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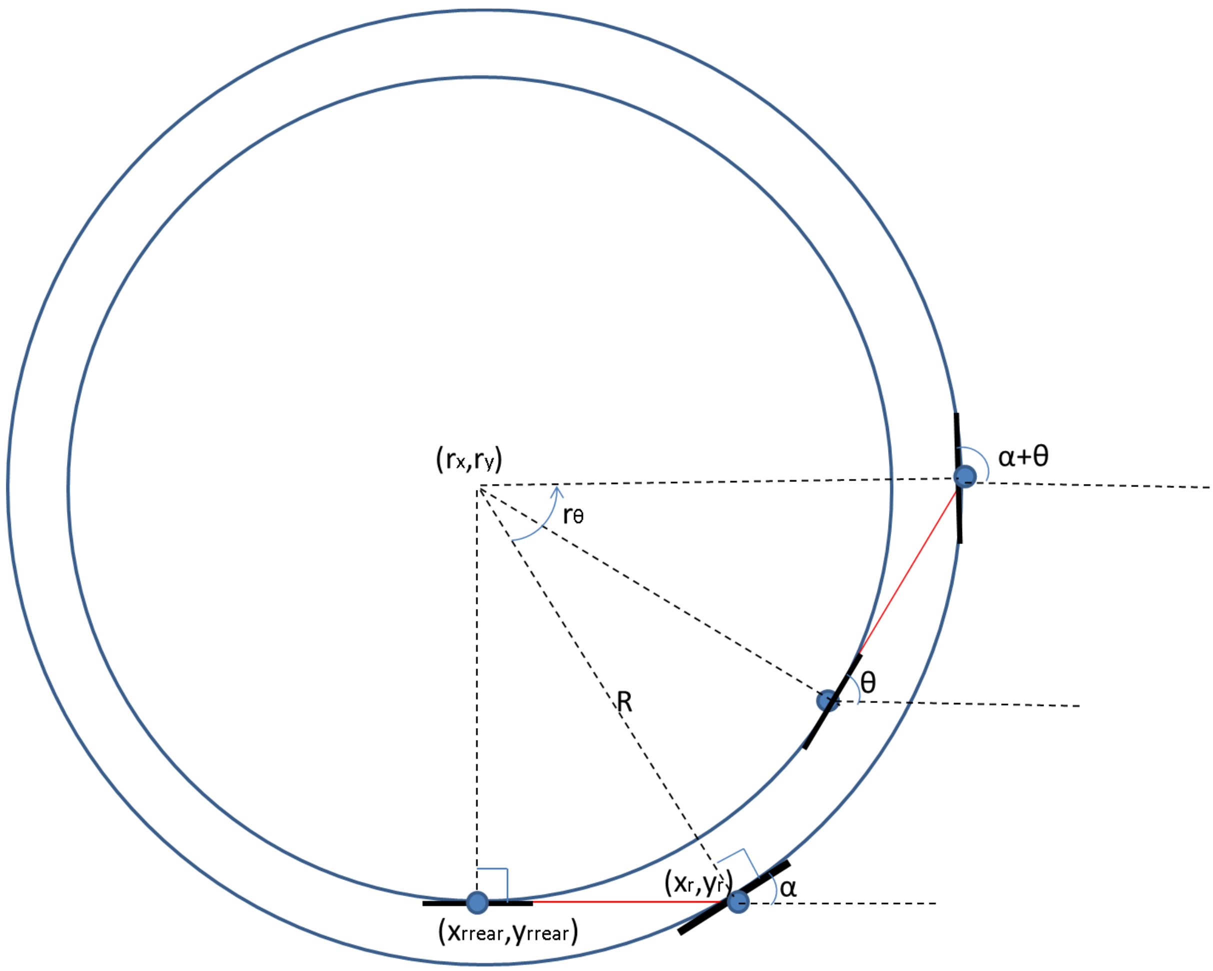
4. Now consider a simple kinematic model of an idealized *bicycle*. Both tires are of diameter d, and are mounted to a frame of length l. The front tire can swivel around a vertical axis, and its steering angle will be demoted α. The rear tire is always parallel to the bicycle frame and cannot swivel.

For the sake of this exercise, the pose of the bicycle shall be defined through three variables: the x-y location of the center of the front tire, and the angular orientation θ (yaw) of the bicycle frame relative to an external coordinate frame. The controls are the forward velocity v of the bicycle, and the steering angle α, which we will assume to be constant during each prediction cycle.

Provide the mathematical prediction model for a time interval Δt, assuming that it is subject to Gaussian noise in the steering angle α and the forward velocity v. The model will have to predict the posterior of the bicycle state after Δt time, starting from a known state. If you cannot find an exact model, approximate it, and explain your approximations.



$\begin{bmatrix} x'_r \\ y'_r \end{bmatrix}$: the predicted bicycle pose,

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$\begin{bmatrix} x_r \\ y_r \end{bmatrix}$: the current bicycle front wheel pose,

$\begin{bmatrix} r_x \\ r_y \end{bmatrix}$: the center of rotation circle,

R : the radius of rotation circle,

$\begin{bmatrix} x_{rrear} \\ y_{rrear} \end{bmatrix}$: the current bicycle rear wheel pose

r_θ : the rotation angle around rotation circle.

$$\begin{bmatrix} x'_r \\ y'_r \end{bmatrix} = \begin{bmatrix} r_x \\ r_y \end{bmatrix} + \begin{bmatrix} \cos r_\theta & -\sin r_\theta \\ \sin r_\theta & \cos r_\theta \end{bmatrix} \begin{bmatrix} x_r - r_x \\ y_r - r_y \end{bmatrix}$$

To get the x_r, y_r, θ_r, we can solve the following two line equation derived from the current front and rear wheels.

$$\begin{cases} r_y - y_r = \tan(\alpha + \theta_r + 90^\circ)(x - x_r) \\ r_y - y_{rrear} = \tan(\theta_r + 90^\circ)(x - x_{rrear}) \end{cases}$$

,where $x_{rrear} = x_r - l\cos\theta, y_{rrear} = y_r - l\sin\theta$

$$R = dist(\begin{bmatrix} x_r \\ y_r \end{bmatrix}, \begin{bmatrix} r_x \\ r_y \end{bmatrix})$$

$$\theta_r = sign(\alpha)\frac{v}{R}\Delta t$$

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5. Consider the kinematic bicycle model from Exercise 4. Implement a sampling function for posterior poses of the bicycles under the same noise assumptions.

For your simulation, you might assume $l = 100cm$, $d = 80cm$, $\Delta t = 1sec$, $|\alpha| \leq 80°$, $v \in [0; 100]cm/sec$. Assume further that the variance of the steering angle is $\sigma_\alpha^2 = 25°^2$ and the variance of the velocity is $\sigma_v^2 = 50cm^2/sec^2 \cdot |v|$. Notice that the variance of the velocity depends on the commanded velocity.

For a bicycle starting at the origin, plot the resulting sample sets for the following values of the control parameters:

problem number	α	v
1	$25°$	$20cm/sec$
2	$-25°$	$20cm/sec$
3	$25°$	$90cm/sec$
4	$80°$	$10cm/sec$
1	$85°$	$90cm/sec$

All your plots should show coordinate axes with units.

Matlab code:

```
function hw4(alpha, v)
    run(25, 20);
    run(-25, 20);
    run(25, 90);
    run(80, 10);
    run(85, 90);
end
function run(alpha, v)
    figure;
    xlabel('x(cm)')
    ylabel('y(cm)')
    for i=1:5000
        sample_bicycle(alpha, v)
    end
    AXIS([-300 300  -300 300])
end


function sample_bicycle(alpha, v)
    x = 0;
    y = 0;
    th = 0;
    dt = 1;
```

```
    maxspeed = 100;
    alpha = deg2rad(alpha);
    l = 100;
    sig_alpha = deg2rad(25);
    sig_v = 50;

    vh = v + sample(sig_v*(sqrt(abs(v))));
    alphah = alpha + sample(sig_alpha);

    % get rx, ry, rth
    xr = x;
    yr = y;
    thr = th;
    x_rrear = xr - l*cos(th);
    y_rrear = yr - l*sin(th);

    t1 = tan(alphah + thr+pi/2);
    t2 = tan(thr+pi/2);
    rx = (-yr+y_rrear + t1*xr-t2*x_rrear) / (t1 - t2);
    ry = yr+t1*(rx-xr);
    R = norm([xr yr] - [rx ry]);
    rth = sign(alpha)* vh / R * dt;
    tmp = [rx ry]' +  [cos(rth) -sin(rth); sin(rth) cos(rth)]*[xr-rx
yr-ry]';
    xrp = tmp(1);
    yrp = tmp(2);
    thrp = thr + xr;
    hold on;
    plot(xrp,yrp,'b.');
    plot(0,0,'r*');
end

function x = sample(sig)
    x = randn()*sig;
end
```

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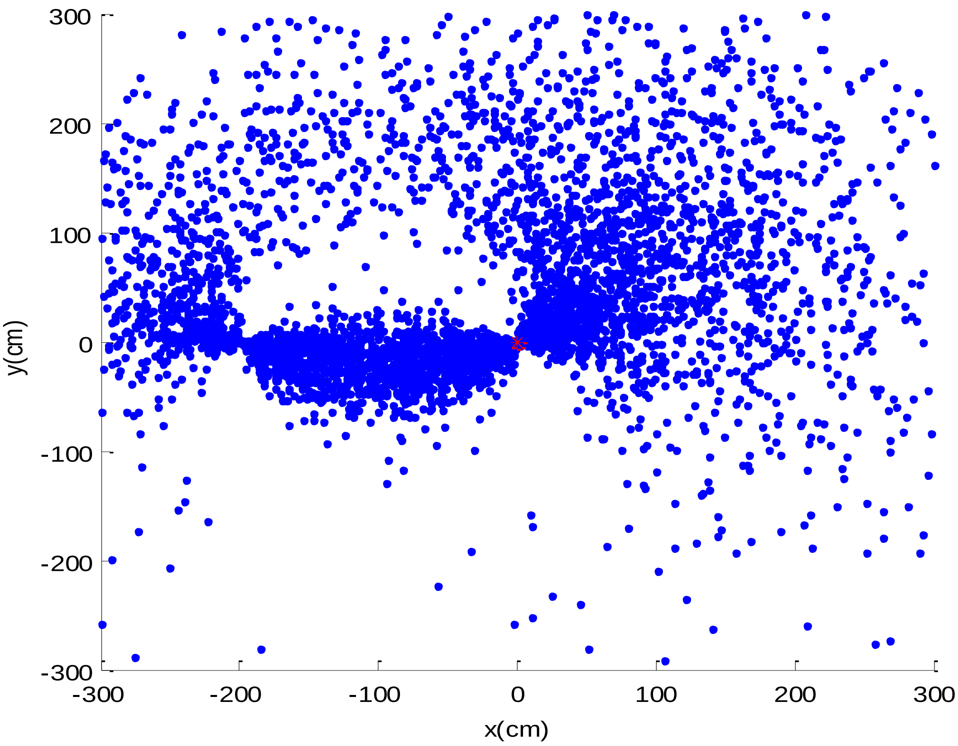
Using the noise specified in the problem, the result is not easy to understand because the noise is too large. Later, I reduce the original noise to 1/10, as the result the plot is much like what we expected. Since the error in velocity is much greater than in rotation, the shape of the posterior sample is thinner and longer. The number of sample point is 5000. In the following plot, red dot denotes the initial point, and the blue dots are the sampled posterior.

Original:

1)



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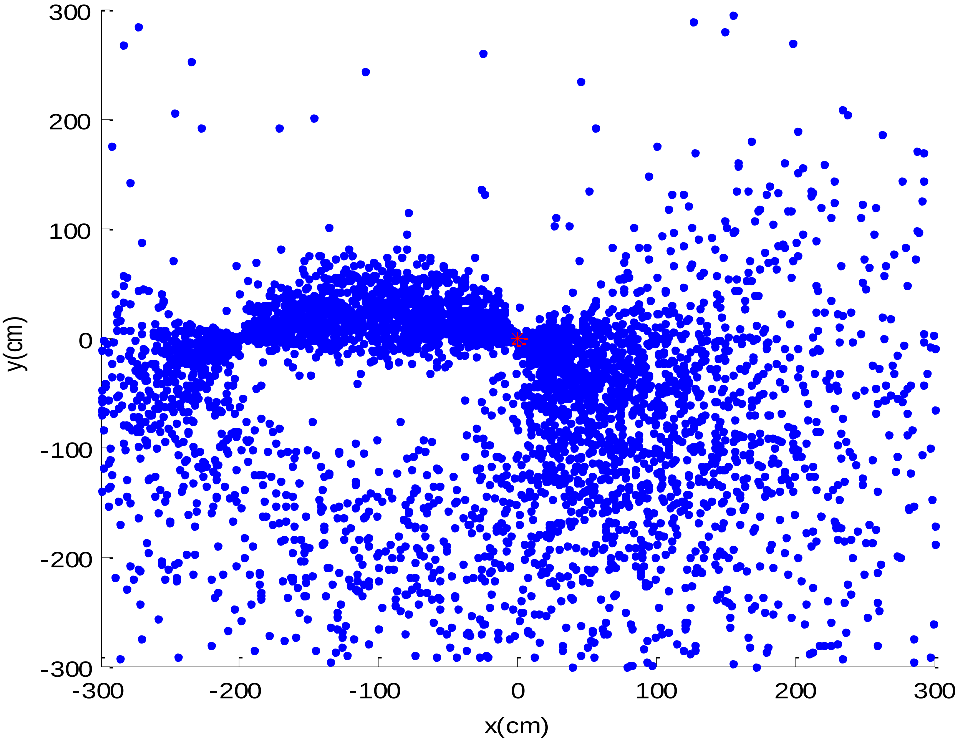
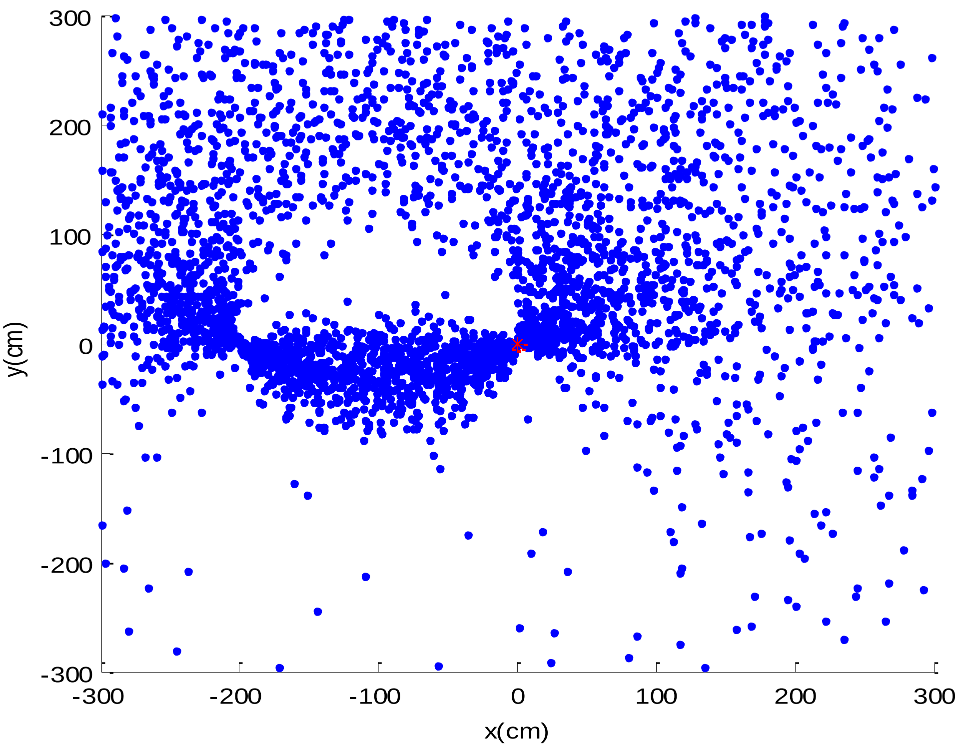
2)



3)



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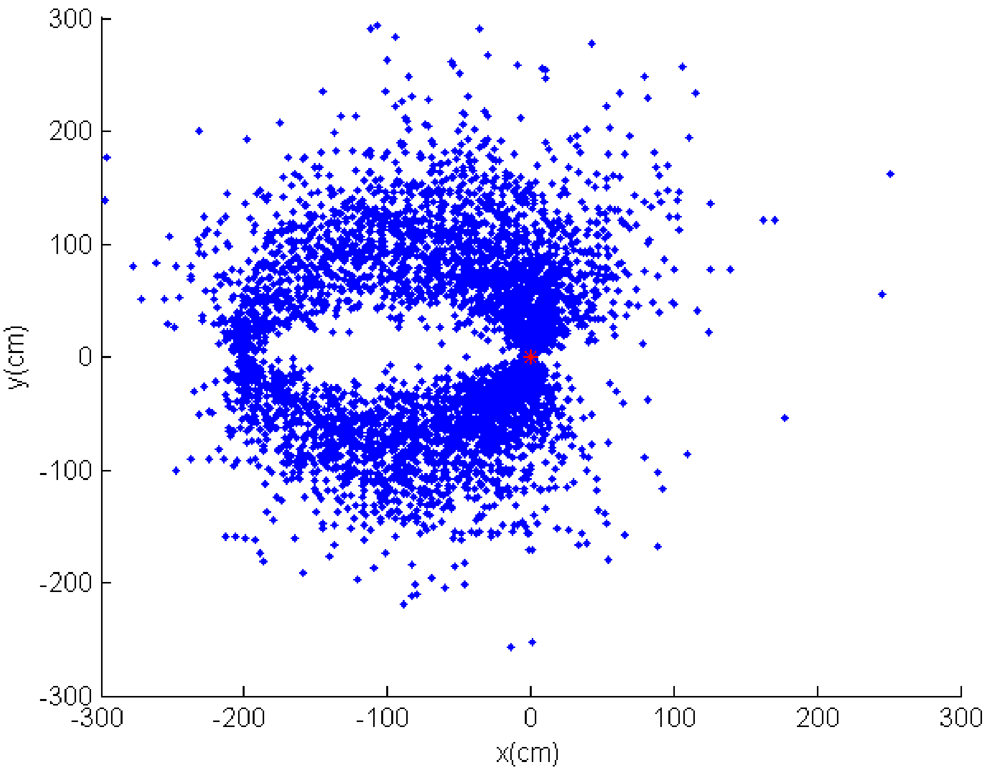
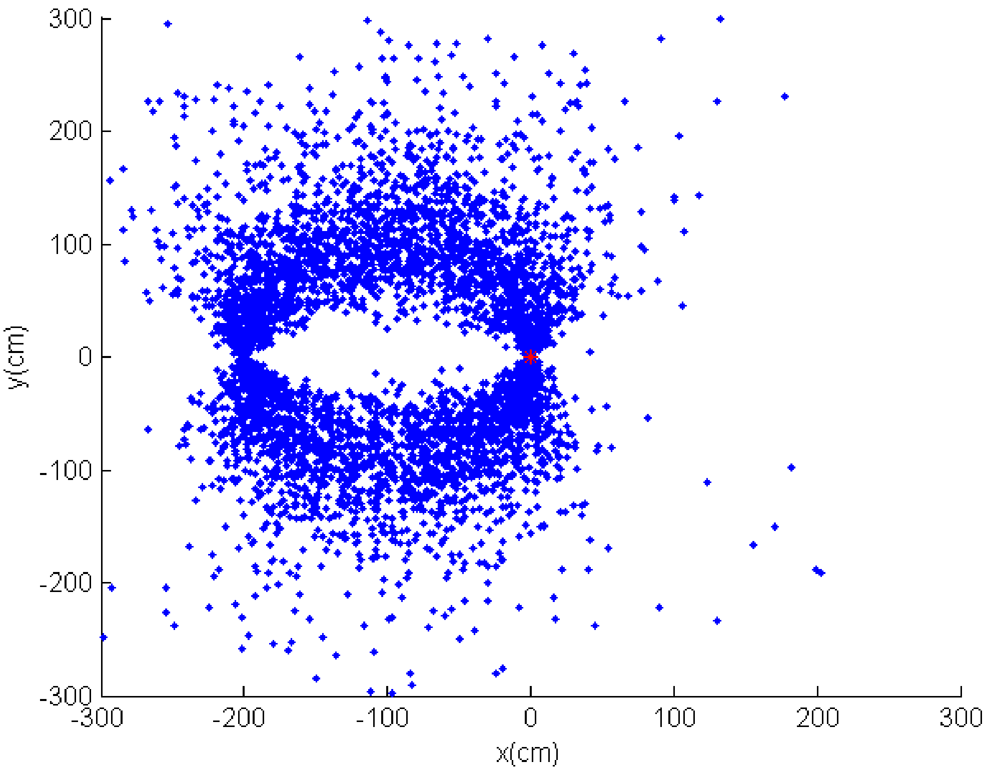
4)



5)



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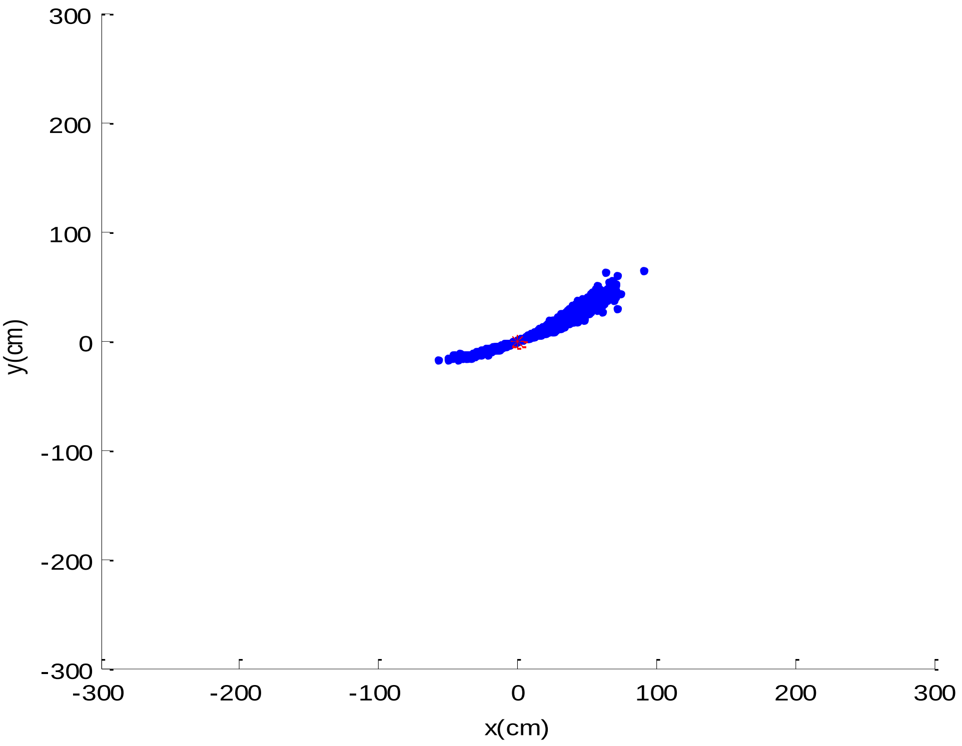
Using 1/10 noise:

1) With $\alpha = 25°, v = 20$, the bicycle generally goes up-right.



2) With $\alpha = -25°, v = 20$, the bicycle generally goes bottom-right.

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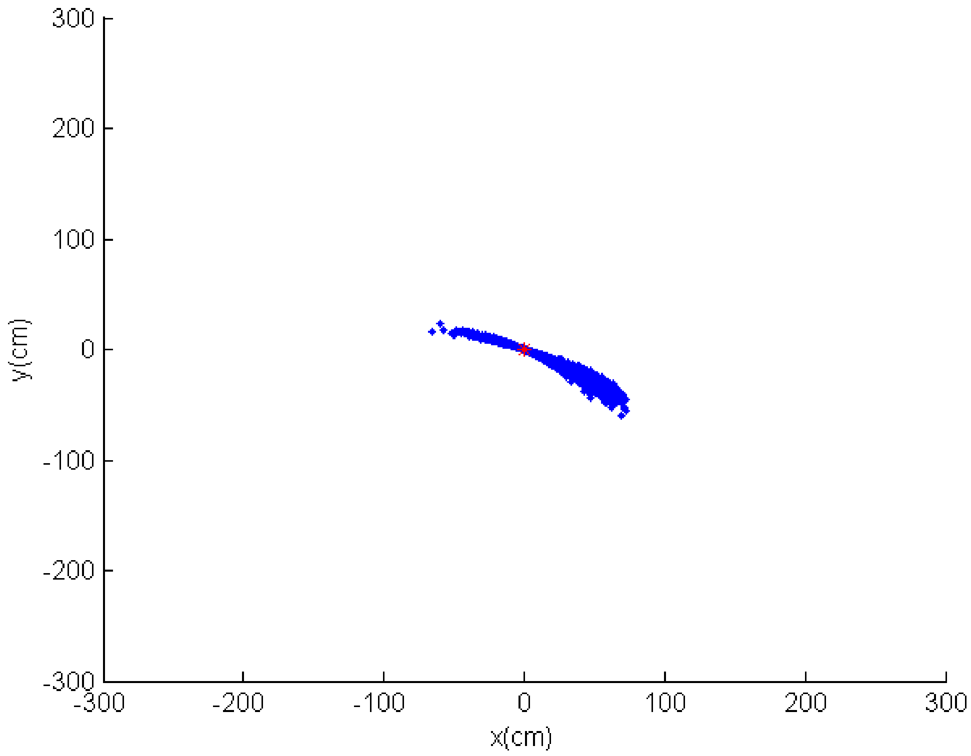
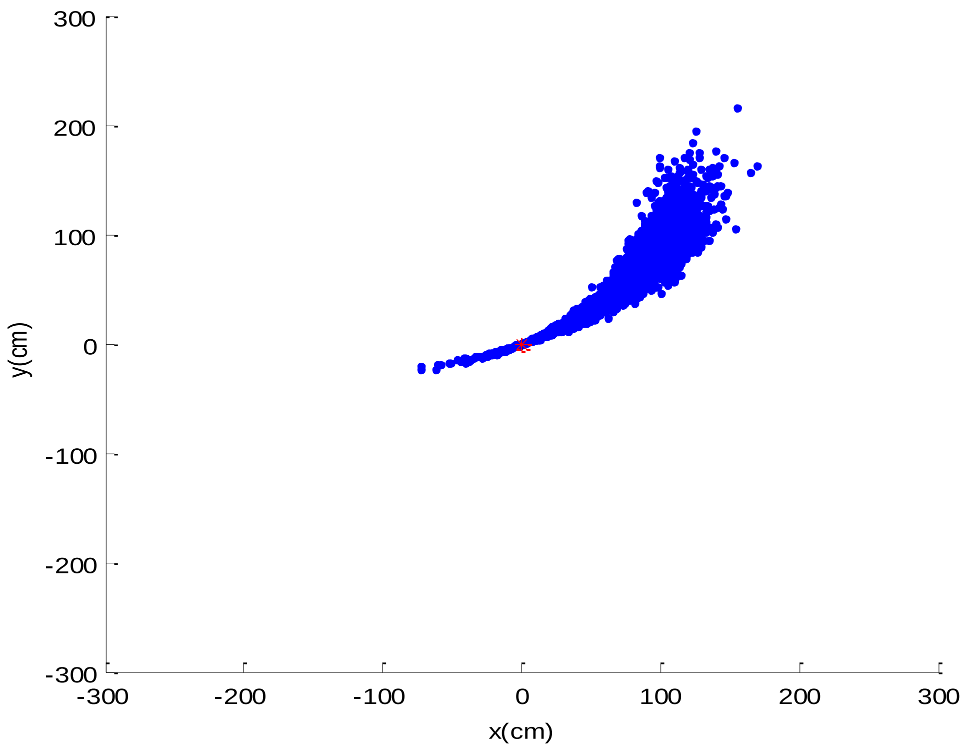


3) With $\alpha = 25°, v = 90$,the result is same as 1) but with larger noise in velocity.



4) With $\alpha = 80°, v = 10$,the result presents a smaller rotation radius.

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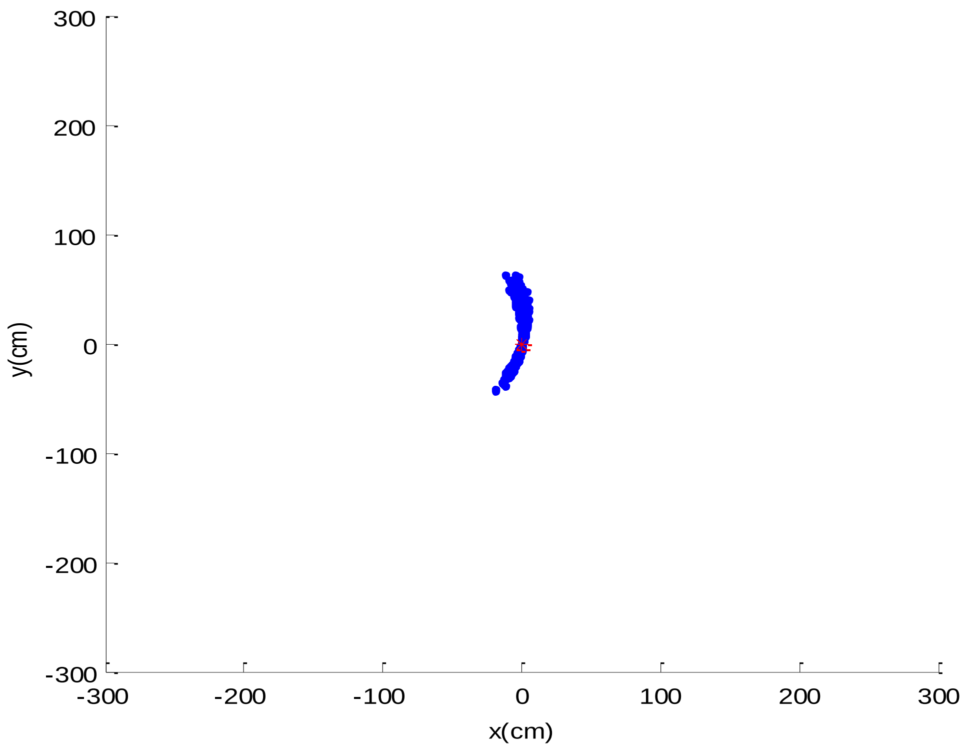

5) With $\alpha = 85°, v = 90$, the result is similar with 4) but with larger noise in velocity.