MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems—Fall 2002

TUTORIAL FOR THE WEEK OF NOVEMBER 4TH - NOVEMBER 8TH

Alex's Office Hours
Monday 3-5pm
Tuesday, 6-8pm

Important Due Dates:

- Problem Set 7 due on Wednesday.
- Lab 2 due this Friday, November 8th (No late submissions accepted).
- Quiz 2 on Thursday, November 14th in Walker
- Quiz 2 Review Sessions on Monday, November 11th and Tuesday, November 12th in 34-101.

Today

1. More Sampling
2. DT processing CT signals

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Sampling in Continuous Time

Recall that sampling a signal, $x(t)$, in time is equivalent to convolving with its Fourier Transform, $X(j\omega)$, with a periodic impulse train in frequency. The impulse train that we convolve with has frequency spacing which is inversely proportional to the sampling period. Recall that there are two main facts we rely on when performing sampling

1. The Fourier Transform of an impulse train is another impulse train

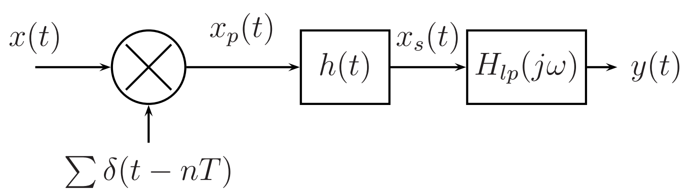
$$\sum_{n=-\infty}^{\infty} \delta\left(t - nT\right) \longleftrightarrow \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - k\frac{2\pi}{T}\right)$$

2. Multiplication in time corresponds to convolution in frequency (with a 2π factor!!)

$$x_1(t)x_2(t) \longleftrightarrow \frac{1}{2\pi} X_1(j\omega) * X_2(j\omega)$$

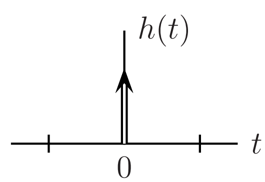
[Example]

There are two additional variation of the vanilla sampling process known as Zero Order Hold and First Order Hold. This problem attempts to illustrate some of consequences of each.

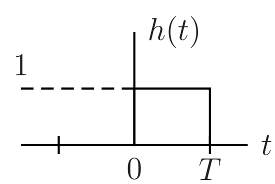


In the above, assume that $x(t)$ is band-limited to $\omega_c = \frac{\pi}{T}$, and that $H_{lp}(j\omega)$ is an ideal low-pass filter with the same cutoff frequency $\omega_c = \frac{\pi}{T}$, and gain T.

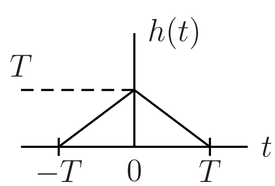
1. Draw $Y(j\omega)$ when $h(t) = \delta(t)$



2. Draw $Y(j\omega)$ when $h(t)$ is as shown below (Zero Order Hold)



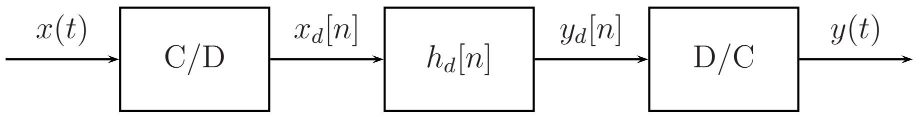
3. Draw $Y(j\omega)$ when $h(t)$ is as shown below (First Order Hold)



Work Page

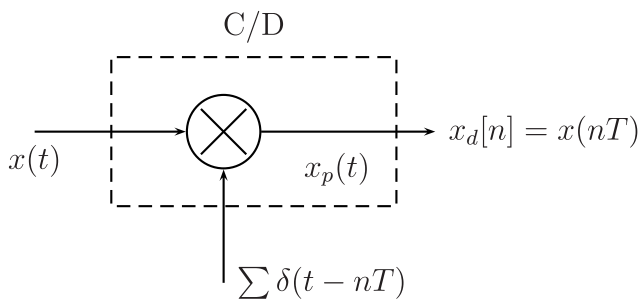
DT processing of CT signals

As we said at the beginning of this chapter, a major motivation for doing sampling is because it is often easier to process discrete-time signals, or sequences, since we can do this with a computer or DSP chip. We will review each of the relevant blocks in such a system.

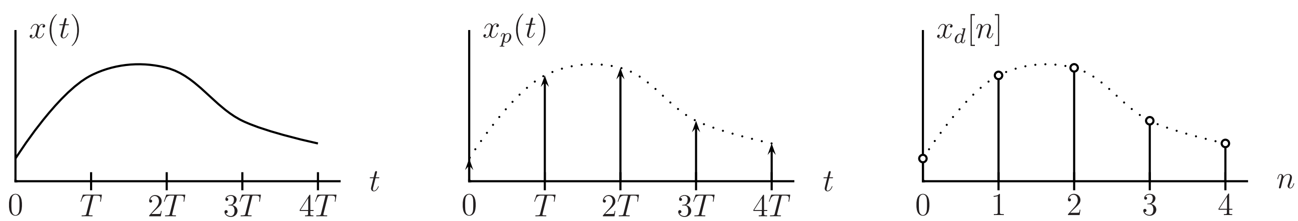


C/D

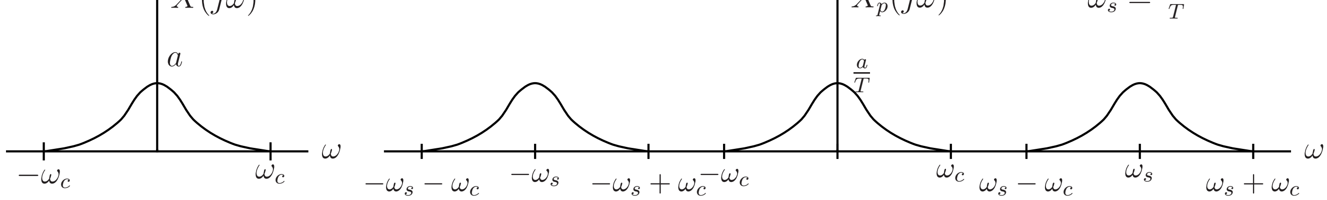
The first step is to convert the continuous time input signal, $x(t)$, into a discrete sequence of samples, $x_d[n]$. As always, we can use pictures to illustrate what is happening.



The associated continuous time signal ($x(t)$), continuous time sampled signal ($x_p(t)$), and the discrete time sampled sequence ($x_d[n]$), look like



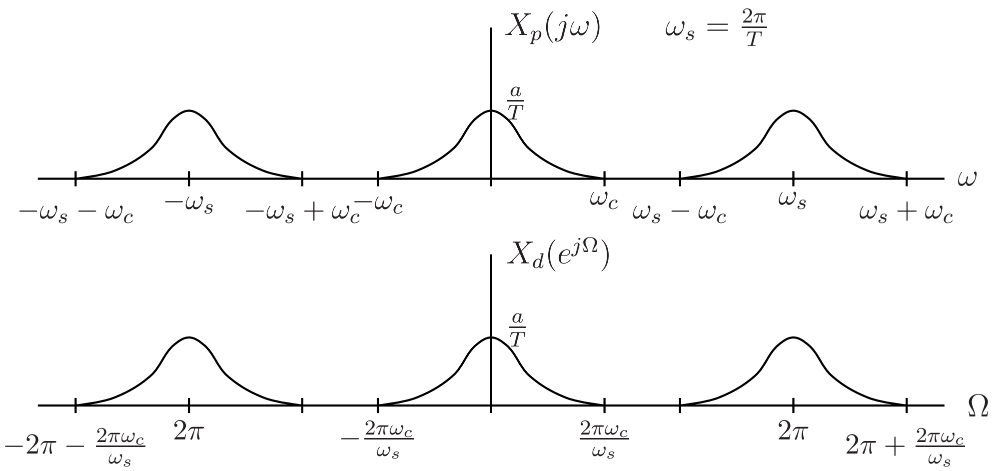
Notice the difference between $x_p(t)$ and $x_d[n]$. The first is a continuous time signal that zero-valued except at integer multiples of T. The second is a discrete sequence which has values at every n, specifically, $x_d[n] = x(nT)$. In the discrete time sequence, all notion of time is lost. If we look at what is happening in the frequency domain, the relationship is more clear.



Notice that the Fourier transform of the periodic signal, $X_p(j\omega)$ is periodic, just as we would expect the DTFT of $x_d[n]$ to be. In order to relate $X_p(j\omega)$ to $X_d(e^{j\Omega})$, we need to map the CT frequency axis (in ω) to the DT frequency axis (in Ω). We use the following periodicity constraint:

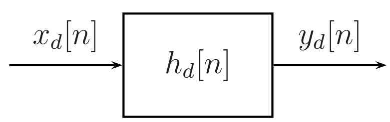
1. $X_p(j\omega)$ is periodic with period ω_s in ω (by construction)
2. $X_d(e^{j\Omega})$ is periodic with period 2π in Ω (by definition)

Therefore, to get the discrete time frequency plot, $X_d(e^{j\Omega})$, we perform a frequency warping which equates $\omega = \omega_s$ to $\Omega = 2\pi$. So $\Omega = \frac{2\pi}{\omega_s}\omega$.

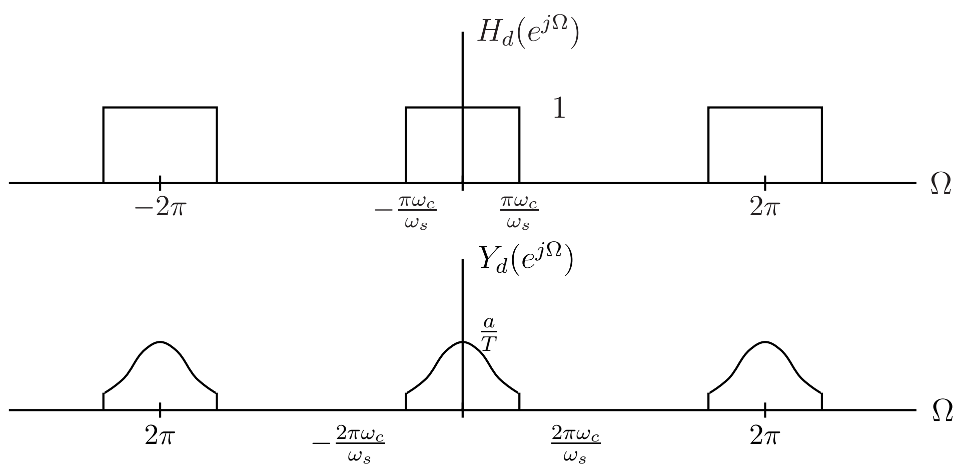


Discrete Time Filtering

The next step is to do our filtering using a discrete time filter, $h_d[n]$, which is something we already know how to do.



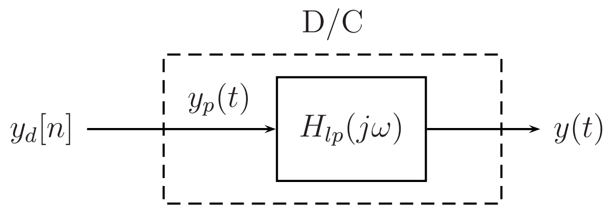
So if our $H_d(e^{j\Omega})$ was just an ideal low pass filter with cutoff at $\Omega = \frac{\pi\omega_c}{\omega_s}$, for example, our frequency picture for $H_d(e^{j\Omega})$ and the resulting $Y_d(e^{j\Omega})$ would look like



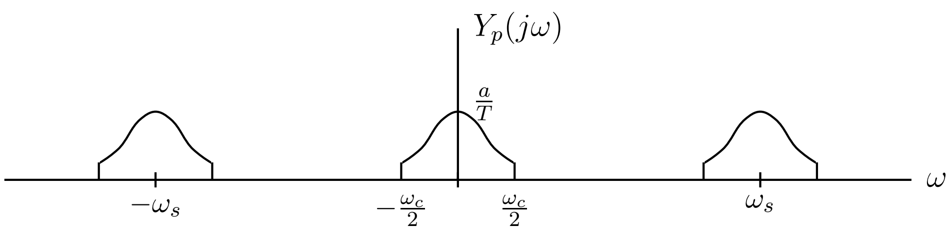
D/C

The final step is to take our discrete time output sequence, $y_d[n]$, convert it back to a continuous time sample train, and then lowpass filter it (or interpolate) to get back the continuous time signal. The D/C converter consists of 2 steps:

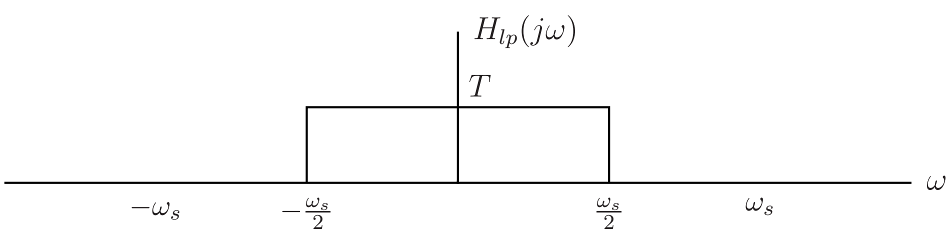
1. Map the DT frequency axis in Ω back to the CT frequency axis in ω.
2. Low pass filter the CT signal with a filter that has height T and cutoff frequency $\frac{\omega_s}{2}$.



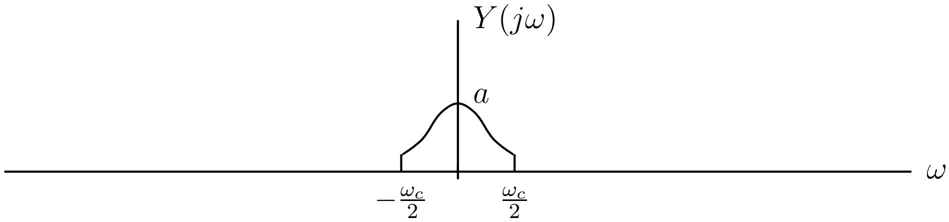
After remapping the frequency axis to get $y_p(t)$ from $y_d[n]$, the Fourier transform for $y_p(t)$ looks like



We then apply $H_{lp}(j\omega)$, which has gain T



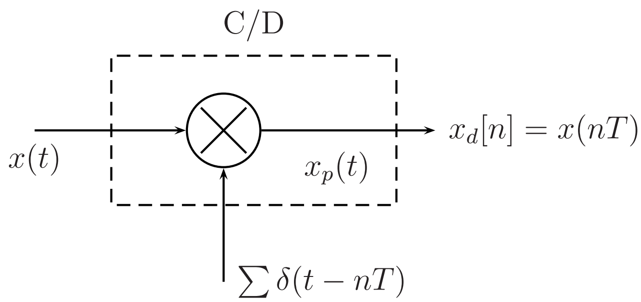
This gives us back our $y(t)$, which has Fourier transform shown below



[Example]

This is a tricky example that illustrates complications that arise when converting between continuous and discrete time.

Suppose we have the following system:

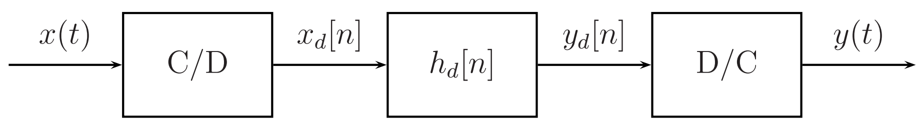


If $x(t)$ is $\cos(\omega_0 t)$, and $\omega_s > 2\omega_0$ then draw a completely labelled sketch of the DTFT of $x_d[n]$.

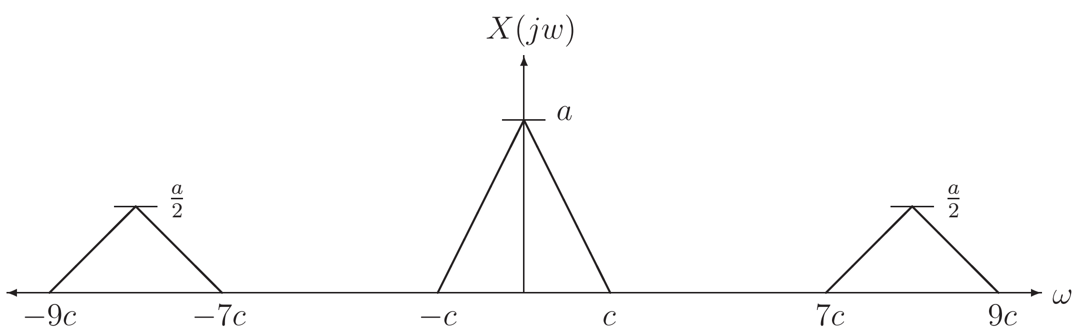
[Example]

The sampling system shown below consists of three major components:

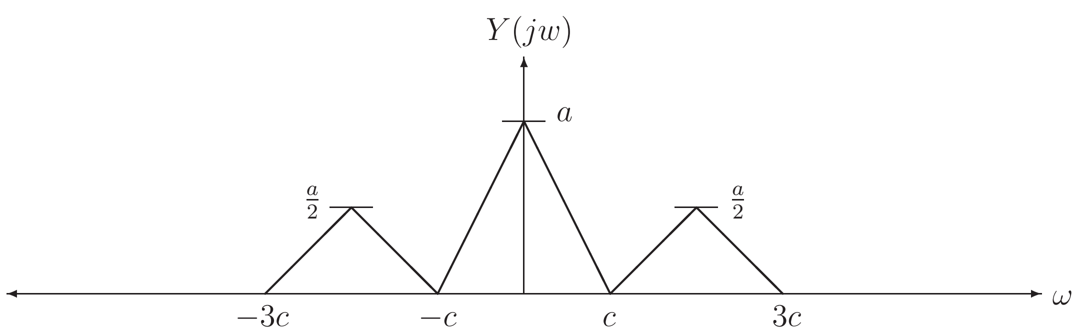
- C/D: a continuous to discrete converter that samples its input with a periodic impulse train with period T and maps the resulting impulses to samples.
- D/C: a discrete to continuous converter that implements ideal band-limited reconstruction for a signal sampled with sampling period T.
- $h_d[n]$: a linear, time-invariant, discrete-time filter that is an ideal lowpass filter whose parameters you are free to choose.



The input $x(t)$ has real-valued Fourier Transform $X(j\omega)$ shown below.



Is it possible to adjust the sampling period T so that the Fourier transform $Y(j\omega)$ of the output $y(t)$ has the following shape?



If yes, determine the sampling period T and the impulse response $h[n]$ as a function of the parameters of $X(j\omega)$. If no, explain why not.

Work Space