Soft Maps Between Surfaces
“Traditional” Mapping

$M_1$

$M_2$
“Traditional” Mapping

\[ \phi : M_1 \rightarrow M_2 \]
“Traditional” Mapping

- Asymmetric
- Ambiguous

\[ \phi : M_1 \rightarrow M_2 \]
Mapping Ambiguities

Global Ambiguity
Mapping Ambiguities

Local Ambiguity
New Idea

Represent maps probabilistically
New Idea

Represent maps probabilistically
Soft Correspondence

$P_{12} \in \text{Prob}(M_1 \times M_2)$

Likelihood of a matched pair
Likelihood of a matched pair

$p_0 \quad p_1$

$(p_0, p_0) \quad (p_1, p_1)$

$p_1 \quad p_1$

$(p_1, p_1)$

$(p_1, p_1')$

$(p_0, p_0)$

$S^1$
Soft Correspondence

\[ P_{12}(U \times V) \in [0, 1] \]

Likelihood of a matched pair
Soft Maps

$P_{U2}(B) \in [0, 1]$
Conditional probability distribution

$P_{1V}(A) \in [0, 1]$
Discretization

\[ M_1 = U_1 \cup \cdots \cup U_{N_1} \]
\[ M_2 = V_1 \cup \cdots \cup V_{N_2} \]

Voronoi cells
Finding Soft Correspondences

\[ E_{map}(A) = E_{\phi}(A) + \lambda E_{cont}(A) + \beta E_s(A) \]

- Descriptor matching
- Continuity
- Sharpness
\[ E_{map}(A) = E_{\phi}(A) + \lambda E_{cont}(A) + \beta E_s(A) \]

\[ E_{\phi}(A) = \sum_{ij} a_{ij} \phi_{ij} \]
Finding Soft Correspondences

\[ E_{\text{map}}(A) = E_{\phi}(A) + \lambda E_{\text{cont}}(A) + \beta E_{s}(A) \]
Finding Soft Correspondences

\[ E_{map}(A) = E_\phi(A) + \lambda E_{cont}(A) + \beta E_s(A) \]

\[ E_s(A) = \sum_{i,j} a_{ij}^2 = \|A\|_{\text{Fro}}^2 \]
$E_{map}(A) = E_\phi(A) + \lambda E_{cont}(A) + \beta E_s(A)$

- Descriptor matching
- Continuity
- Sharpness
Finding Soft Correspondences

\[ E_{map}(A) = E_{\phi}(A) + \lambda E_{cont}(A) + \beta E_s(A) \]

- **Descriptor matching**
- **Continuity**
- **Sharpness**

Linear program!
Finding Soft Correspondences

\[ E_{map}(A) = E_{\phi}(A) + \lambda E_{cont}(A) + \beta E_s(A) \]
Finding Soft Correspondences

\[ E_{map}(A) = E_\phi(A) + \lambda E_{cont}(A) + \beta E_s(A) \]

Sharp transition

Want this point
Finding Soft Correspondences

\[
\min E_\phi(A) + \lambda E_{cont}(A)
\]
Finding Soft Correspondences

\[
\min \ E_\phi(A) + \lambda E_{cont}(A)
\]

such that
Finding Soft Correspondences

\[
\min \ E_\phi(A) + \lambda E_{\text{cont}}(A)
\]

such that

Primal
Finding Soft Correspondences

\[
\min_{A} E_{\phi}(A) + \lambda E_{cont}(A) \rightarrow E_{\phi}(A^*)
\]

such that
Finding Soft Correspondences

\[ E_{\text{map}}(A) = E_{\text{avg}}(A) + \theta E_{\text{cont}}(A) + \bar{\theta} E_{s}(A) \]

such that

\[ E_{\text{avg}}(A_{\phi}) = \phi \]

Dual
Finding Soft Correspondences

\[ \text{max} \quad E_\phi(A^\star) \]

such that

\[ \lambda \]

Dual
Finding Soft Correspondences

\[ \max \quad E_\phi(A^*) \]

such that

\[ \lambda = \ldots \]

Dual
Finding Soft Correspondences

\[
\min \lambda
\]

such that

\[
E_\phi(A^*) = \lambda
\]
Finding Soft Correspondences

\[ \min_{\Lambda^*} E_{\phi}^A(\Lambda^*) \]
Finding Soft Correspondences

See Appendix for details.
Continuous soft maps
PCA Analysis

Basis for probability measures
PCA Analysis

Basis for probability measures
Projection onto Basis

Validation improves
Projection onto Basis

Sharpness decreases
Future Work

- Computing denser maps
- Applications to graphics and other fields
- Map collections and composition
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Soft Maps Between Surfaces

Questions?