Rely: Verifying Quantitative Reliability for Programs that Execute on Unreliable Hardware

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Image Scaling

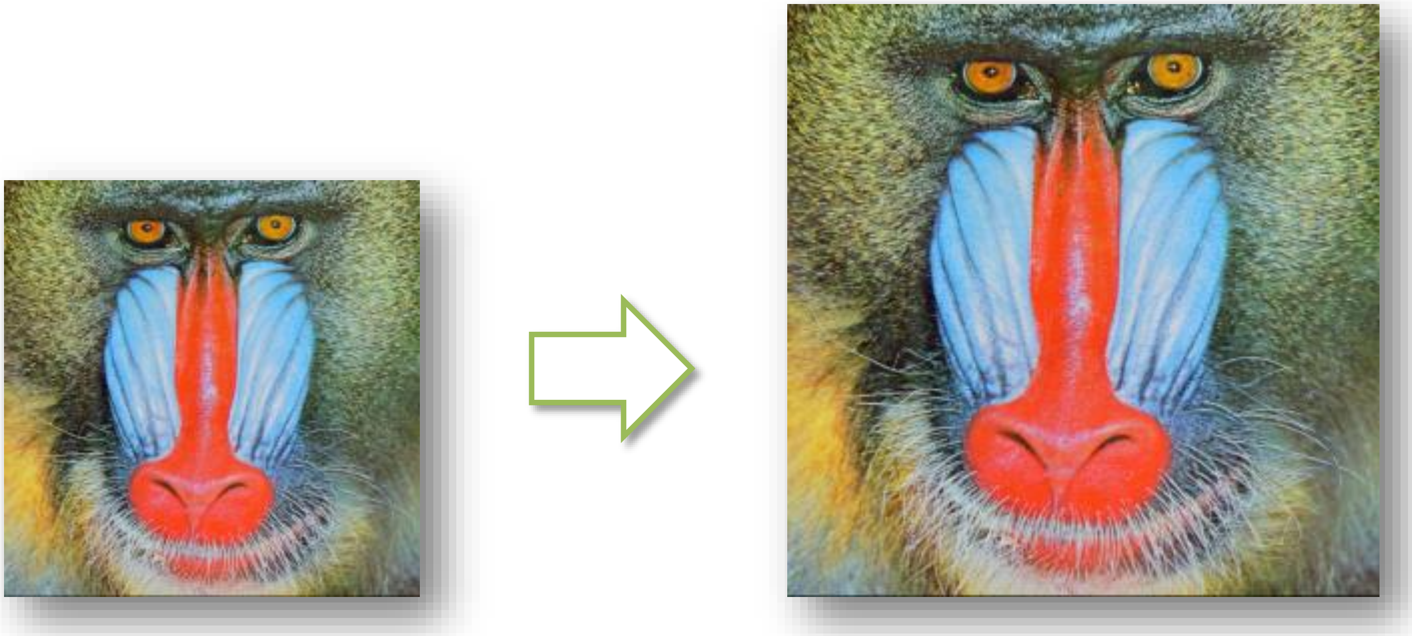
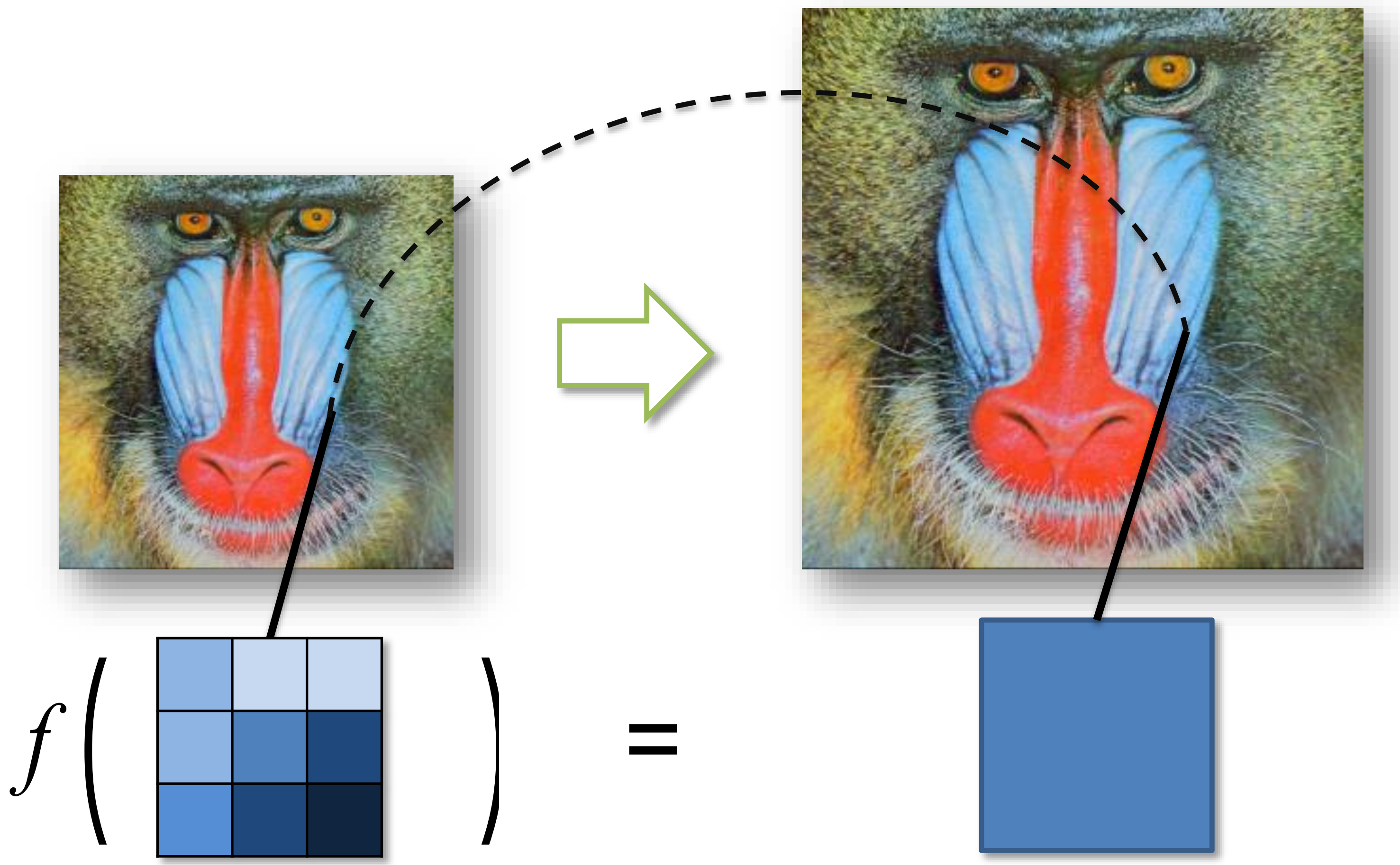


Image Scaling Kernel: Bilinear Interpolation



Bilinear Interpolation

```
int bilinear_interpolation(int i,  int j,
             int src[][], int dest[][])
{
    int i_src = map_y(i, src, dest),
        j_src = map_x(j, src, dest);

    int up   = i_src - 1, down  = i_src + 1,
        left = j_src - 1, right = j_src + 1;

    int val = src[up][left]    + src[up][right] +
              src[down][right] + src[down][left];

    return 0.25 * val;
}
```

Bilinear Interpolation

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int bilinear_interpolation(int i,  int j,
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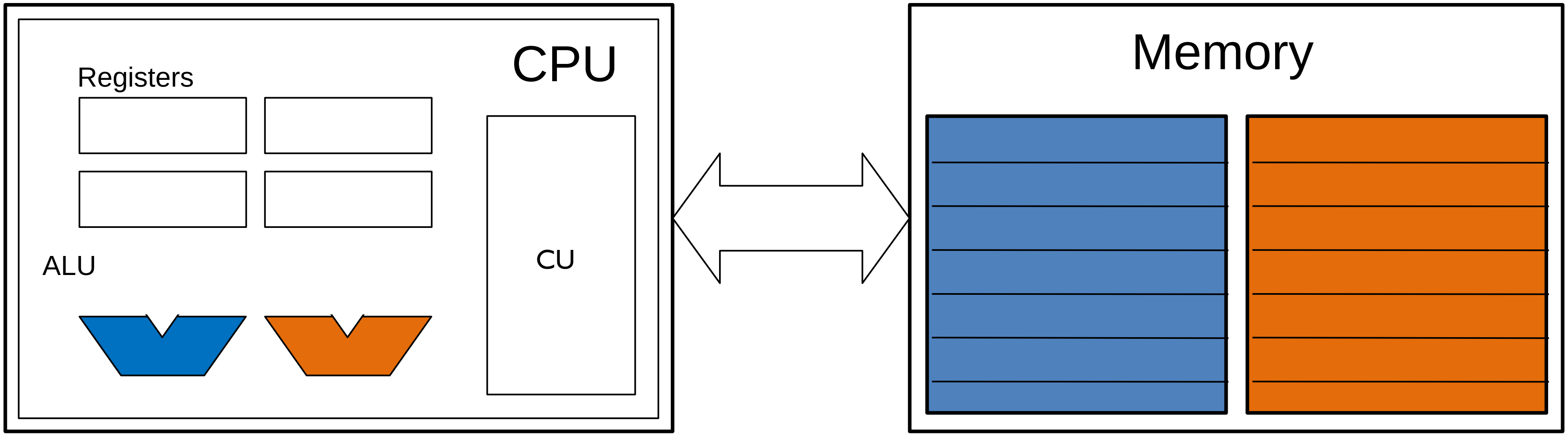
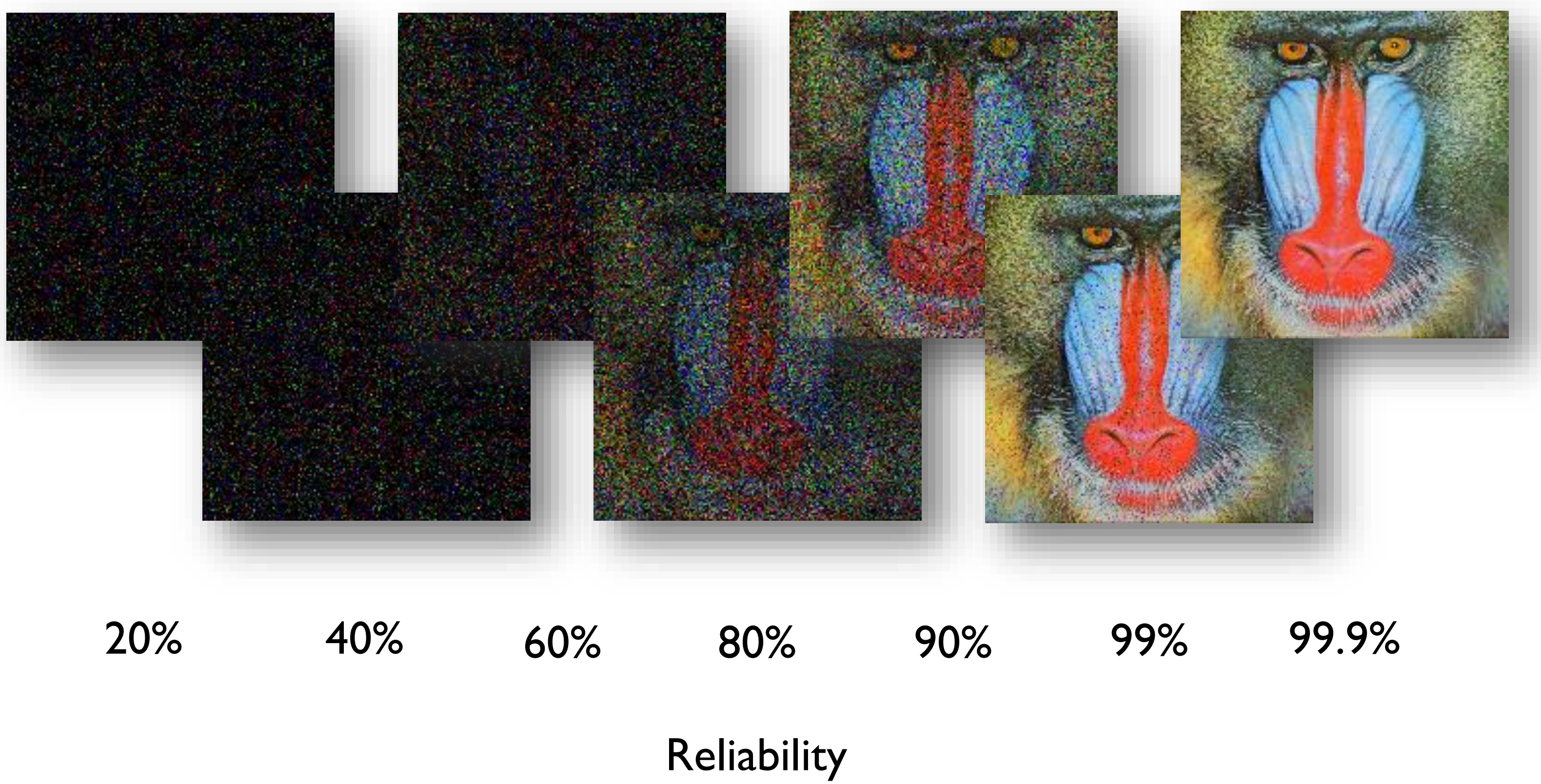
Unreliable Hardware



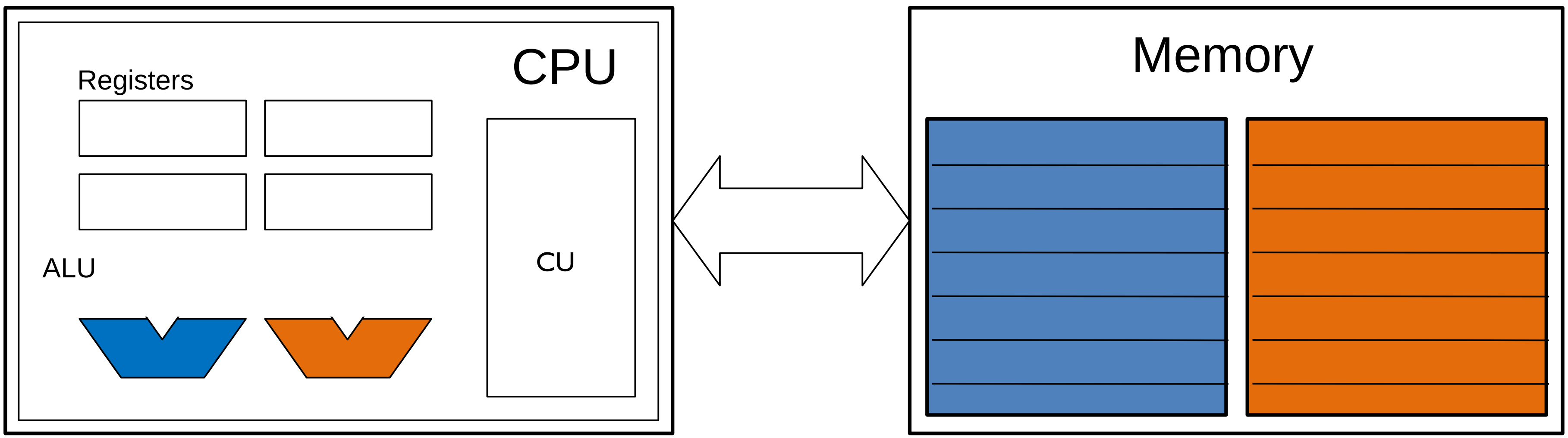
Unreliable Units (ALUs and Memories)

- May produce incorrect results
- Faster, smaller, and lower power

Image Scaling with Approximate Bilinear Interpolation



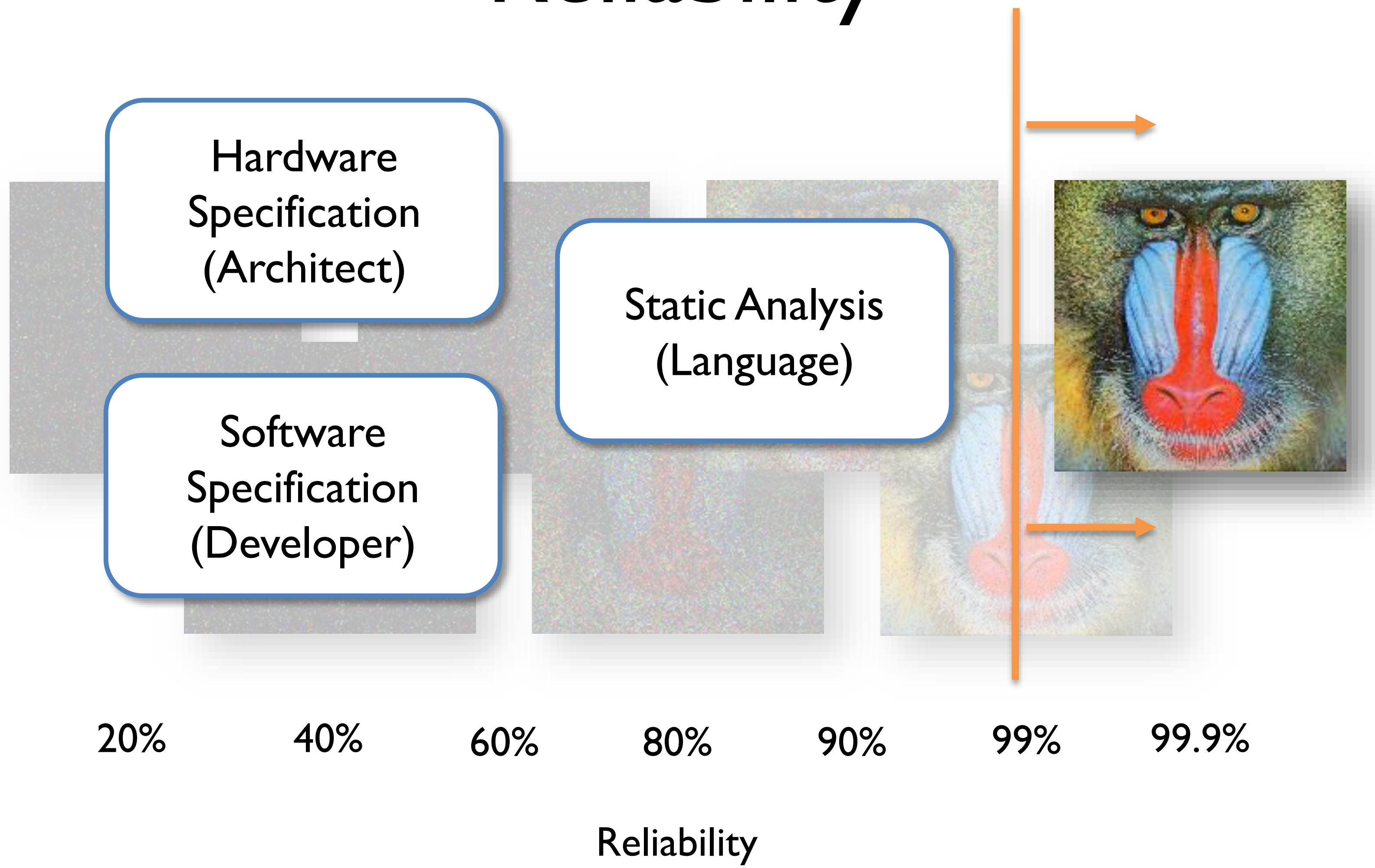
Unreliable Hardware



Necessitates

- Hardware Specification: probability operations execute correctly
- Software Specification: required reliability of computations
- Analysis: verify software satisfies its specification on hardware

Rely: a Language for Quantitative Reliability



Hardware Specification

```
hardware {
  operator (+) = 1 - 10^-7;
  operator (-) = 1 - 10^-7;
  operator (*) = 1 - 10^-7;
  operator (<) = 1 - 10^-7;
  memory urel {rd = 1 - 10^-7, wr = 1};
}
```

Approximate Bilinear Interpolation in Rely

```
int bilinear_interpolation(int i,  int j,
                           int src[][], int dest[][])
{
    int i_src = map_y(i, src, dest),
        j_src = map_x(j, src, dest);

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    int val = src[up][left]     +. src[up][right] +.
              src[down][right] +. src[down][left];

    return 0.25 *. val;
}
```

Unreliable Operations: executed on unreliable ALUs

Approximate Bilinear Interpolation in Rely

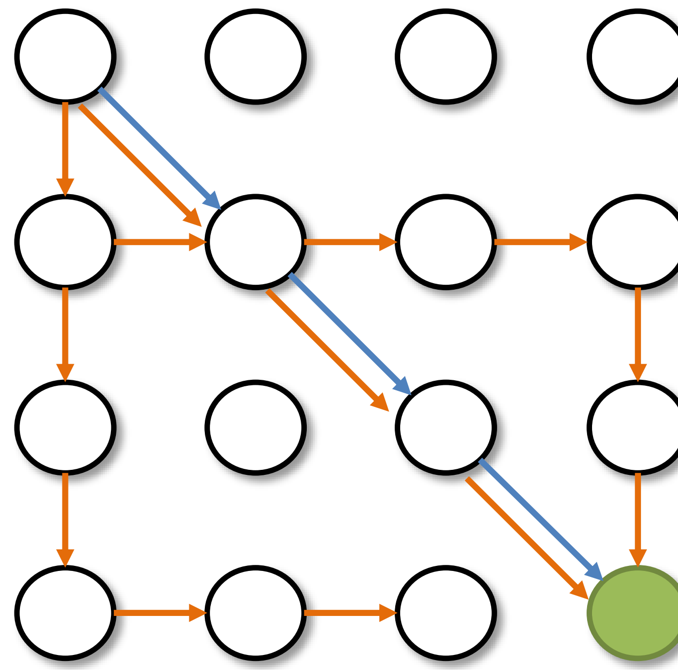
```
int bilinear_interpolation(int i,  int j,
                int in urel src[][], int in urel dest[][])
{
    int i_src = map_y(i, src, dest),
        j_src = map_x(j, src, dest);

    int up   = i_src - 1, down  = i_src + 1,
        left = j_src - 1, right = j_src + 1;

    int in urel val = src[up][left]    +. src[up][right] +.
                      src[down][right] +. src[down][left];

    return 0.25 *. val;
}
```

Unreliable Memories: stored in unreliable SRAM/DRAM

What is reliability?

Semantics of Reliability

- Reliable Hardware
  - One Execution
- Unreliable Hardware
  - Multiple Executions
- Reliability
  - Probability unreliable execution reaches same state
  - Or, $R(\{x, y\})$ = probability over distribution of states that x and y (only) have correct values.



Semantics of Reliability

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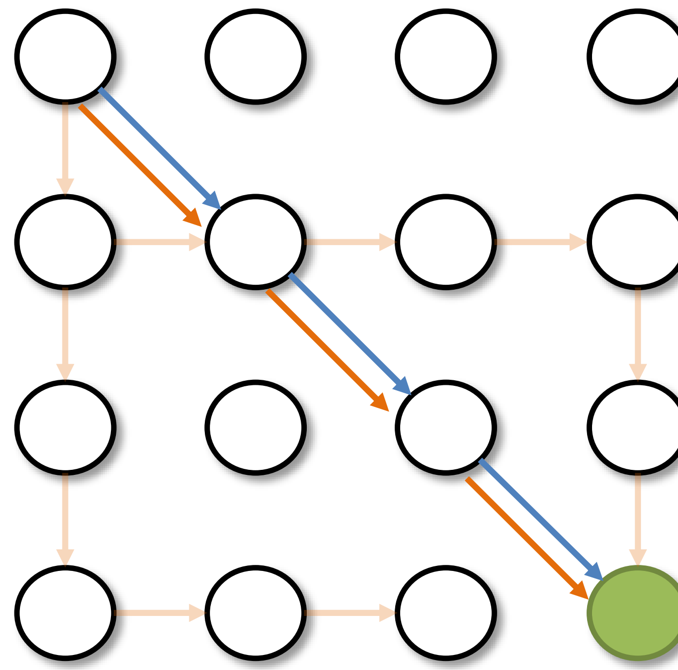


Approximate Bilinear Interpolation Reliability Specification

```
int
bilinear_interpolation(int i, int j,
          int in urel src[][], int in urel dest[][]);
```

Approximate Bilinear Interpolation Reliability Specification

```
int<.99>
bilinear_interpolation(int i, int j,
           int in urel src[][], int in urel dest[][]);
```

- Reliability of output is a function of reliability of inputs

Approximate Bilinear Interpolation Reliability Specification

```
int<.99 * R(i, j, src, dest)>
bilinear_interpolation(int i, int j,
          int in urel src[][], int in urel dest[][]);
```

- Reliability of output is a function of reliability of inputs
- The term `R(i, j, src, dest)` abstracts the **joint reliability** of the function's inputs on entry

Approximate Bilinear Interpolation Reliability Specification

```
int<.99 * R(i, j, src, dest)>
bilinear_interpolation(int i, int j,
          int in urel src[][], int in urel dest[][]);
```

- Reliability of output is a function of reliability of inputs
- The term `R(i, j, src, dest)` abstracts the **joint reliability** of the function's inputs on entry
- Coefficient .99 bounds reliability degradation

How does Rely verify reliability?

Rely's Analysis Framework

- Precondition generator for statements

{ Precondition }

s

{ Postcondition }

$$\bigwedge_i r_i * R(\{x_{i1}, \ldots, x_{in}\}) \leq r'_i * R(\{x'_{i1}, \ldots, x'_{im}\})$$

Specification $0.9 * R(\{\mathrm{x}\}) \leq 0.99 * R(\{\mathrm{y}\})$ Computation

Assignment Rule

$$\{r_1 * R(\{x_1 \ldots, x_n\}) \leq r_2 * rel(e) * wr(x') * R(\{x'_1, \ldots, x'_m\} \cup fv(e))\}$$

x' = e

$$\{r_1 * R(\{x_1 \ldots, x_n\}) \leq r_2 * R(\{x'_1, \ldots, x', \ldots, x'_m)\}$$

Assignment Rule

$$\{r_1 * R(\{x_1 \dots, x_n\}) \leq r_2 * rel(e) * wr(x') * R(\{x'_1, \dots, x'_m\} \cup fv(e))\}$$

Unmodified

x’ = e

$$\{r_1 * R(\{x_1 \dots, x_n\}) \leq r_2 * R(\{x'_1, \dots, x', \dots, x'_m)\}$$

Assignment Rule

$$\{r_1 * R(\{x_1 \dots, x_n\}) \leq r_2 * rel(e) * wr(x') * \boldsymbol{R(\{x'_1, \dots, x'_m\} \cup fv(e))}\}$$

x' = e

$$\{r_1 * R(\{x_1 \dots, x_n\}) \leq r_2 * \boldsymbol{R(\{x'_1, \dots, x', \dots, x'_m)\}}$$

Standard Substitution

Assignment Rule

$$\{r_1 * R(\{x_1 \ldots, x_n\}) \leq r_2 * \boldsymbol{rel(e)} * \boldsymbol{wr(x')} * R(\{x'_1, \ldots, x'_m\} \cup fv(e))\}$$

x' = e

$$\{r_1 * R(\{x_1 \ldots, x_n\}) \leq r_2 * R(\{x'_1, \ldots, x', \ldots, x'_m)\}$$

- $rel(e) * wr(x')$ is the probability the expression and write execute correctly

Verifying the Reliability of Bilinear Interpolation

```
int<.99 * R(i,j,src,dest)>
bilinear_interpolation(int i,  int j,
         int in urel src[][], int in urel dest[][])
{
    int i_src = map_y(i, src, dest),
        j_src = map_x(j, src, dest);

    int up   = i_src - 1, down  = i_src + 1,
        left = j_src - 1, right = j_src + 1;

    int in urel val = src[up][left]    +. src[up][right] +.
                      src[down][right] +. src[down][left];

    return 0.25 *. val;
}
```

Verifying the Reliability of Bilinear Interpolation

1. Generate postcondition from return statement

```
return 0.25 *. val;
```

$$.99 * R(src, i, j, dest) \leq rd(val) * op(*.) * R(val)$$

2. Work backwards to produce verification condition

$$.99 * R(src, i, j, dest) \leq \underbrace{rd(val) * op(*.)}_{\text{Reliability of return}} * \underbrace{rd(src)^4 * op(+.)^3 * wr(val)}_{\text{Reliability of sum of neighbors}} * R(src, i, j, dest)$$

3. Use hardware specification to replace reliabilities

$$.99 * R(src, i, j, dest) \leq (1 - 10^{-7}) * (1 - 10^{-7}) * (1 - 10^{-7})^4 * (1 - 10^{-7})^3 * 1.0 * R(src, i, j, dest)$$

Verifying the Reliability of Bilinear Interpolation

1. Generate postcondition from return statement

```
return 0.25 *. val;
```

⇨

$$.99 * R(src, i, j, dest) \leq rd(val) * op(*.) * R(val)$$

2. Work backwards to produce verification condition

$$.99 * R(src, i, j, dest) \leq rd(val) * op(*.) * rd(src)^4 * op(+.)^3 * wr(val) * R(src, i, j, dest)$$

3. Use hardware specification to replace reliabilities

$$.99 * R(src, i, j, dest) \leq .999999 * R(src, i, j, dest)$$

4. Discharge Verification Condition

Verification Condition Checking Insight

$$\boxed{\bigwedge_i r_i * \underline{R(\{x_{i1}, \dots, x_{in}\})} \leq r'_i * \underline{R(\{x'_{i1}, \dots, x'_{im}\})}}$$

Computing full joint distributions is intractable and input distribution dependent

$$\{x'_1, \dots, x'_m\} \subseteq \{x_1, \dots, x_n\} \rightarrow$$
$$R(\{x_1, \dots, x_n\}) \leq R(\{x'_1, \dots, x'_m\})$$

Conjunct Checking

- A conjunct is implied by a pair of constraints

$$\frac{r_1 \leq r_2 \qquad \{x'_1, \ldots, x'_m\} \subseteq \{x_1, \ldots, x_n\}}{r_1 * R(\{x_1, \ldots, x_n\}) \leq r_2 * R(\{x'_1, \ldots, x'_m\})}$$

- Decidable, efficiently checkable, and **input distribution agnostic**

Verification Condition Checking for Approximate Bilinear Interpolation

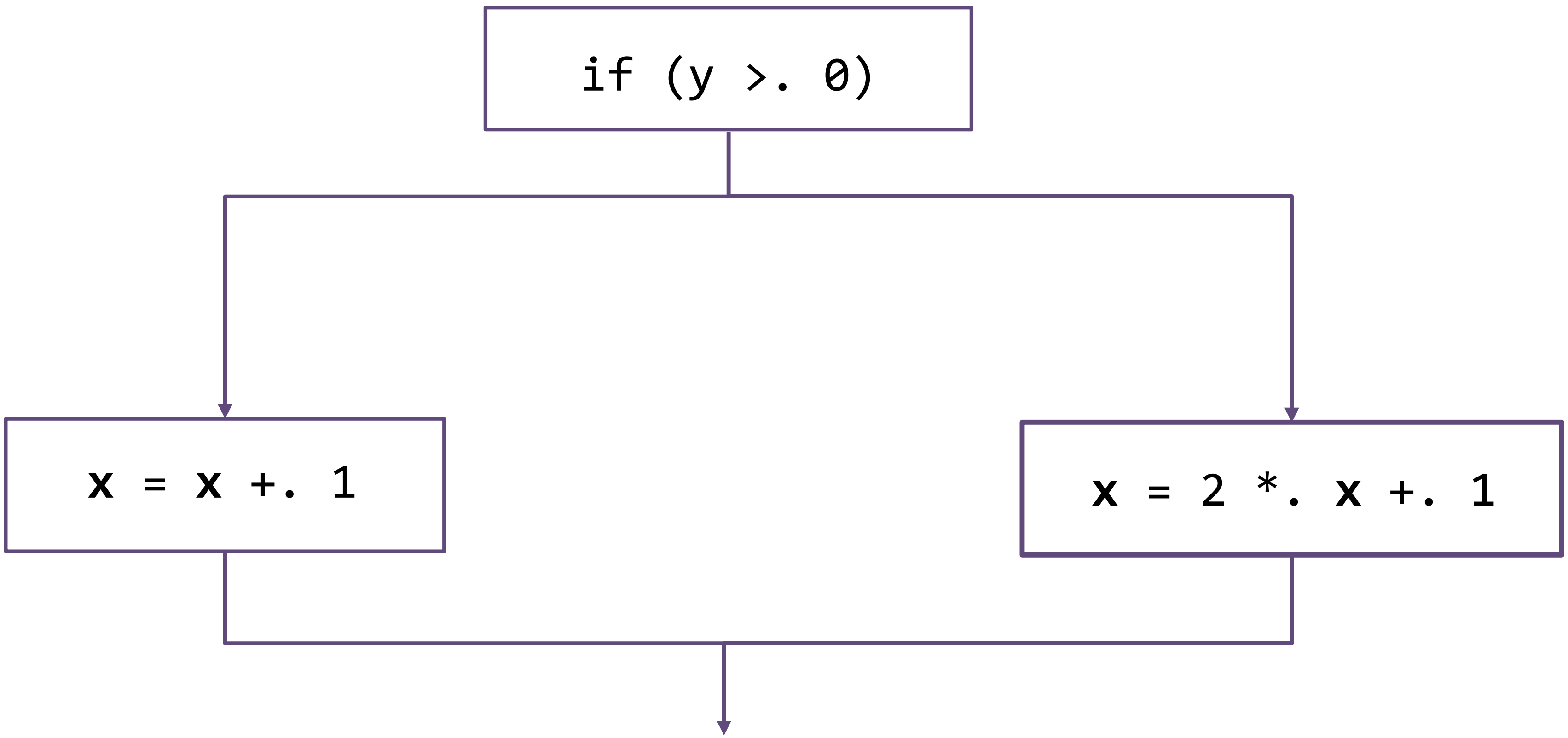
Hardware Specification

Data Dependences

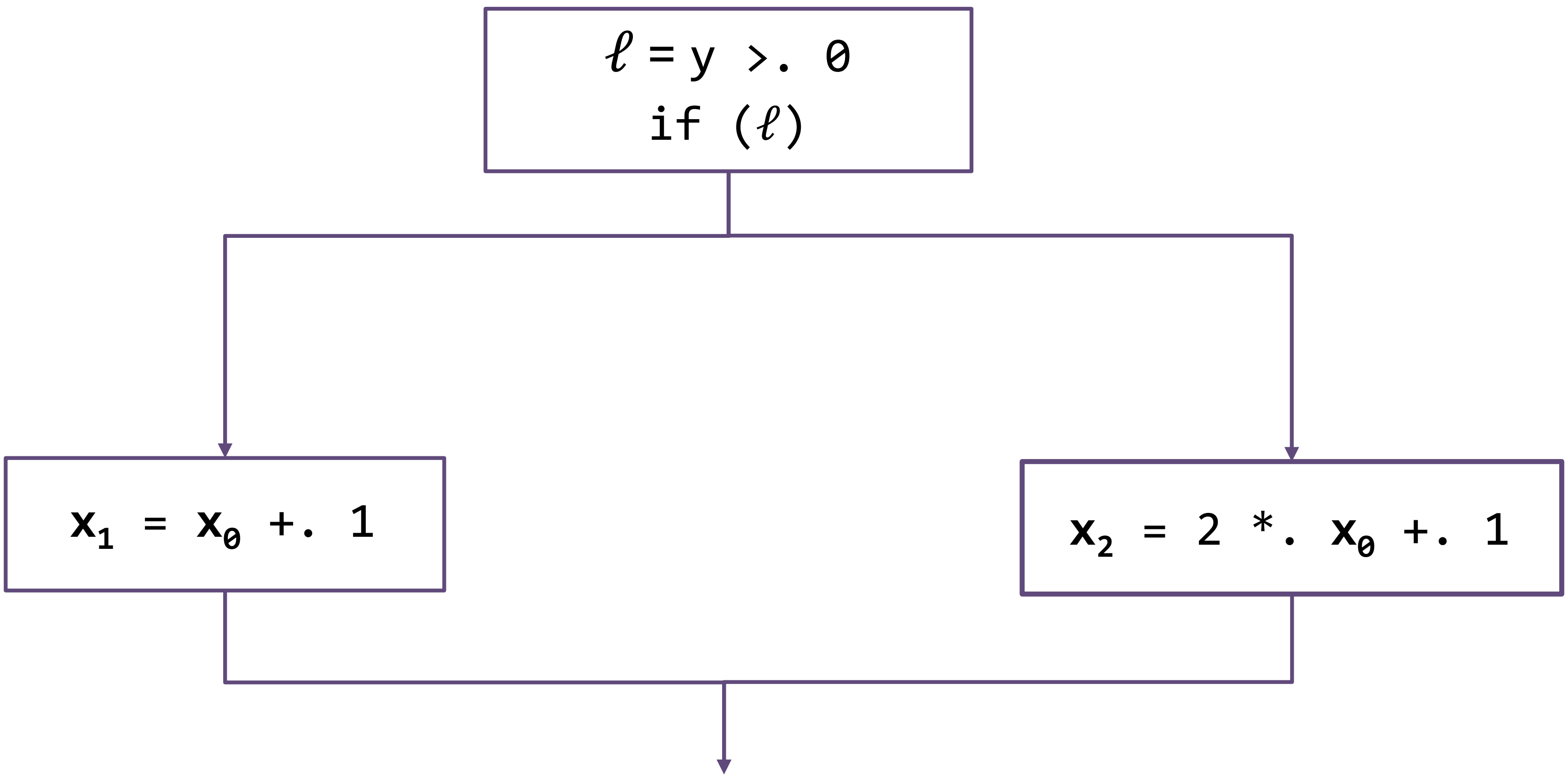
$$\frac{.99 \leq .999999 \qquad \{\mathtt{src}, \mathtt{i}, \mathtt{j}, \mathtt{dest}\} \subseteq \{\mathtt{src}, \mathtt{i}, \mathtt{j}, \mathtt{dest}\}}{.99 * R(\{\mathrm{src}, \mathrm{i}, \mathrm{j}, \mathrm{dest}\}) \leq .999999 * R(\{\mathrm{src}, \mathrm{i}, \mathrm{j}, \mathrm{dest}\})}$$

What about…programs?
(conditionals, loops, and functions)

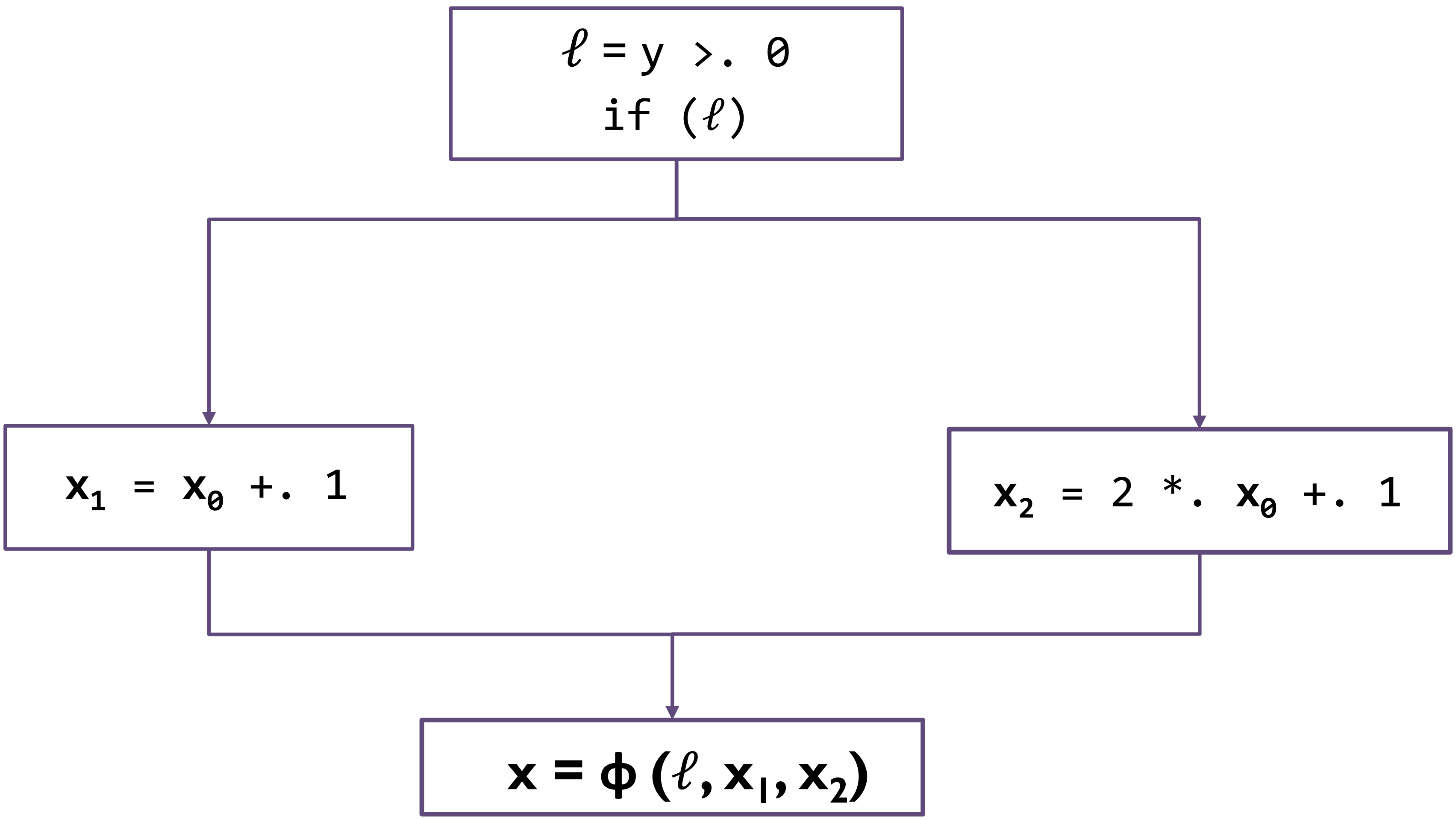
Conditionals



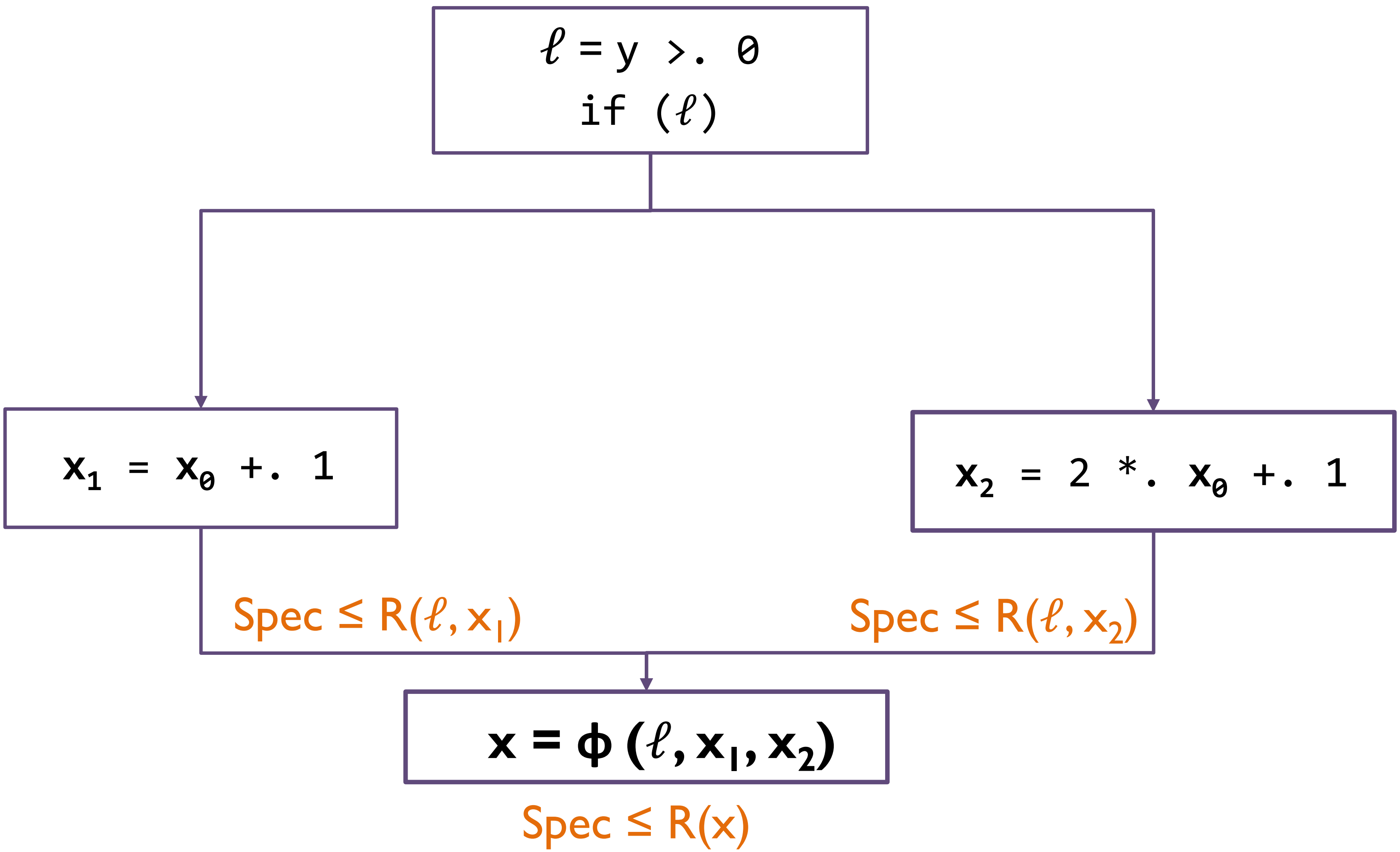
Conditionals



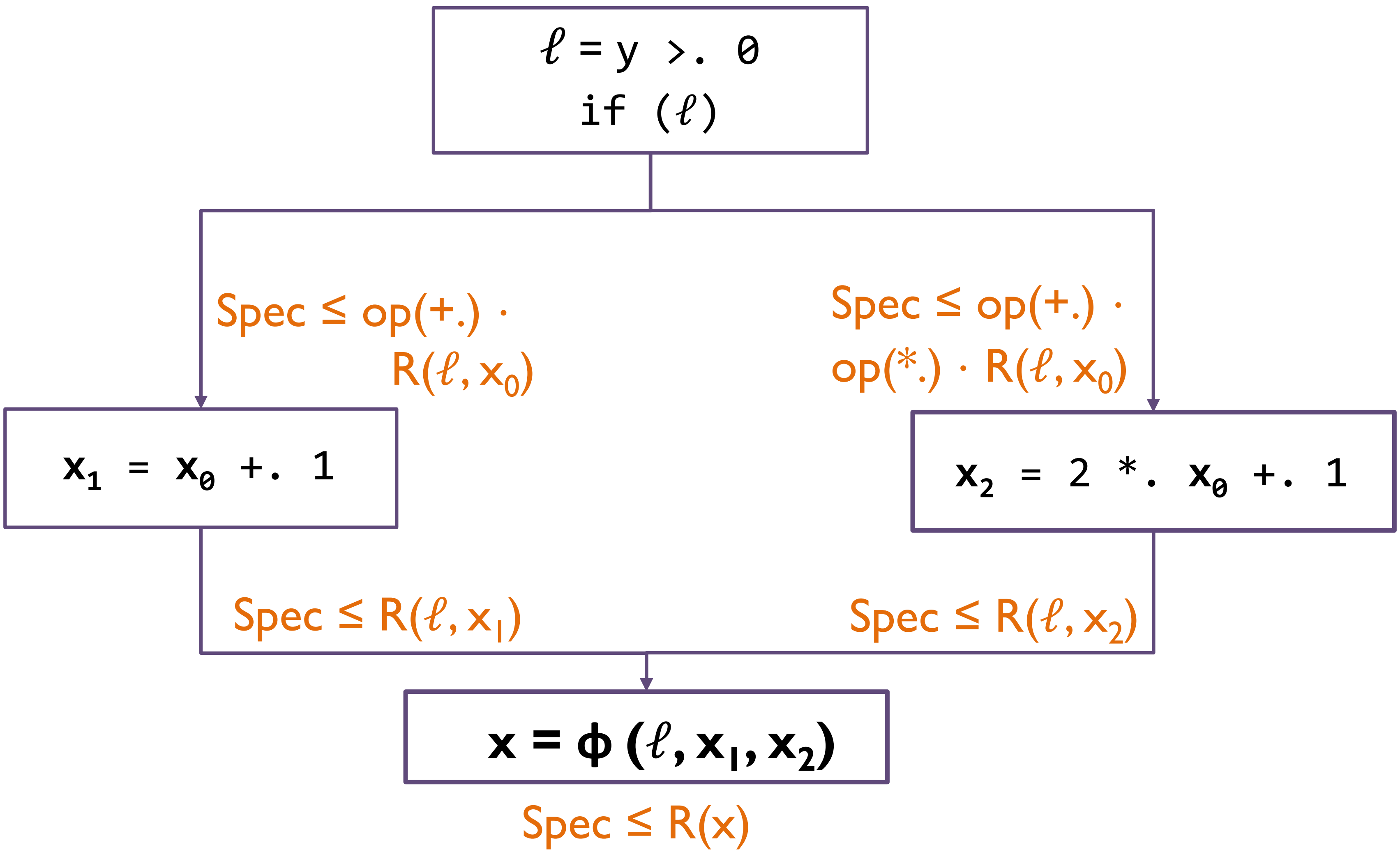
Conditionals



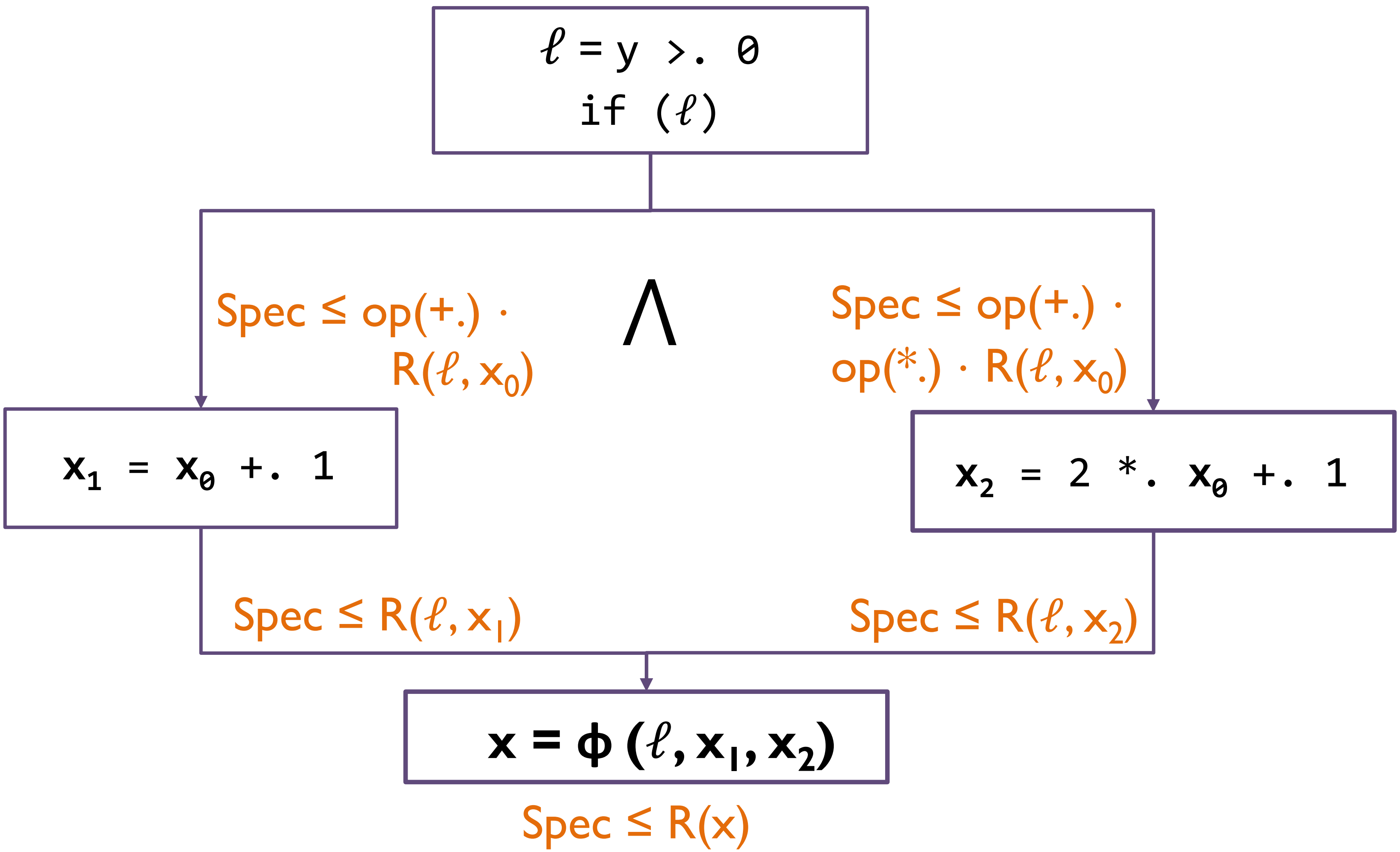
Conditionals



Conditionals



Conditionals



Conditionals

$\text{Spec} \leq \text{op}(+.) \cdot \text{op}(>.) \cdot R(x_0, y) \quad \wedge$

$\text{Spec} \leq \text{op}(+.) \cdot \text{op}(*.) \cdot \text{op}(>.) \cdot R(x_0, y)$

```
ℓ = y >. 0
if (ℓ)
```

$\text{Spec} \leq \text{op}(+.) \cdot R(\ell, x_0)$

$\wedge$

$\text{Spec} \leq \text{op}(+.) \cdot \text{op}(*.) \cdot R(\ell, x_0)$

```
x1 = x0 +. 1
```

```
x2 = 2 *. x0 +. 1
```

$\text{Spec} \leq R(\ell, x_1)$

$\text{Spec} \leq R(\ell, x_2)$

$\mathbf{x} = \phi\,(\ell, \mathbf{x}_1, \mathbf{x}_2)$

$\text{Spec} \leq R(x)$

Simplification

$\text{Spec} \leq op(+.) \cdot op(>.) \cdot R(x_0, y) \quad \wedge$
$\text{Spec} \leq op(+.) \cdot op(*.) \cdot op(>.) \cdot R(x_0, y)$

```
ℓ = y >. 0
if (ℓ)
```

$\text{Spec} \leq op(+.) \cdot R(\ell, x_0)$

$\wedge$

$\text{Spec} \leq op(+.) \cdot op(*.) \cdot R(\ell, x_0)$

```
x1 = x0 +. 1
```

```
x2 = 2 *. x0 +. 1
```

$\text{Spec} \leq R(\ell, x_1)$

$\text{Spec} \leq R(\ell, x_2)$

$\mathbf{x = \phi(\ell, x_1, x_2)}$

$\text{Spec} \leq R(x)$

Simplification

$\cancel{\text{Spec} \leq op(+.) \cdot op(>.) \cdot R(x_0, y) \quad \wedge}$

$\text{Spec} \leq op(+.) \cdot op(*.) \cdot op(>.) \cdot R(x_0, y)$

```
ℓ = y >. 0
if (ℓ)
```

$\cancel{\text{Spec} \leq op(+.) \cdot R(\ell, x_0)}$

$\wedge$

$\text{Spec} \leq op(+.) \cdot op(*.) \cdot R(\ell, x_0)$

```
x₁ = x₀ +. 1
```

```
x₂ = 2 *. x₀ +. 1
```

$\text{Spec} \leq R(\ell, x_1)$

$\text{Spec} \leq R(\ell, x_2)$

$\mathbf{x = \phi\,(\ell, x_1, x_2)}$

$\text{Spec} \leq R(x)$

Loops

- Reliability of loop-carried, unreliably updated variables decreases monotonically

```
int sum = 0;
for (int i = 0; i < n; i = i + 1) {
   sum = sum +. a[i];
}
```

R(sum) depends on n unreliable adds

- **Finitely Bounded Loops:** bounded decrease
- **Unbounded loops:** conservative result is 0

Functions

- Verification is modular (assume/guarantee)

$\texttt{int}\langle r_f * R(\mathrm{x})\rangle\ \texttt{f(x);}$

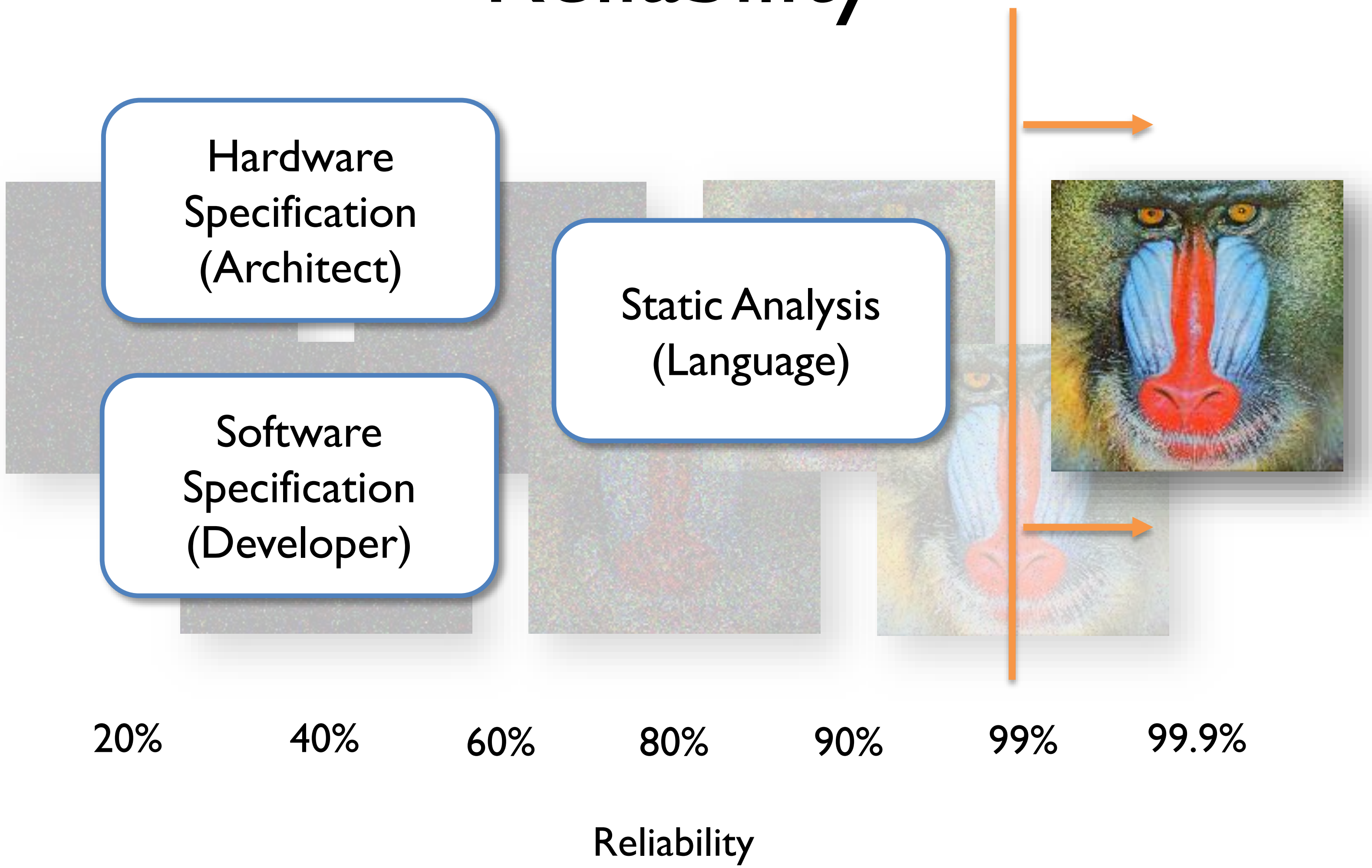
$$r_1 * R(X) \le r_2 * r_f * R(Y \setminus \{\mathrm{y}\} \cup \{\mathrm{x}\})$$

```
y = f(x);
```

$$r_1 * R(X) \le r_2 * R(Y)$$

- Recursion similar to loops: unreliably updated variables naturally have 0 reliability

Rely: a Language for Quantitative Reliability



Evaluation

- **Experiment #1:** verify specifications
  - How does the analysis behave?

Benchmarks

- **newton:** zero-finding using Newton's method
- **secant:** zero-finding using Secant Method
- **coord:** Cartesian to polar coordinate converter
- **search_ref:** motion estimation
- **mat_vec:** matrix-vector multiply
- **hadamard:** frequency-domain pixel-block difference metric

Experiment #1: Results

Benchmark	LOC	Time (ms)	Conjuncts	
			w/o	with
newton	21	8	82	1
secant	30	7	16356	2
coord	36	19	20	1
search_ref	37	348	36205	3
matvec	32	110	1061	4
hadamard	87	18	3	3

Observation: small number of conjuncts with simplification

Evaluation

- **Experiment #2:** application scenarios
    - How to use reliabilities?

Checkable Computations

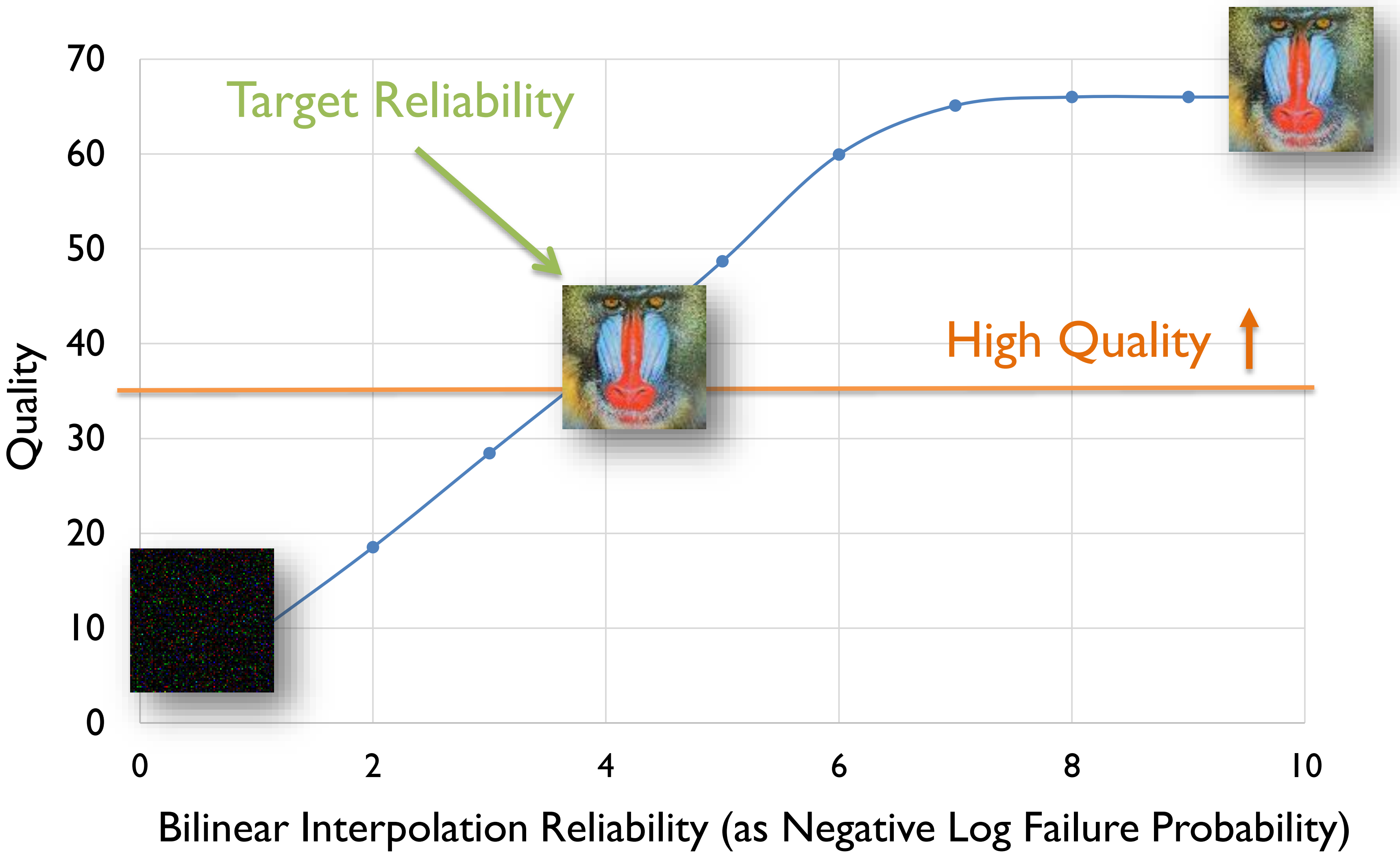
- A **simple checker** can validate whether the program produced a correct result

- Execution time optimization:

$$T_{reliable} \textbf{ vs.}$$

$$T_{unreliable} + T_{checker} + (1 - \mathbf{r}) \cdot T_{reliable}$$

Approximate Computations



Other Concerns for Unreliable Hardware

Safety: does the program always produce a result?

– no failures or ill-defined behaviors

[Misailovic et al. ICSE '10; Carbin et al. ISSTA '10; Sidiroglou et al. FSE'11; Carbin et al., PLDI '12; Carbin et al., PEPM '13]

Accuracy: is result accurate enough?

– small expected error

[Rinard ICS'06; Misailovic et al.,ICSE '10; Hofffmann at al. ASPLOS '11; Misailovic et al. SAS '11; Sidiroglou et al. FSE'11; Zhu et al. POPL '12; Misailovic et al. RACES '12]

Takeaway

- Separating approximate computation isn't enough
- Acceptability of results depends on reliability

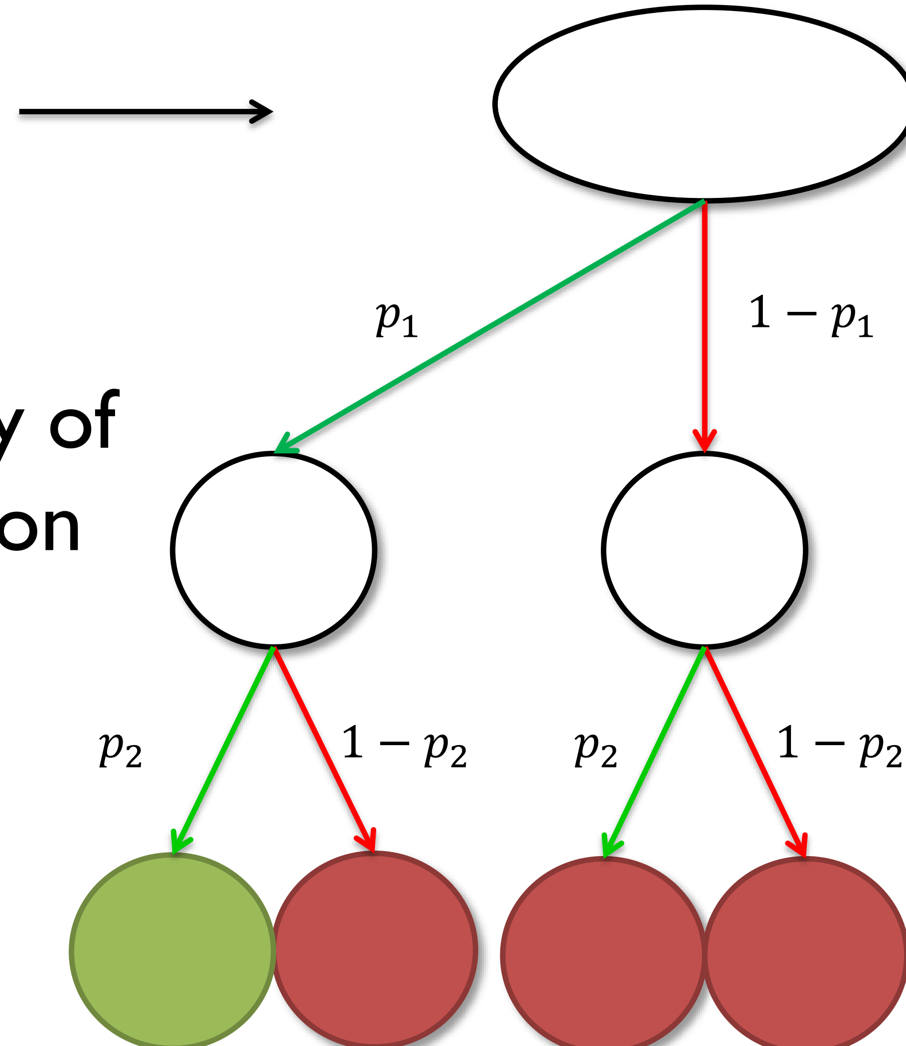
Rely

- Architect provides hardware specification
- Developer provides software specification
- Rely provides verified reliability guarantee

Backup Slides

Semantic Model

- Execution of e is a stochastic process



- Independent probability of failure for each operation
- Reliability is probability of fully reliable path

Semantic Formalization

- Probabilistic transition system

$$\langle s, \sigma \rangle \xrightarrow{p} \langle s', \sigma' \rangle$$

See paper for inference rules!

- Set of possible executions on unreliable hardware gives distributions of states

$$\phi \in \Sigma \rightarrow \mathbb{R} \qquad \langle s, \phi \rangle \Rightarrow \phi$$

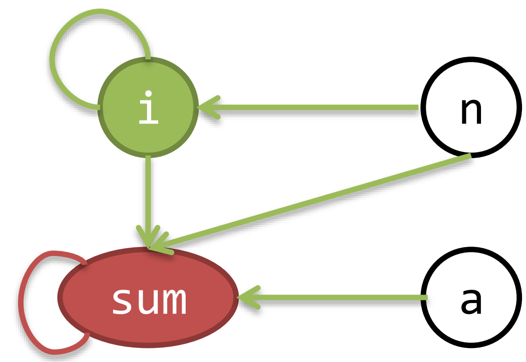
- Predicates defined over distributions

$$[\![P]\!] \in \wp(\Phi)$$

Identifying Reliably Update Variabes

- Reliably updated vs. unreliably updated variables

```
int sum = 0;
for (int i = 0; i < n; i = i + 1)
{
    sum = sum +. a[i];
}
```



- Dependence graph gives classification
- Reliably updated variables have same reliability