# 6.946 Assignment 2

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## 8.1: Chain rule

$F(x,y) = x^2y^3$
$G(x,y) = (F(x,y), y)$
$H(x,y) = F(F(x,y), y)$

(a) $\partial_0 F(x,y) = 2xy^3$
$\partial_1 F(x,y) = 3x^2y^2$

(b) $\partial_0 F(F(x,y),y) = 2x^2y^6$
$\partial_1 F(F(x,y),y) = 3x^4y^8$

(c) $\partial_0 G(x,y) = (2xy^3, 0)$
$\partial_1 G(x,y) = (3x^2y^2, 1)$

(d) $DF(a,b) = (2ab^3, 3a^2b^2)$
$DG(3,5) = ((750,0),(675,1))$
$DH(3a^2, 5b^3) = (210937500a^6b^27, 284765625a^8b^24)$

## 8.2: Computing derivatives

Representation 1: Identify procedure arguments with function's arguments.

```
(define (f1 x y)
  (* (square x) (cube y)))

(define (g1 x y)
  (up (f1 x y) y))

(define (h1 x y)
  (f (f x y) y))
```

Representation 2: Represent the function arguments as slots of a tuple datastructure.

```
(define (f2 v)
  (let ((x (ref v 0))
        (y (ref v 1)))
    (* (square x) (cube y))))

(define (g2 v)
  (let ((x (ref v 0))
        (y (ref v 1)))
    (up (f2 v) y)))

(define h2 (compose f2 g2))
```

(a)
```
(print-expression ((D f1) 'x 'y))
(show-expression ((D f2) (up 'x 'y)))
; (down (* 2 x (expt y 3)) (* 3 (expt x 2) (expt y 2)))
```

$$\begin{bmatrix} 2xy^3 \\ 3x^2y^2 \end{bmatrix}$$

(b)
```
(print-expression ((D f1) (f1 'x 'y) 'y))
(show-expression ((D f2) (up (f2 (up 'x 'y)) 'y)))
; (down (* 2 (expt x 2) (expt y 6)) (* 3 (expt x 4) (expt y 8)))
```

$$\begin{bmatrix} 2x^2y^6 \\ 3x^4y^8 \end{bmatrix}$$

(c)
```
(print-expression ((D g1) 'x 'y))
(show-expression ((D g2) (up 'x 'y)))
; (down (up (* 2 x (expt y 3)) 0)
;       (up (* 3 (expt x 2) (expt y 2)) 1))
```

$$\begin{bmatrix} \begin{pmatrix} 2xy^3 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 3x^2y^2 \\ 1 \end{pmatrix} \end{bmatrix}$$

(d)
```
(print-expression ((D f1) 'a 'b))
(show-expression ((D f2) (up 'a 'b)))
; (down (* 2 a (expt b 3)) (* 3 (expt a 2) (expt b 2)))
```

$$\begin{bmatrix} 2ab^3 \\ \\ 3a^2b^2 \end{bmatrix}$$

```
(print-expression ((D g1) 3 5))
(show-expression ((D g2) (up 3 5)))
; (down (up 750 0) (up 675 1))
```

$$\begin{bmatrix} \begin{pmatrix} 750 \\ \\ 0 \end{pmatrix} \\ \begin{pmatrix} 675 \\ \\ 1 \end{pmatrix} \end{bmatrix}$$

```
(print-expression ((D h1) (* 3 (square 'a)) (* 5 (cube 'b))))
(show-expression ((D h2) (up (* 3 (square 'a)) (* 5 (cube 'b)))))
; (down (* 210937500 (expt a 6) (expt b 27))
;       (* 284765625 (expt a 8) (expt b 24)))
```

$$\begin{bmatrix} 210937500a^6b^{27} \\ \\ 284765625a^8b^{24} \end{bmatrix}$$