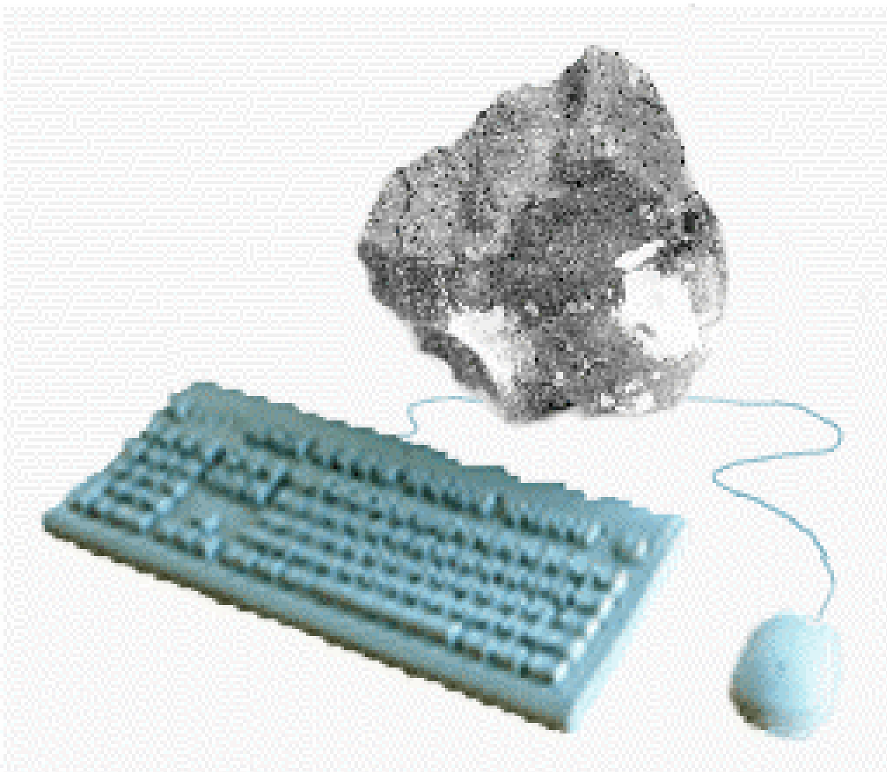
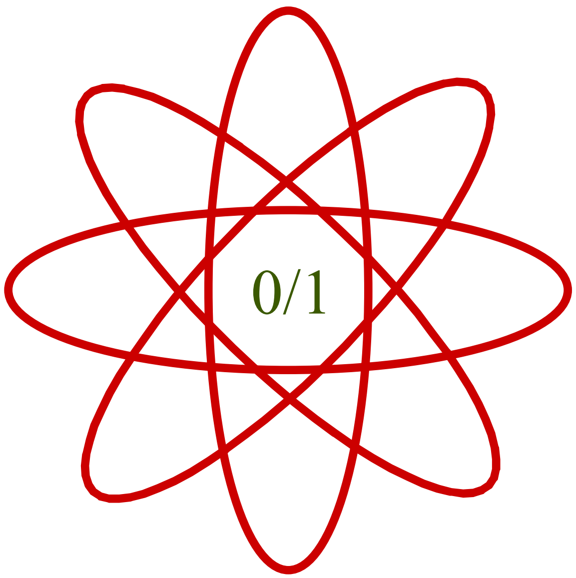
Computing Beyond Silicon Summer School

Physics becomes the computer



Norm Margolus

Physics becomes the computer



Emulating Physics

- » *Finite-state, locality, invertibility, and conservation laws*

Physical Worlds

- » *Incorporating comp-universality at small and large scales*

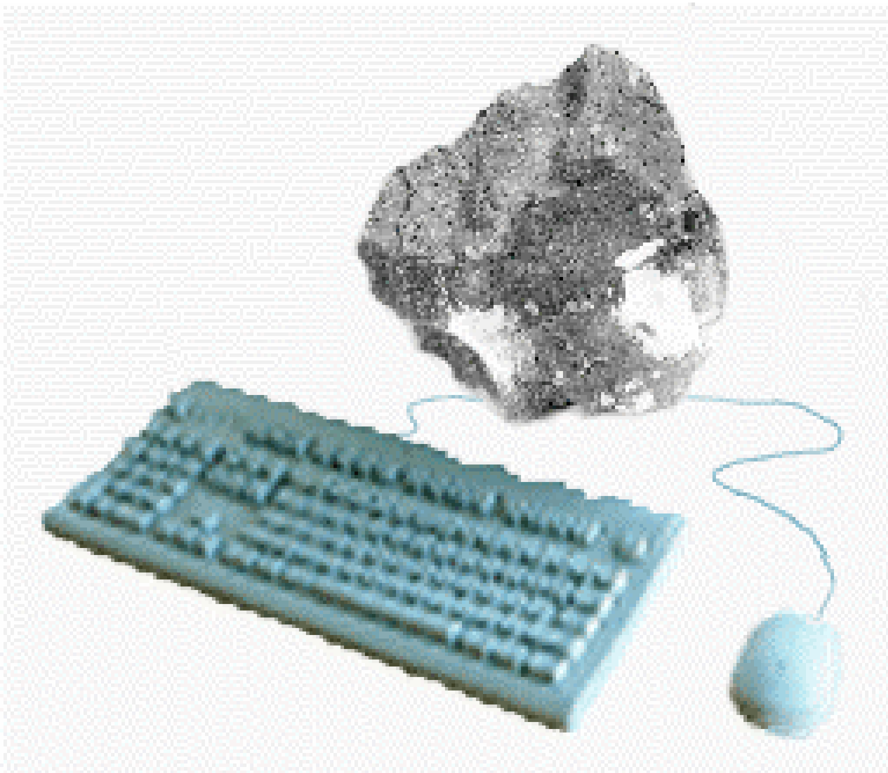
Spatial Computers

- » *Architectures and algorithms for large-scale spatial computations*

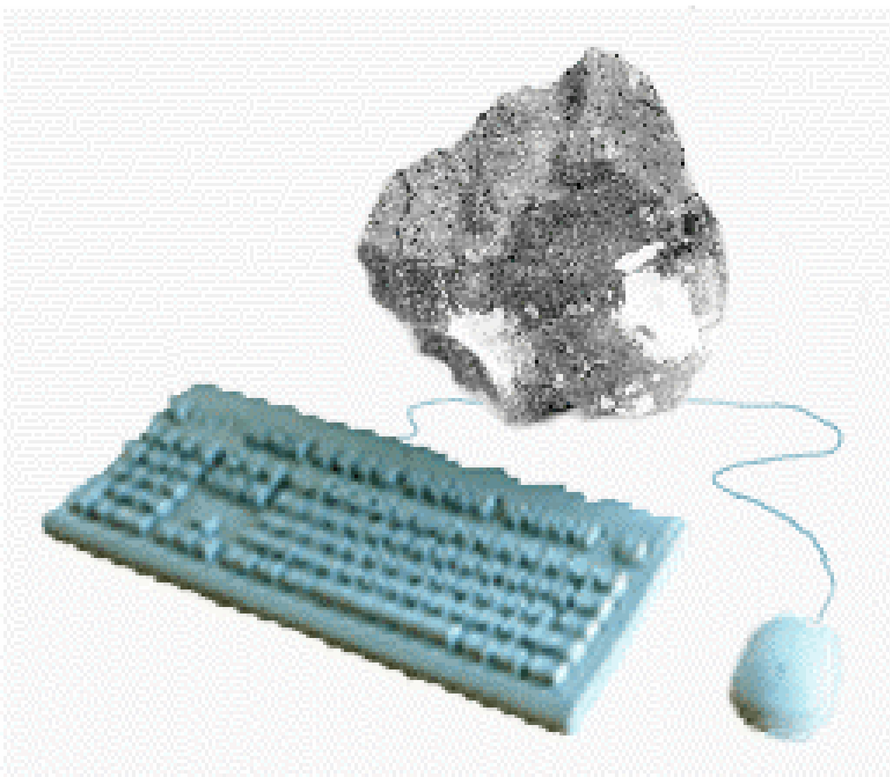
Nature as Computer

- » *Physical concepts enter CS and computer concepts enter Physics*

Looking at nature as a computer



Looking at computation as physics



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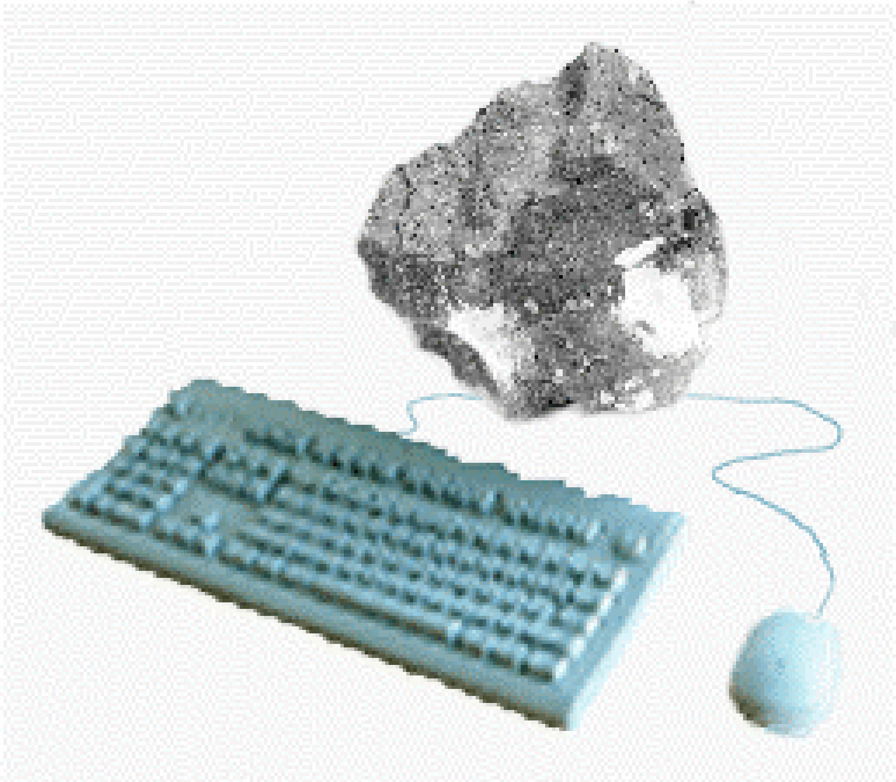

Introduction

As we zoom in on a digital image,

Introduction

As we zoom in on a digital image, we begin to notice that there isn't an infinite amount of resolution:

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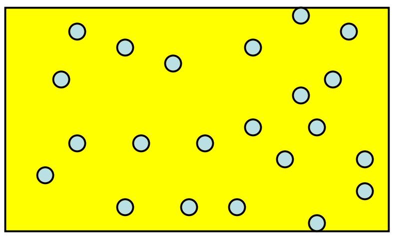
As we zoom in on a digital image, we begin to notice that there isn't an infinite amount of resolution: *We begin to see the pixels.*

Introduction



Something similar happens in nature. A box full of particles doesn't have an infinite number of possible configurations:

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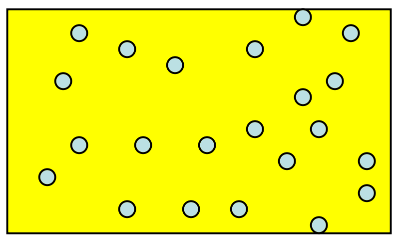
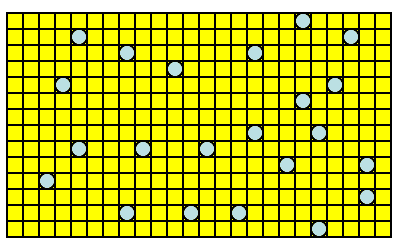
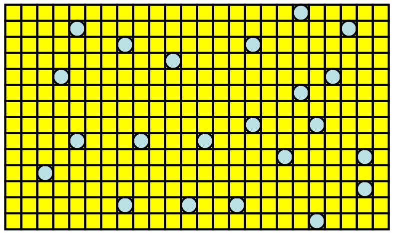




Something similar happens in nature. A box full of particles doesn't have an infinite number of different configurations: *the number of distinct configurations is finite.*

Introduction



Similarly, the rate at which a finite system can transition from one distinct state to another is also finite.

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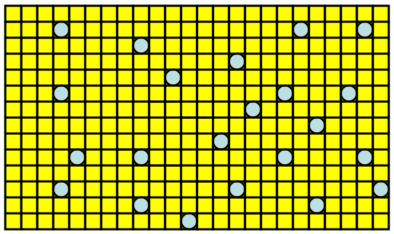

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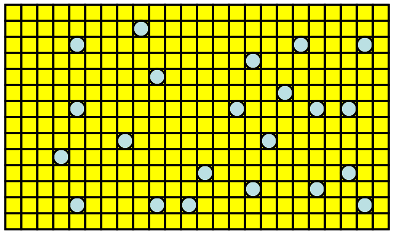

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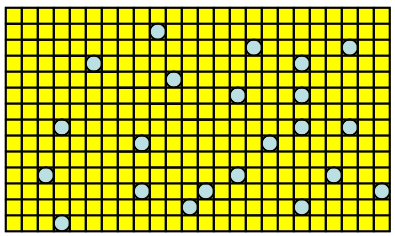

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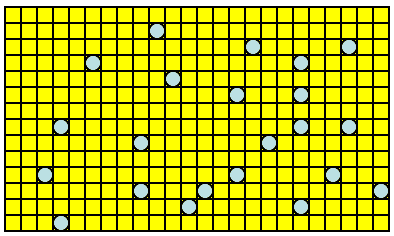
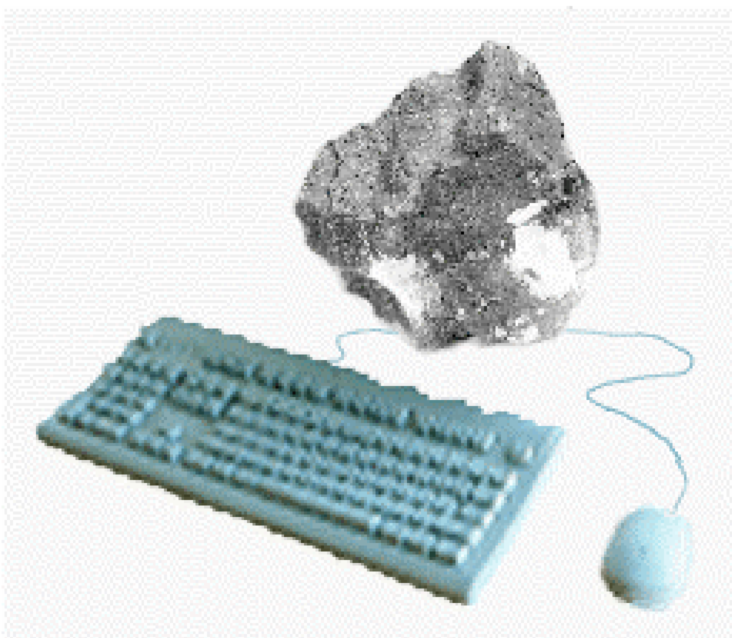
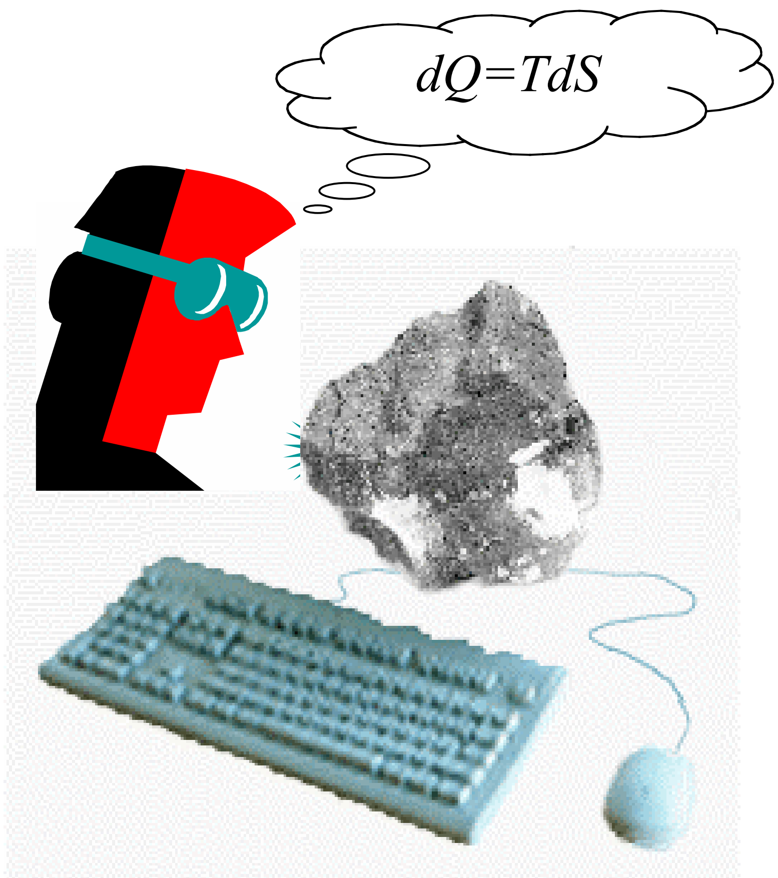




Similarly, the rate at which a finite system can transition from one distinct state to another is also finite. *Thus a finite physical system is much like a computer.*

Introduction



- Physics studies macro properties of finite information systems
- Basic quantities such as Entropy and Energy are informational:

$Entropy_{MAX}=Info_{MAX}$

$KineticE_{MAX}=Ops_{MAX}$

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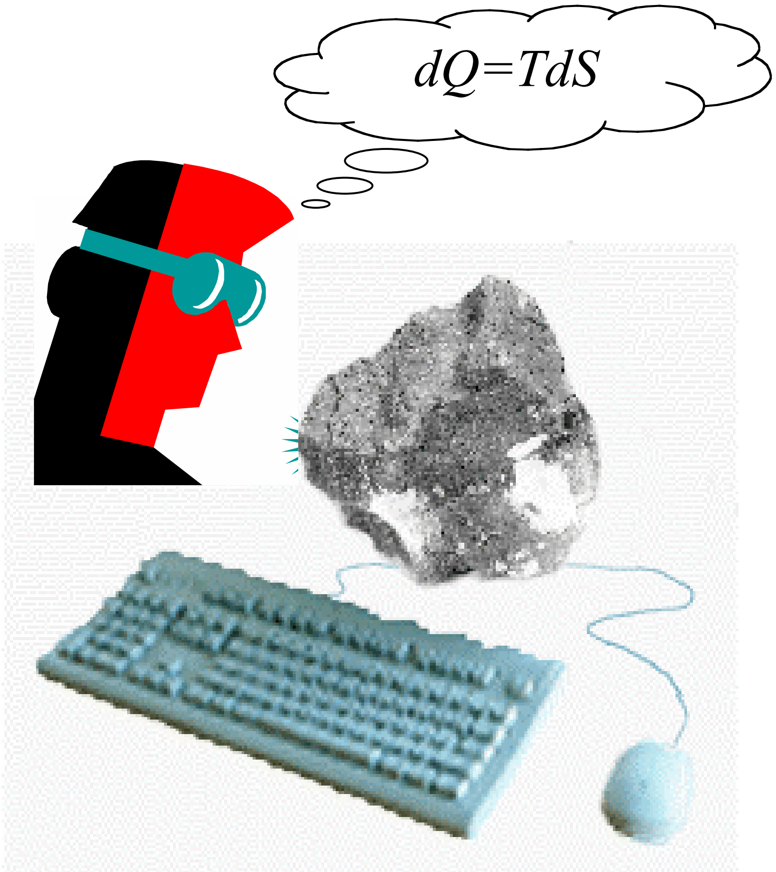


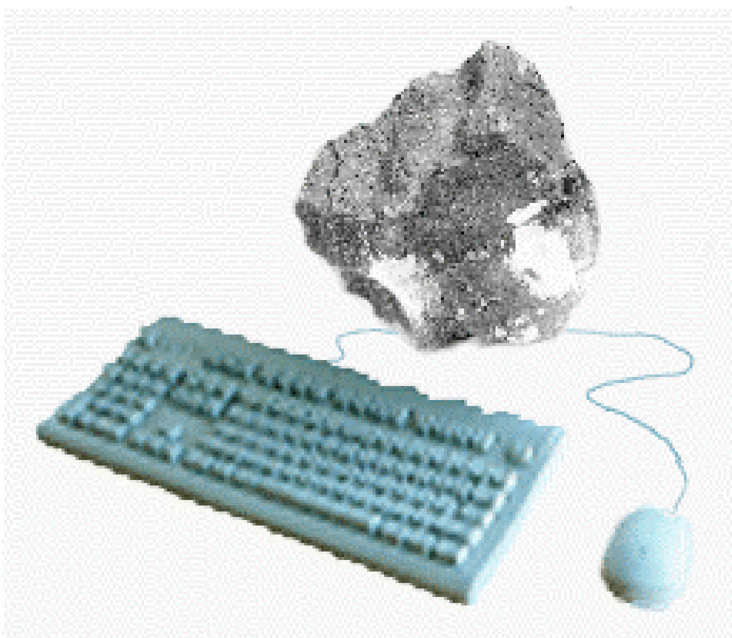
- Physics studies macro properties of finite information systems
- Basic quantities such as Entropy and Energy are informational:

$Entropy_{MAX}=Info_{MAX}$

$KineticE_{MAX}=Ops_{MAX}$

(1996, with Levitin)

In this talk…



Review:

- info (Entropy) in physics

Discuss:

- statistical description of computation ($\rightarrow$ QM)
- energy and action in comp
- what does QM add?
- physics as computation

What is Info?

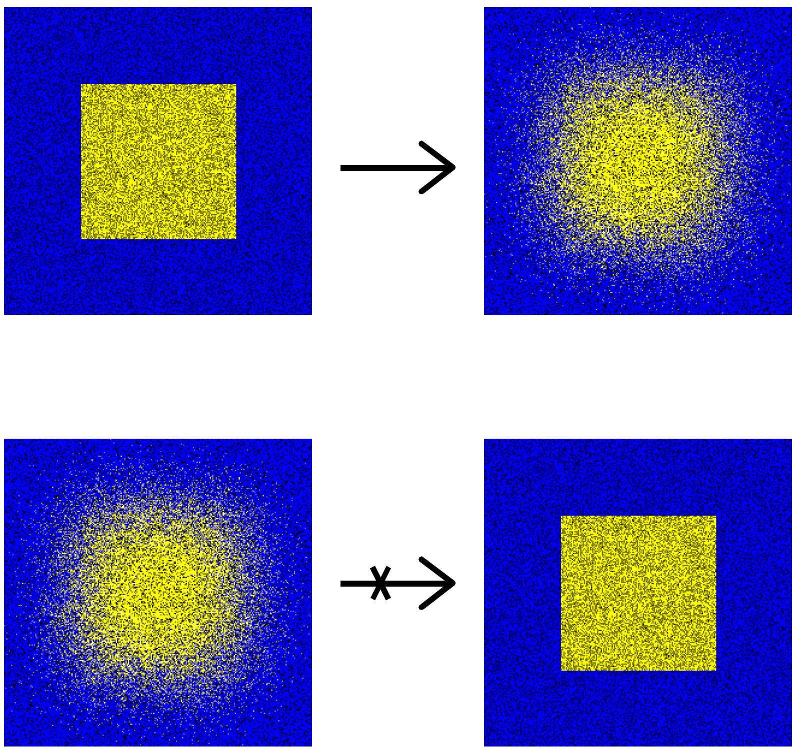
$$\text{Info} = -\sum_i p_i \log p_i$$

‰ equally probable states,

$$\text{Info} = -\sum_{i=1}^{\Omega} \frac{1}{\Omega} \log \frac{1}{\Omega}$$
$$= \log \Omega$$

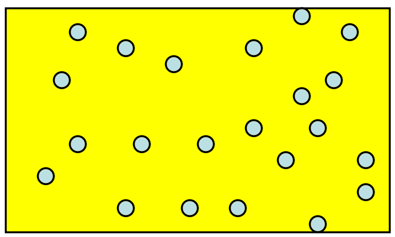
- number of bits system can hold, given its constraints
- system with 2^n possible states can represent n bits
- focus on classical info:
  - » survives in macro limit
  - » substitute micro dynamics when QM is invisible
  - » ordinary macro quantities have classical info interp

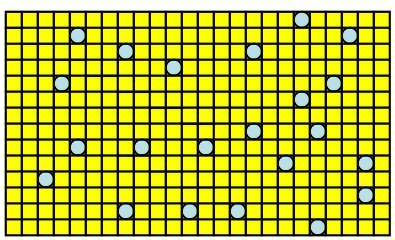
What is Entropy?



- Formal parameter in thermo (irreversibility)
- Boltzmann and Gibbs understood as counting
- Mixing neat $\rightarrow$ mess
- Mixing mess $\rightarrow$ mess
- Entropy is log of #states that fit with constraints

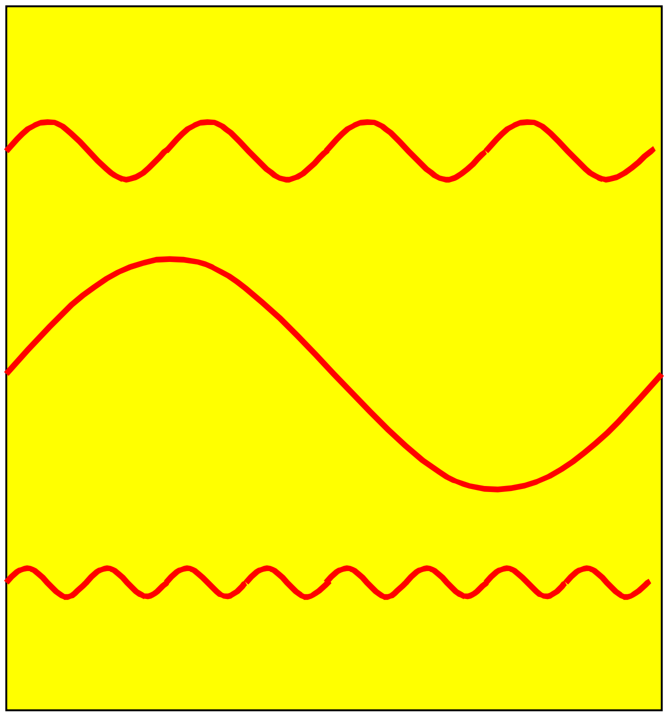
Classical Entropy





- For particles in a box, can introduce some coarseness
- This allows relative probabilities to be calculated
- (Also do the same thing for momentum)

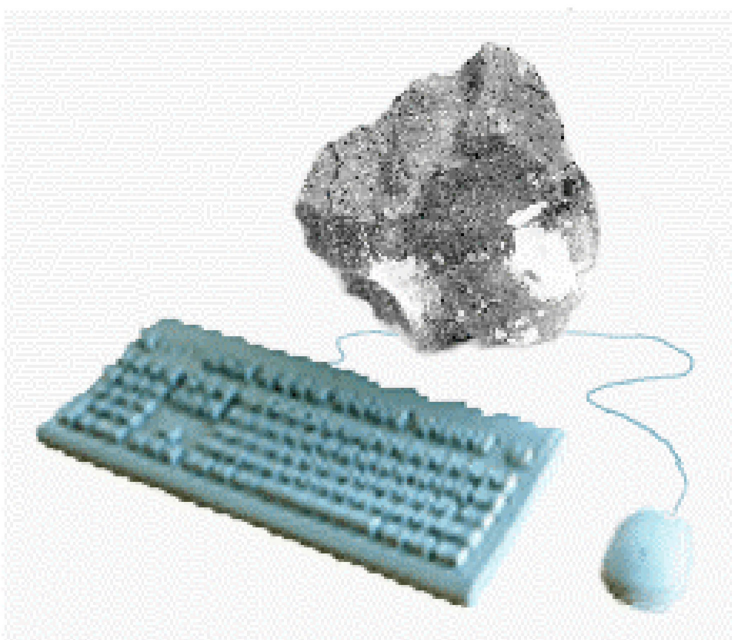
Infinite Entropy?



EM radiation in a cavity (periodic boundaries)

- Thermo of EM radiation in cavity led to QM
- General state is a superposition of waves with integer num peaks
- Any amplitude, can put unit of energy into any wave *(infinite info!)*
- Planck proposed $E = nh\nu$ *(finite info!)*

Looking at nature as a computer



- *With QM, every finite system has finite state*
- Dynamics of finite state systems is familiar
- Develop QM from computer viewpoint!
- Begin by discussing computer logic in statistical situations

Looking at computation as physics

- *With QM, every finite system has finite state*
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- Develop QM from computer viewpoint
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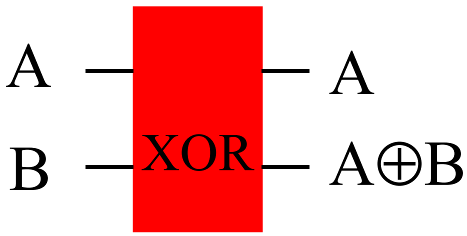
Statistical Dynamics

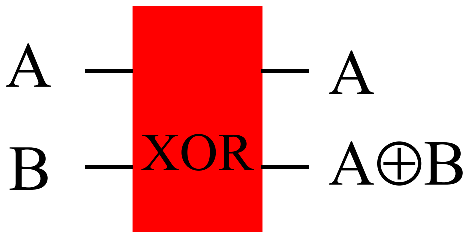


$$U_{\mathrm{XOR}}|00\rangle = |00\rangle$$
$$U_{\mathrm{XOR}}|01\rangle = |01\rangle$$
$$U_{\mathrm{XOR}}|10\rangle = |11\rangle$$
$$U_{\mathrm{XOR}}|11\rangle = |10\rangle$$

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$
$$\rightarrow$$
$$a|00\rangle + b|01\rangle + c|11\rangle + d|10\rangle$$

- To give a complete dynamics, we say what happens to each state in a fixed time
- Weighted sum of states *(superposition)* describes an *ensemble*
- Probability of initial state applies to corresponding final state

Statistical Dynamics



$$U_{\mathrm{XOR}}|00\rangle = |00\rangle$$
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$$U_{\mathrm{XOR}}|11\rangle = |10\rangle$$

$$\sqrt{a}|00\rangle + \sqrt{b}|01\rangle + \sqrt{c}|10\rangle + \sqrt{d}|11\rangle$$
$$\rightarrow$$
$$\sqrt{a}|00\rangle + \sqrt{b}|01\rangle + \sqrt{c}|11\rangle + \sqrt{d}|10\rangle$$

- Better to use square roots of probabilities *(amplitudes)*
- Evolution preserves *vector length*
- Lets us analyze system in other bases

Energy Basis

$$U_\tau : |X_0\rangle \rightarrow |X_1\rangle \rightarrow \cdots \rightarrow |X_{N-1}\rangle \rightarrow |X_0\rangle$$

$$|E_0\rangle = \frac{1}{\sqrt{N}}\left(|X_0\rangle + |X_1\rangle + \cdots + |X_{N-1}\rangle\right)$$

$$U_\tau|E_0\rangle = \frac{1}{\sqrt{N}}\left(|X_1\rangle + |X_2\rangle + \cdots + |X_0\rangle\right)$$

$$= |E_0\rangle$$

- Suppose U_τ represents one clock period of a reversible computer
- Add together all configs in orbit
- This state has equal prob for any config
- Time evolution leaves this state unchanged!

Energy Basis

$$U_\tau|0\rangle = |1\rangle, \qquad U_\tau|1\rangle = |0\rangle$$

$$|E_0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \qquad |E_1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$U_\tau|E_0\rangle = |E_0\rangle, \qquad U_\tau|E_1\rangle = -|E_1\rangle$$

- *Example:* suppose computer only has one bit, and U_τ just flips it.
- Form new 2-state basis by adding and subtracting configs
- Magnitudes of amplitudes of energy states don’t change with time

Energy Basis

$$U_\tau : |\mathrm{X}_0\rangle \to |\mathrm{X}_1\rangle \to \cdots \to |\mathrm{X}_{N-1}\rangle \to |\mathrm{X}_0\rangle$$

$$|E_n\rangle = \frac{1}{\sqrt{N}} \sum_m e^{2\pi i n m/N} |\mathrm{X}_m\rangle,$$

$$U_\tau |E_n\rangle = \frac{1}{\sqrt{N}} \sum_m e^{2\pi i n m/N} |\mathrm{X}_{m+1}\rangle$$

$$= e^{-2\pi i n/N} |E_n\rangle$$

$$\langle E_j | E_k \rangle = \frac{1}{N} \sum_{m,m'} e^{2\pi i (km - jm')/N} \langle \mathrm{X}_{m'} | \mathrm{X}_m \rangle$$

$$= \frac{1}{N} \sum_m e^{2\pi i m(k-j)/N} = \delta_{j,k}$$

- *In general:* use complex amplitudes to form new orthogonal basis
- $|a\rangle$ is like a column vector of components
- $\langle a|$ is like a row vector of complex conjugates

Energy Basis

$$U_\tau : |\mathrm{X}_0\rangle \to |\mathrm{X}_1\rangle \to \cdots \to |\mathrm{X}_{N-1}\rangle \to |\mathrm{X}_0\rangle$$

$$|E_n\rangle = \frac{1}{\sqrt{N}} \sum_m e^{2\pi inm/N} |\mathrm{X}_m\rangle,$$

$$|\mathrm{X}_m\rangle = \frac{1}{\sqrt{N}} \sum_n e^{-2\pi inm/N} |E_n\rangle.$$

$$U_\tau |E_n\rangle = \frac{1}{\sqrt{N}} \sum_m e^{2\pi inm/N} |\mathrm{X}_{m+1}\rangle$$
$$= e^{-2\pi in/N} |E_n\rangle$$

For a cycle:

$$2\pi = 2\pi \frac{n}{N} \times \frac{\tau_n}{\tau}$$

- Energy basis is Fourier Transform of config basis
- $|E_n\rangle$ cycles with a frequency of $\nu_n = \nu(n/N)$, where $\nu = 1/\tau$
- We will call $h\nu_n$ the Energy of the state $|E_n\rangle$, i.e. $E_n = h\nu_n$

Energy Basis

$$|E_n\rangle = \frac{1}{\sqrt{N}}\sum_m e^{2\pi inm/N}|\mathrm{X}_m\rangle,$$

$$|\mathrm{X}_m\rangle = \frac{1}{\sqrt{N}}\sum_n e^{-2\pi inm/N}|E_n\rangle.$$

e.g., $|\Psi_0\rangle = \alpha|E_j\rangle + \beta|E_k\rangle,$

$$|\Psi_\tau\rangle = \alpha e^{-2\pi ij/N}|E_j\rangle + \beta e^{-2\pi ik/N}|E_k\rangle$$

$$E = |\alpha|^2 E_j + |\beta|^2 E_k$$

For $|\mathrm{X}_m\rangle,$ energies are

$$0, \frac{h\nu}{N}, 2\frac{h\nu}{N}, 3\frac{h\nu}{N}, \ldots, (N-1)\frac{h\nu}{N}$$

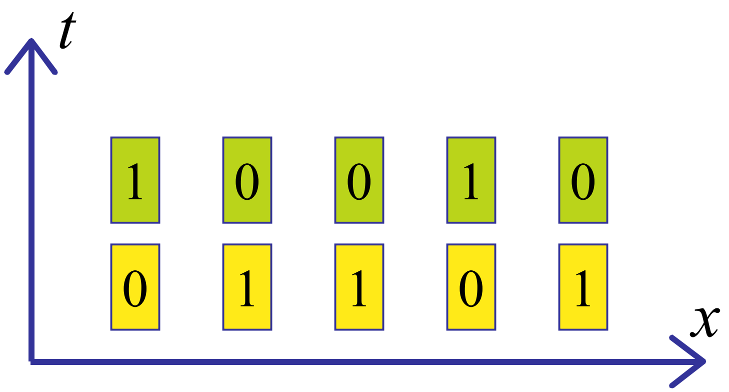
so $E = \frac{h\nu}{2}$

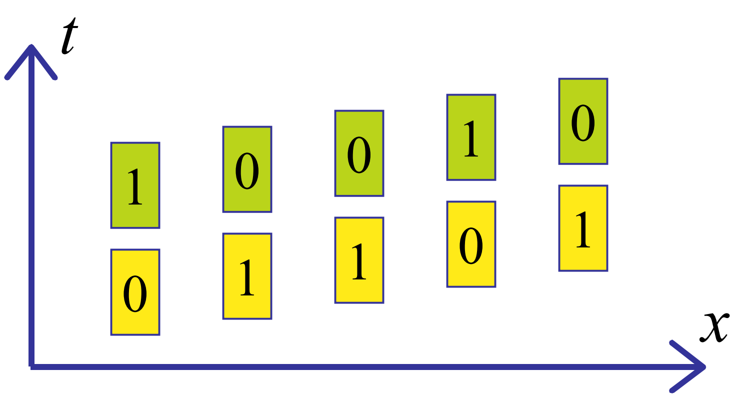
$$U_\tau : |\mathrm{X}_0\rangle \rightarrow |\mathrm{X}_1\rangle \rightarrow \cdots \rightarrow |\mathrm{X}_{N-1}\rangle \rightarrow |\mathrm{X}_0\rangle$$

- Interpret coefficients in energy basis as probs
- Energy of any state is independent of time
- $|\mathrm{X}_n\rangle$ is composed of equally spaced energies, $E_n = nh\nu_1$
- $E=h\nu/2$, or $\nu=2E/h$

What is Energy?

$$U_\tau : |X_0\rangle \rightarrow |X_1\rangle \rightarrow \cdots \rightarrow |X_{N-1}\rangle \rightarrow |X_0\rangle$$

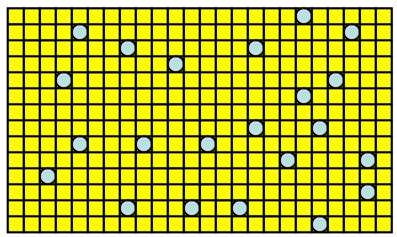




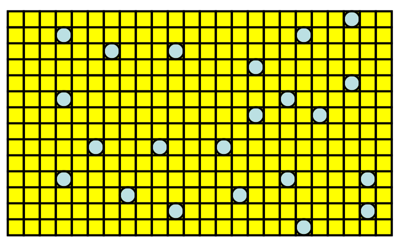
- *ν=2E/h*, so *energy* is rate of change of configurations
- CA lattice can change one spot at a time for reversible rules
- Should count changes as *bit changes* (i.e., energy is extensive!)

What is Energy?

$t:$



$t+\tau_l:$



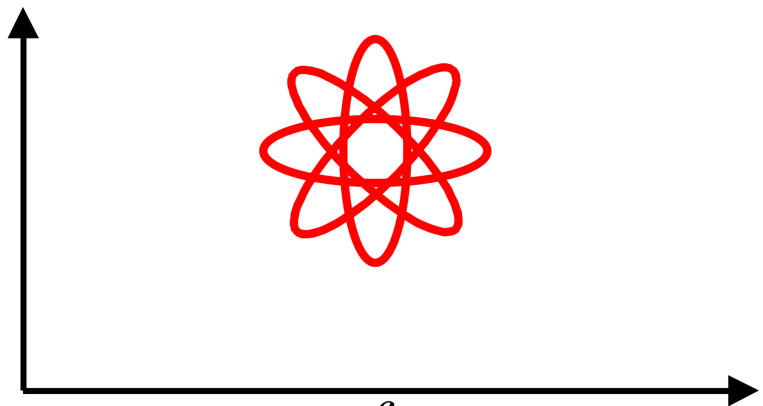
$$M = \text{num particles}$$

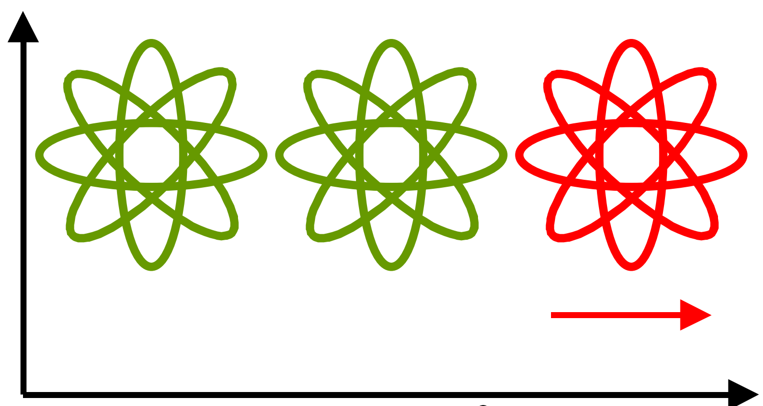
$$h\nu_l = \text{particle energy}$$

$$\nu_\Delta = 2M\nu_l = 2E/h$$

- *Conservation Law:* number of ones constant
- Constrains number of spots that can change in lattice update period τ_l
- Focus on energy of the spots that can change
- If each particle is assigned an energy $h\nu_l$ max change is still $2E/h$

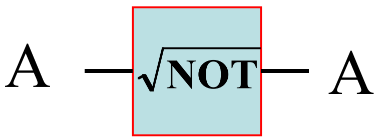
What is Action?





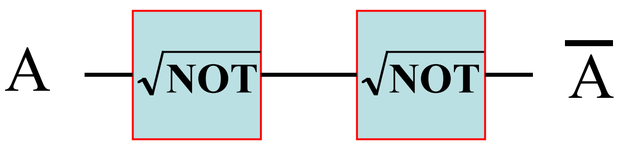
- $\nu = 2E/h$, so $\Omega(t) = 2Et/h$
- *Action* is amount of evolution (total ops for ideal computation)
- Number of comp events in rest frame is rel scalar
- Comp energy must transform like rel energy: $2E_r t_r/h = 2(Et\text{-}px)/h$
- If $x/t=c$, then $E=cp$ so that $Et=px$ (comp stops)

What does QM add?



$$U_{\sqrt{\mathrm{NOT}}}|0\rangle = \tfrac{1}{\sqrt{2}}|0\rangle - \tfrac{1}{\sqrt{2}}|1\rangle$$

$$U_{\sqrt{\mathrm{NOT}}}|1\rangle = \tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle$$



$$U_{\sqrt{\mathrm{NOT}}}U_{\sqrt{\mathrm{NOT}}}|0\rangle =$$

$$U_{\sqrt{\mathrm{NOT}}}\left(\tfrac{1}{\sqrt{2}}|0\rangle - \tfrac{1}{\sqrt{2}}|1\rangle\right) = -|1\rangle$$

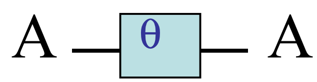
$$U_{\sqrt{\mathrm{NOT}}}U_{\sqrt{\mathrm{NOT}}}|1\rangle =$$

$$U_{\sqrt{\mathrm{NOT}}}\left(\tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle\right) = +|0\rangle$$

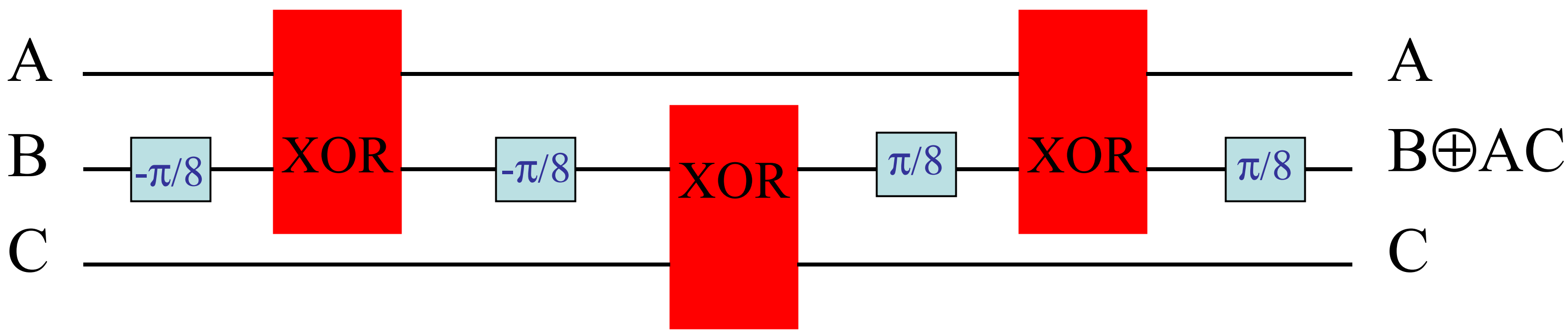
- Stat Comp is special case: QM allows some new kinds of operations
- *Any invertible evolution which preserves vector length is okay*
- Probabilities can *come and go!*
- Only need to add extra single-bit operations
- $\nu_{\Delta}=2(E-E_{\mathrm{min}})/h$

XOR + $\sqrt[4]{\text{NOT}}$ are universal!

$$U_\theta|0\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle$$
$$U_\theta|1\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$



$$\theta = \pi/2: \quad U_\theta = U_{\text{NOT}}$$
$$\theta = \pi/4: \quad U_\theta = U_{\sqrt{\text{NOT}}}$$



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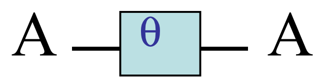

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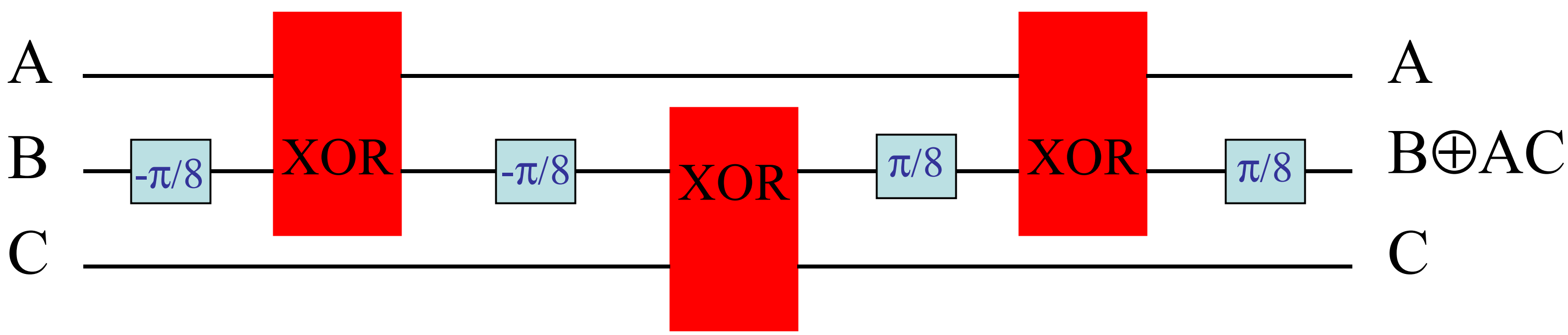

No prob-abilities

Superposition of different configurations

No prob-abilities

XOR + $\sqrt[4]{\text{NOT}}$ are universal!

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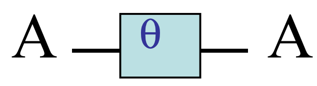
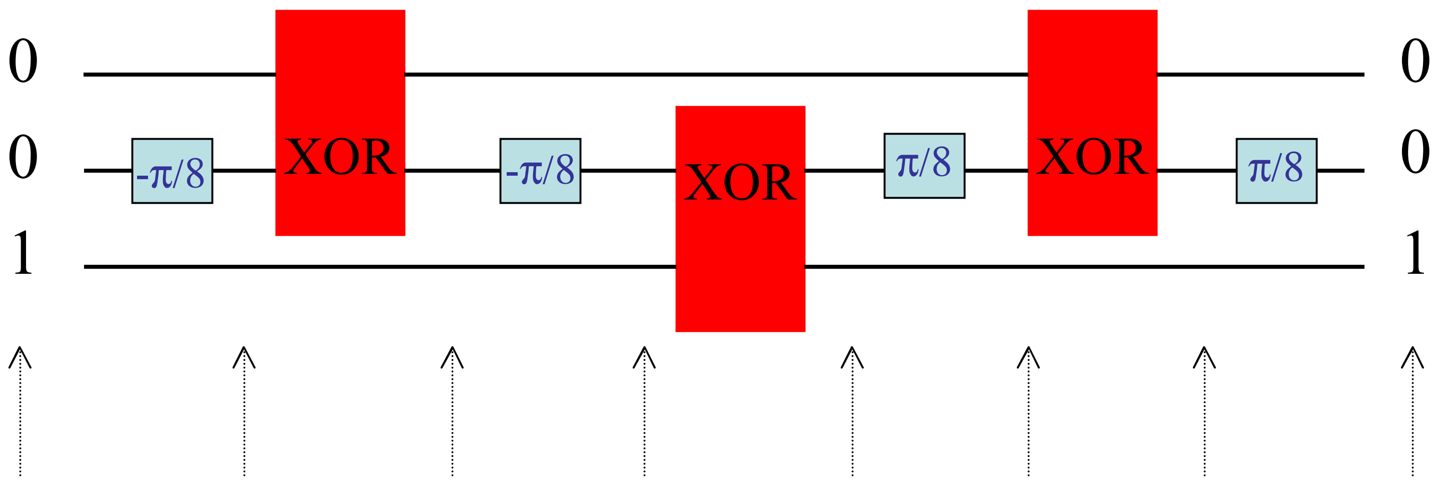
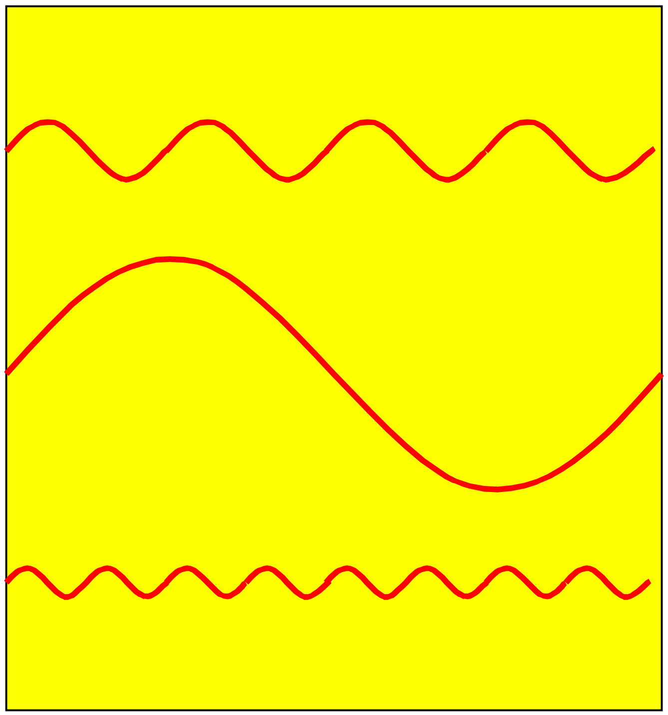


$$c = \cos(\pi/8)$$
$$s = \sin(\pi/8)$$



$|001\rangle$ → $c|001\rangle + s|011\rangle$ → $c|001\rangle + s|011\rangle$ → $\frac{1}{\sqrt{2}}|001\rangle + \frac{1}{\sqrt{2}}|011\rangle$ → $\frac{1}{\sqrt{2}}|001\rangle + \frac{1}{\sqrt{2}}|011\rangle$ → $c|001\rangle + s|011\rangle$ → $c|001\rangle + s|011\rangle$ → $|001\rangle$

What does QM add?



- No *new kinds* of computations; at most *reduces effort required*
- Distinction is basis dependent
- Fundamental Q: If speedup is exponential, then distinction is real!

What does this mean?

Classical: $$\sqrt{a}|0\rangle + \sqrt{b}|1\rangle \underset{\text{NOT}}{\rightarrow} \sqrt{b}|0\rangle - \sqrt{a}|1\rangle$$

Quantum: $$\sqrt{a}|0\rangle + \sqrt{b}|1\rangle \underset{\sqrt{\text{NOT}}}{\rightarrow} \frac{\sqrt{a}+\sqrt{b}}{\sqrt{2}}|0\rangle + \frac{\sqrt{a}-\sqrt{b}}{\sqrt{2}}|1\rangle$$

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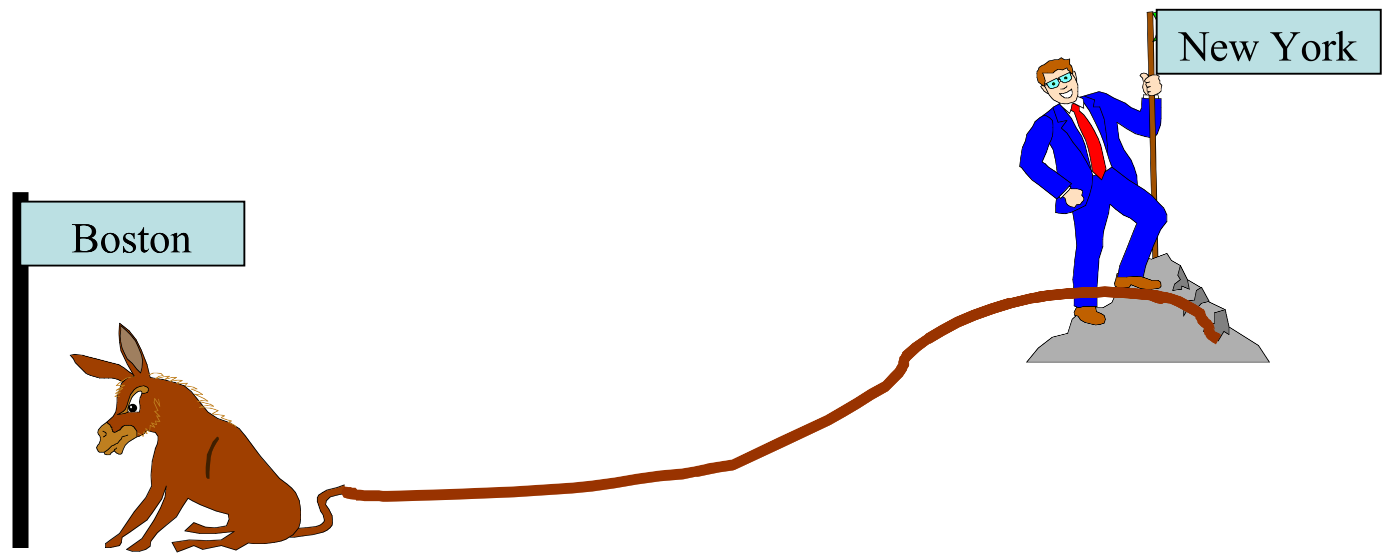
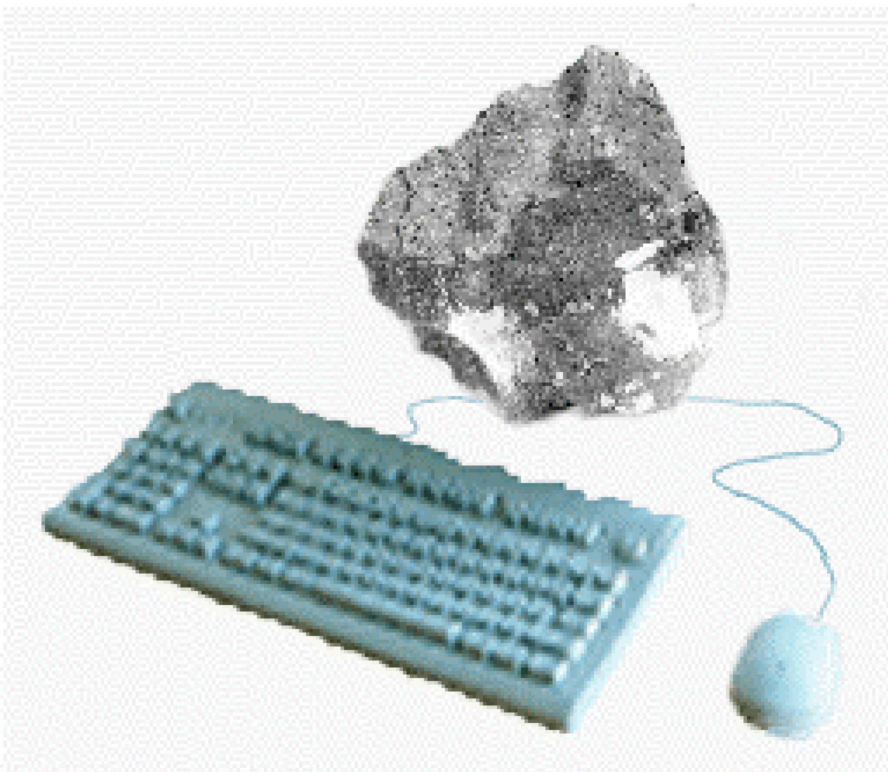


Conclusions



- On a large scale, often can't tell if micro finite-state is QM or CM
- *Entropy*, *Energy* and *Action* all have comp meaning: others must
- Significant for comp and for physics

for more information, see http://www.ai.mit.edu/people/nhm/looking-at-nature.pdf