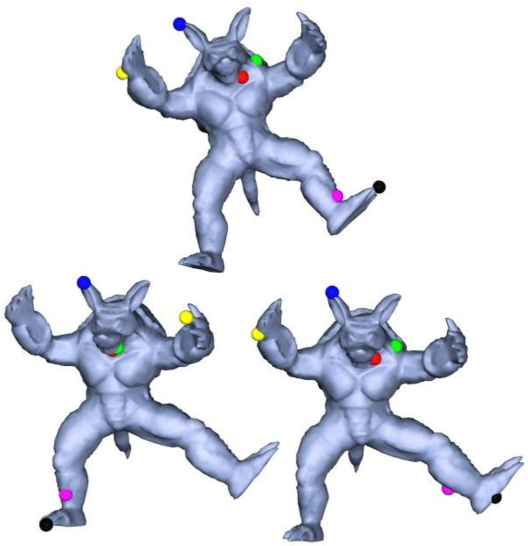


Cycle consistency measured by displacement around loop.

An Optimization Approach to Improving Collections of Shape Maps

Nguyen et al. 2011

Mapping



Iterate:

1. Compute error of each three-cycle.
2. Assign errors to edges in map graph by solving an LP distributing cycle error.
3. Replace bad edges with composition.

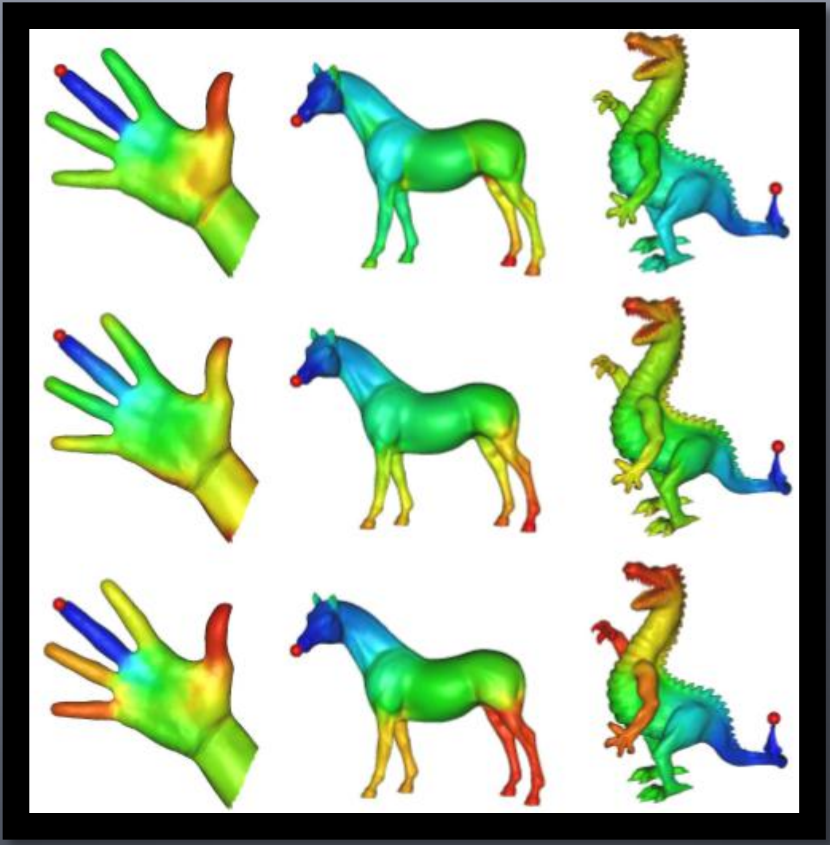
An Optimization Approach to Improving Collections of Shape Maps
Nguyen et al. 2011

Navigating Shape Collections

Exploration of Continuous Variability in Collections of 3D Shapes
Ovsjanikov et al. 2011

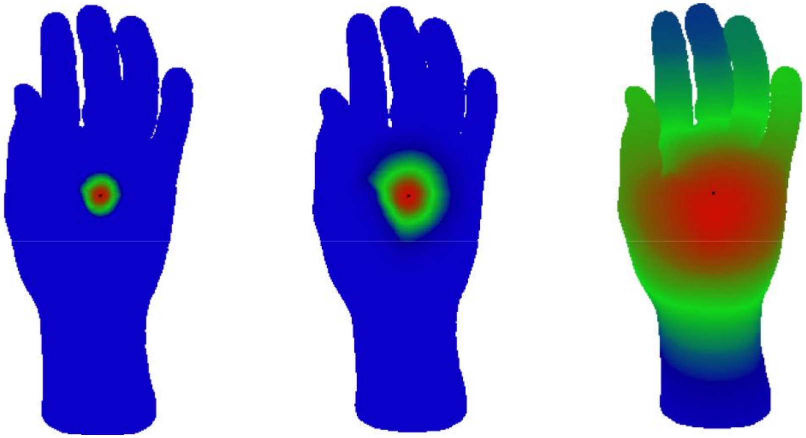
Part V:

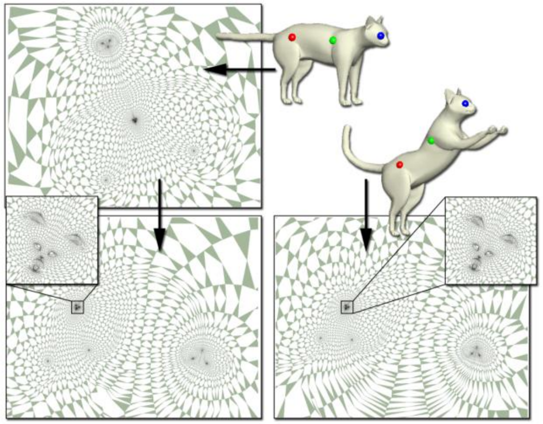
Conclusion

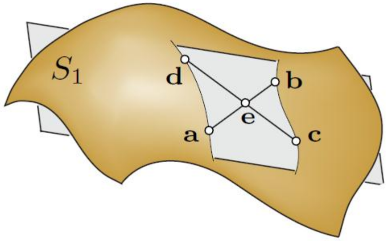


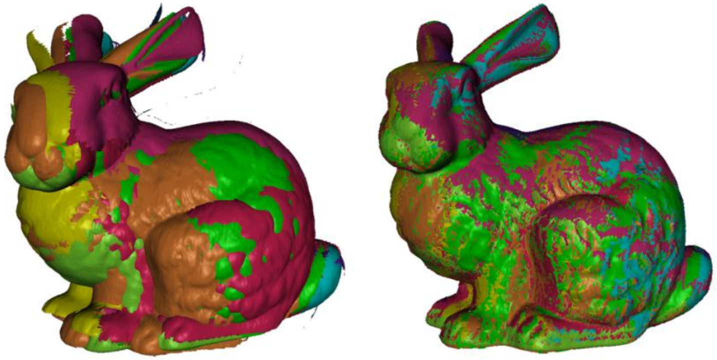
http://www.stanford.edu/~huangqx/siga11_jointseg.pdf

We've Covered a Lot of Ground









We've Covered a Lot of Ground

Summarized approaches to

- Local descriptors
- Shape understanding
- Correspondence
- Shape collections

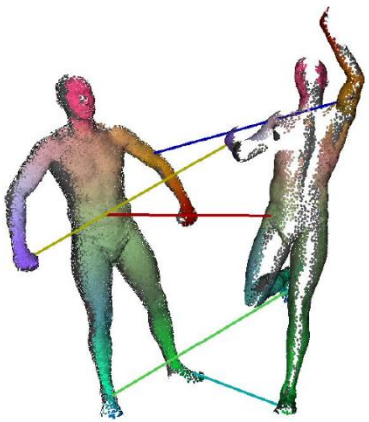
We've Covered a Lot of Ground

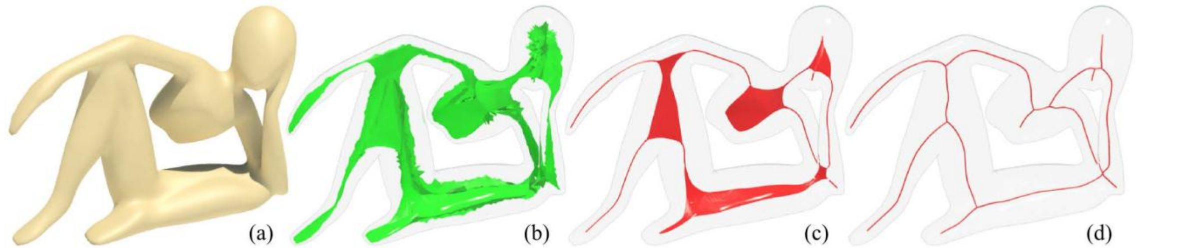
Summarized approaches to

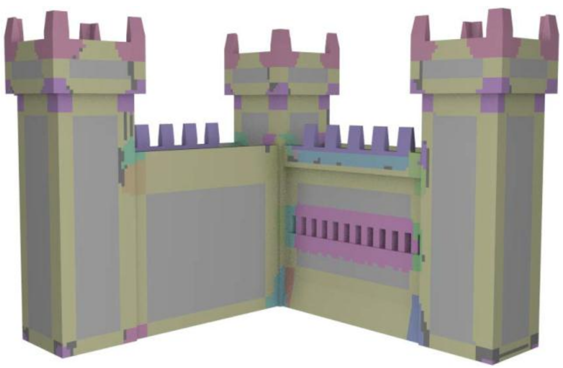
- Local descriptors
- Shape understanding
- Correspondence
- Shape collections

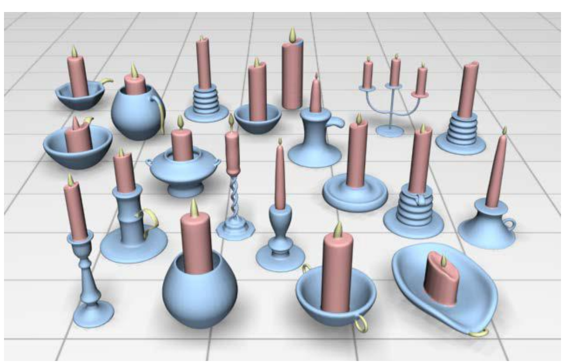
Barely scratched the surface!

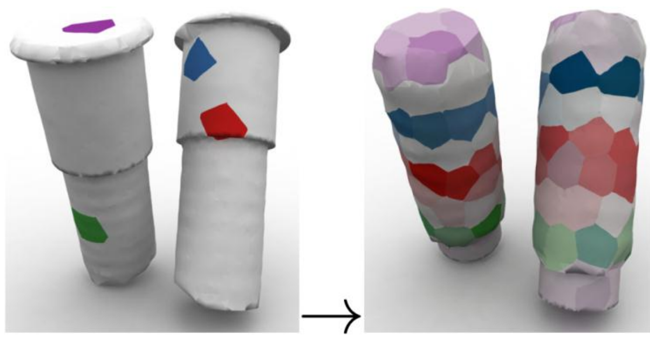
At SGP 2012...

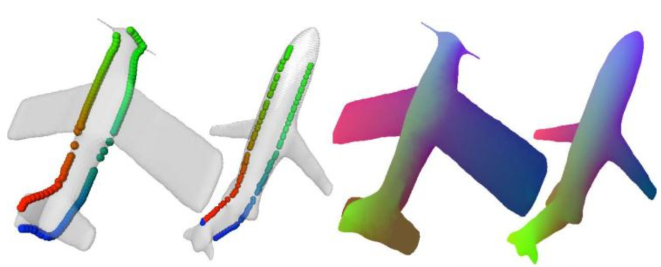


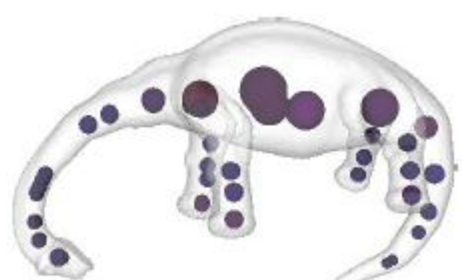


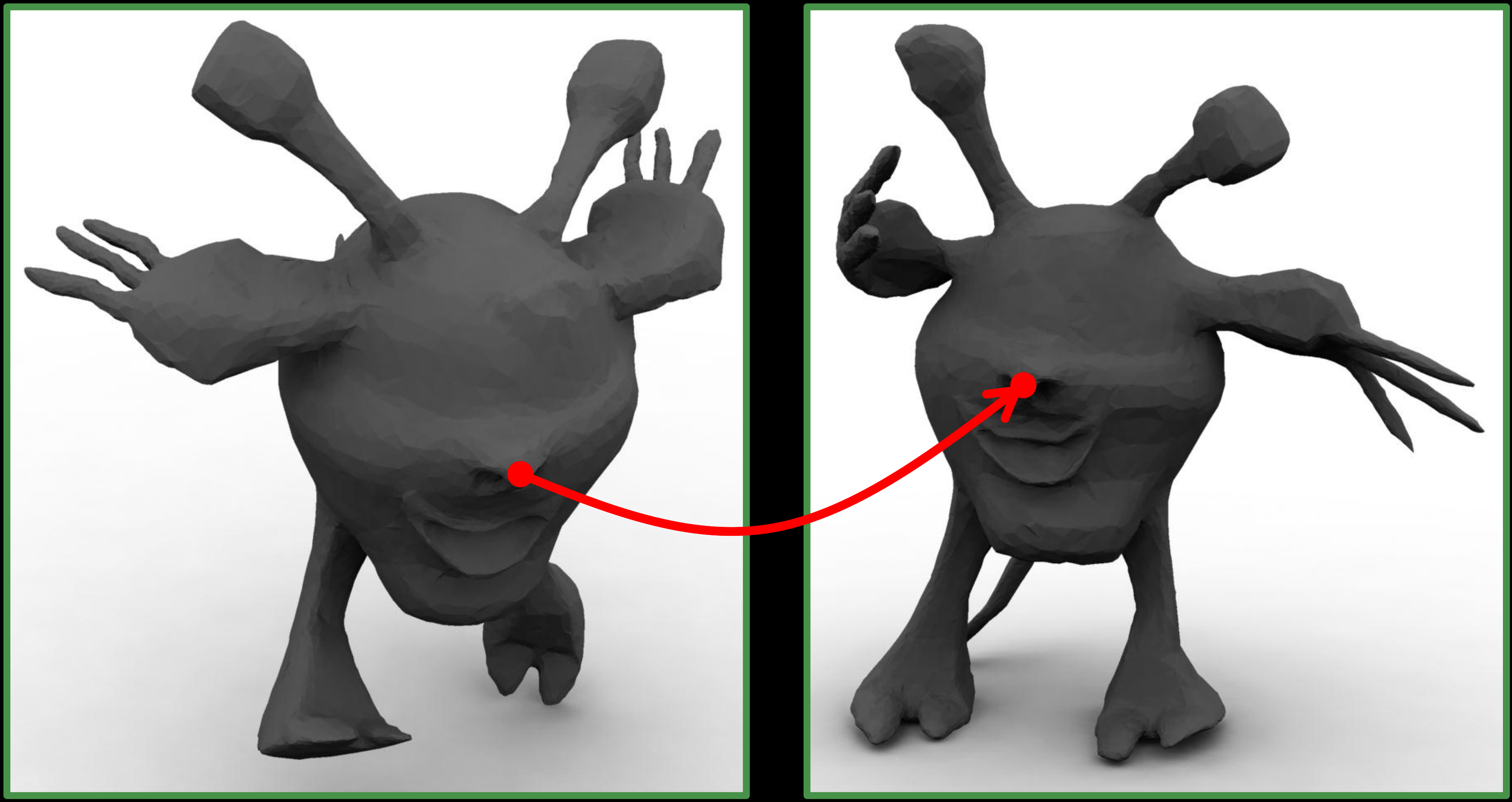












Shape Analysis and Correspondence

Questions?