2.

	sunny	cloudy	rainy
Day1	1	0	0
Day2	0.8	0.2	0
Day3	0.8*0.8+0.2*0.4 =0.72	0.8*0.2+0.2*0.4 =0.24	0.8*0+0.2*0.2 =0.04
Day4	0.72*0.8+0.24*0.4+ 0.04*0.2 =0.68	0.72*0.2+0.24*0.4+ 0.04*0.6 =0.264	0.72*0+0.24*0.2+ 0.04*0.2 =0.056

(a)

$$P(d2 = cloudy | d1 = sunny) = 0.2$$
$$P(d3 = cloudy | d1 = sunny) = 0.24$$
$$P(d4 = rainy | d1 = sunny) = 0.056$$

(b)

```
function A = genW(n)
    t = [.8 .2 .0; .4 .4 .2; .2 .6 .2];
    A = zeros(n,1);
    A(1) = 1;
    for i = 2:n
        r = rand();
        if r < t(A(i-1),1)
            A(i) = 1;   % sunny
        elseif r < t(A(i-1),2)+t(A(i-1),1)
            A(i) = 2;   % cloudy
        else
            A(i) = 3;   % rainy
        end
    end
end
```

(c)

```
function x=staticP(A)
    n = size(A,1);
    ntype = 3;
    x = [sum(A ==1), sum(A ==2), sum(A==3)];
```

```
    x = x/n;
end
```

\>> ps=staticP(genW(2000000))

ps =

0.6431   0.2855   0.0714

$$P(sunny) \approx 0.6431$$
$$P(cloudy) \approx 0.2855$$
$$P(rainy) \approx 0.0714$$

(d)

$$\mathbf{T} = \begin{bmatrix} 0.8 & 0.4 & 0.2 \\ 0.2 & 0.4 & 0.6 \\ 0 & 0.2 & 0.2 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} P_{sunny} \\ P_{cloudy} \\ P_{rainy} \end{bmatrix}$$

By solving

$$\mathbf{Tx} = \mathbf{T}$$
$$\|\mathbf{x}\|_1 = 1$$

We can get :

$$P_{sunny} = \frac{9}{14} = 0.6428, P_{cloudy} = \frac{4}{14} = 0.2857, P_{rainy} = \frac{1}{14} = 0.0714$$

(e)

$$H(X) \equiv \sum_{x \in \mathcal{A}_X} P(x) \log \frac{1}{P(x)},$$

H(X) = -0.6431*log(0.6431) - 0.2855*log(0.2855) - 0.0714*log(0.0714) = 1.1978

(f)

```
ps =[9 4 1]/14;
T = [.8 .2 .0; .4 .4 .2; .2 .6 .2];
r = [ps' ps' ps'] .* T';
r(1,:) = r(1,:)./sum(r(1,:));
r(2,:) = r(2,:)./sum(r(2,:));
r(3,:) = r(3,:)./sum(r(3,:));
```

r =

0.5714   0.2857   0.1429
0.1667   0.3333   0.5000
0   0.5000   0.5000

today\yesterday	sunny	cloudy	rainy
sunny	0.5714	0.2857	0.1429

cloudy	0.1667	0.3333	0.5000
rainy	0	0.5000	0.5000

$$P(yesterday|today) = \frac{P(today|yesterday)P(yesterday)}{\sum P(today|yesterday)P(yesterday)}$$

(g)

Markov assumption says the current state only depends on the previous state. However, the dynamic transition function does not violate the rule.

3.

(a)

Using the bayes filter implemented with matlab code, $P(d5 = sunny) = 0.4$

$$Bel(x_t) = \eta\ P(z_t\,|\,x_t)\ \int P(x_t\,|\,u_t, x_{t-1})\ Bel(x_{t-1})\ dx_{t-1}$$

```
x = [1 0 0];
Pzx = [.6,.4,.0;.3,.7,.0;0,0,1]';
Pxx = [.8 .2 .0; .4 .4 .2; .2 .6 .2]';
z = [0,2,2,3,1];

for i=2:5
    xp = zeros(1,3);
    for j=1:3
        for k=1:3
            xp(j) = xp(j) + Pzx(z(i),j)*Pxx(j,k)*x(i-1,k);
        end
    end
    xp = xp/sum(xp);
    x = [x;xp];
end
```

x =

1.0000	0	0
0.6957	0.3043	0
0.5977	0.4023	0
0	0	1.0000
0.4000	0.6000	0

(where x(i,j) = P(j^{th} weather on i^{th} day), 1^{st} weather is sunny, 2^{nd} is cloudy, 3^{rd} is rainy.)

(b)

1) Only the data available to the day is used:

Using the above program we can generate the possibility,

x =

1.0000	0	0
0.8889	0.1111	0
0.8718	0.1282	0
0	0	1.0000

(where x(i,j) = P(jth weather on ith day), 1st weather is sunny, 2nd is cloudy, 3rd is rainy.)

Therefore, the most likely weather on 2nd day is sunny, on 3rd day is sunny, and 4th day is rainy.

2) Data from future is available:

Because the measurement for rainy is always perfect, we can infer that the 4th day must be rainy. After that, we can infer that 3rd day must be cloudy by using the result from 2.f. and perfect measurement for rainy. Then we left two choices, (2nd = cloudy) or (2nd = sunny).

$$P(d2 = sunny | d1, d3) = \eta P(d1 | d2) P(d3 | d2) P(d2) = \eta * 0.5714 * 0.2 * 0.6428 = \eta 0.0732$$

$$P(d2 = cloudy | d1, d3) = \eta P(d1 | d2) P(d3 | d2) = \eta * 0.167 * 0.4 * 0.2857 = \eta 0.019$$

So, day 2 is more likely to be sunny.

(c)

From the previous inference from both passed and future data, there are only two possibilities: (cloudy, cloudy, rain) or (sunny, cloudy, rain). By normalizing the probability,

$$P(d2 = sunny, d3 = cloudy, d4 = rainy) = \eta 0.0732 = 0.7939$$

$$P(d2 = cloudy, d3 = cloudy, d4 = rainy) = \eta 0.019 = 0.2061$$