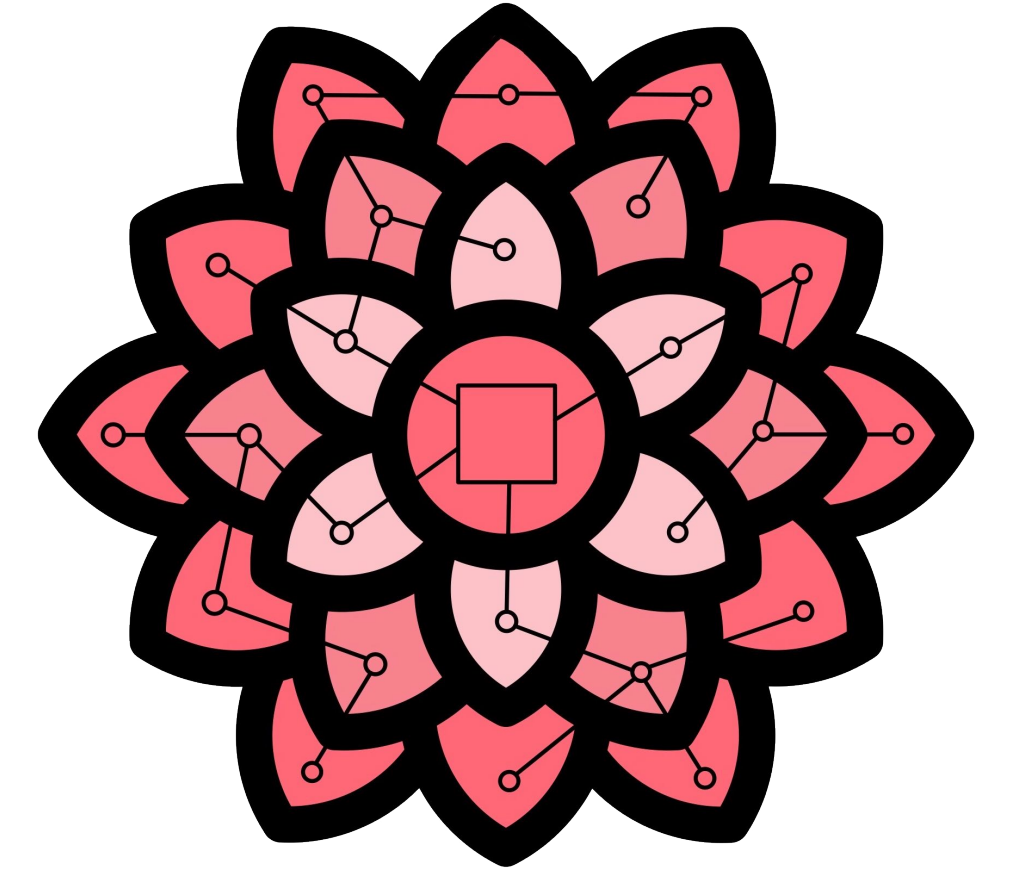


Predictable Accelerator Design

with Time-Sensitive Affine Types



Rachit Nigam, Sachille Atapattu, Samuel Thomas, Zhijing Li, Theodore Bauer,
Yuwei Ye, Apurva Koti, Adrian Sampson, Zhiru Zhang

capra.cs.cornell.edu/dahlia





Rachit Nigam

- Second year PhD
- Computer Architect by day
- Programming languages by night



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- Second year PhD
- Computer Architect by day
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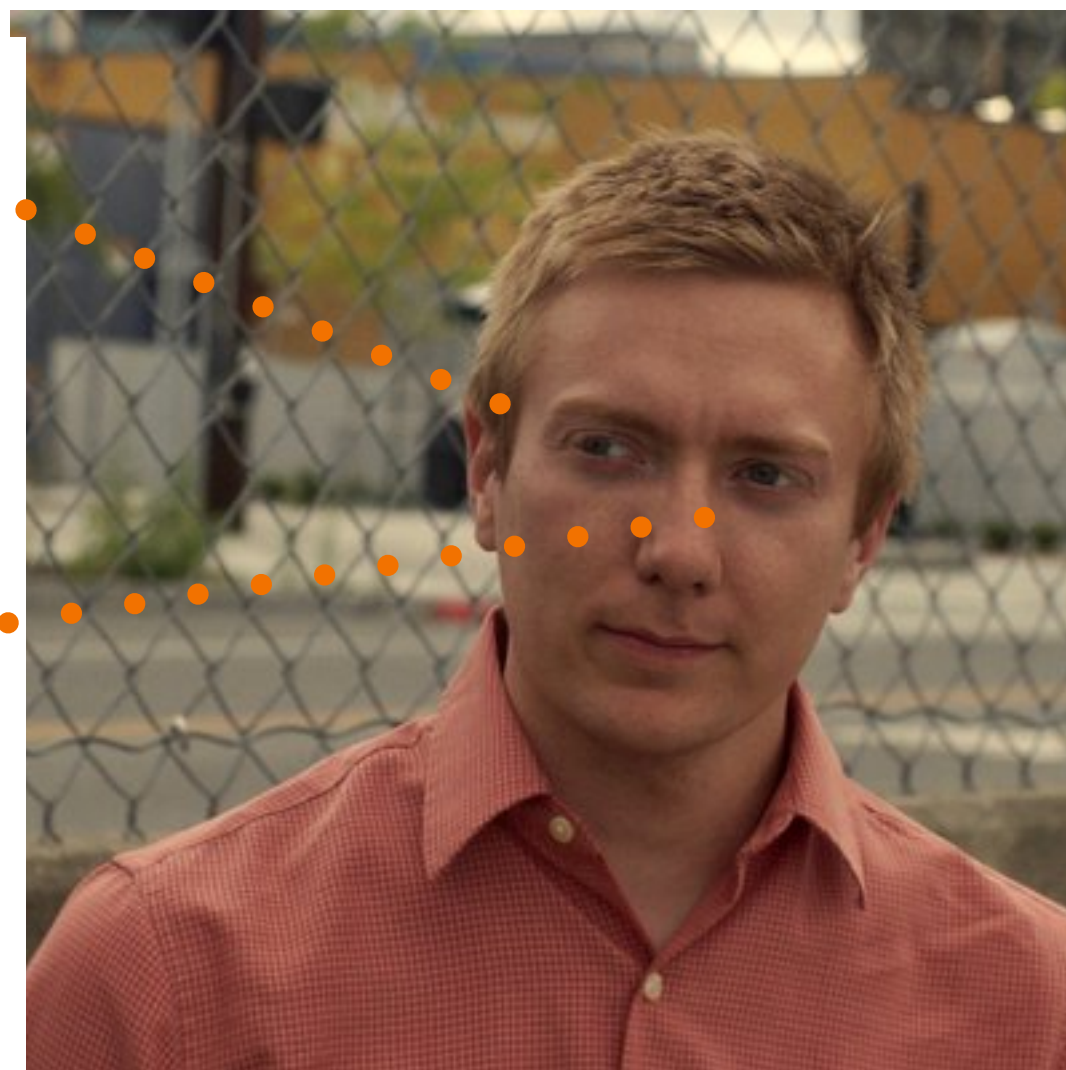
Adrian Sampson

- Nth year PhD
- PL & Compilers & Architecture



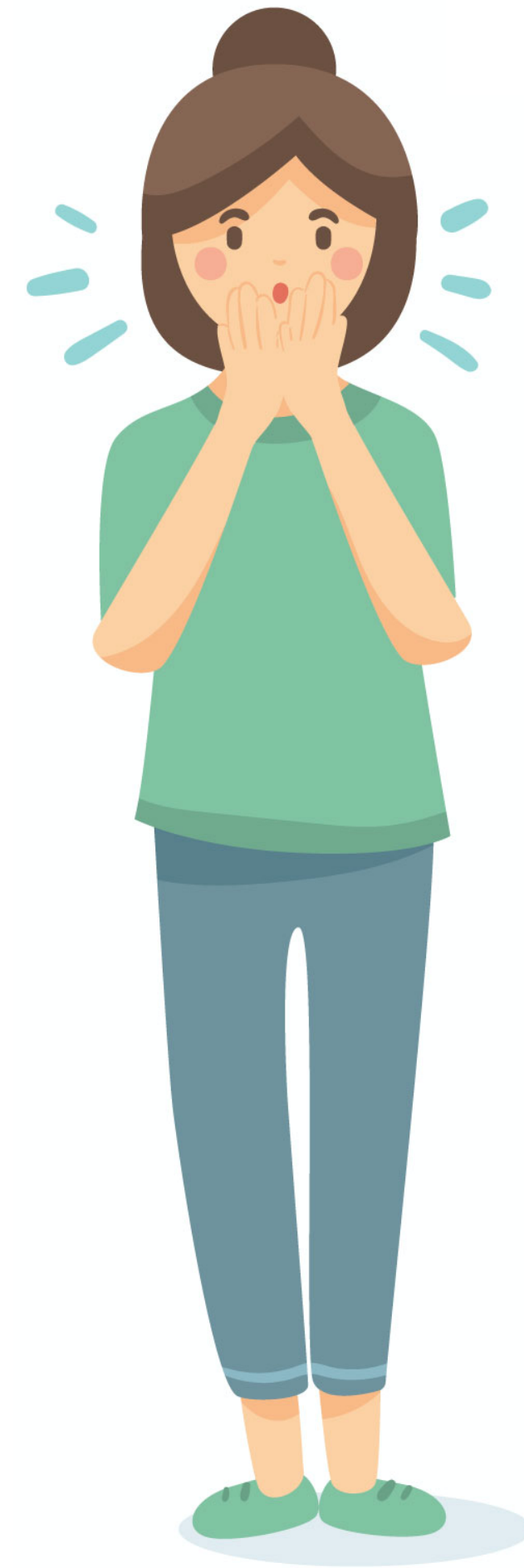
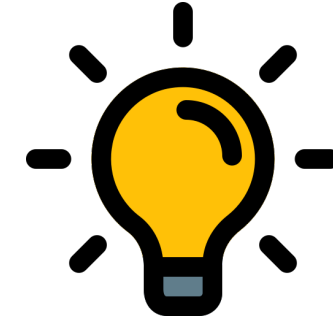
**Rachit
Nigam**

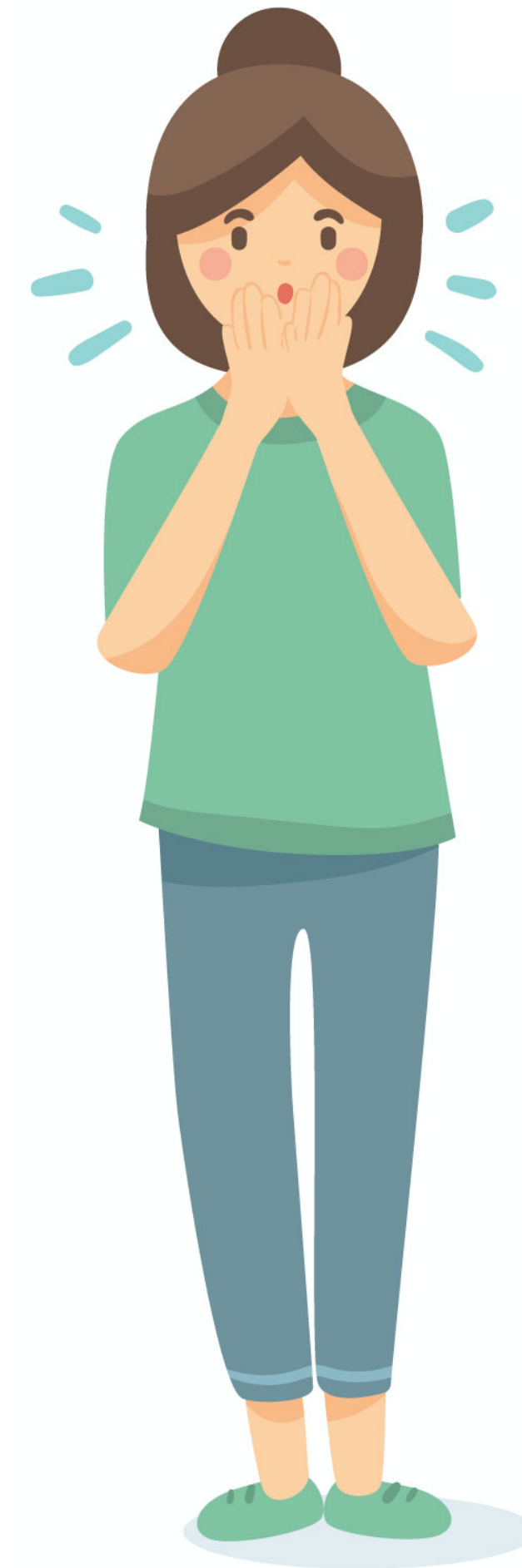
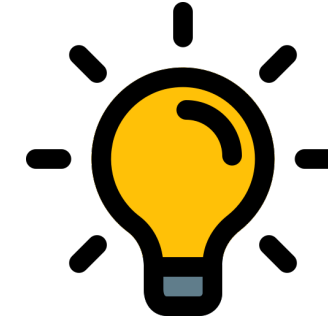
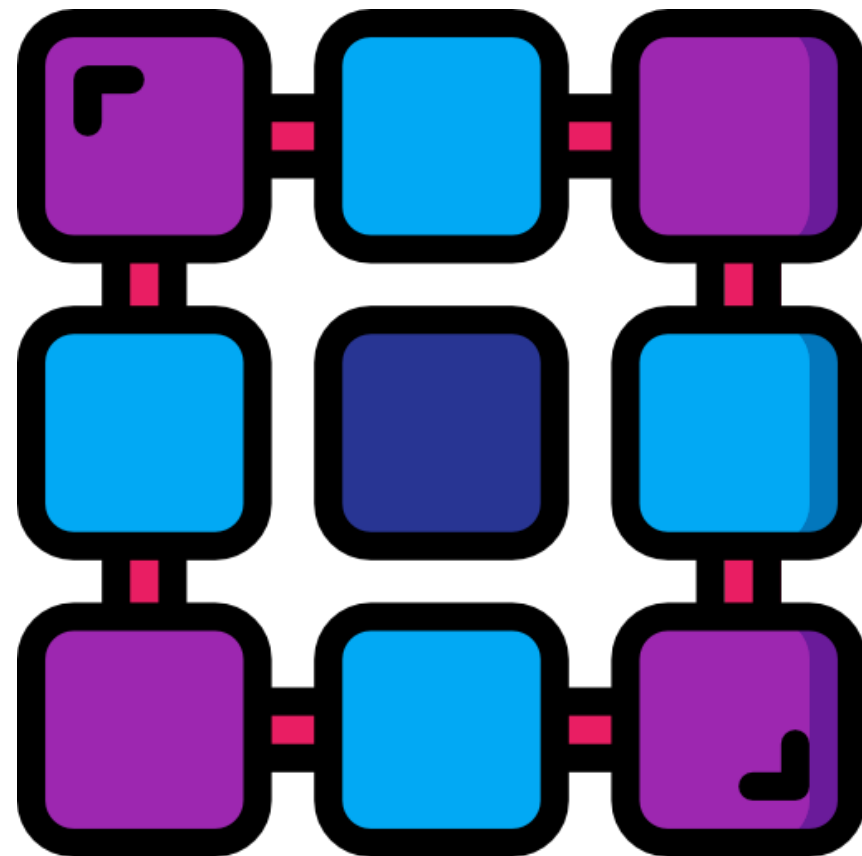
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**Adrian
Sampson**

- Nth year PhD
- PL & Compilers & Architecture

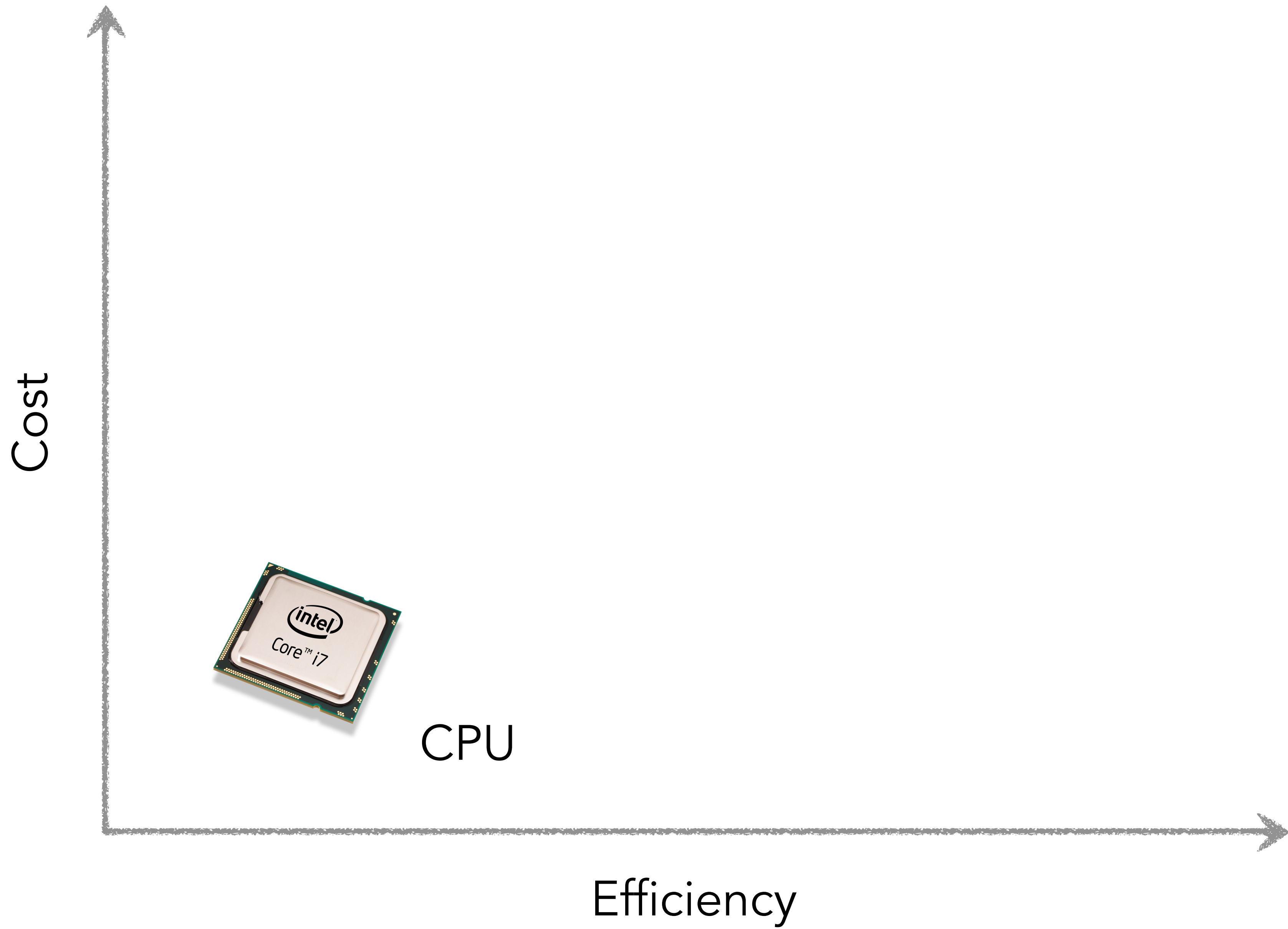


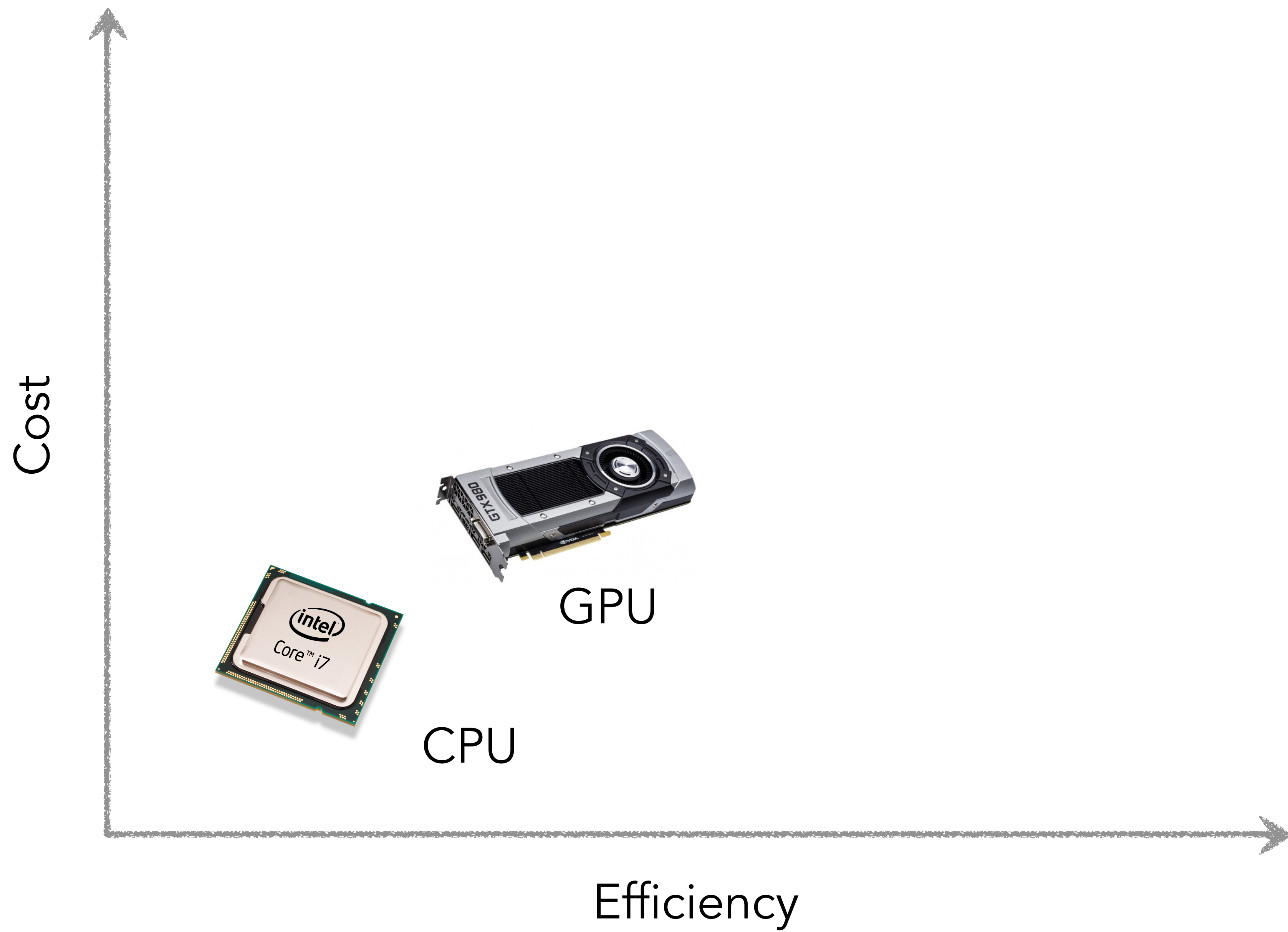


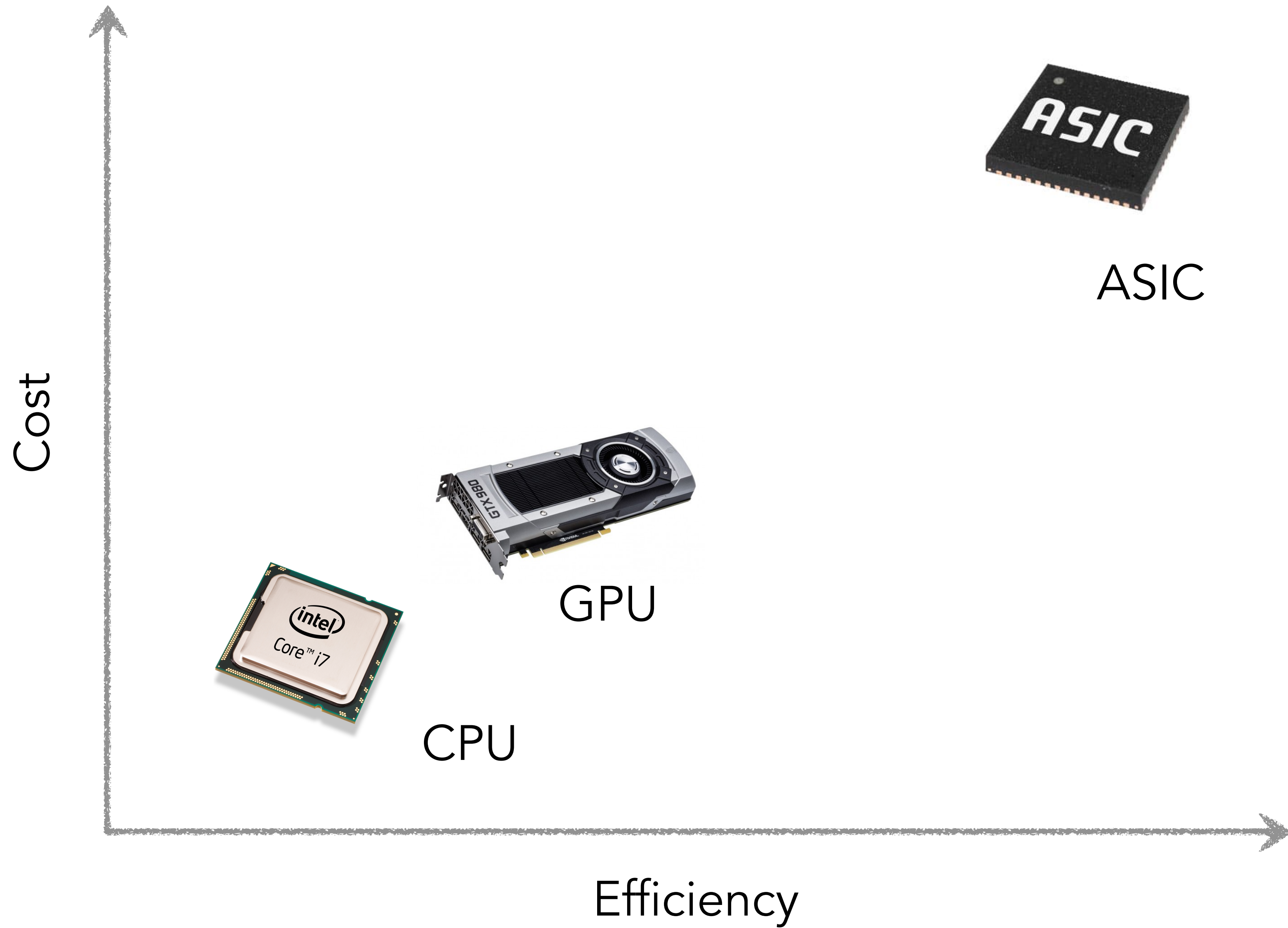


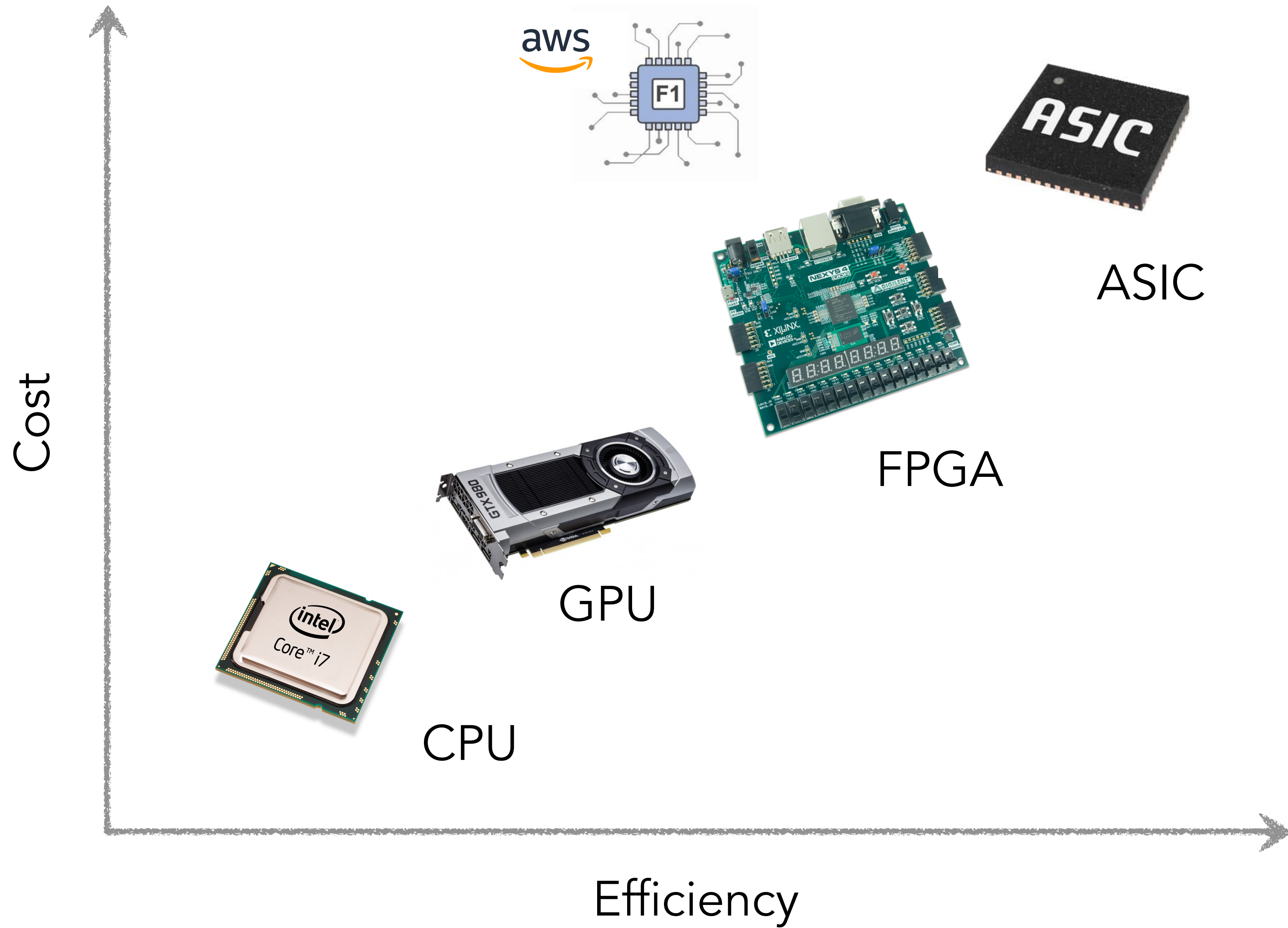
Cost

Efficiency

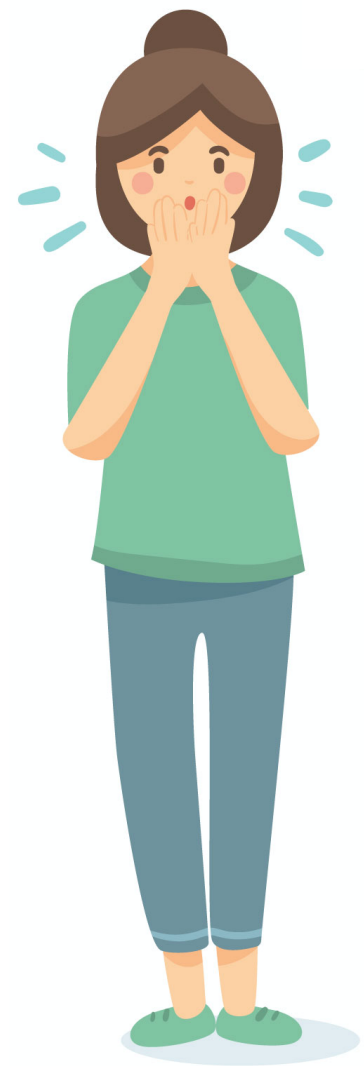
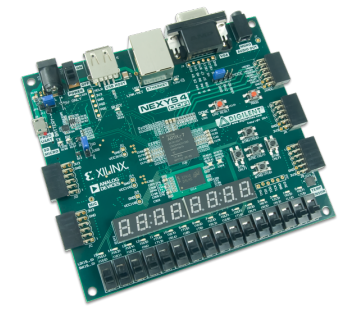




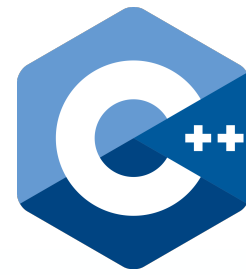
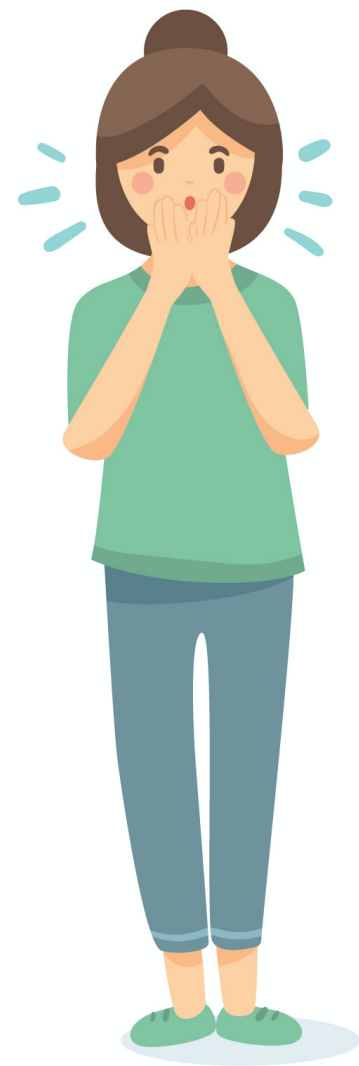
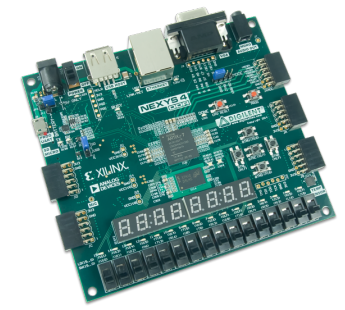




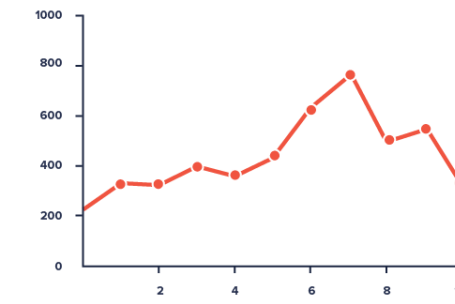
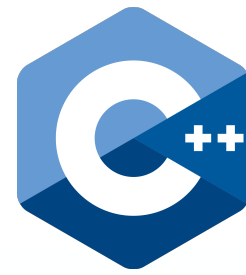
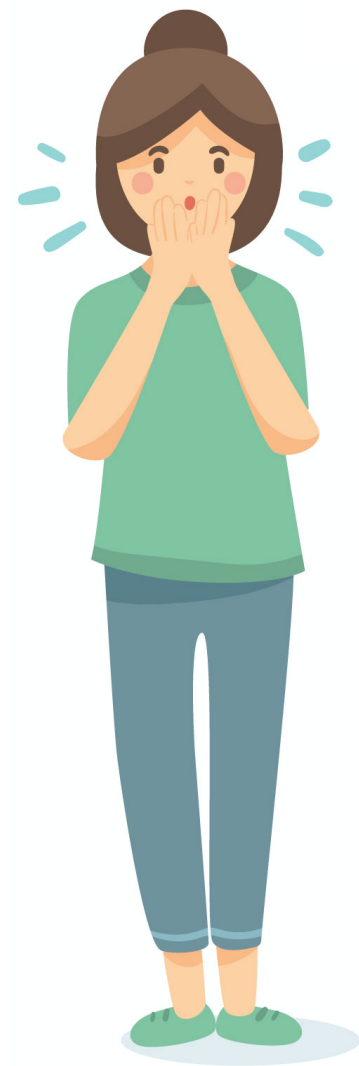
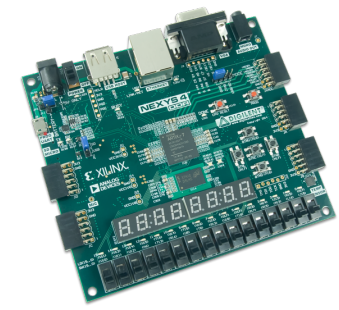
Ada's Journey



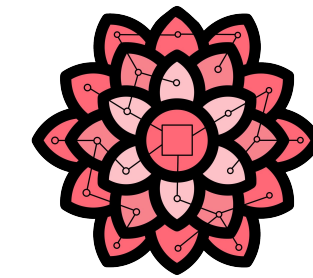
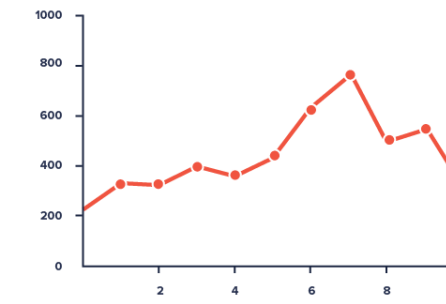
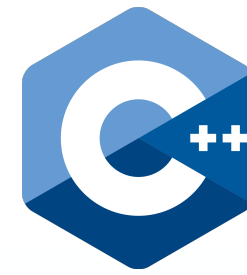
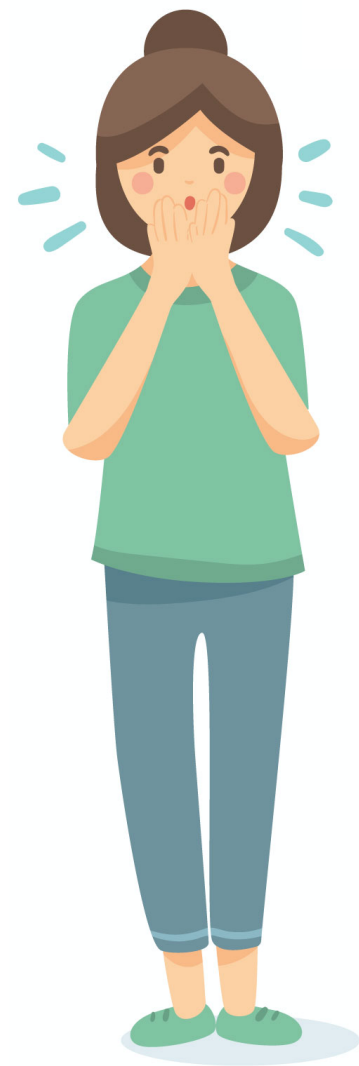
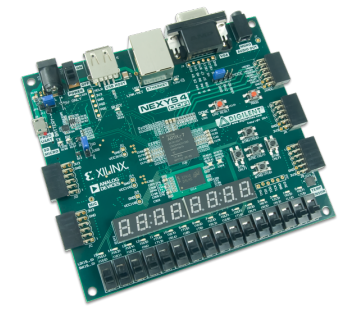
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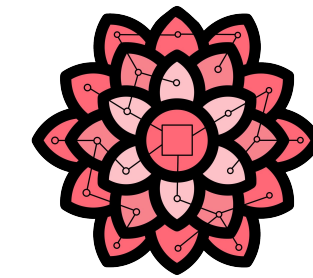
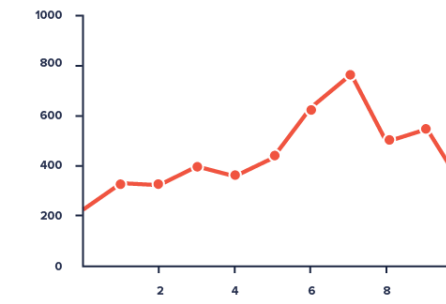
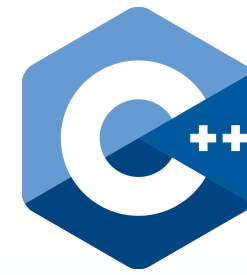
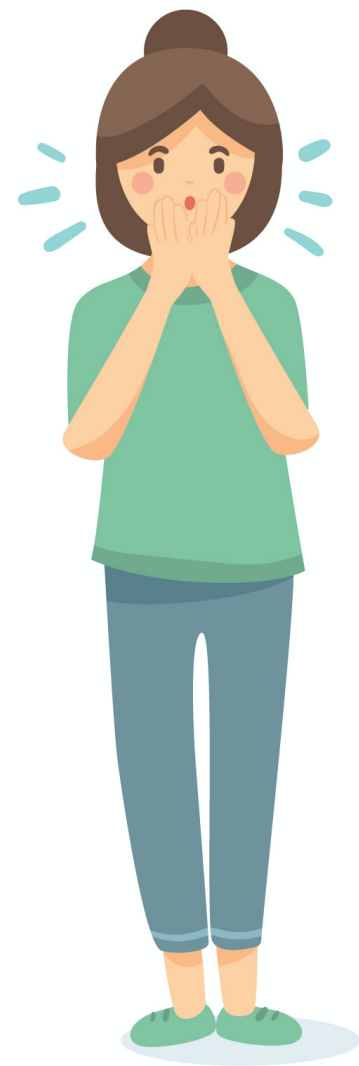
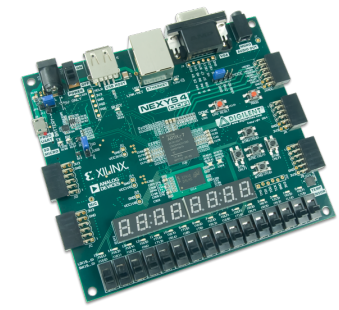
Ada's Journey



Ada's Journey

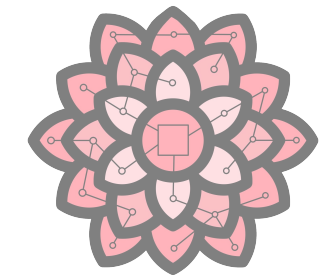
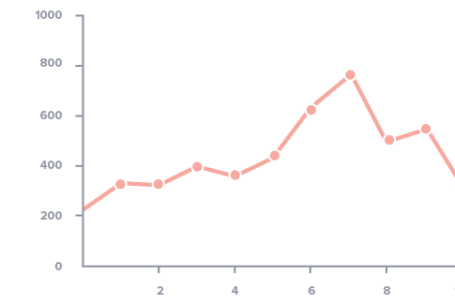
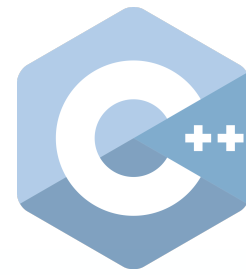
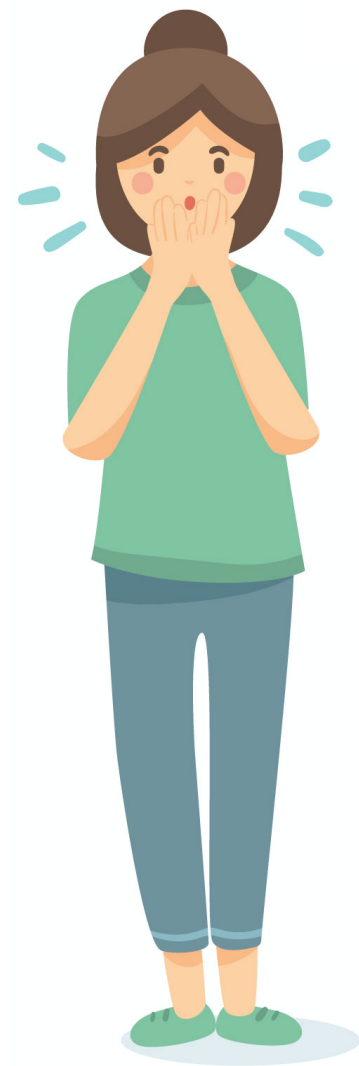
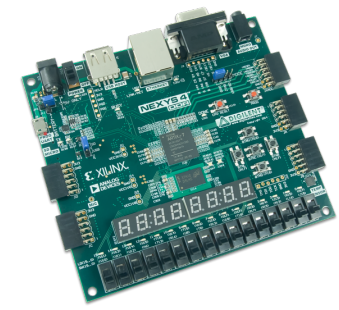


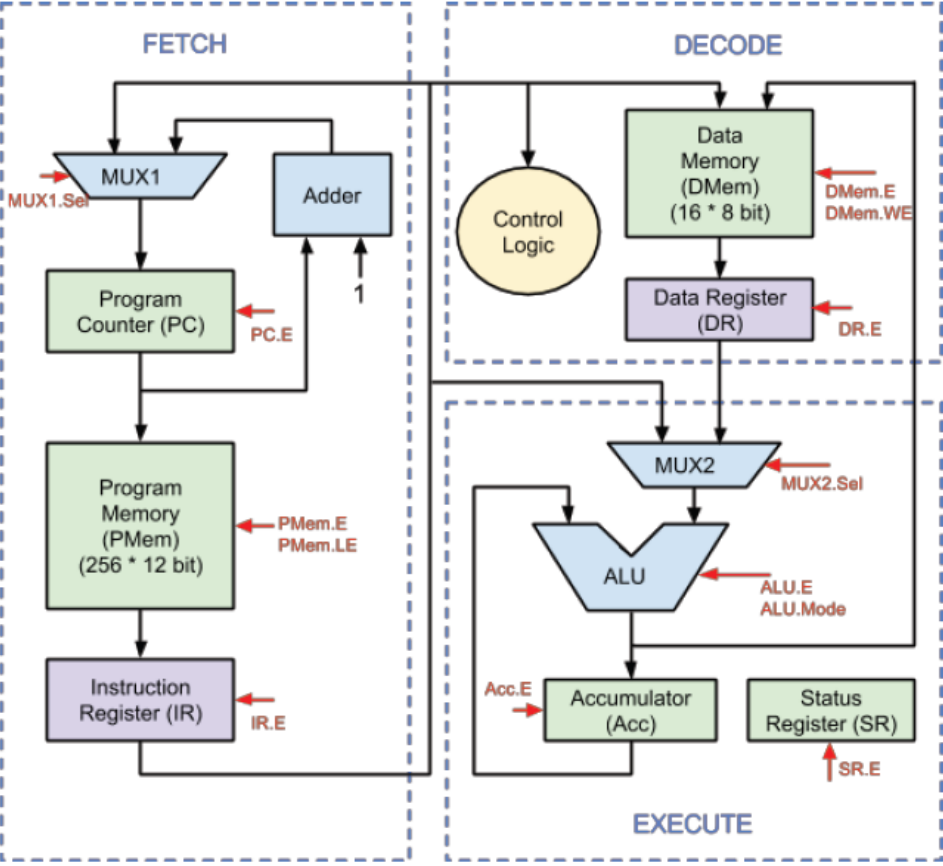
Ada's Journey



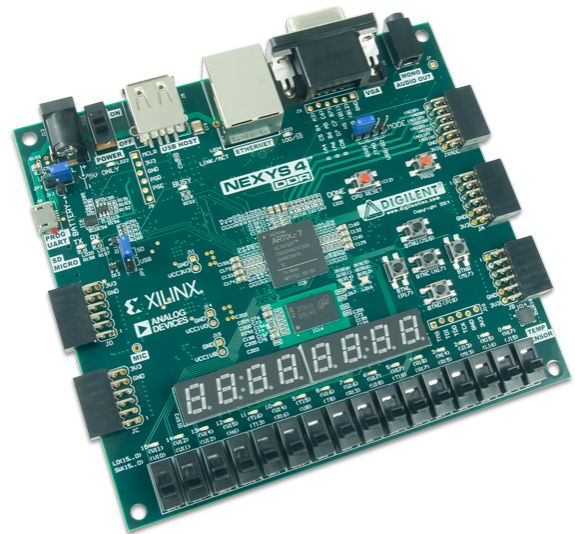
Our Research

Ada's Journey

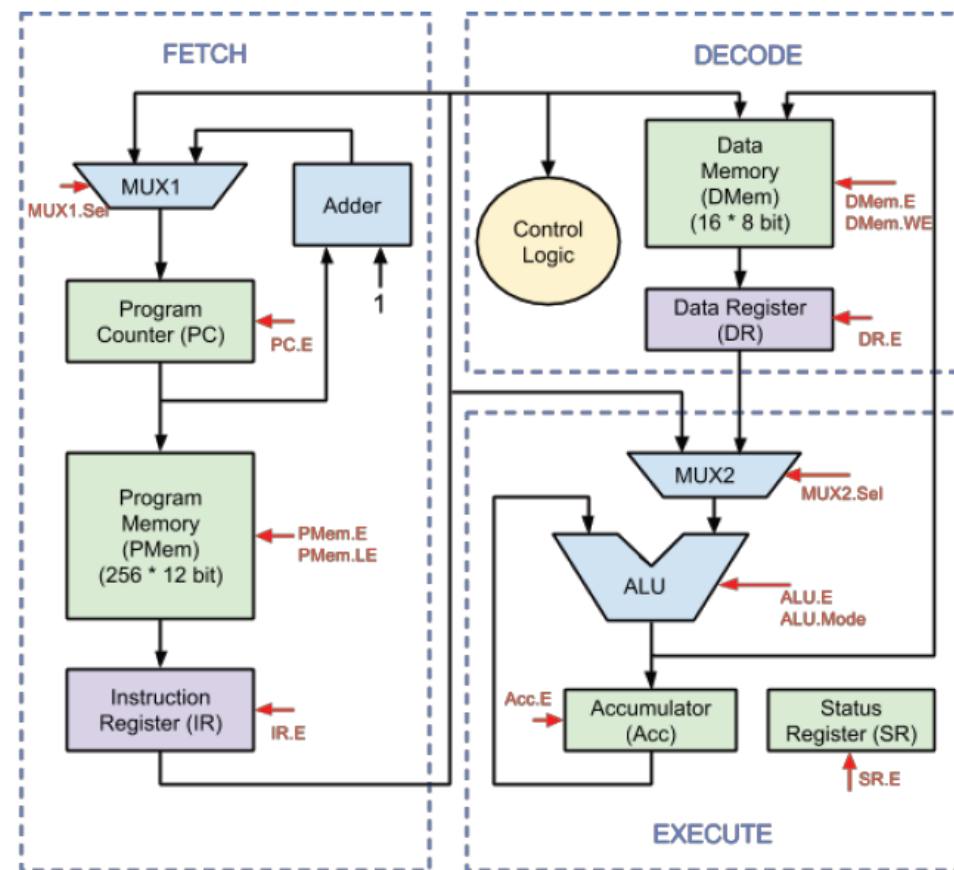




Hardware Design



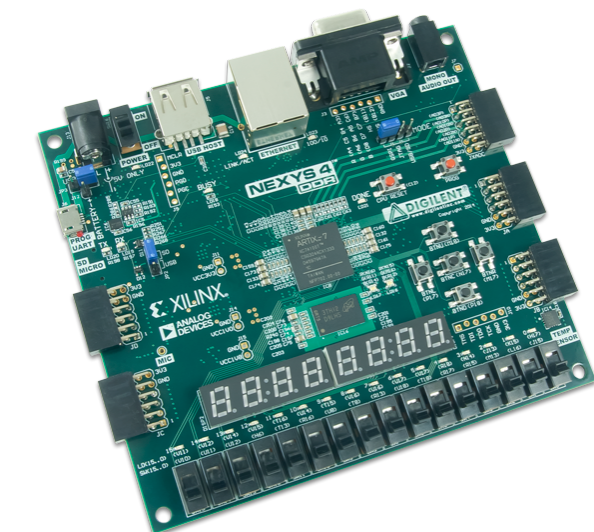
FPGA



Hardware Design



Traditional HDLs



FPGA



Traditional HDLs

```
module ctr (input          up_down,
            clk,
            rstn,
            output reg [2:0] out);

    always @ (posedge clk)
        if (!rstn)
            out <= 0;
        else begin
            if (up_down)
                out <= out + 1;
            else
                out <= out - 1;
        end
    endmodule
```



Traditional HDLs

```
module ctr (input          up_down,
            clk,
            rstn,
            output reg [2:0] out);

    always @ (posedge clk)
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                out <= out - 1;
        end
endmodule
```

- Time is real



Traditional HDLs

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always @ (posedge clk)
  if (!rstn)
    out <= 0;
  else begin
    if (up_down)
      out <= out + 1;
    else
      out <= out - 1;
  end
endmodule
```

- Time is real
- **Concurrent** semantics



Traditional HDLs

```
neptune_metadata_unpack #(
    .TUSER_W (TUSER_W)
) neptune_metadata_unpack_inst (
    // input
    .metadata      (s_axis_tuser),
    // output
    .std_md        (neptune_std_md),
    .mem_instr_0   (),
    .mem_instr_1   (),
    .mem_instr_2   (),
    .egress_ts     (),
    .fpm           (fpm),
    .reserved      ()
);
```

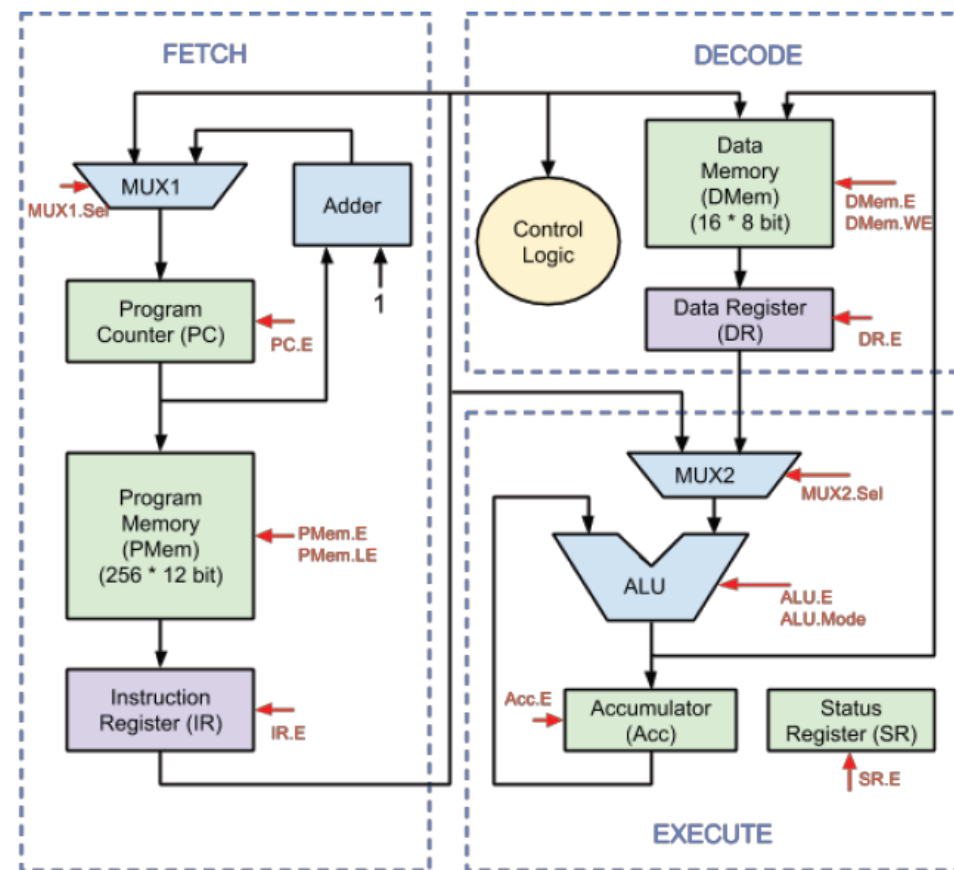
- Time is real
- Concurrent semantics



Traditional HDLs

```
neptune_metadata_unpack #(
    .TUSER_W (TUSER_W)
) neptune_metadata_unpack_inst (
    // input
    .metadata      (s_axis_tuser),
    // output
    .std_md        (neptune_std_md),
    .mem_instr_0   (),
    .mem_instr_1   (),
    .mem_instr_2   (),
    .egress_ts     (),
    .fpm           (fpm),
    .reserved      ()
);
```

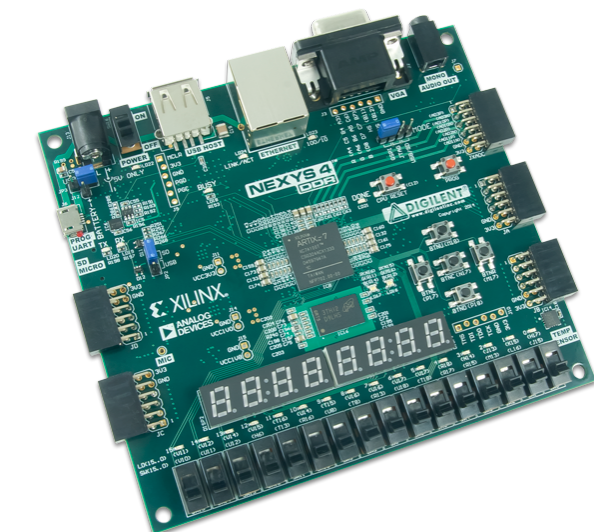
- Time is real
- Concurrent semantics
- **Wires** and hardware modules



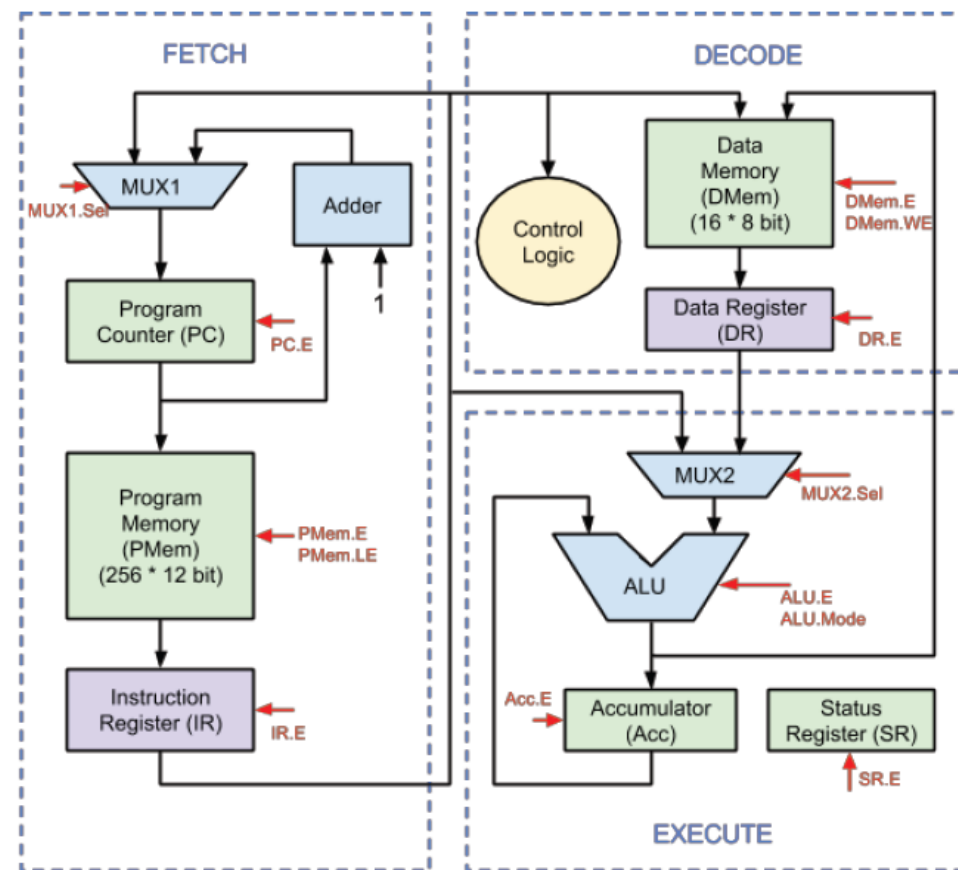
Hardware Design



Traditional HDLs



FPGA



Hardware Design



bluespec

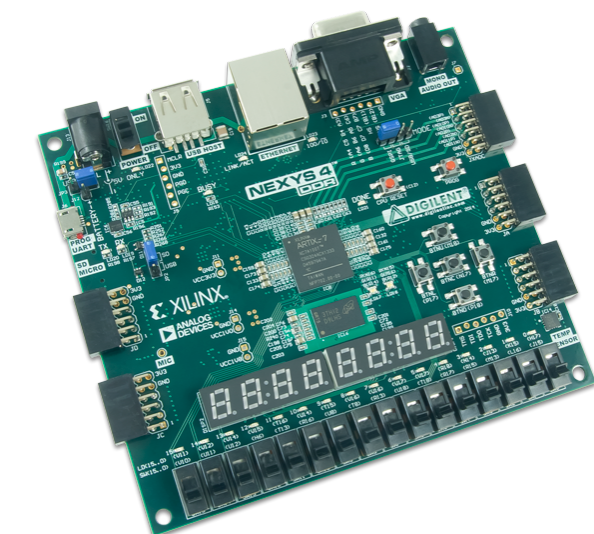
CHISEL

PyMTL

Modern HDLs

SystemVerilog

Traditional HDLs



FPGA

Modern HDLs

```
// Generalized FIR filter parameterized by the convolution coefficients
class FirFilter(bitWidth: Int, coeffs: Seq[UInt]) extends Module {

  // Register the output
  val zs = Reg(Vec(coeffs.length, UInt(bitWidth.W)))
  zs(0) := io.in
  for (i <- 1 until coeffs.length) {
    zs(i) := zs(i-1)
  }

  // Do the multiplies
  val products = VecInit.tabulate(coeffs.length)(i => zs(i) * coeffs(i))

  // Sum the products
  val sum = io.in0 + products.reduce(_ + _)

  io.out := sum
}
```


Modern HDLs

```
// Generalized FIR filter parameterized by the convolution coefficients
class FirFilter(bitWidth: Int, coeffs: Seq[UInt]) extends Module {

  val zs = Reg(Vec(coeffs.length, UInt(bitWidth.W)))
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    // Do the multiplies
    val products = VecInit.tabulate(coeffs.length)(i => zs(i) * coeffs(i))

  }
}
```

High level constructs!

Modern HDLs

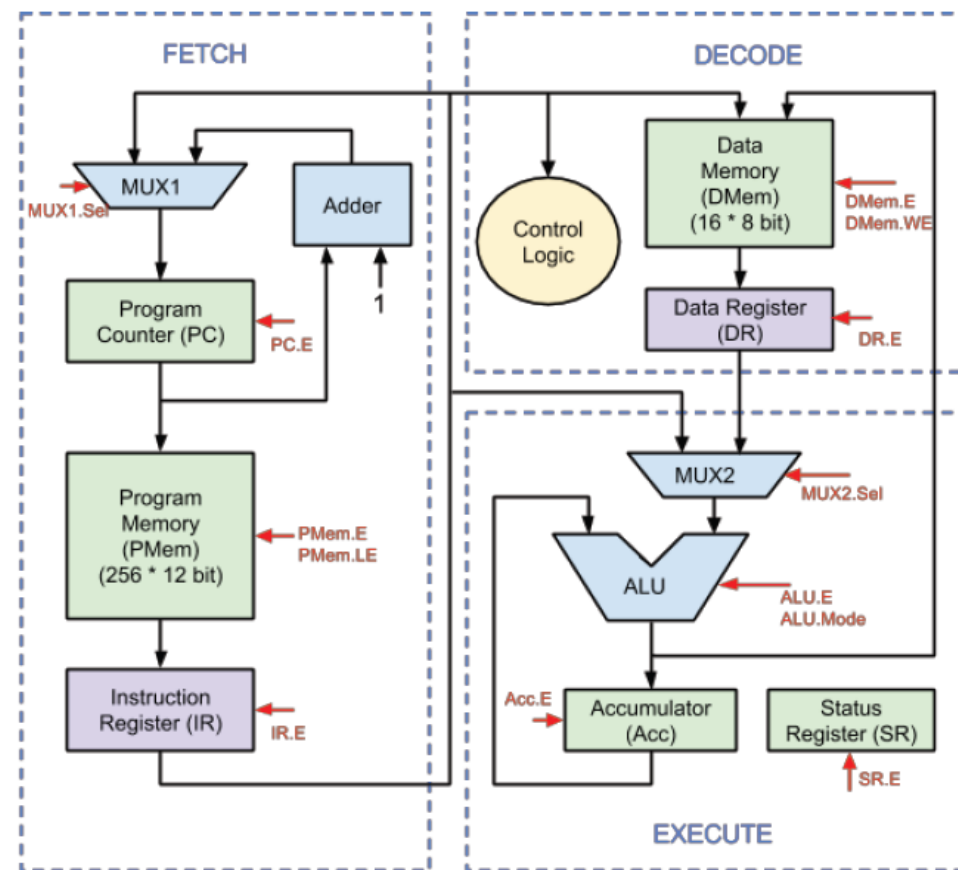
```
// Generalized FIR filter parameterized by the convolution coefficients
class FirFilter(bitWidth: Int, coeffs: Seq[UInt]) extends Module {
  val io = IO(new Bundle {
    val in = Input(UInt(bitWidth.W))
    val out = Output(UInt(bitWidth.W))
  })
  // Create the serial-in, parallel-out shift register
  val zs = Reg(Vec(coeffs.length, UInt(bitWidth.W)))
  zs(0) := io.in
  for (i <- 1 until coeffs.length) {
    zs(i) := zs(i-1)
  }

  // Do the multiplies
  val products = VecInit.tabulate(coeffs.length)(i => zs(i) * coeffs(i))

  // Sum up the products
  io.out := products.reduce(_ + _)
}
```

High level constructs!

... **elaborated** into circuits



Hardware Design



bluespec

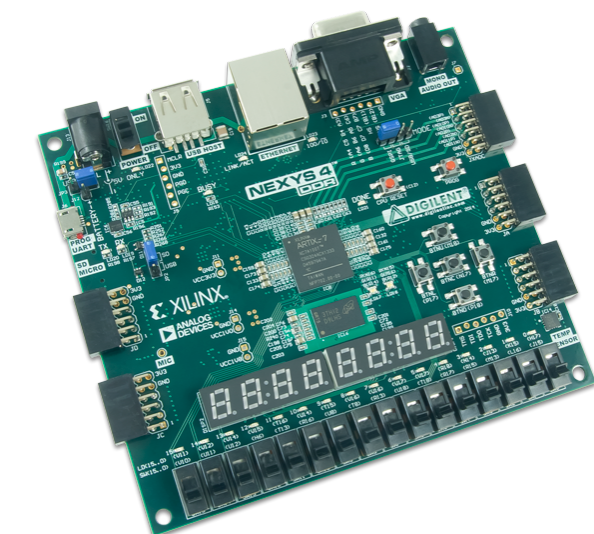
CHISEL

PyMTL

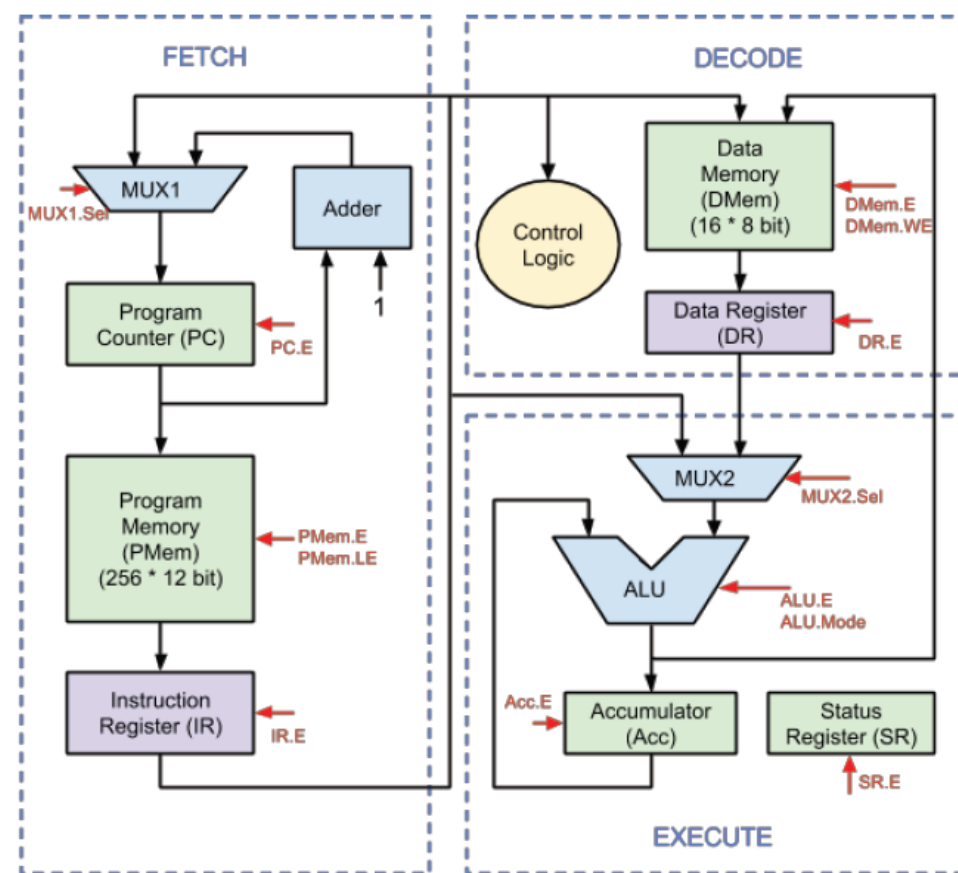
Modern HDLs

SystemVerilog

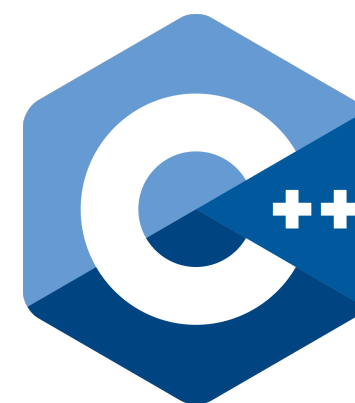
Traditional HDLs



FPGA



Hardware Design



High-Level
Synthesis



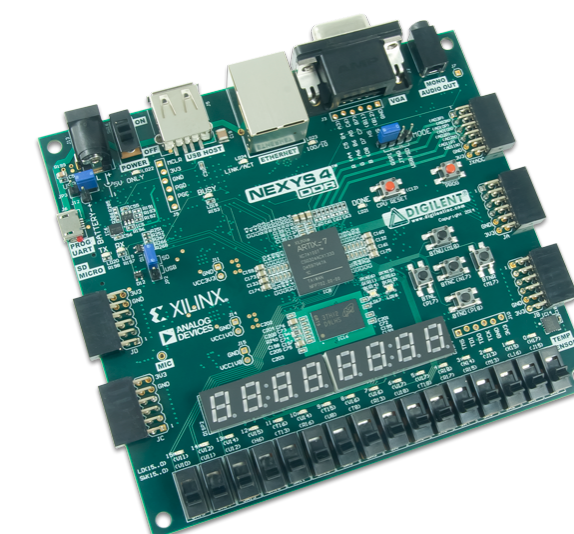
Traditional HDLs

bluespec

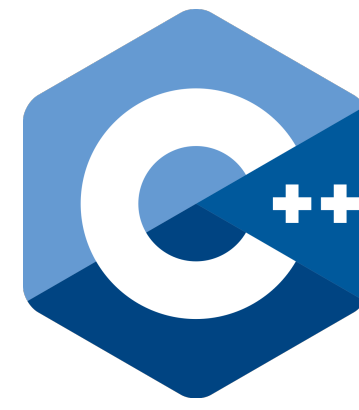
CHISEL



Modern HDLs

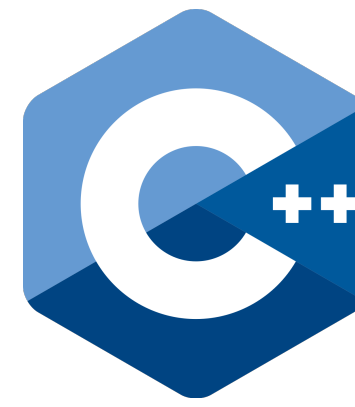


FPGA



High-Level Synthesis

```
int m1[512], m2[512], sum;  
for (int i = 0; i < 512; i++) {  
    sum += m1[i] * m2[i]  
}
```



High-Level Synthesis

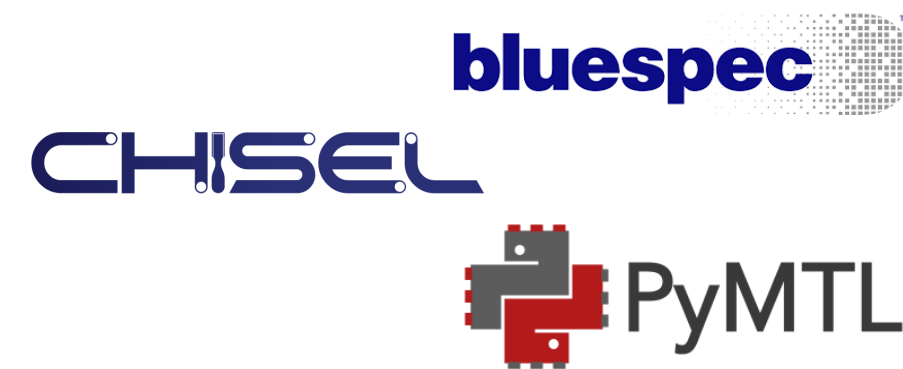
```
int m1[512], m2[512], sum;  
for (int i = 0; i < 512; i++) {  
    sum += m1[i] * m2[i]  
}
```

Software programs that are
compiled to hardware designs



High-Level Synthesis

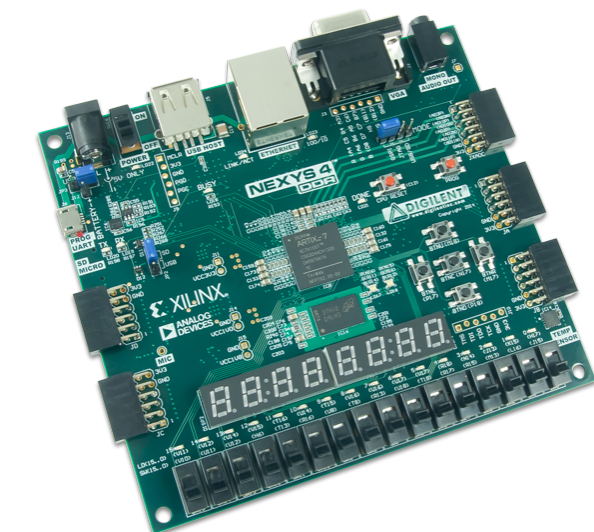
Level of abstraction



Modern HDLs



Traditional HDLs

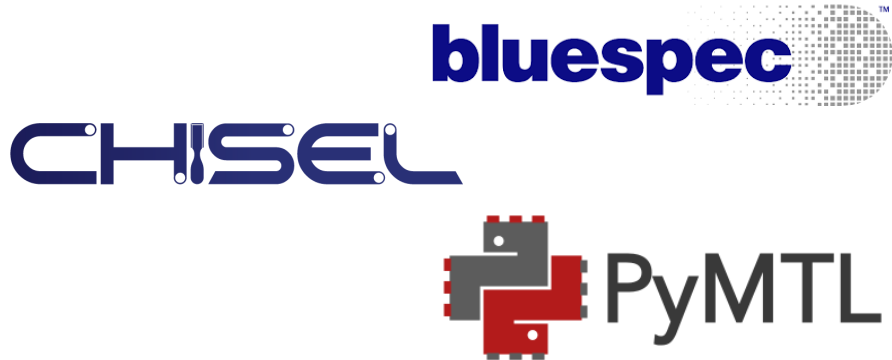


FPGA



High-Level Synthesis

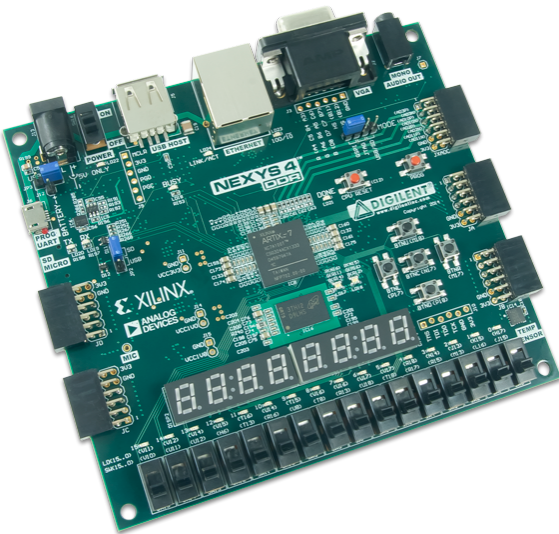
Level of abstraction



Modern HDLs



Traditional HDLs



FPGA

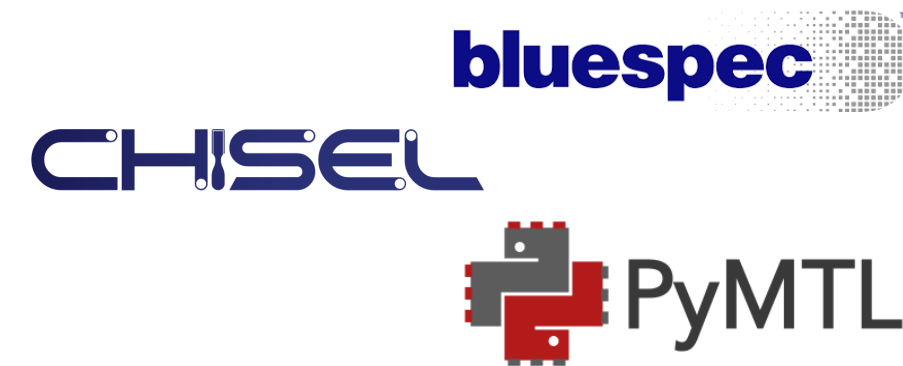
Circuit Specification



Computational Specification

High-Level Synthesis

Level of abstraction

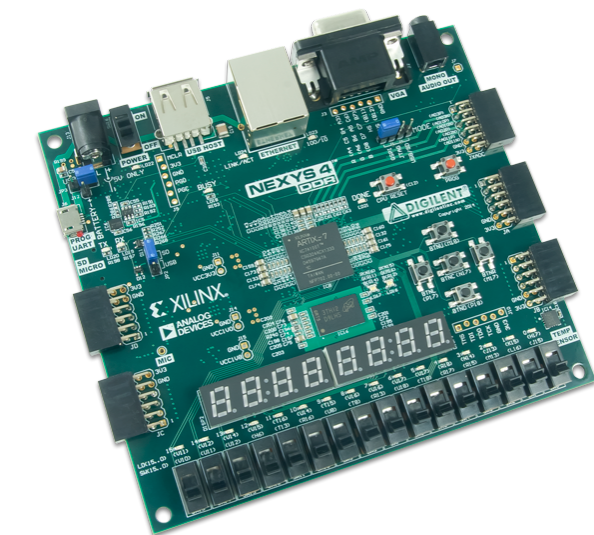


Modern HDLs



Traditional HDLs

Circuit Specification



FPGA

loops

method calls

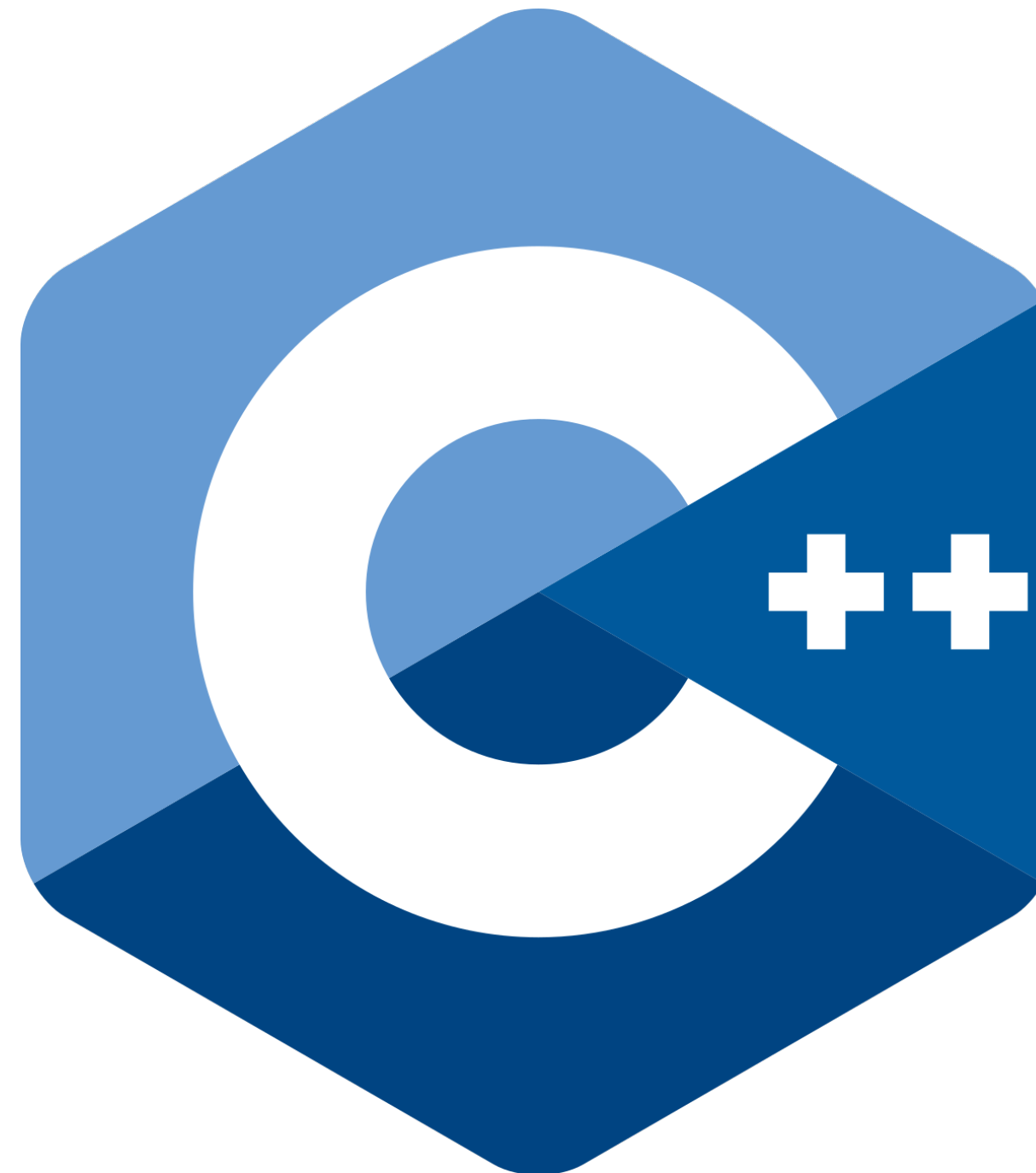
arrays

break

malloc

functions

conditionals



loops

~~method calls~~

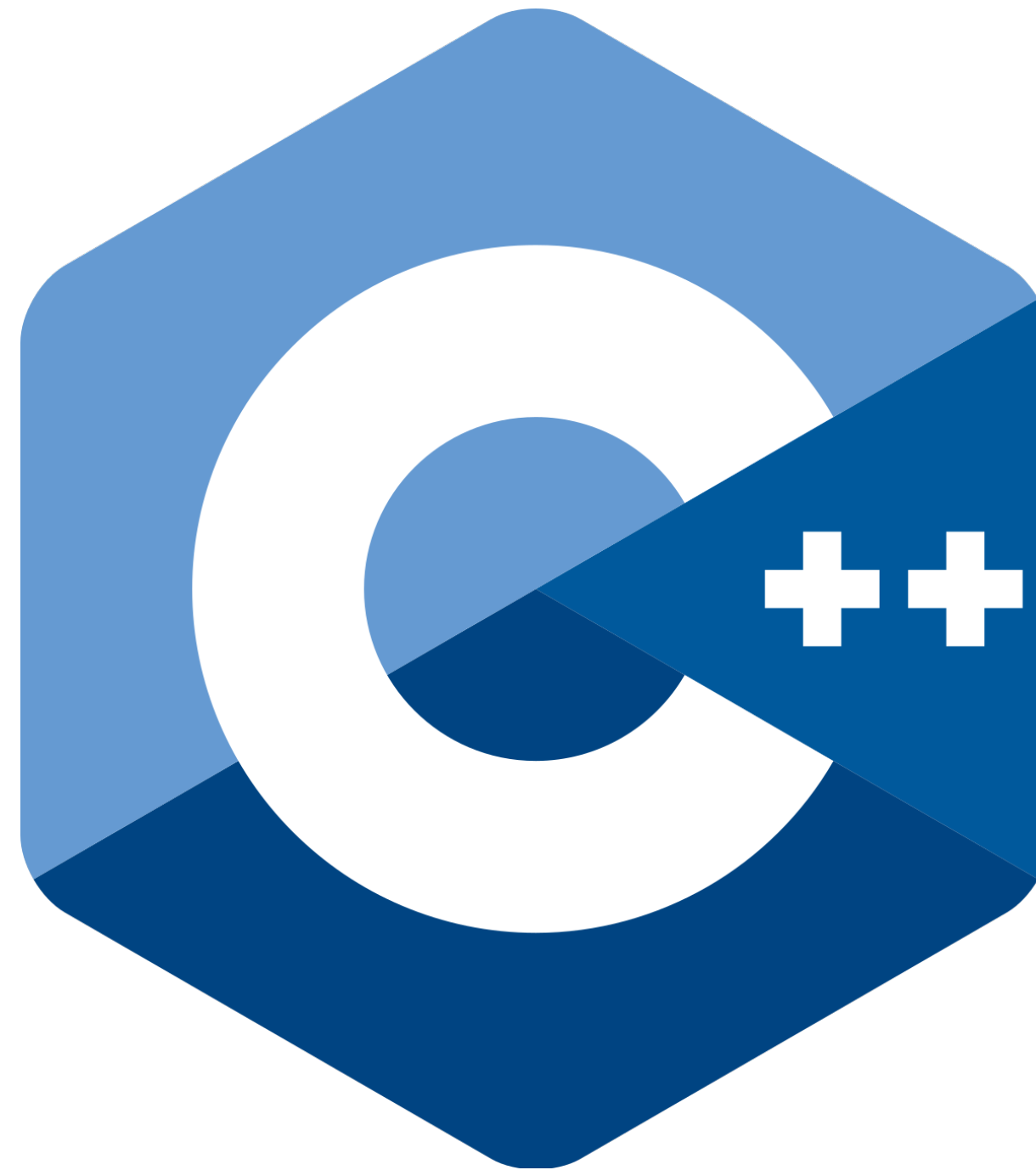
arrays

~~break~~

~~malloc~~

functions*

conditionals



loops

~~method calls~~

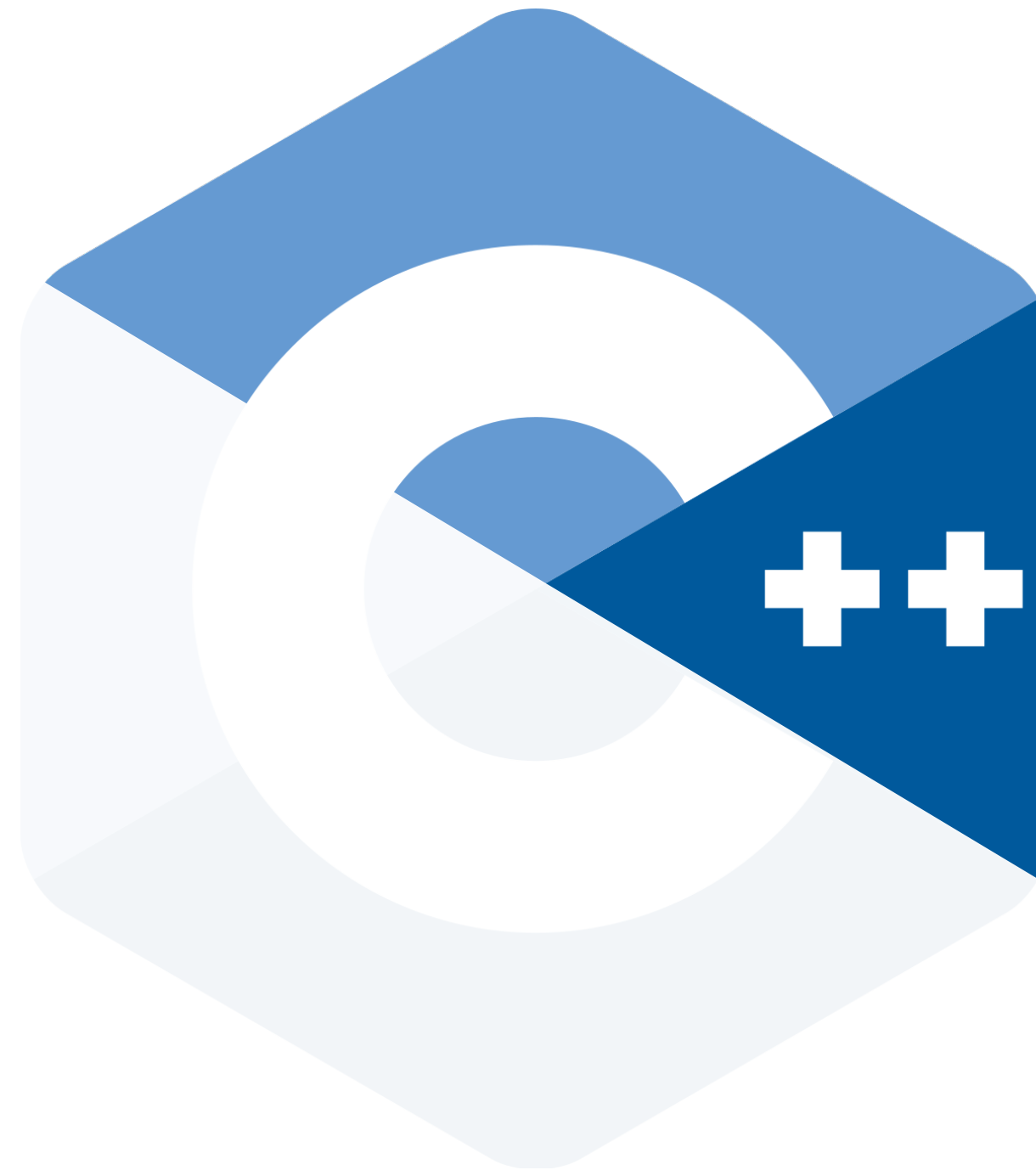
arrays

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functions*

conditionals



loops

~~method calls~~

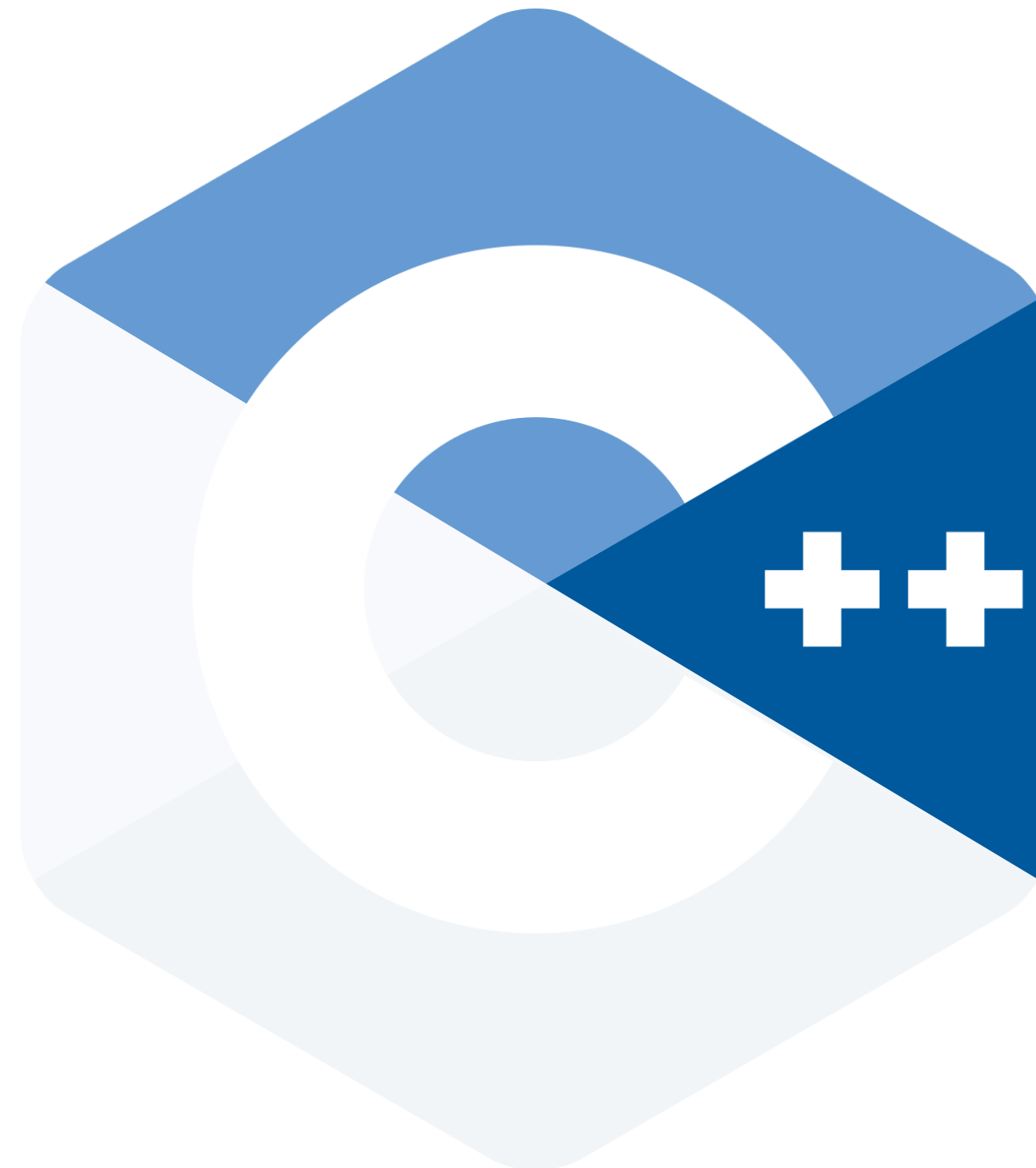
arrays

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~~malloc~~

functions*

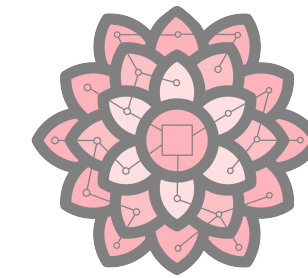
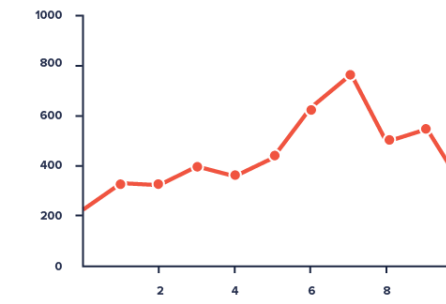
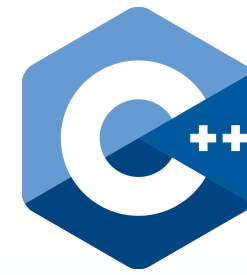
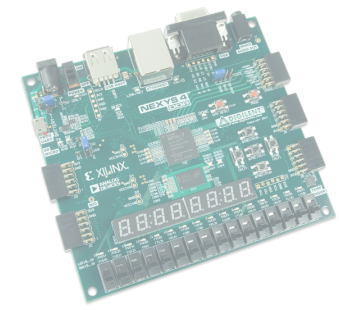
conditionals



**High-performance
hardware designs**



Ada's Journey



Super secret™ accelerator

Hardware

Super secret™ accelerator

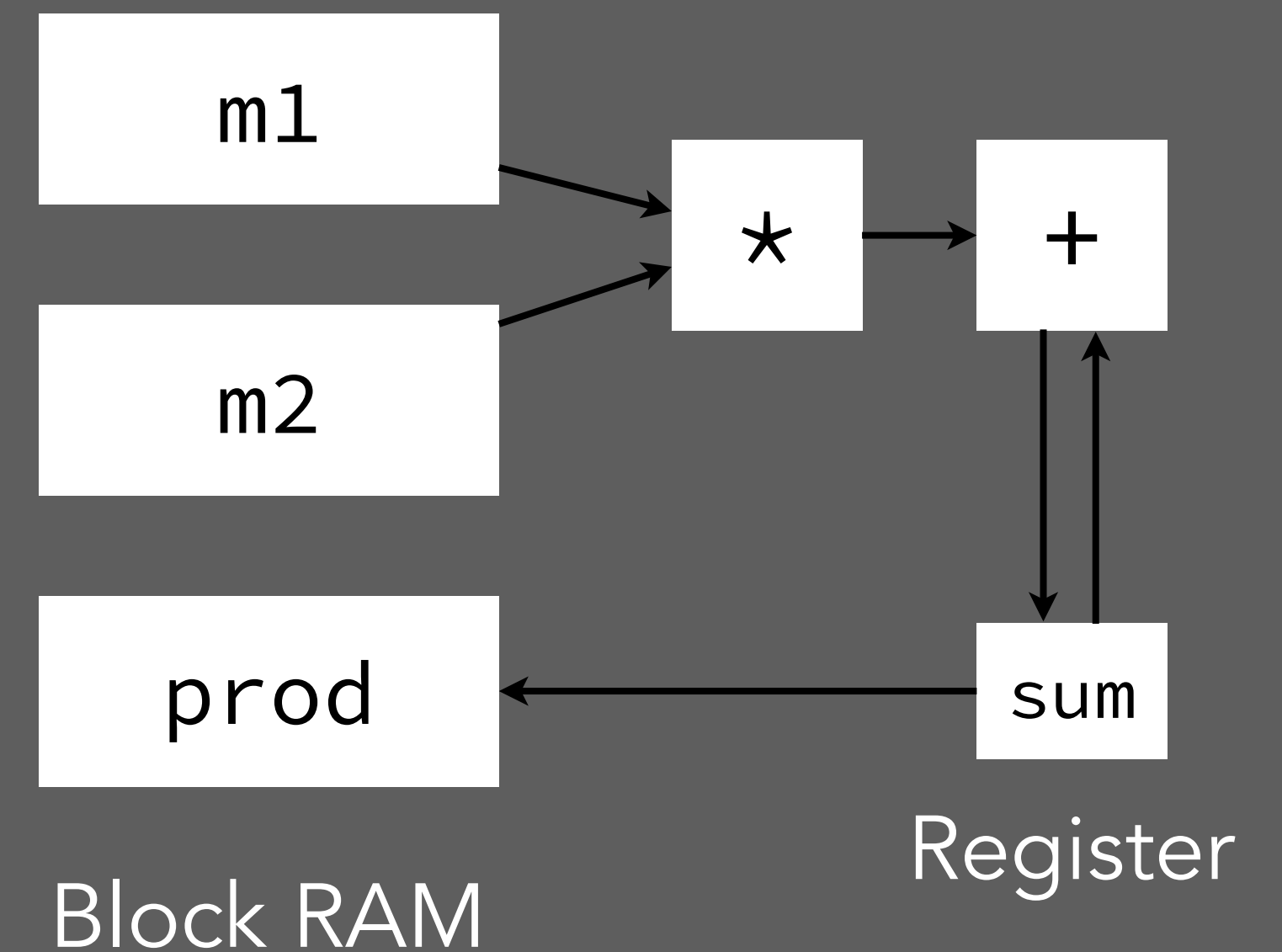
Hardware

```
int m1[512][512];
int m2[512][512];
int prod[512][512];
for (int i = 0; i < 512; i++) {
    for (int j = 0; j < 512; j++) {
        int sum = 0;
        for (int k = 0; k < 512; k++) {
            sum += m1[i][k] * m2[k][j];
        }
        prod[i][j] = sum;
    }
}
```


Super secret™ accelerator

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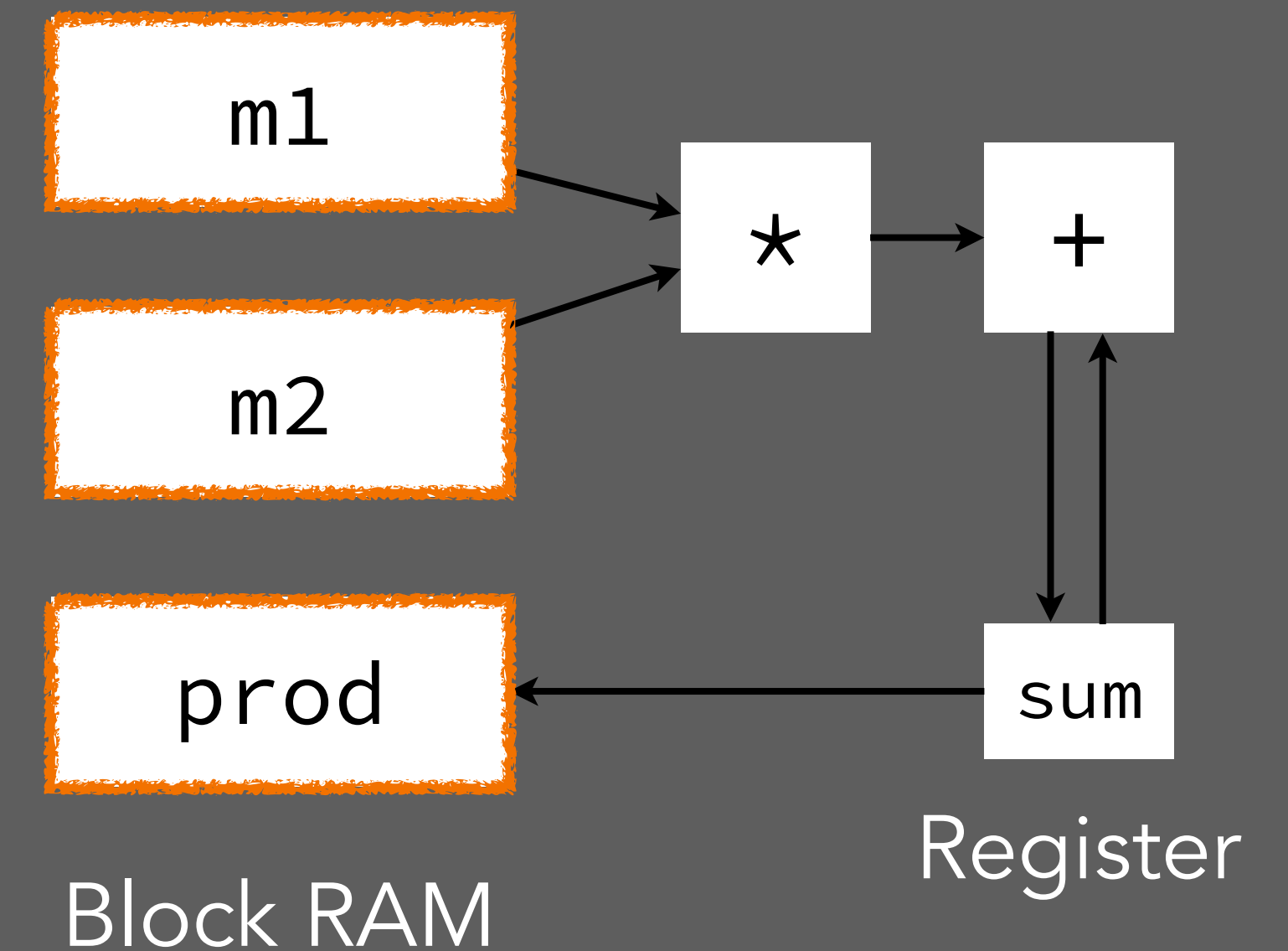
Hardware



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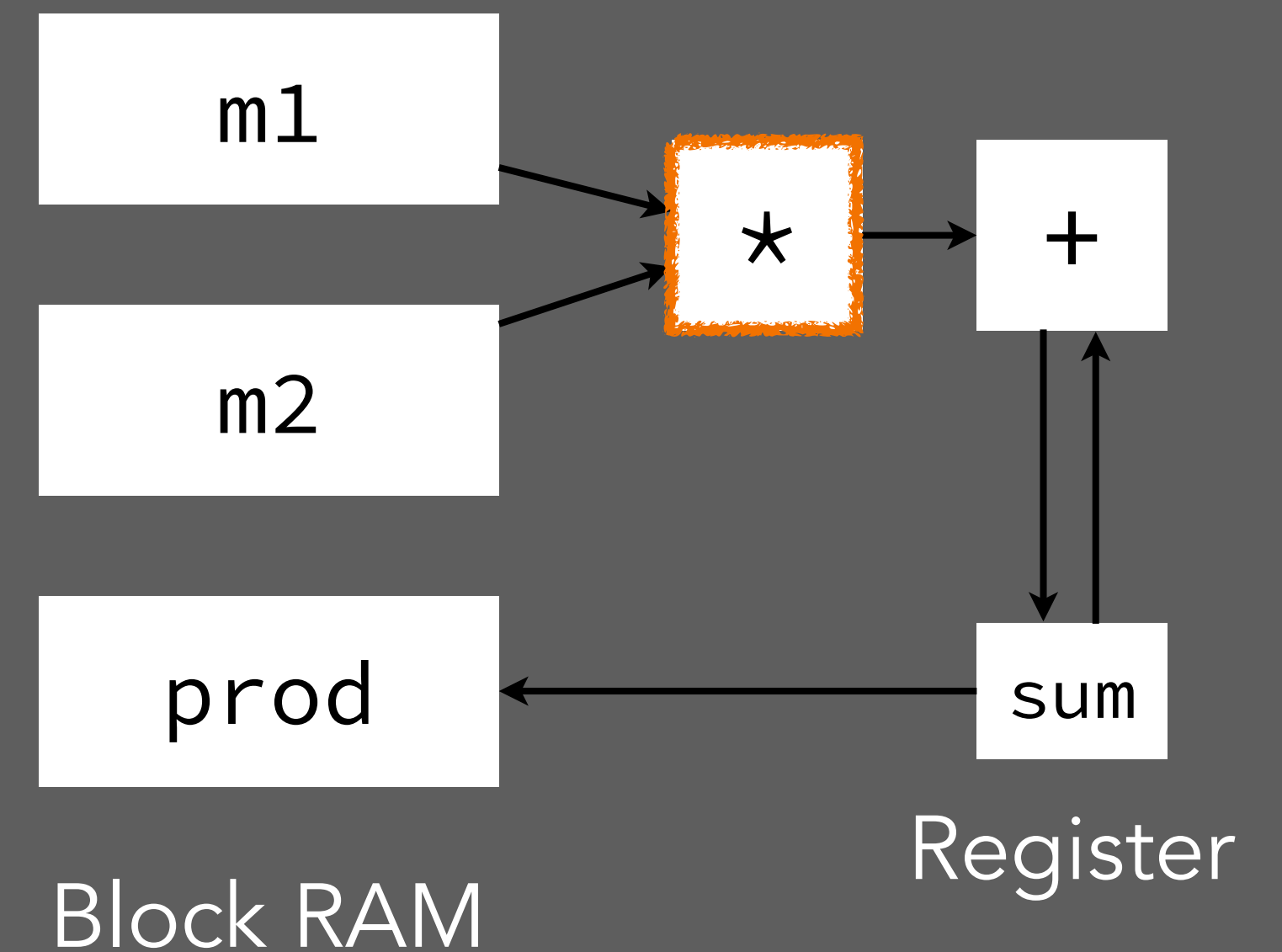
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        for (int k = 0; k < 512; k++) {
            sum += m1[i][k] * m2[k][j];
        }
        prod[i][j] = sum;
    }
}
```

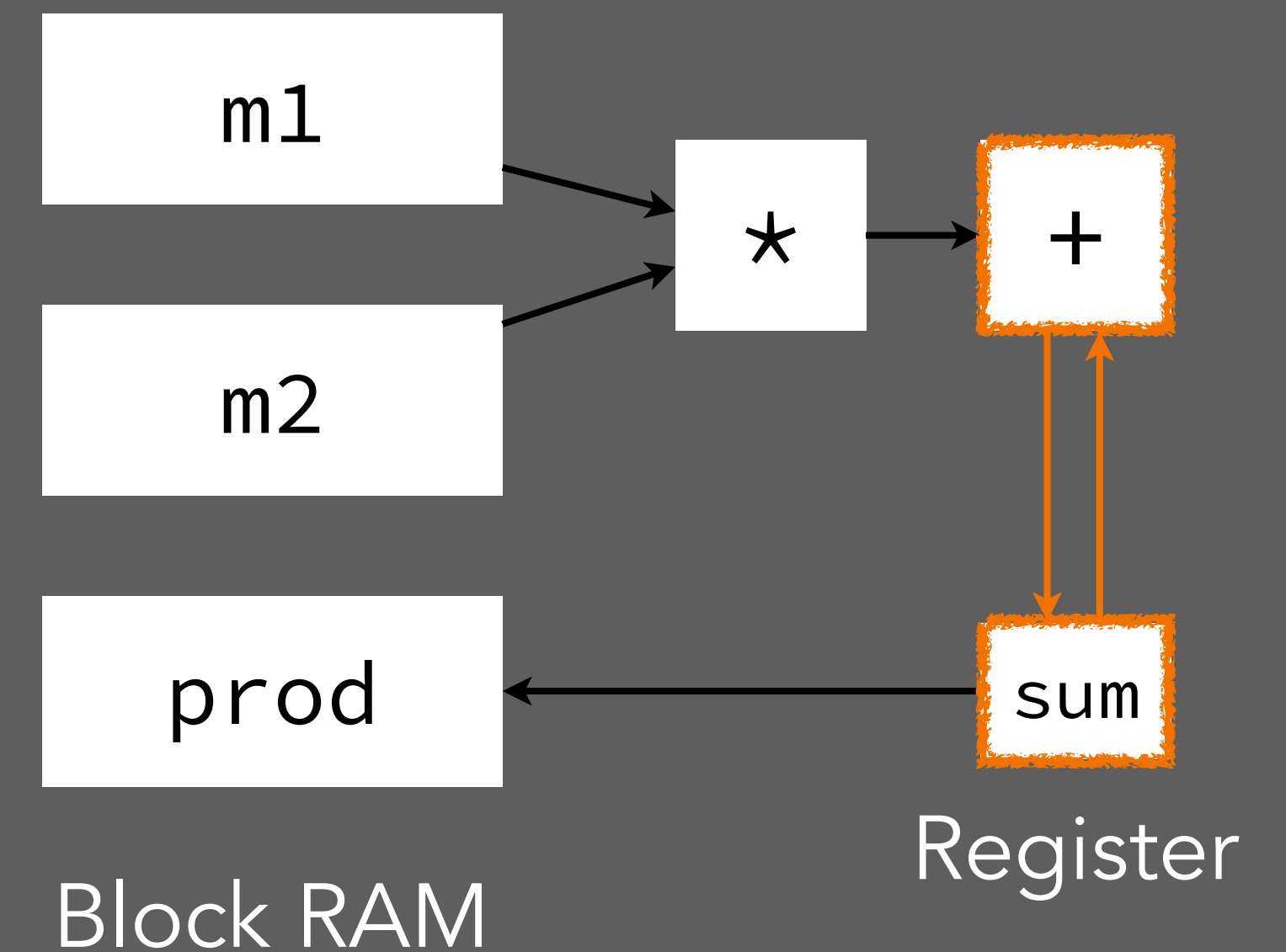
Hardware



Super secret™ accelerator

```
int m1[512][512];
int m2[512][512];
int prod[512][512];
for (int i = 0; i < 512; i++) {
    for (int j = 0; j < 512; j++) {
        int sum = 0;
        for (int k = 0; k < 512; k++) {
            sum += m1[i][k] * m2[k][j];
        }
        prod[i][j] = sum;
    }
}
```

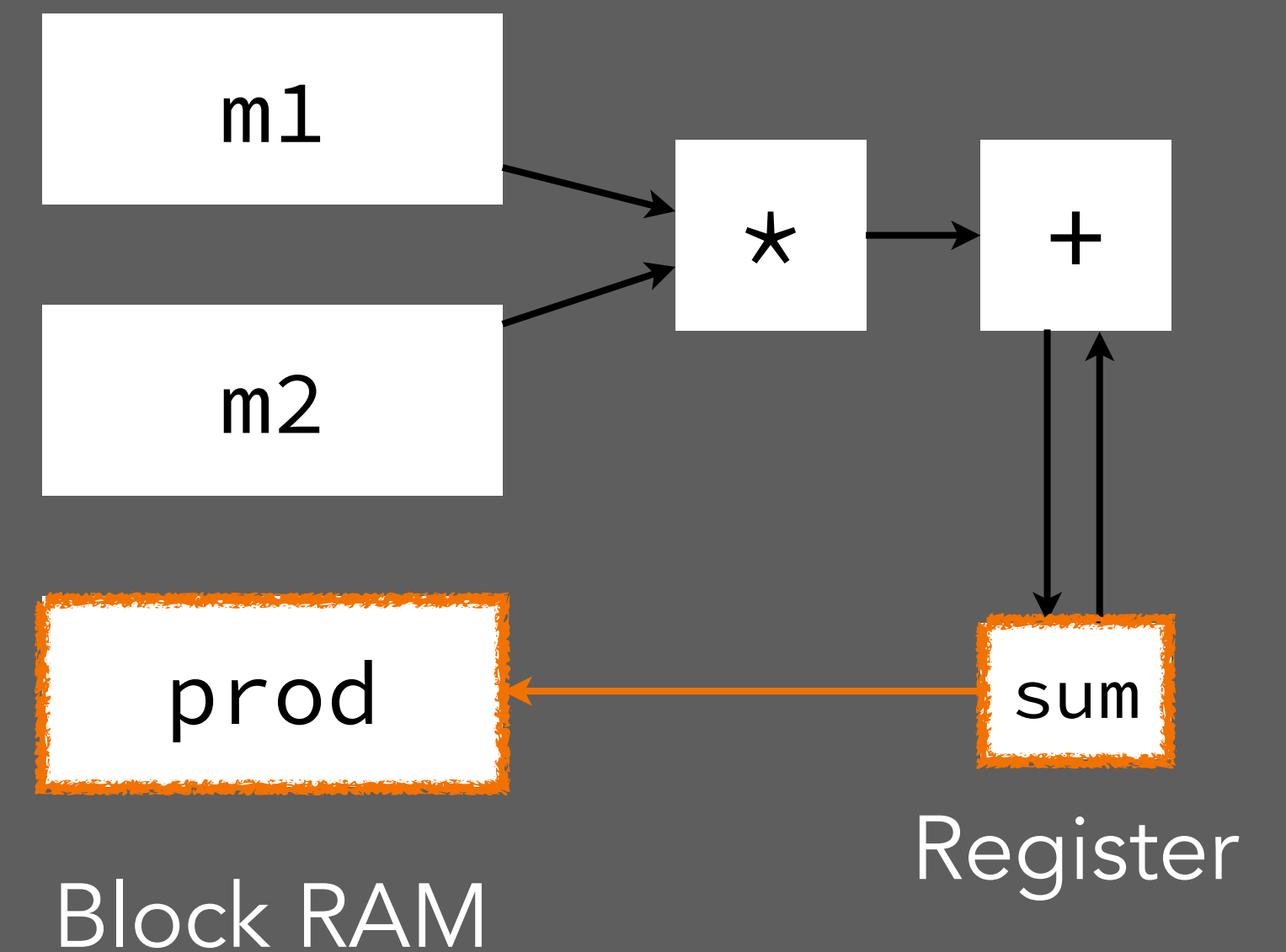
Hardware



Super secret™ accelerator

```
int m1[512][512];
int m2[512][512];
int prod[512][512];
for (int i = 0; i < 512; i++) {
    for (int j = 0; j < 512; j++) {
        int sum = 0;
        for (int k = 0; k < 512; k++) {
            sum += m1[i][k] * m2[k][j];
        }
        prod[i][j] = sum;
    }
}
```

Hardware

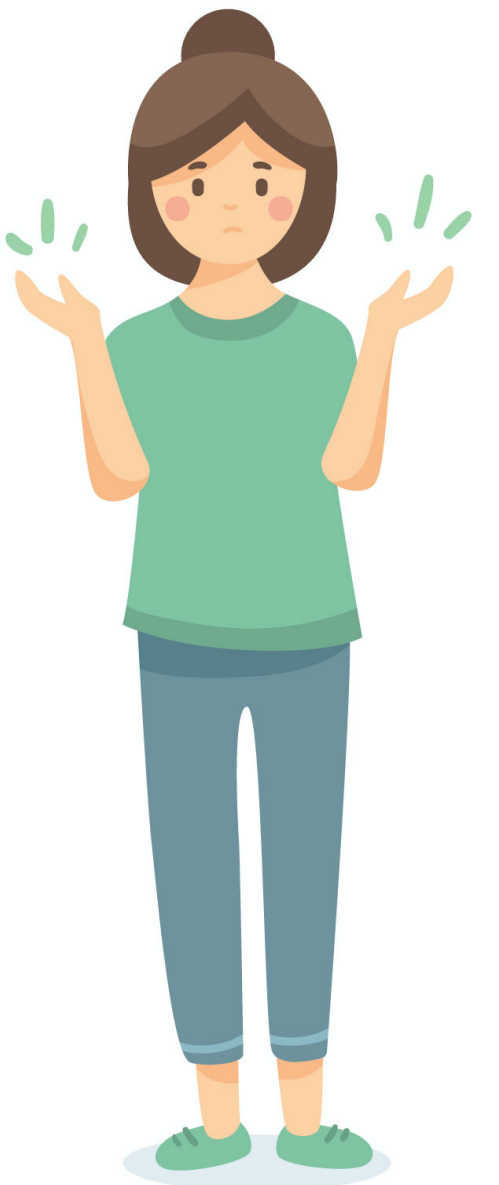




reaching up to **35.9 effective TFLOPS** for a large GRU over hundreds of timesteps. This represents an approximate **two orders of magnitude** advantage over the Titan Xp. This is

Darwin: A Genomics Co-processor Provides up to **15,000× acceleration on long read assembly**

*ranking candidate documents. Under high load, the large-scale reconfigurable fabric improves the ranking throughput of each server **by a factor of 95%** for a fixed latency distribution—or, while maintaining equivalent throughput, reduces the tail latency by 29%.*

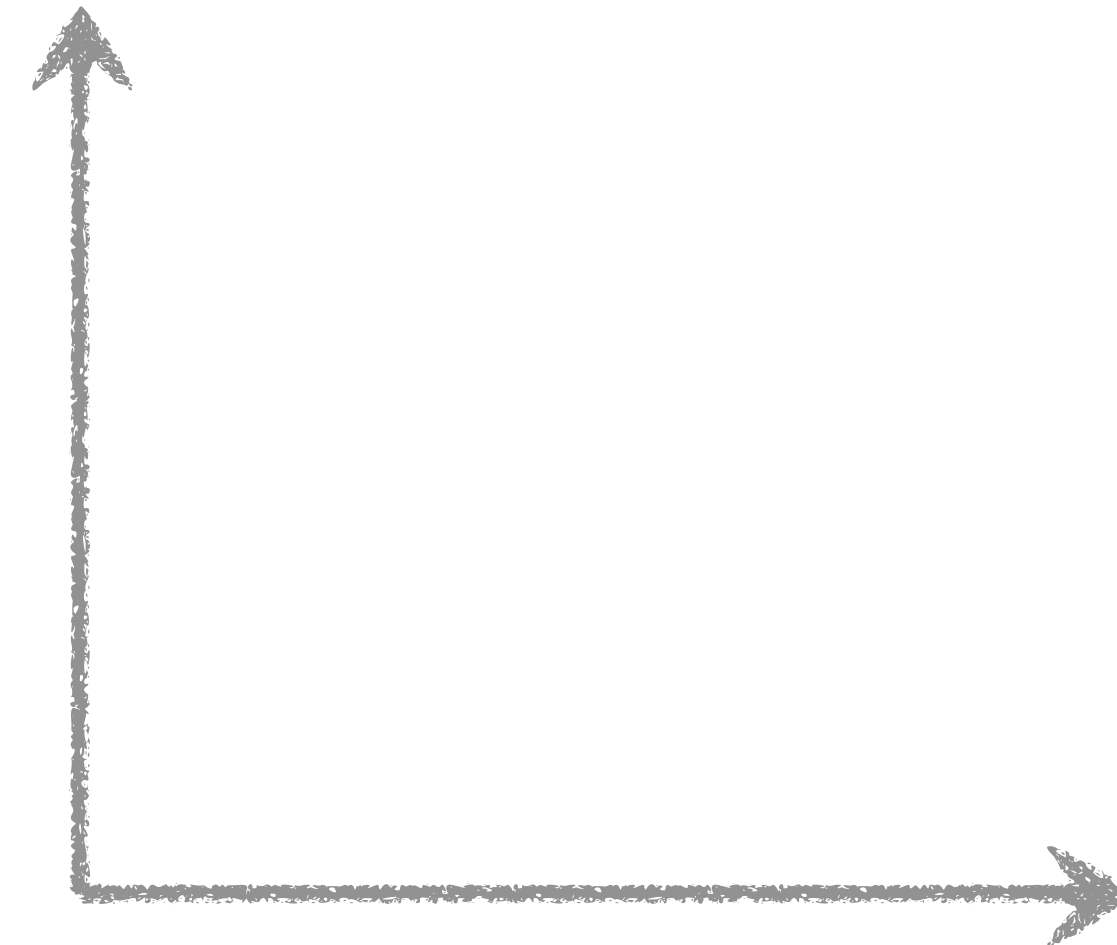


Super secret™
accelerator



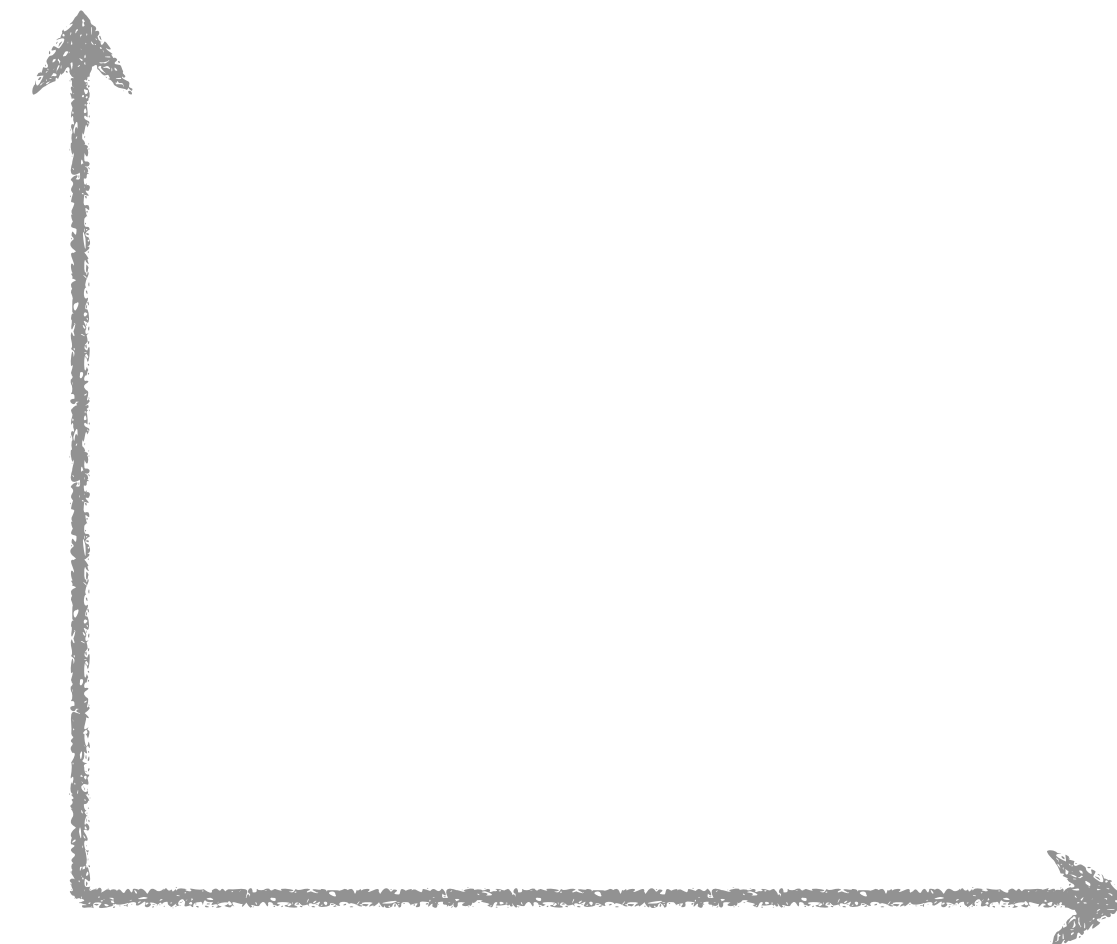
Resources on the FPGA

Area



Parallelism

Latency

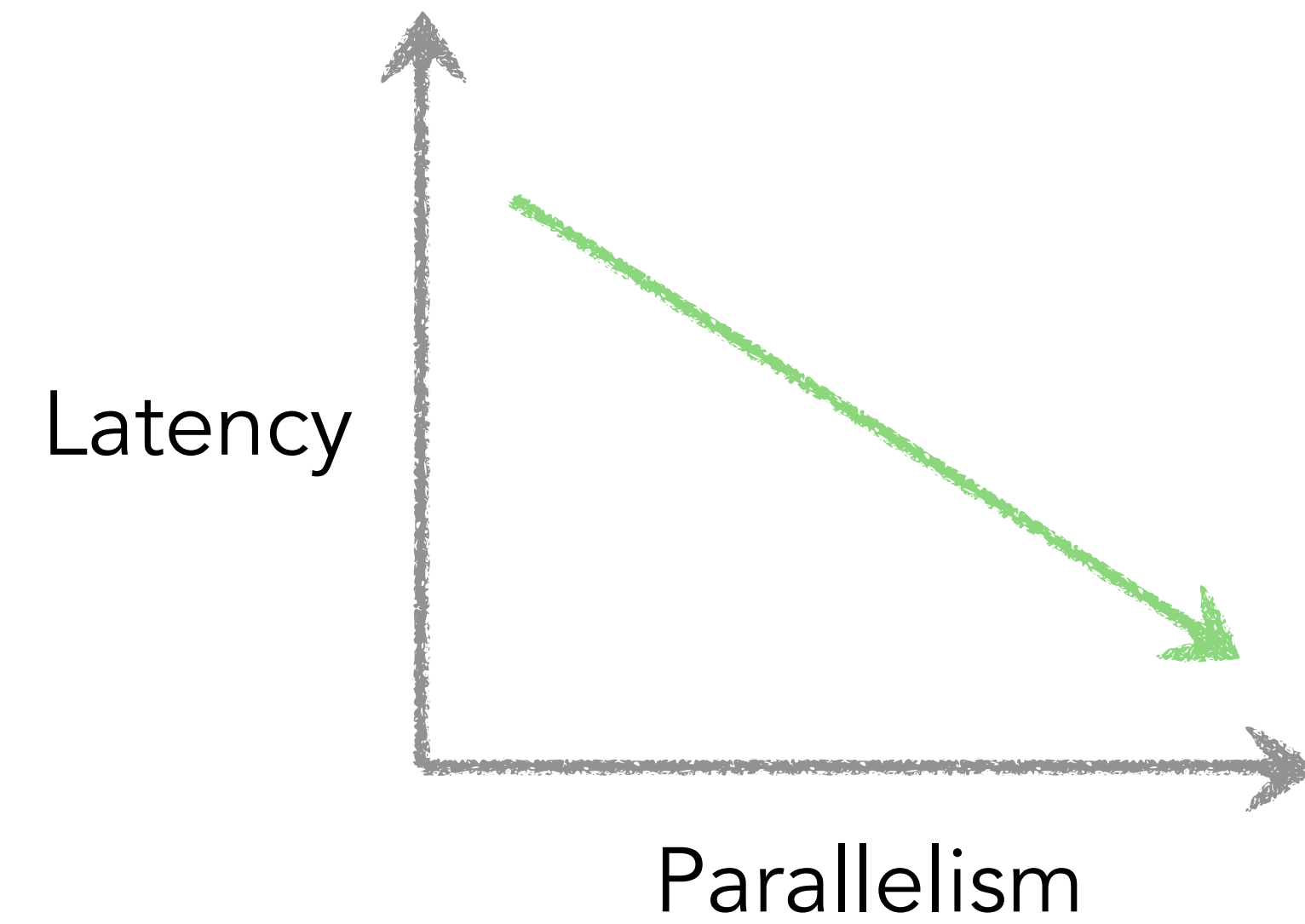
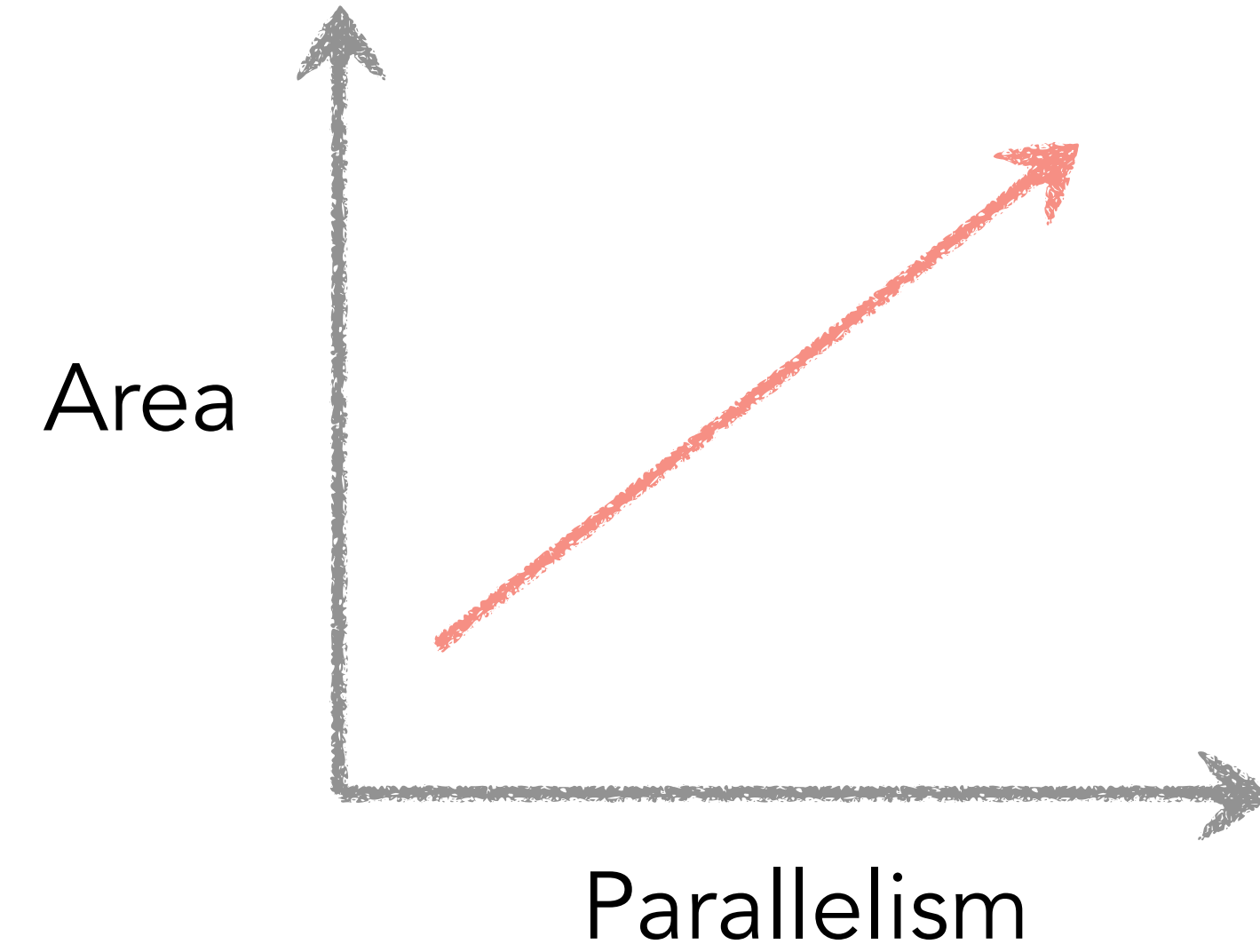


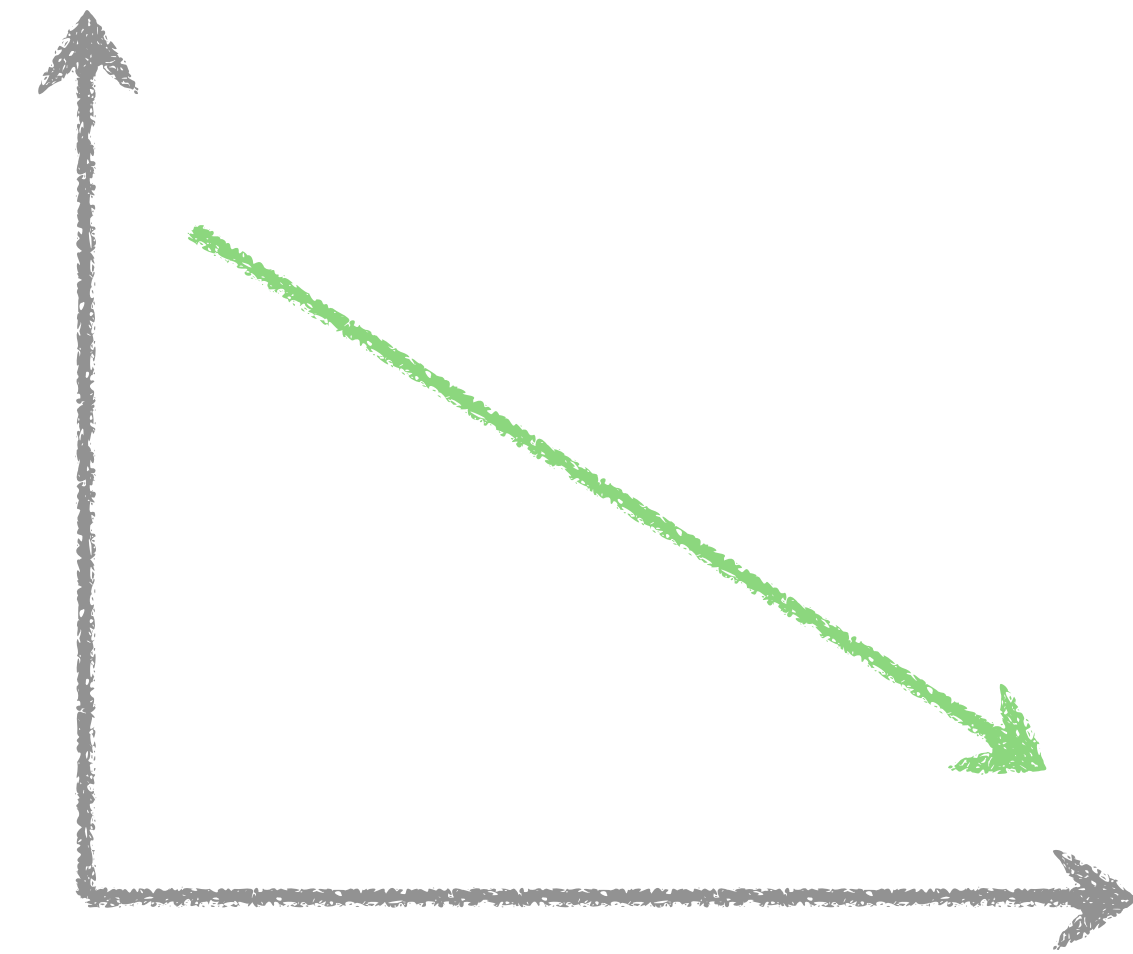
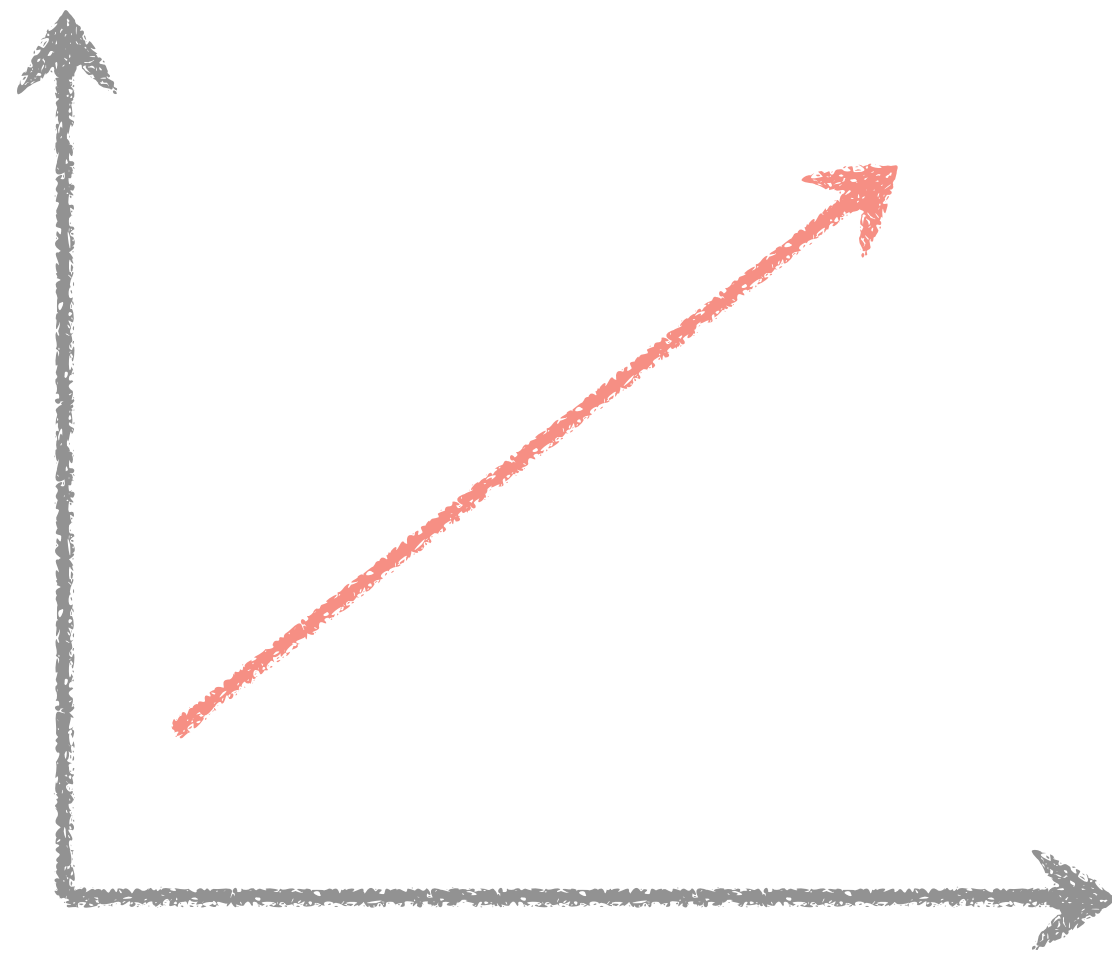
Parallelism

Super secret™
accelerator



Resources on the FPGA





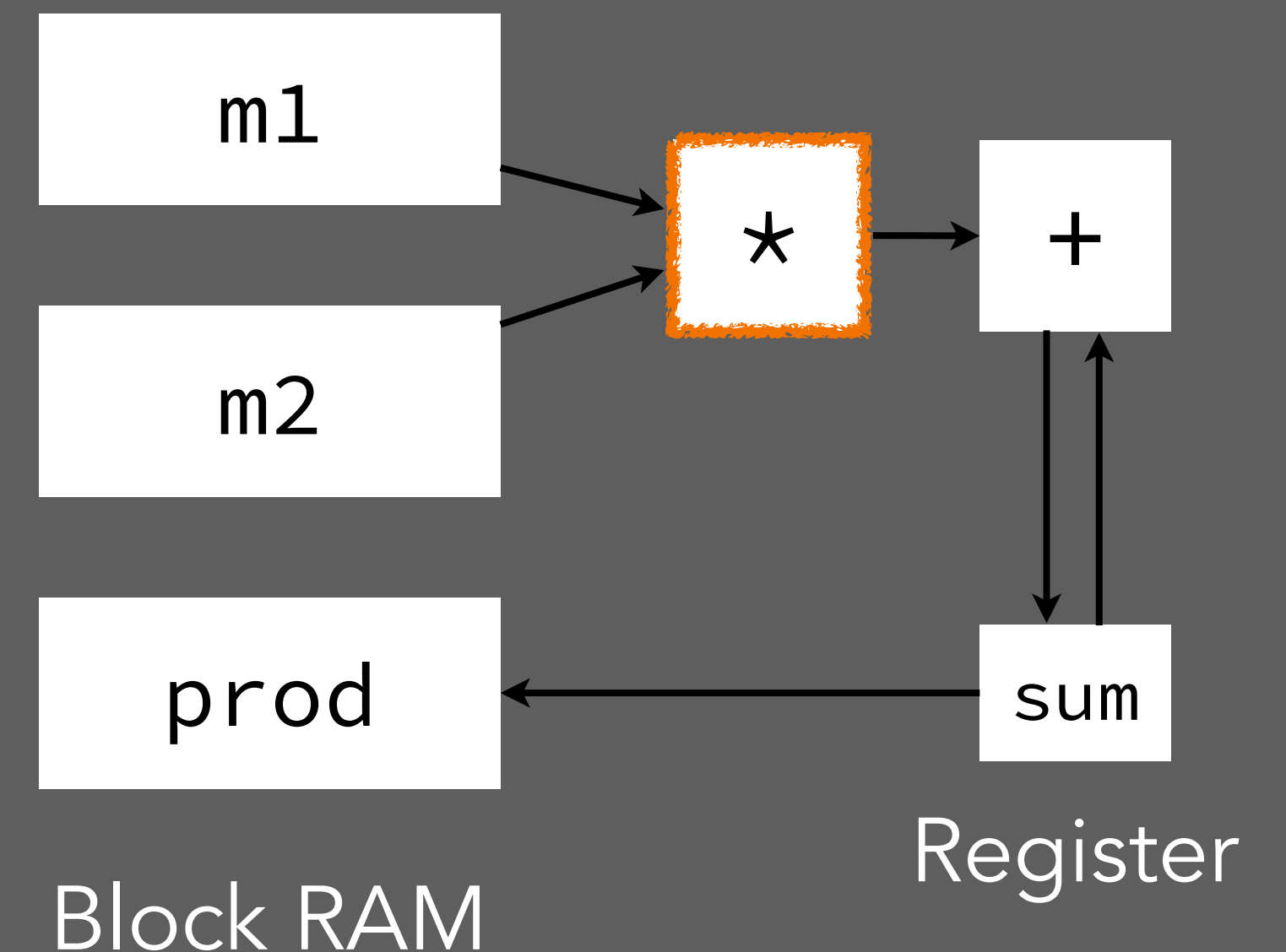
Area-Latency trade-offs

Create more processing elements to extract parallelism

Super secret™ accelerator

```
int m1[512][512];
int m2[512][512];
int prod[512][512];
for (int i = 0; i < 512; i++) {
    for (int j = 0; j < 512; j++) {
        int sum = 0;
        for (int k = 0; k < 512; k++) {
            sum += m1[i][k] * m2[k][j];
        }
        prod[i][j] = sum;
    }
}
```

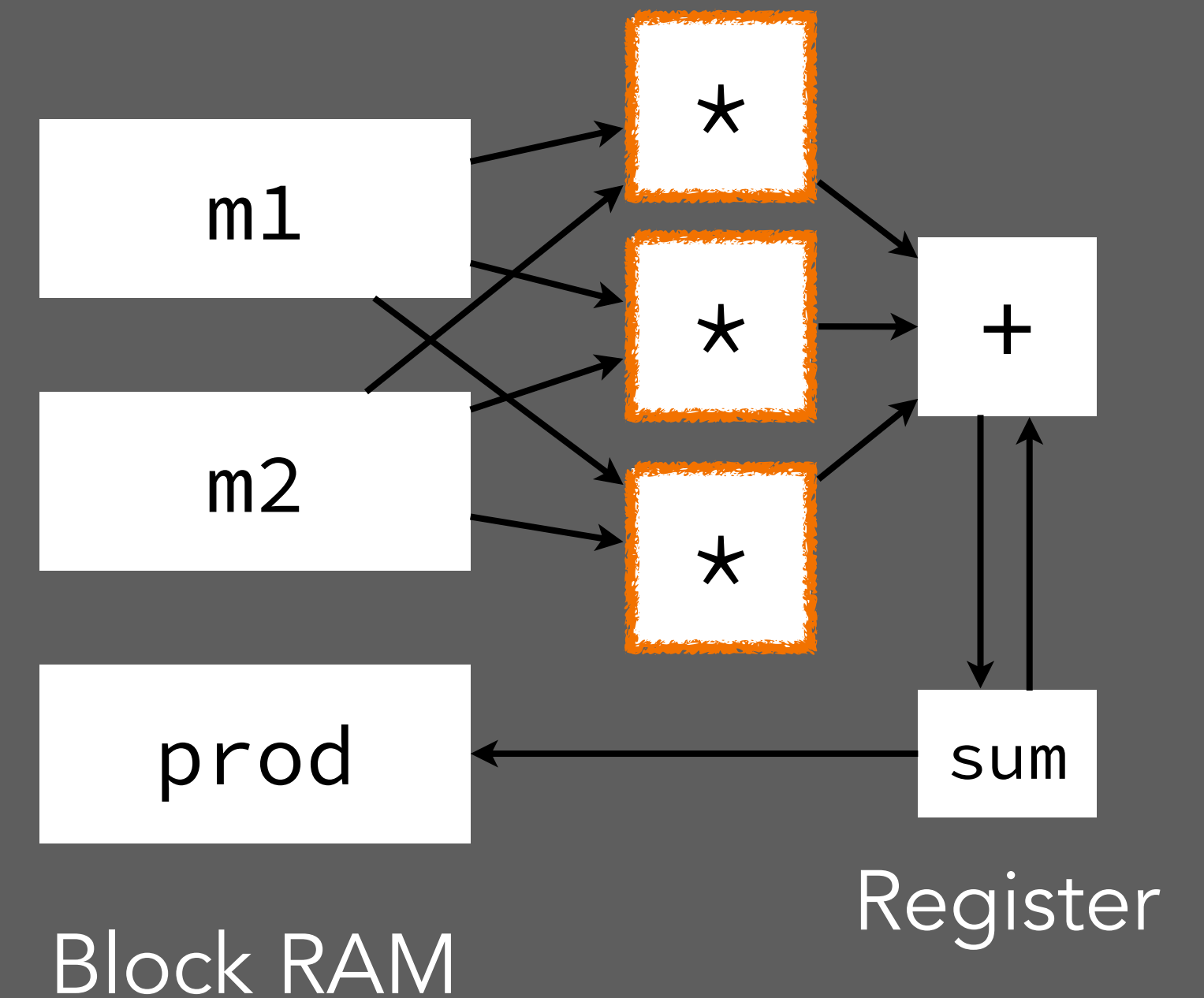
Hardware



Super secret™ accelerator

```
int m1[512][512];
int m2[512][512];
int prod[512][512];
for (int i = 0; i < 512; i++) {
    for (int j = 0; j < 512; j++) {
        int sum = 0;
        for (int k = 0; k < 512; k++) {
            sum += m1[i][k] * m2[k][j];
        }
        prod[i][j] = sum;
    }
}
```

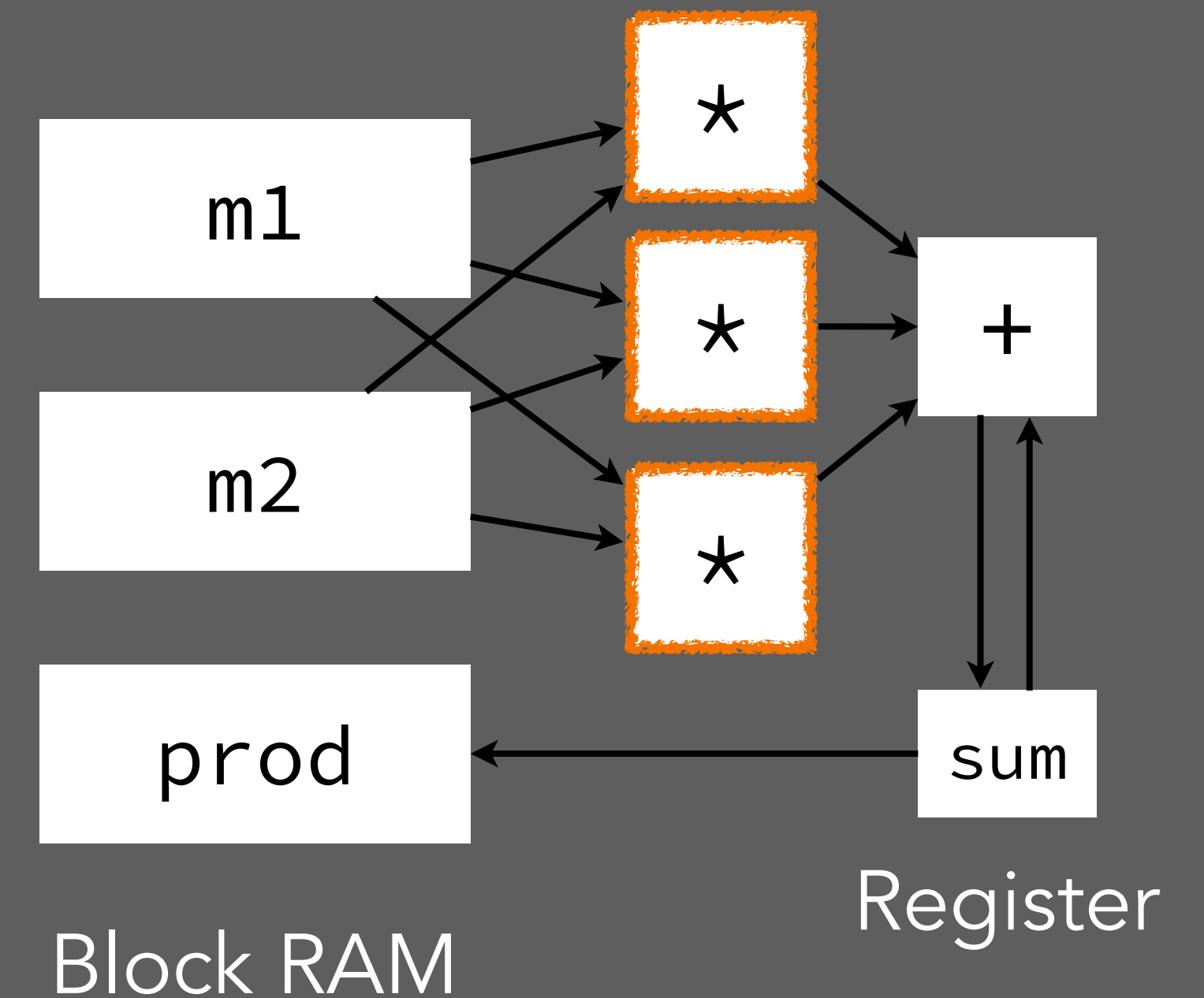
Hardware

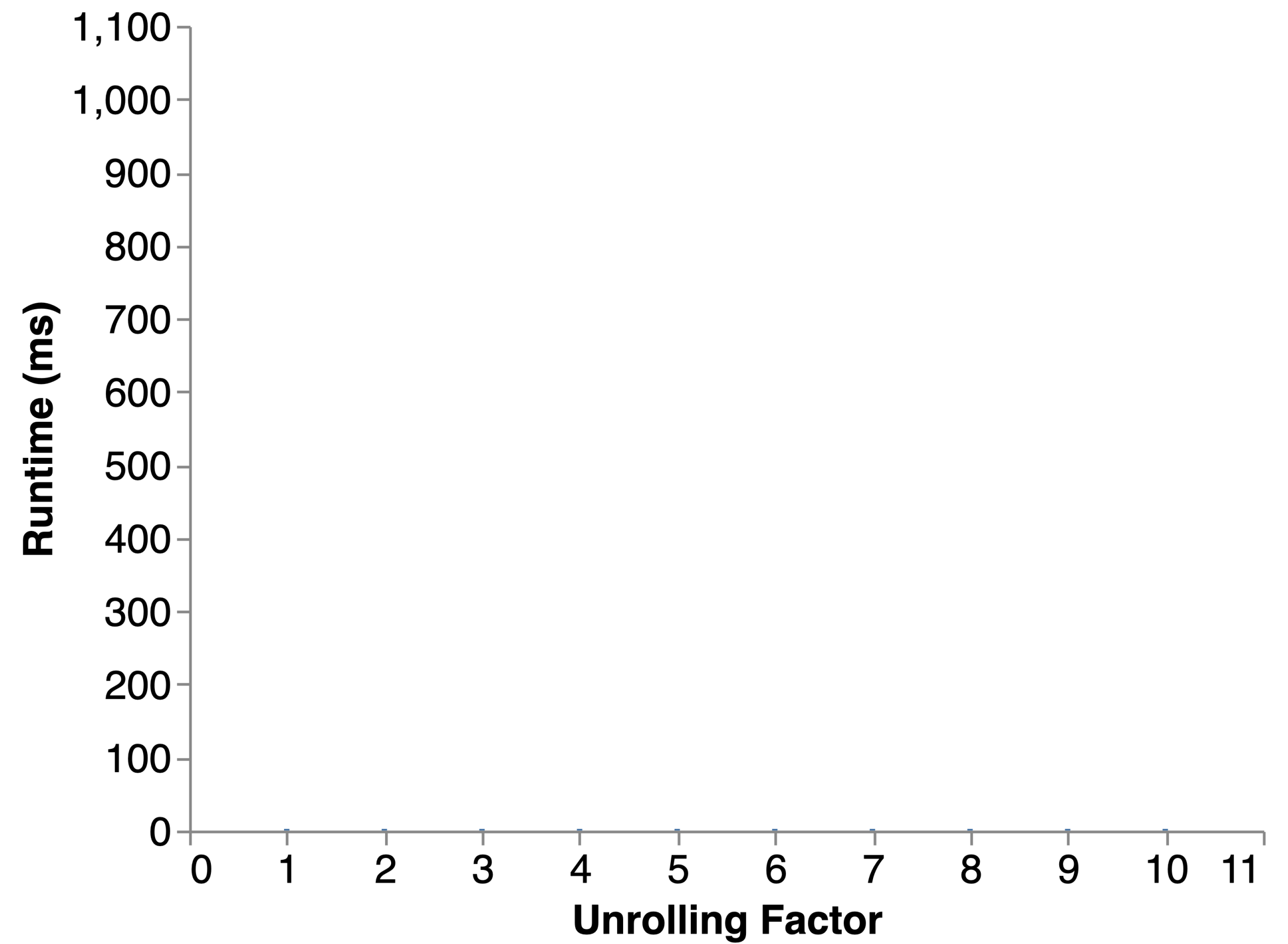
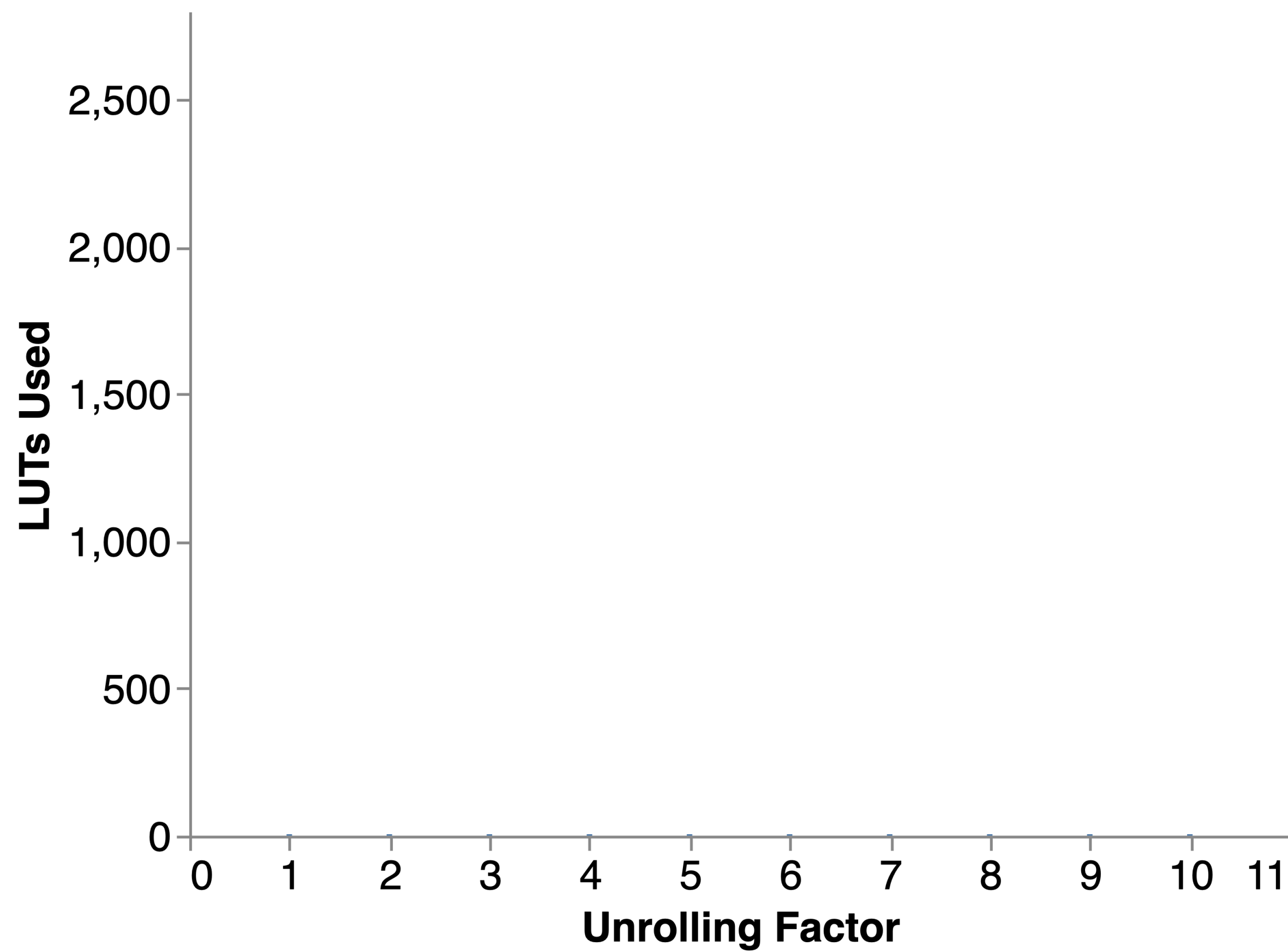


Super secret™ accelerator

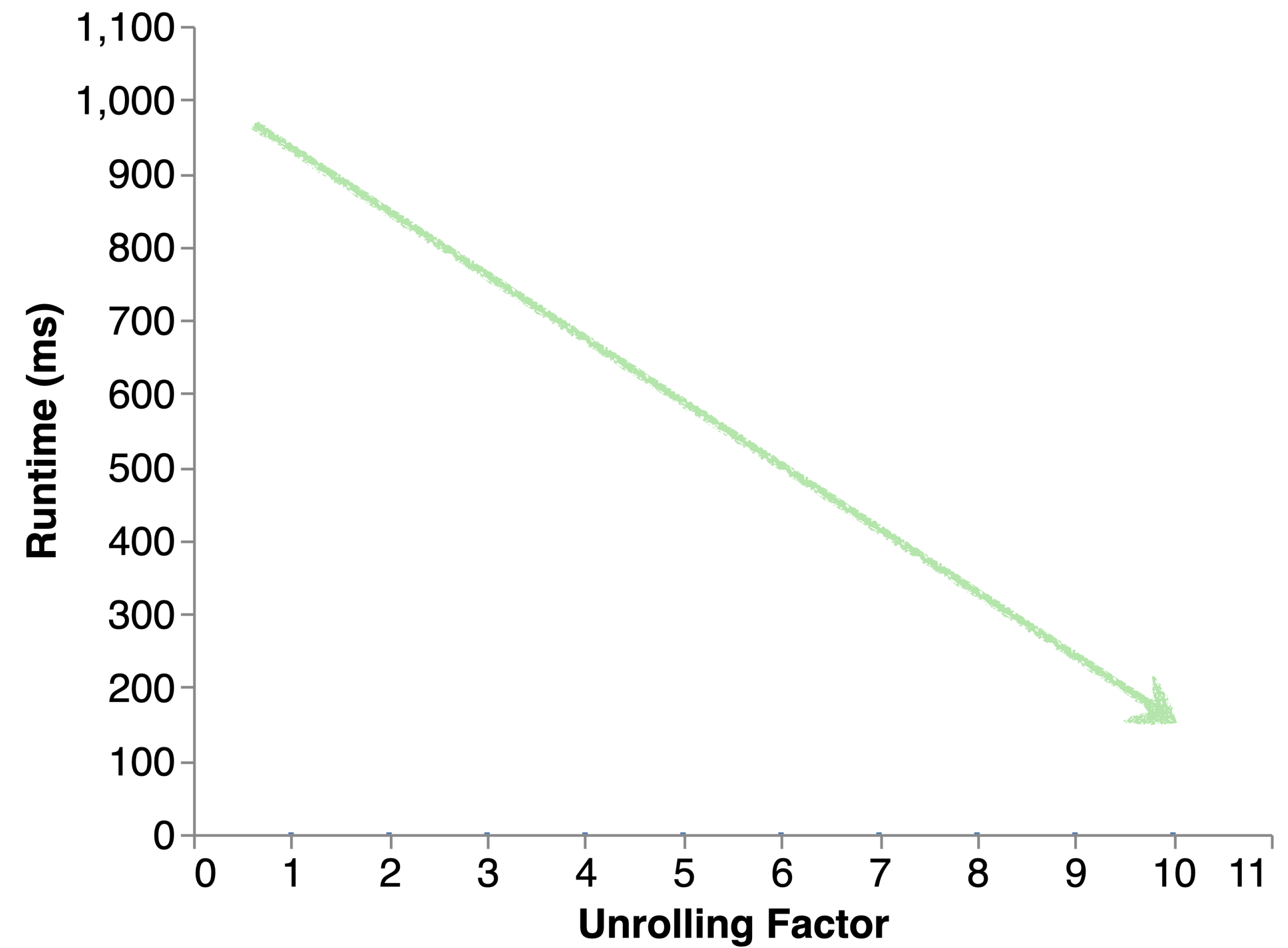
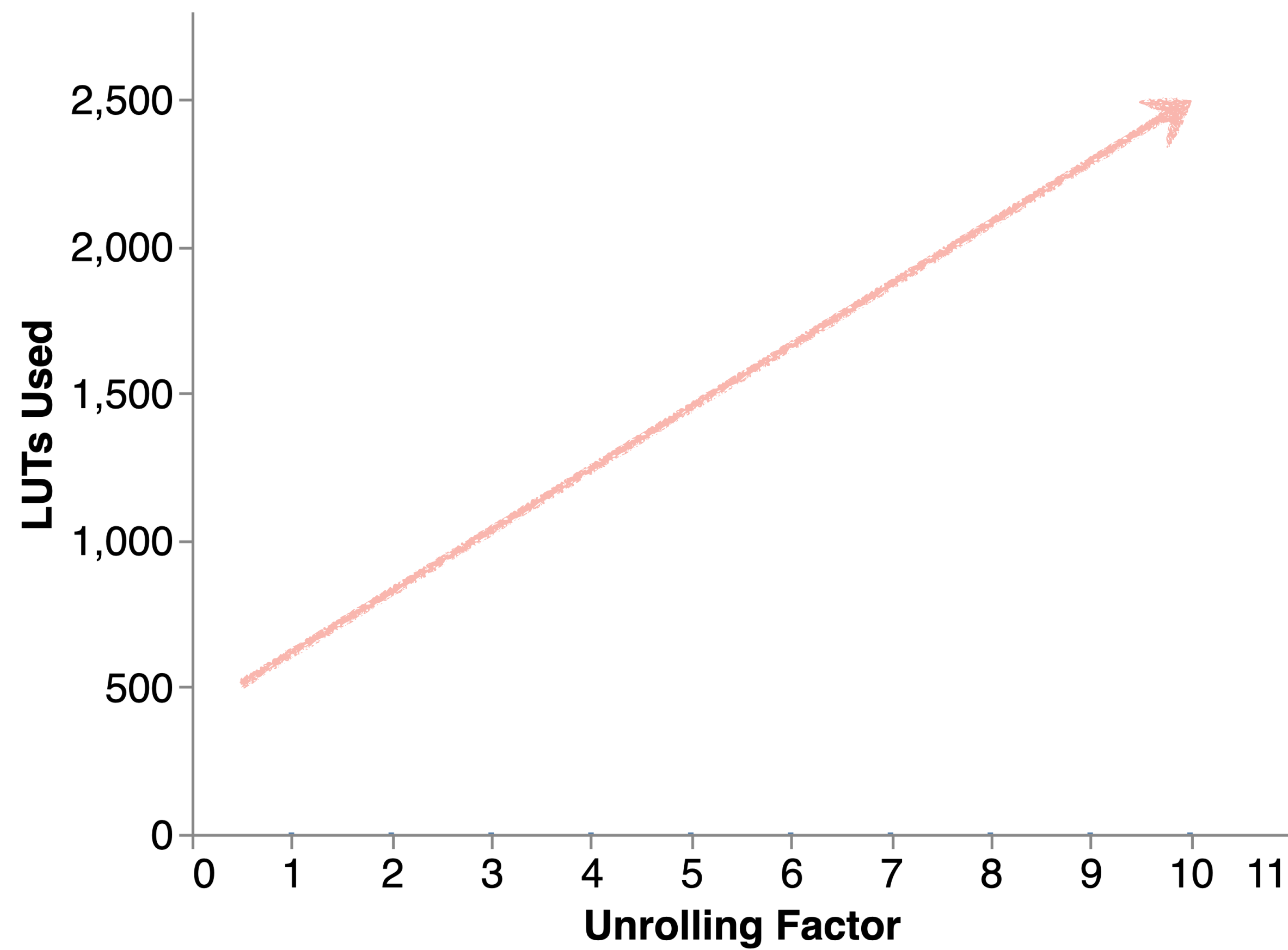
```
int m1[512][512];
int m2[512][512];
int prod[512][512];
for (int i = 0; i < 512; i++) {
    for (int j = 0; j < 512; j++) {
        int sum = 0;
        for (int k = 0; k < 512; k++) {
            #pragma HLS UNROLL factor=3
            sum += m1[i][k] * m2[k][j];
        }
        prod[i][j] = sum;
    }
}
```

Hardware

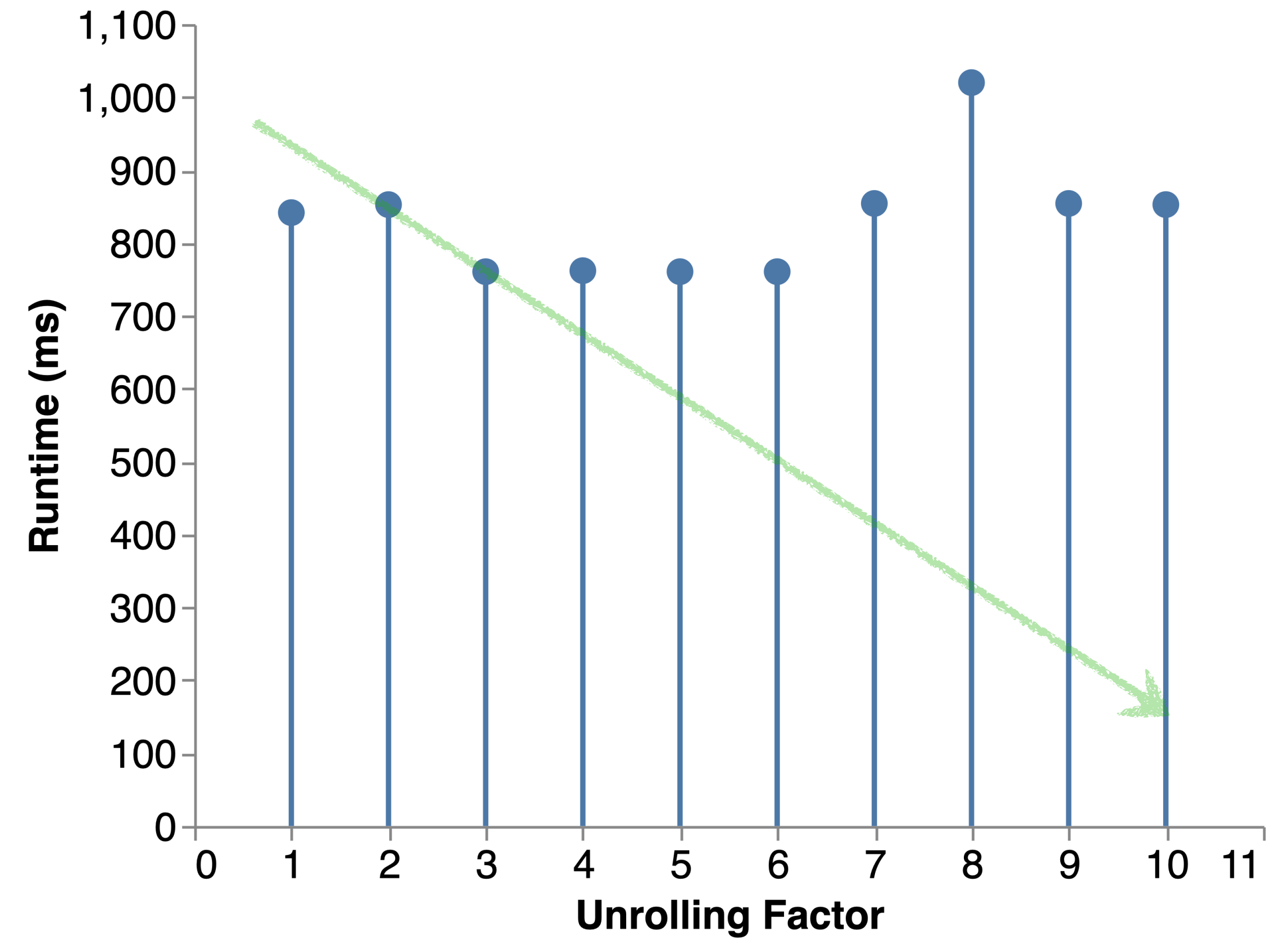
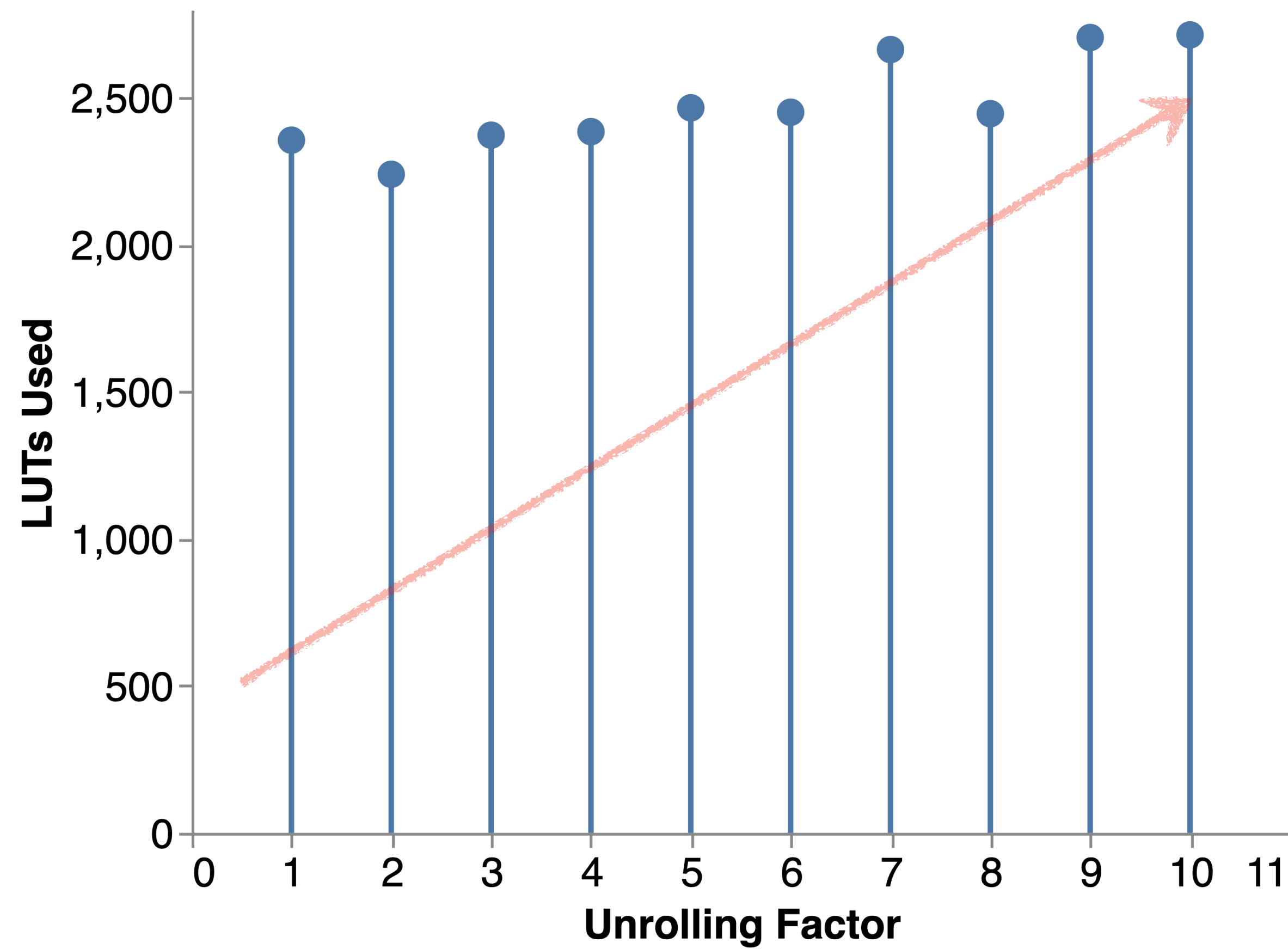




Area-Latency trade-offs



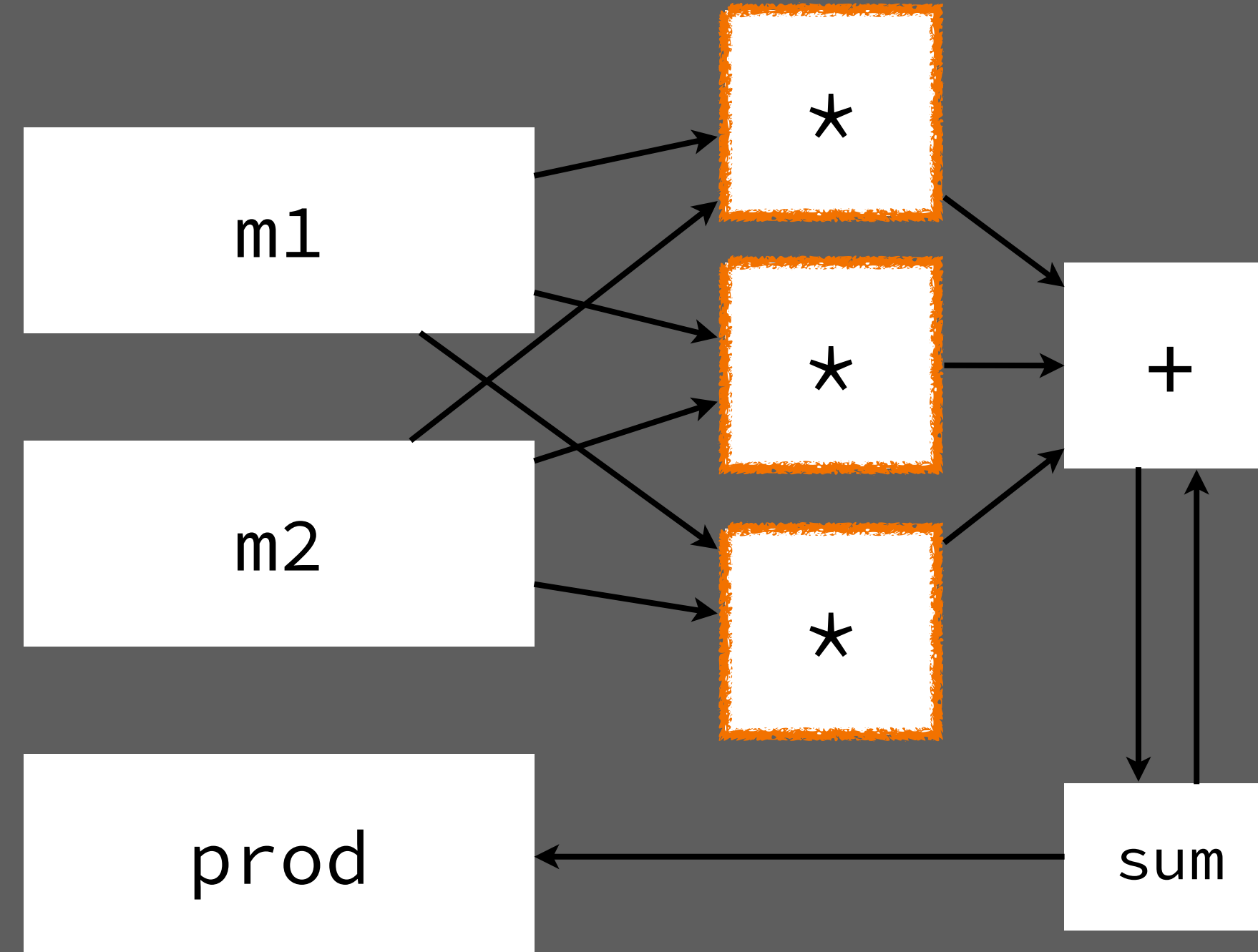
Area-Latency trade-offs



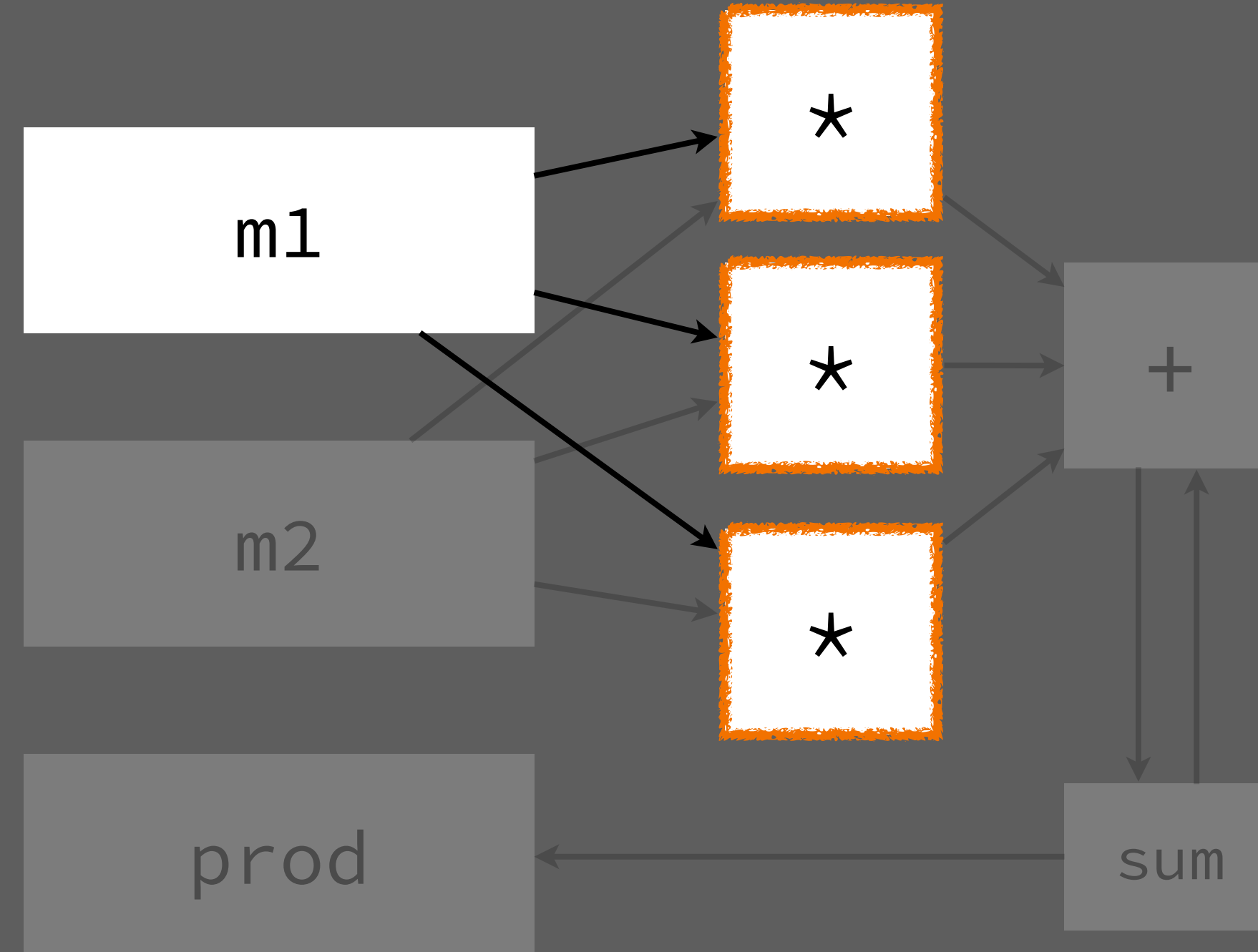
Area-Latency trade-offs



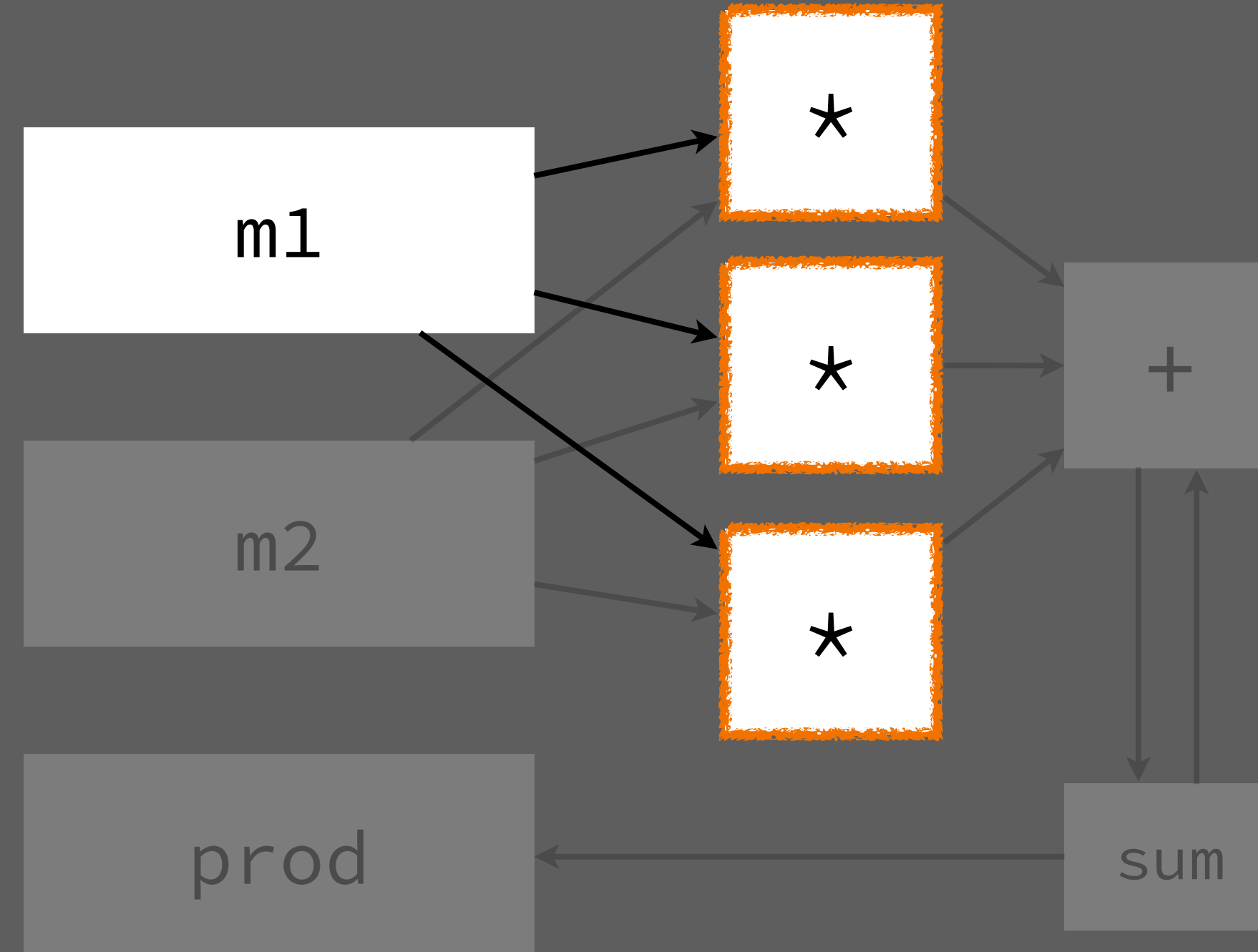
Hardware



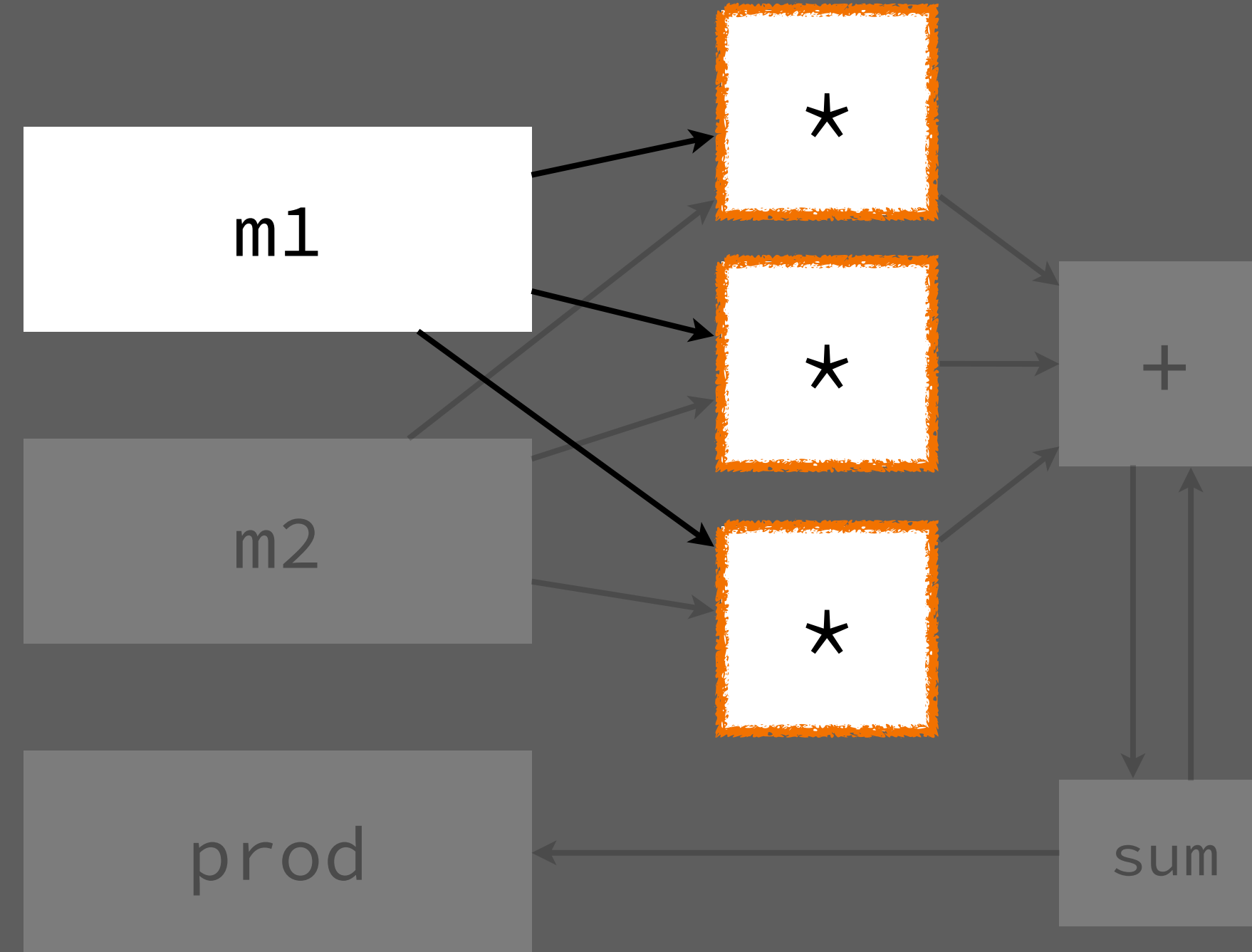
Hardware



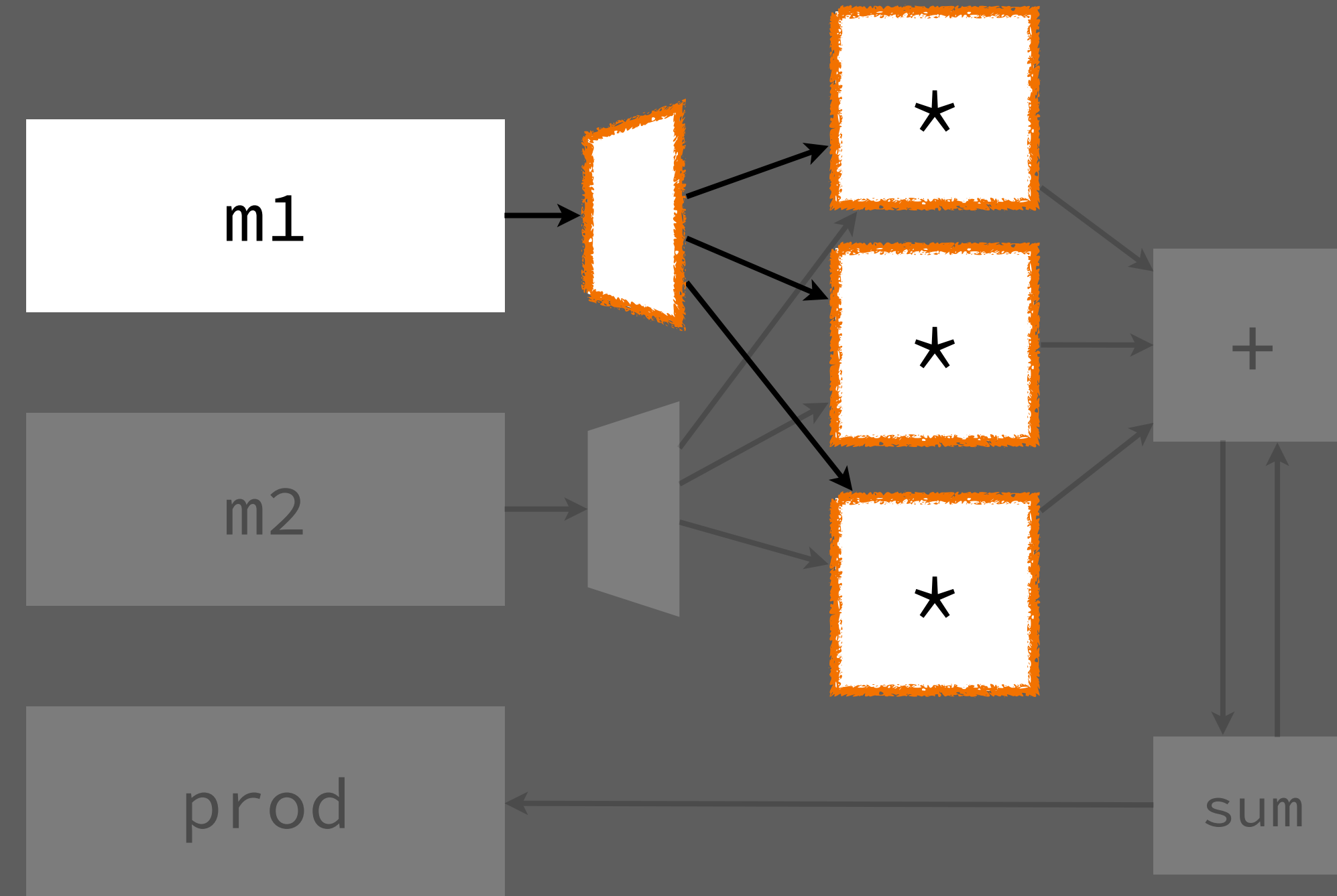
Hardware



Hardware

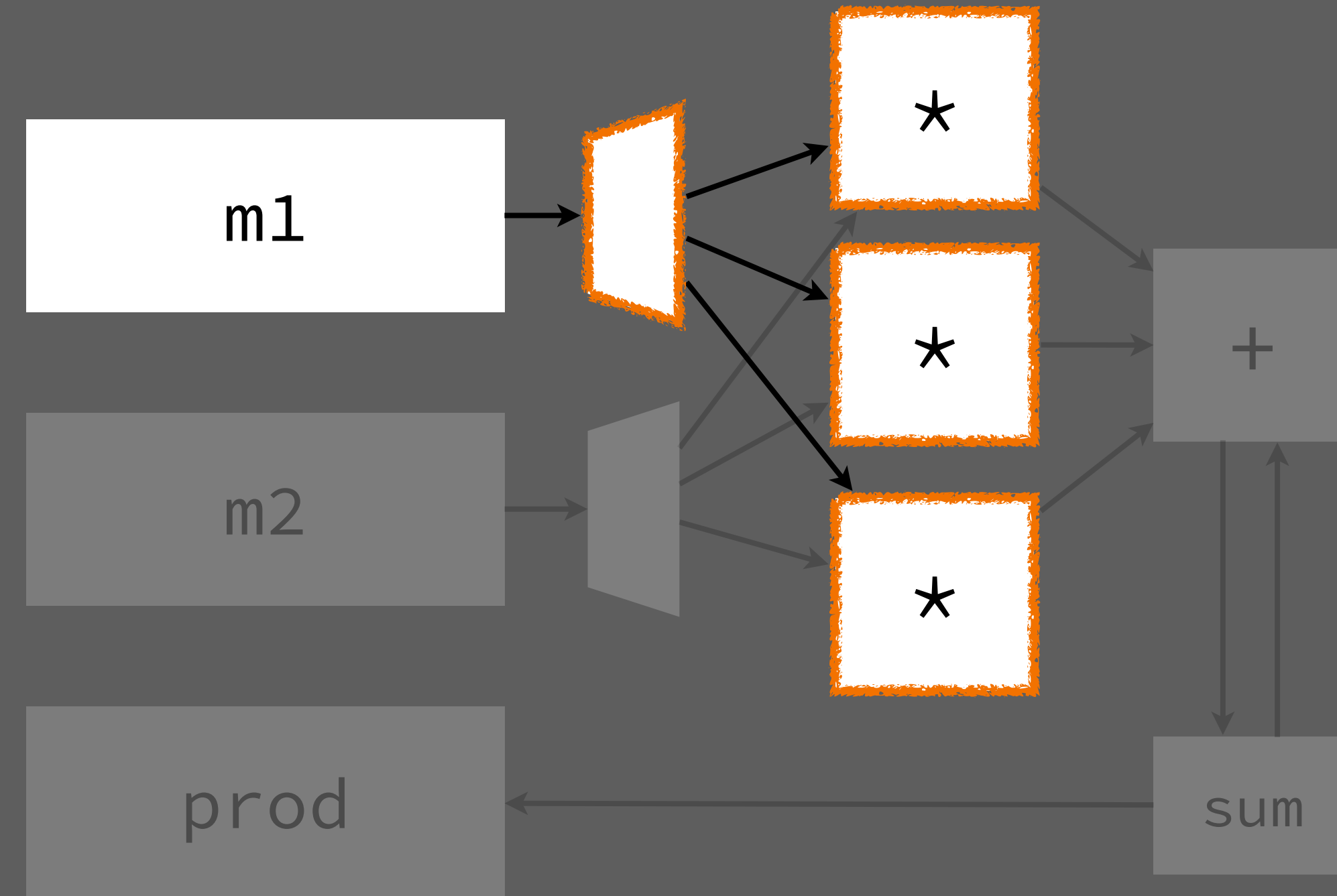


Hardware

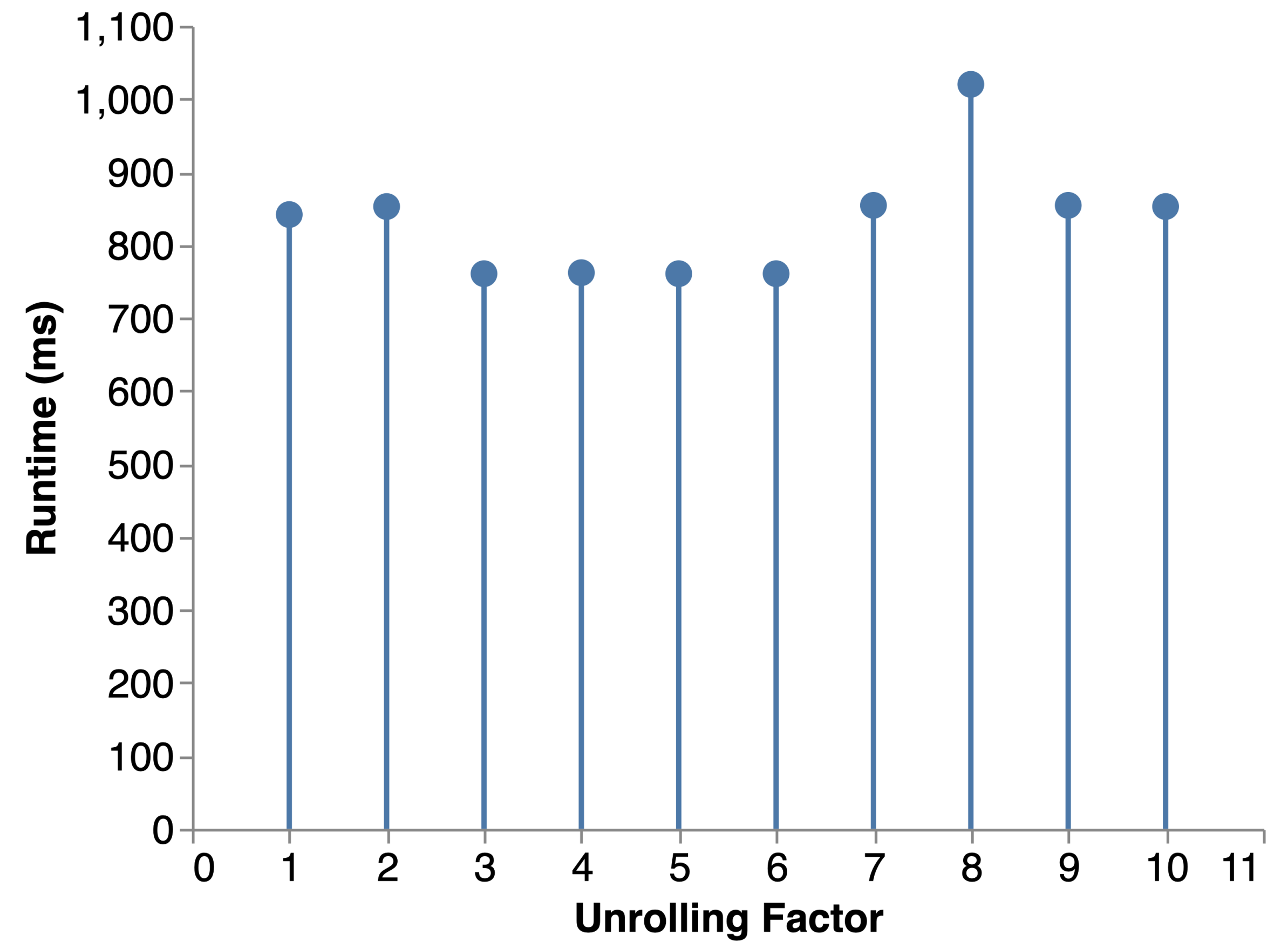
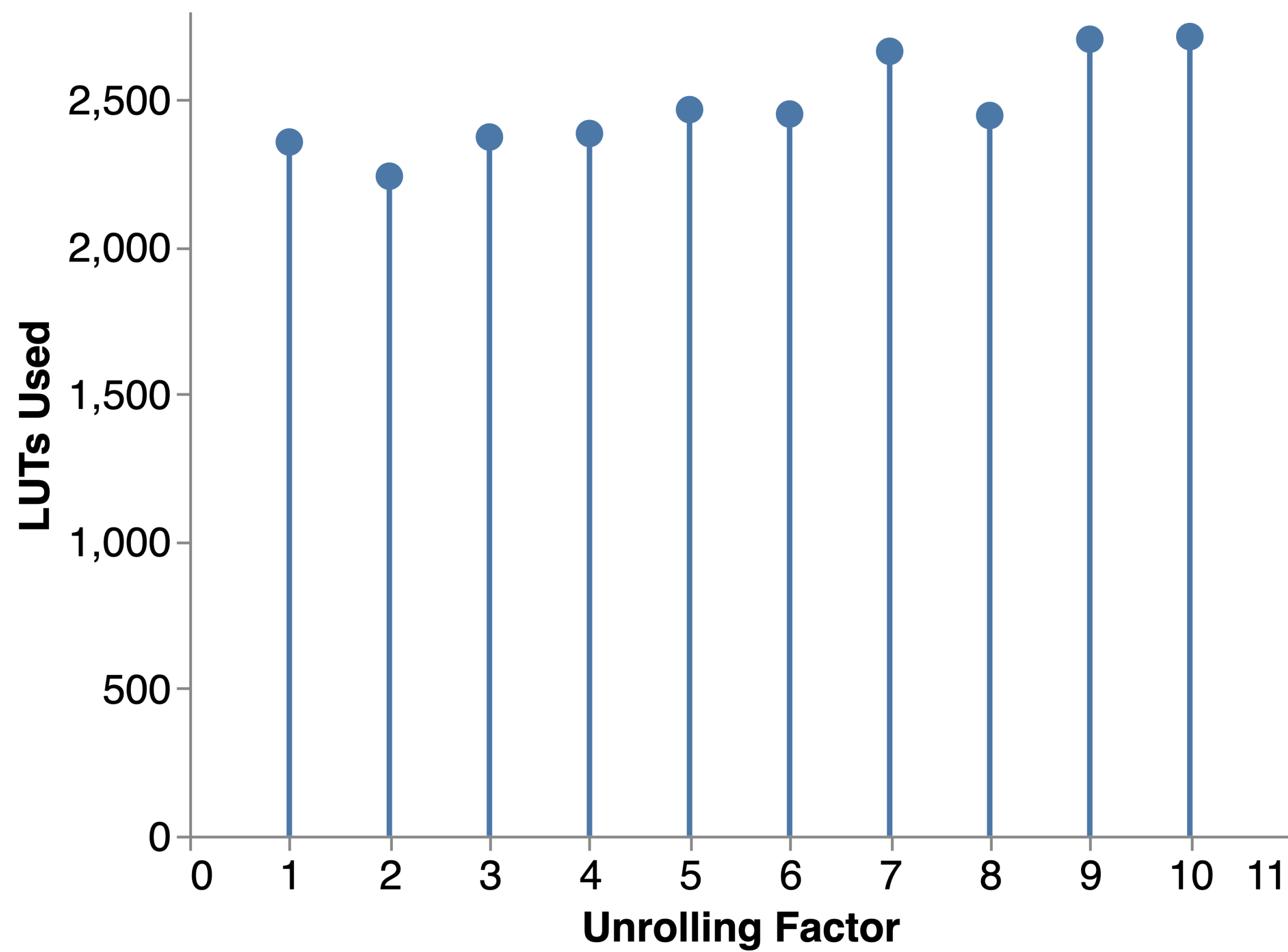


Multiplexers

Hardware

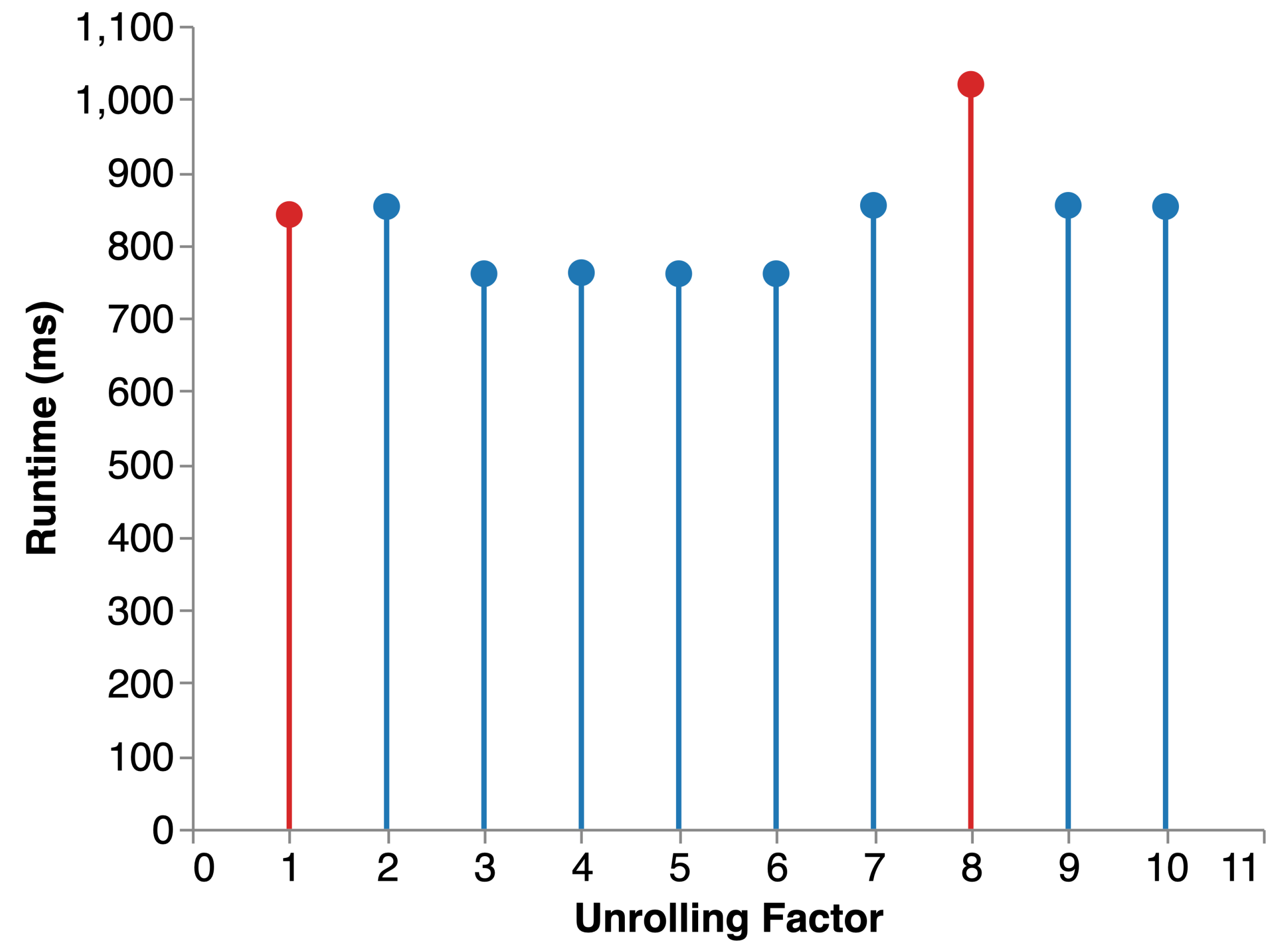
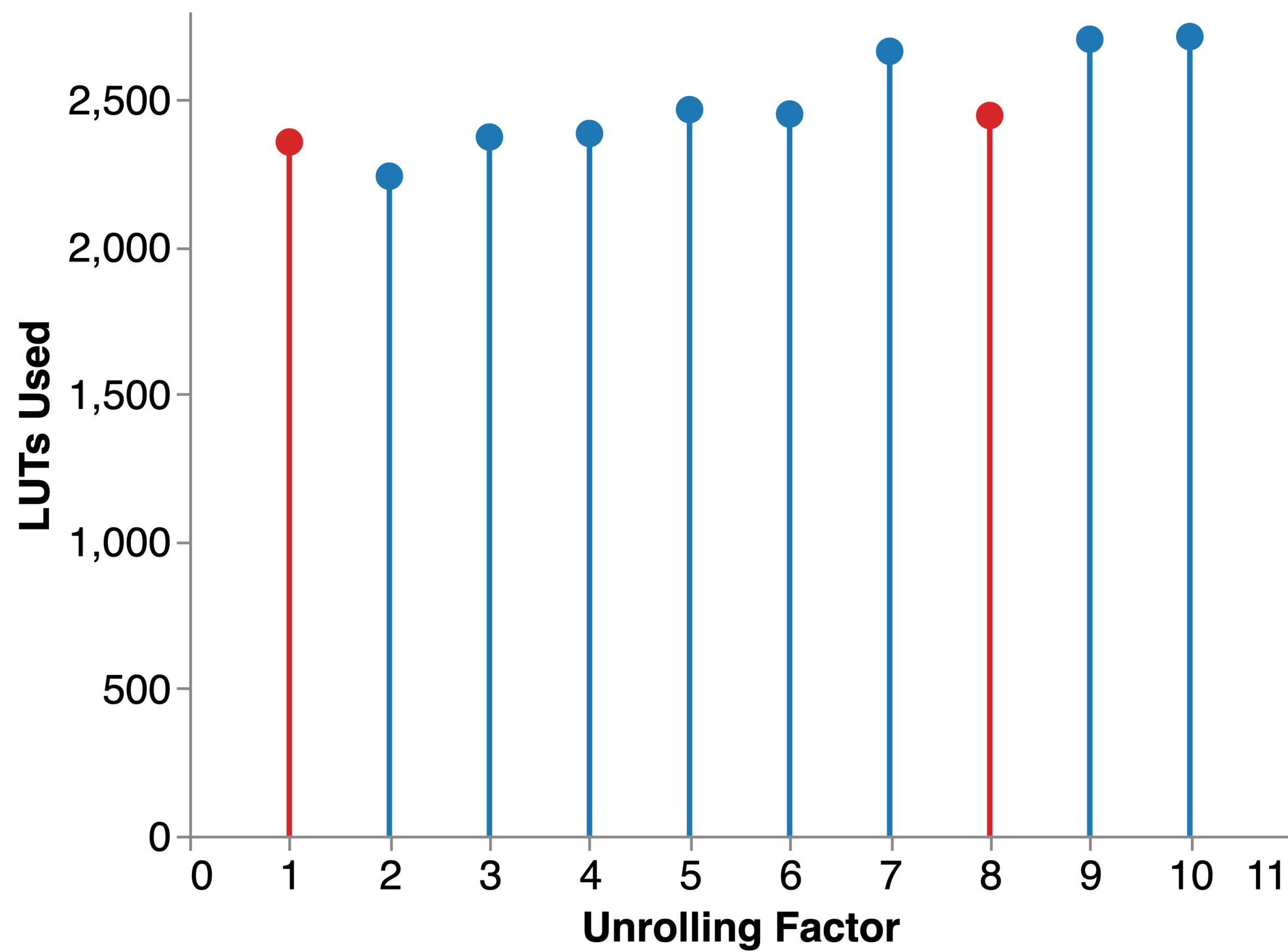


Multiplexers



Area-Latency trade-offs





Area-Latency trade-offs



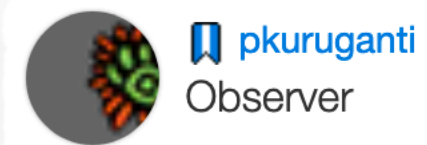
Un·pre·dict·a·ble

adjective

Behaving in a way that is not easily
predicted.



2012.1 Vivado HLS - Mutually exclusive memory access is not implemented with MUX on addresses, and reports: "@W [SCHED-69] Unable to schedule 'load' operation on array 'x' due to limited resources (II = 1) "



09-21-2017 02:38 PM

1,879 Views

Registered: 02-21-2013

SdAccle 2017.2 + KCU1500: [XOCC 204-69] Unable to schedule 'load/store' operation



02-06-2019 04:47 AM - edited 02-06-2019 04:49 AM

1,973 Views

Registered: 02-06-2019

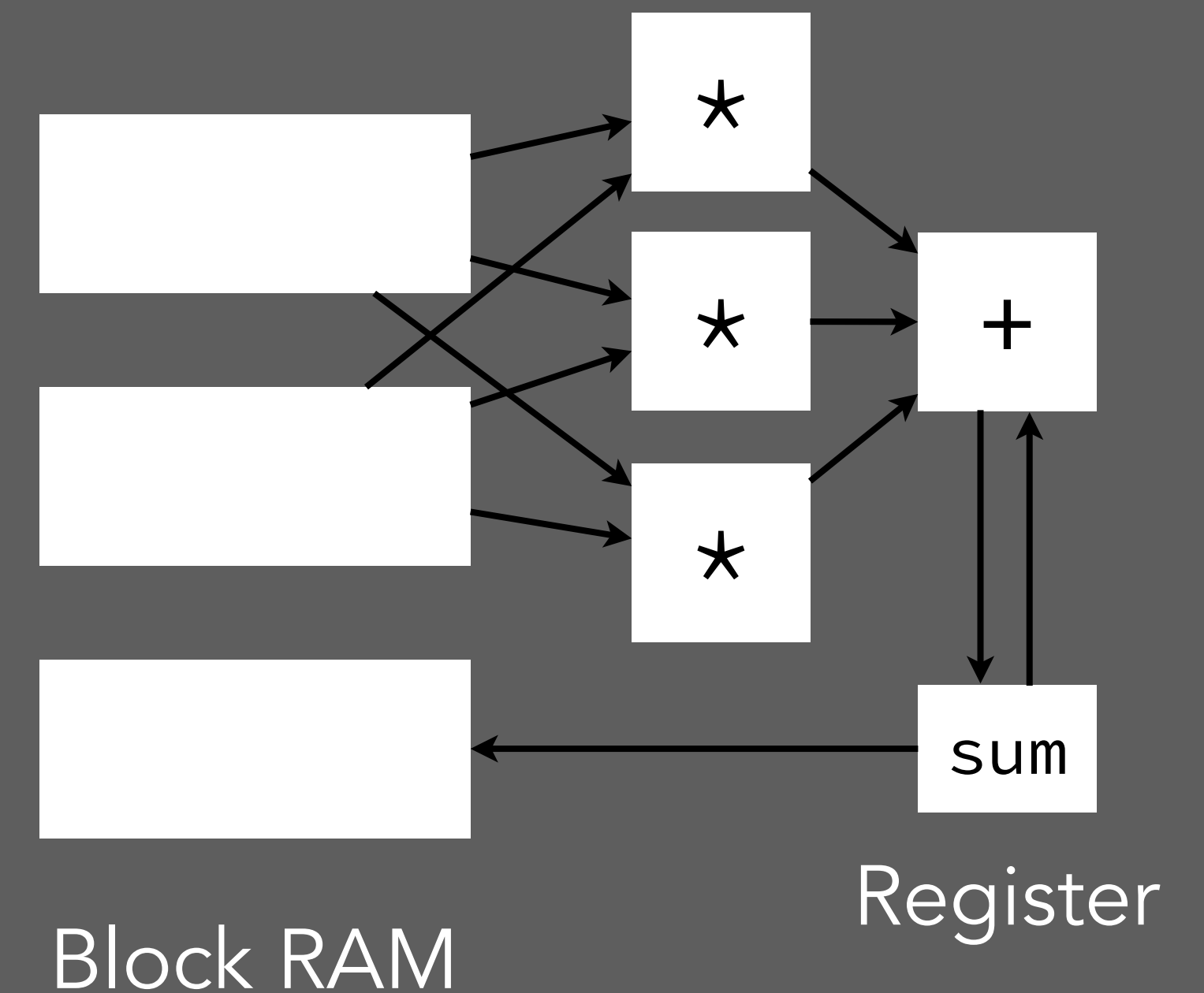
Unable to schedule 'load' operation



Super secret™ accelerator

```
int m1[512][512];
#pragma PARTITION factor=3
int m2[512][512];
#pragma PARTITION factor=3
int prod[512][512];
for (int i = 0; i < 512; i++) {
    for (int j = 0; j < 512; j++) {
        int sum = 0;
        for (int k = 0; k < 512; k++) {
            #pragma HLS UNROLL factor=3
            sum += m1[i][k] * m2[k][j];
        }
        prod[i][j] = sum;
    }
}
```

Hardware

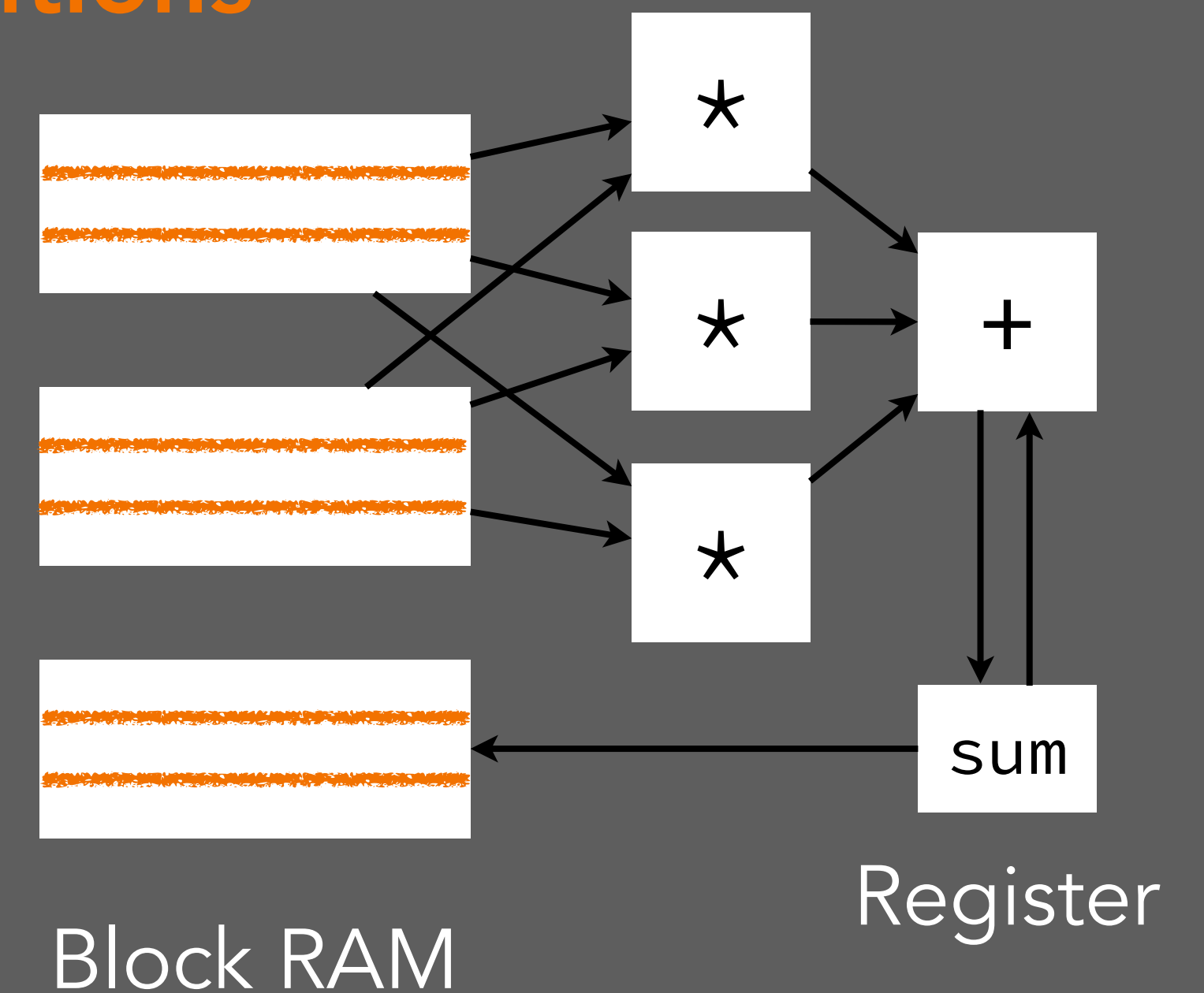


Super secret™ accelerator

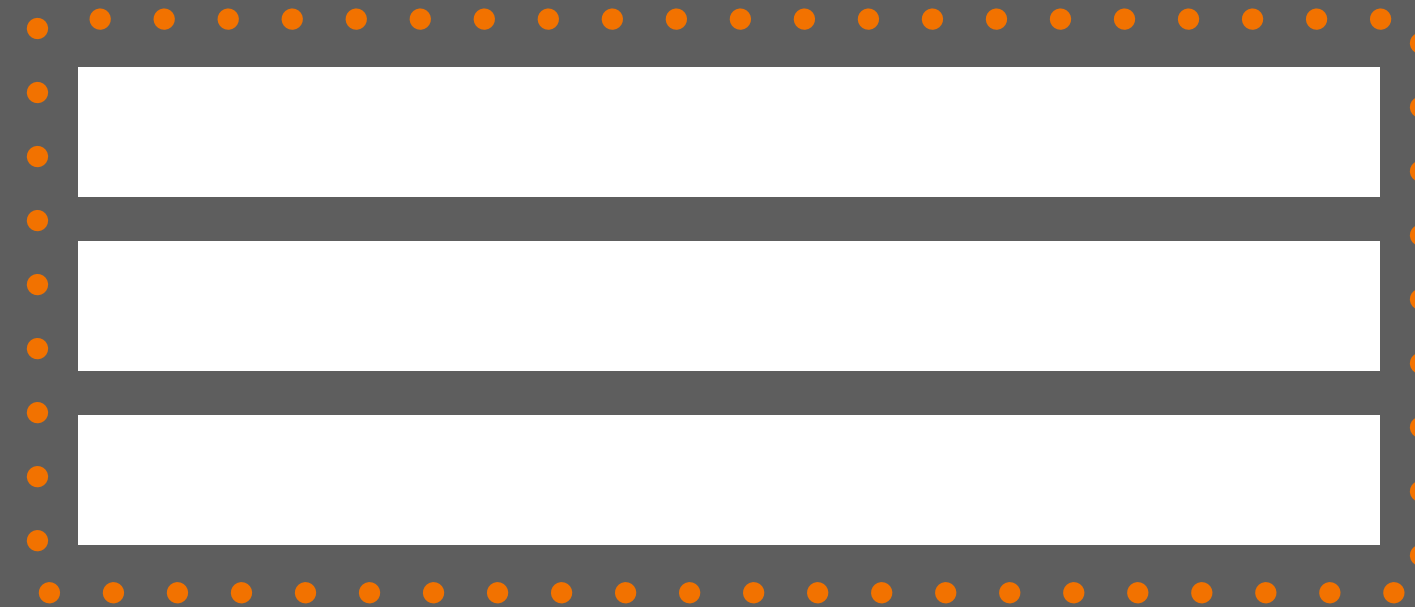
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int m1[512][512];
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int m2[512][512];
#pragma PARTITION factor=3
int prod[512][512];
for (int i = 0; i < 512; i++) {
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        int sum = 0;
        for (int k = 0; k < 512; k++) {
            #pragma HLS UNROLL factor=3
            sum += m1[i][k] * m2[k][j];
        }
        prod[i][j] = sum;
    }
}
```

Hardware

Memory Partitions



Hardware



Logical
Memory

Hardware

Physical
Partitions

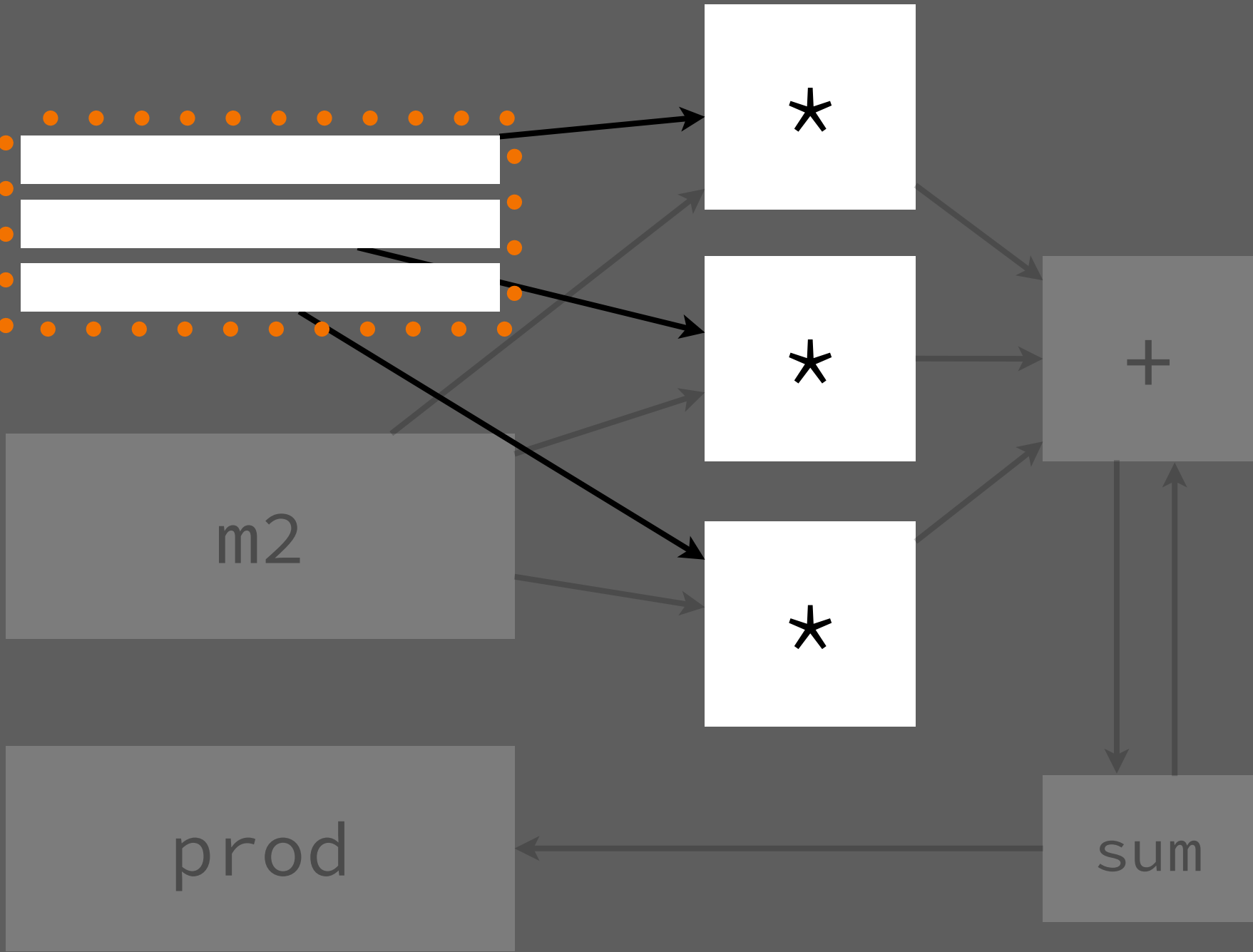


Logical
Memory

Generated hardware



Memory
Partitions

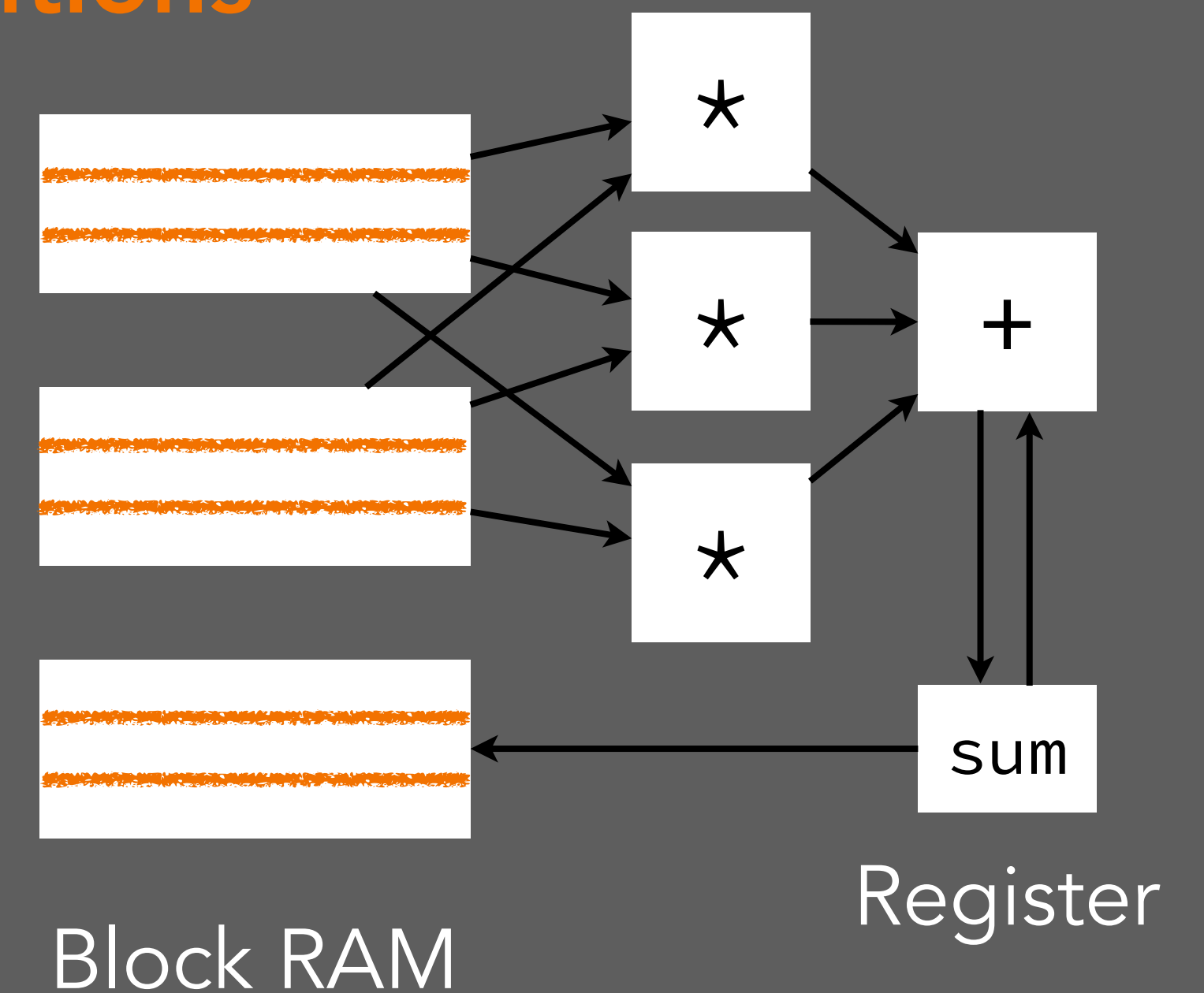


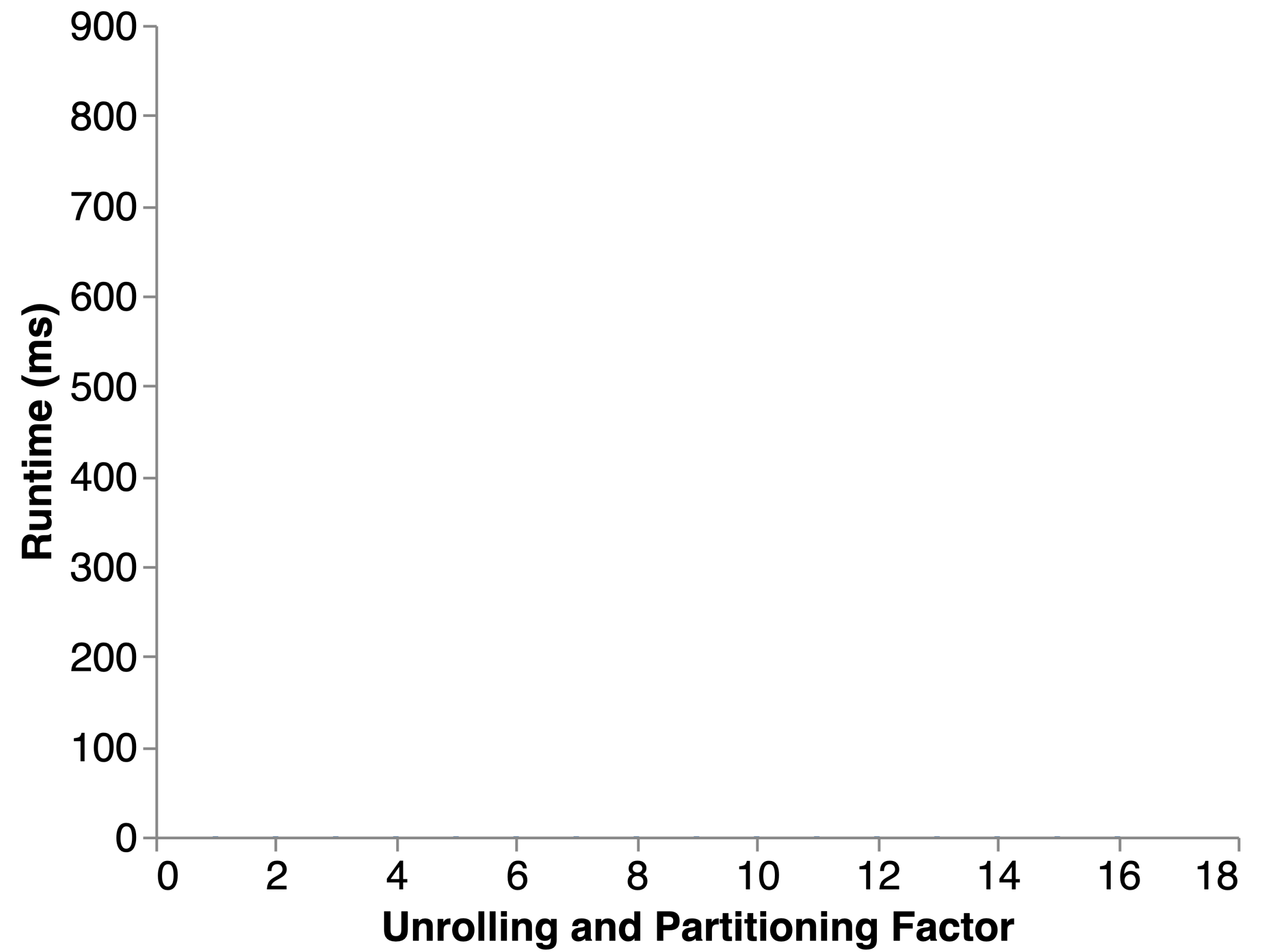
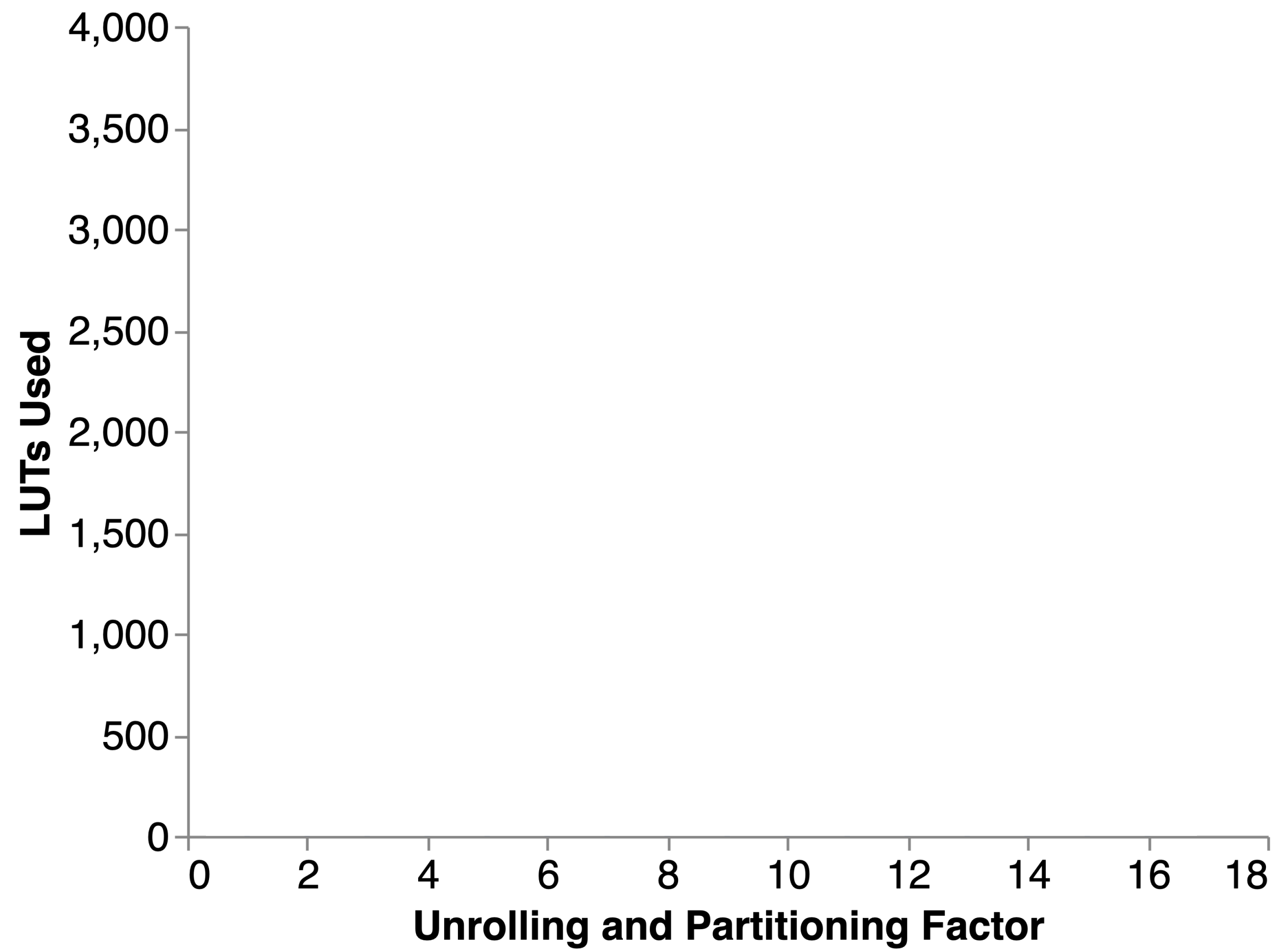
Super secret™ accelerator

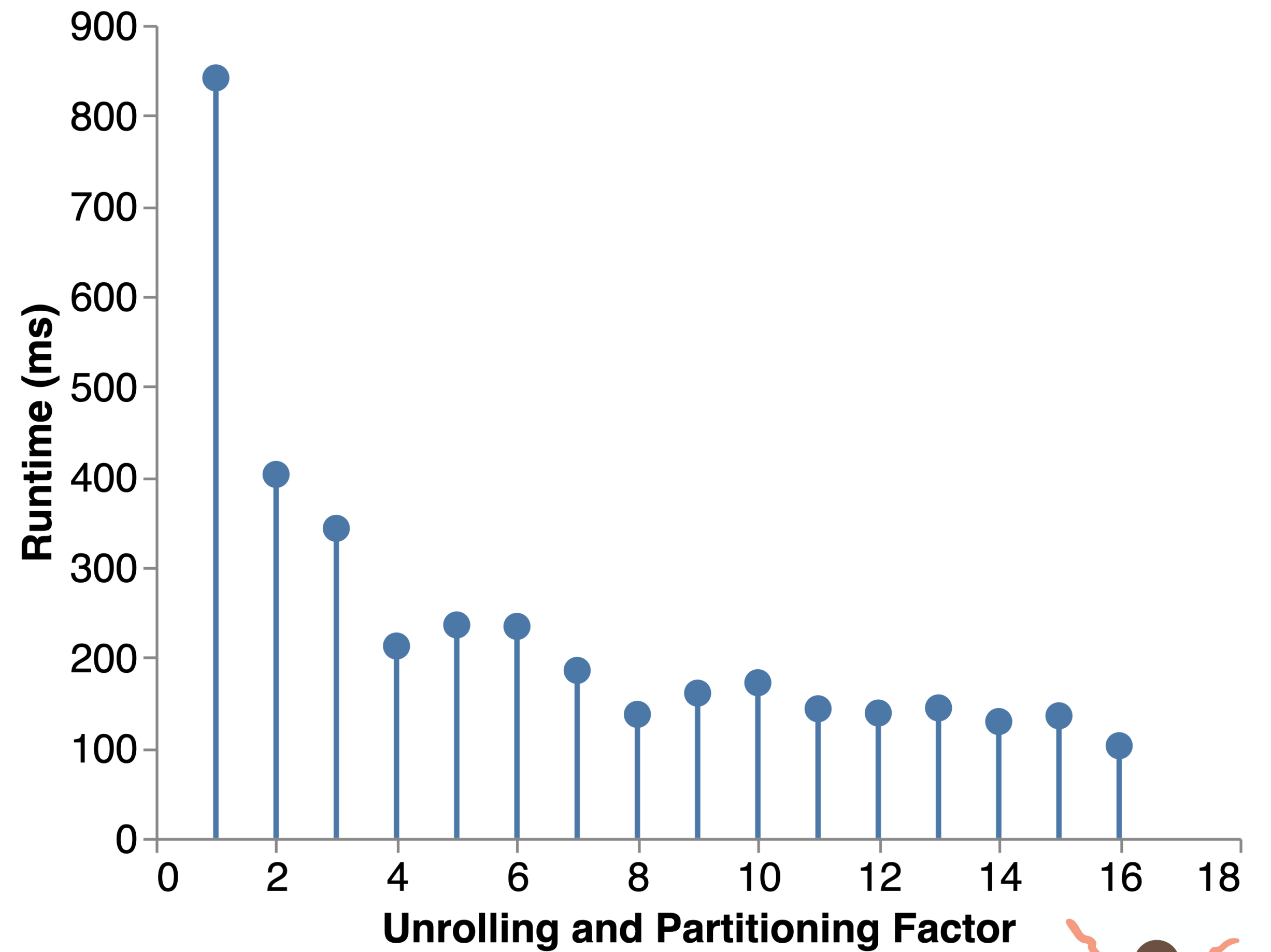
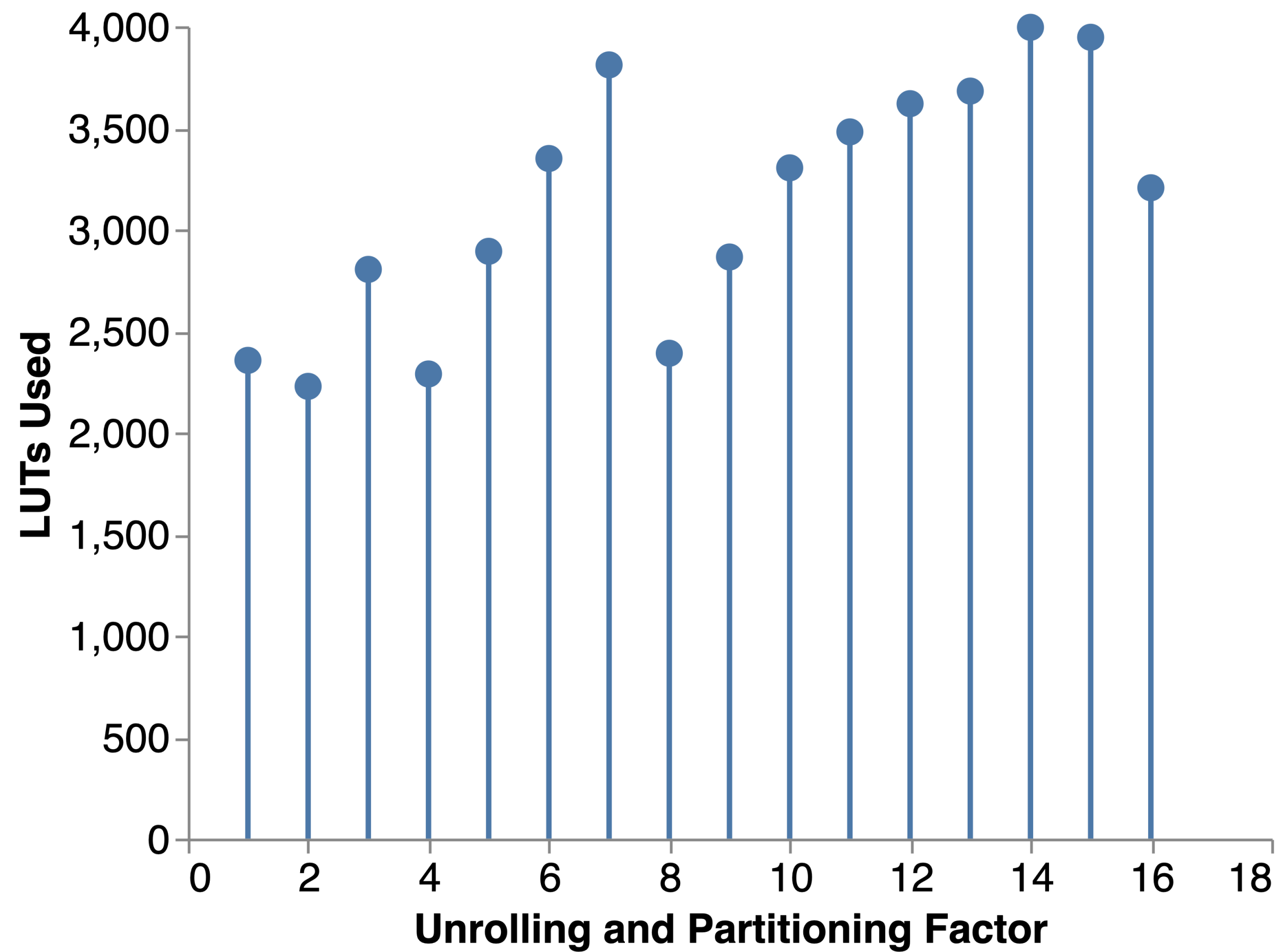
```
int m1[512][512];
#pragma PARTITION factor=3
int m2[512][512];
#pragma PARTITION factor=3
int prod[512][512];
for (int i = 0; i < 512; i++) {
    for (int j = 0; j < 512; j++) {
        int sum = 0;
        for (int k = 0; k < 512; k++) {
            #pragma HLS UNROLL factor=3
            sum += m1[i][k] * m2[k][j];
        }
        prod[i][j] = sum;
    }
}
```

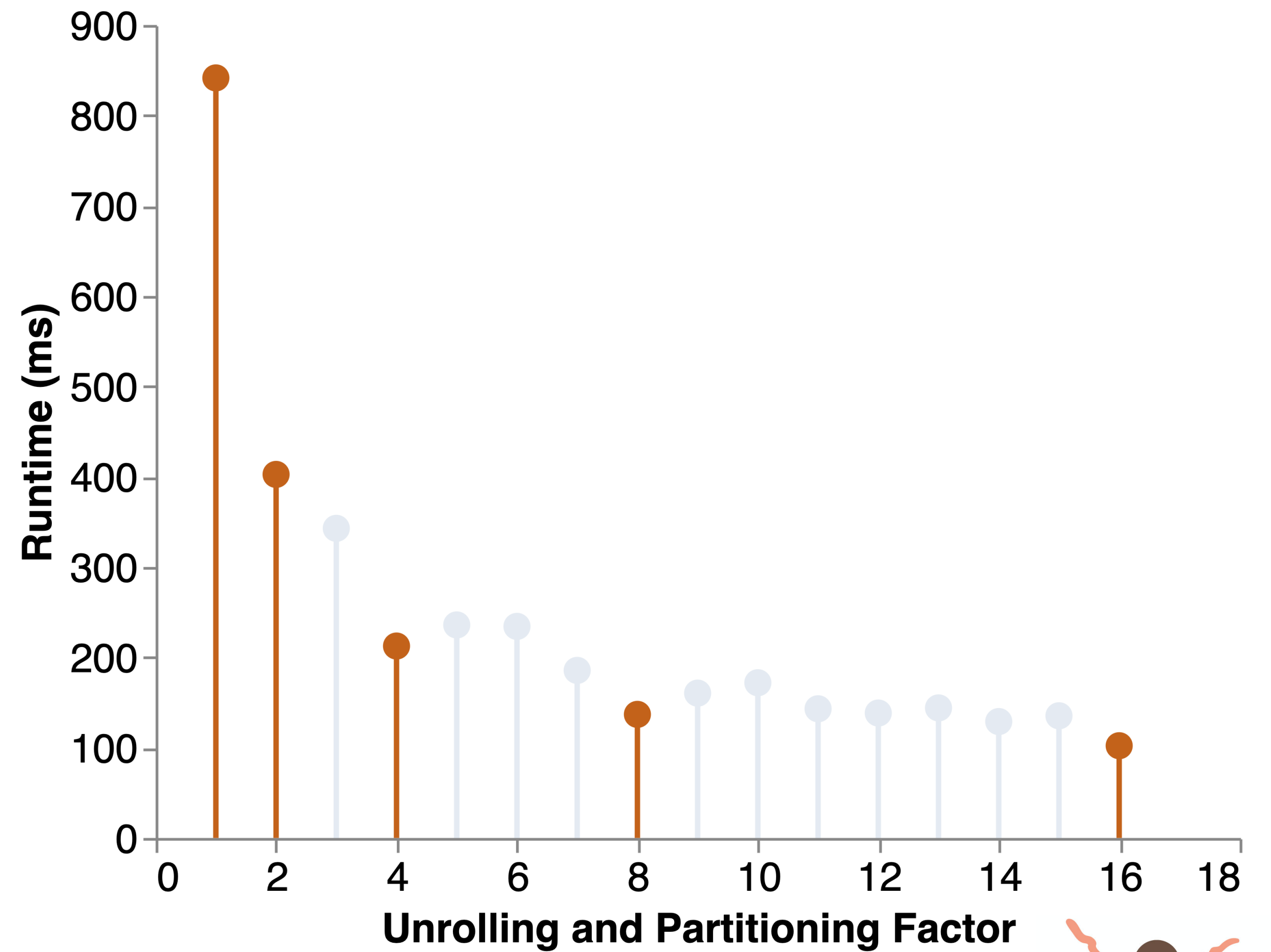
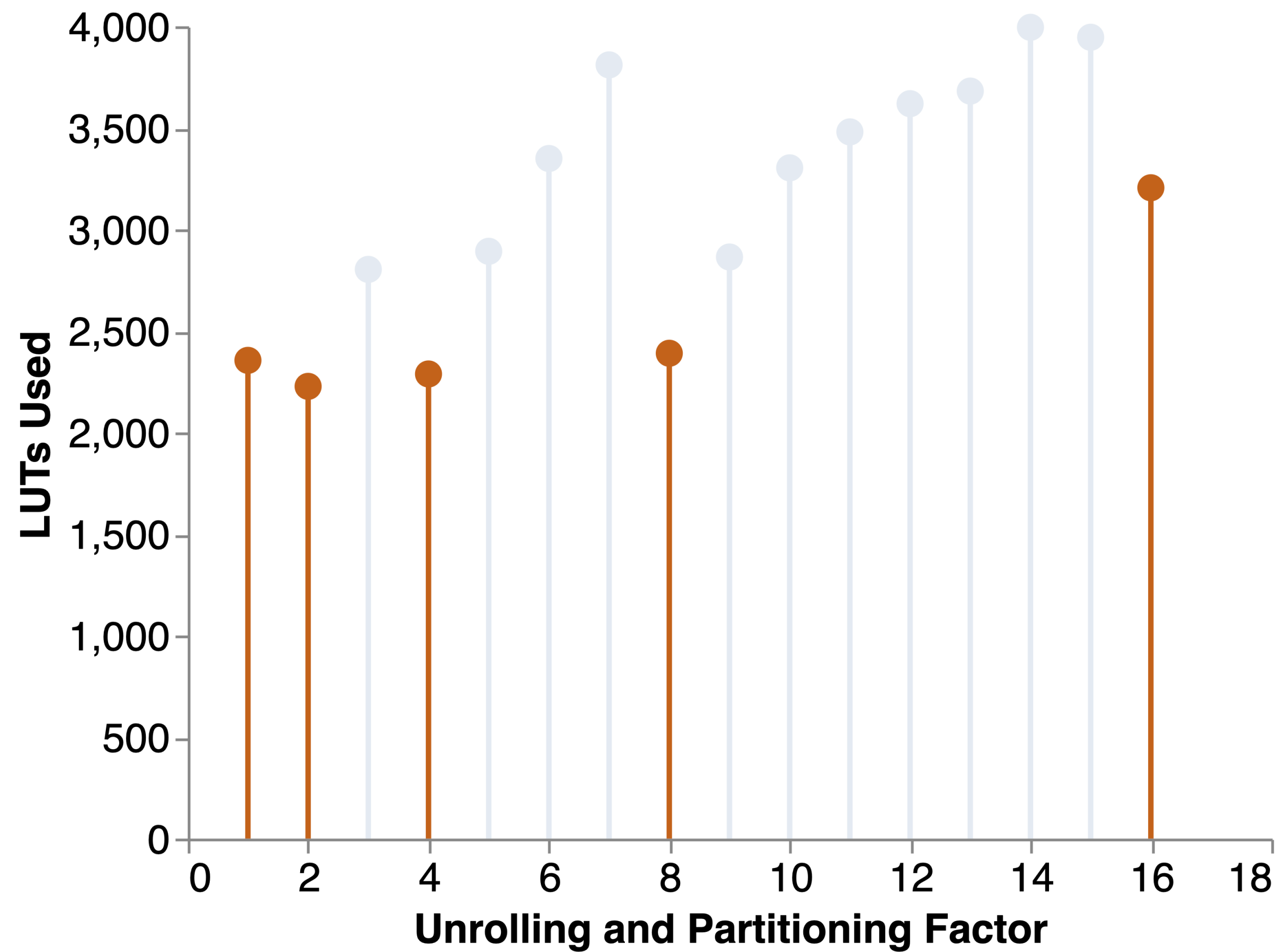
Hardware

Memory Partitions







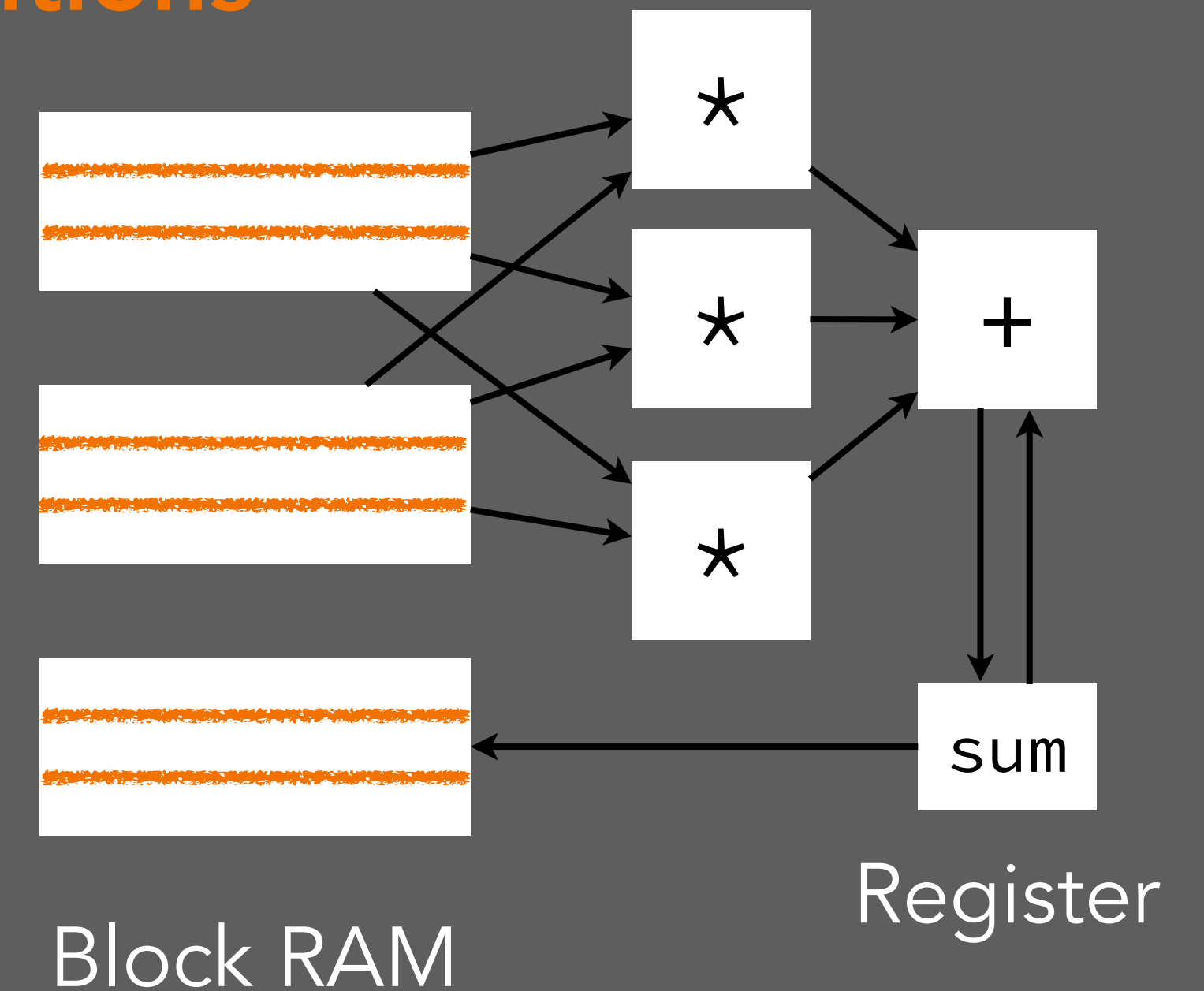


Super secret™ accelerator

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for (int i = 0; i < 512; i++) {
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        int sum = 0;
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            #pragma HLS UNROLL factor=3
            sum += m1[i][k] * m2[k][j];
        }
        prod[i][j] = sum;
    }
}
```

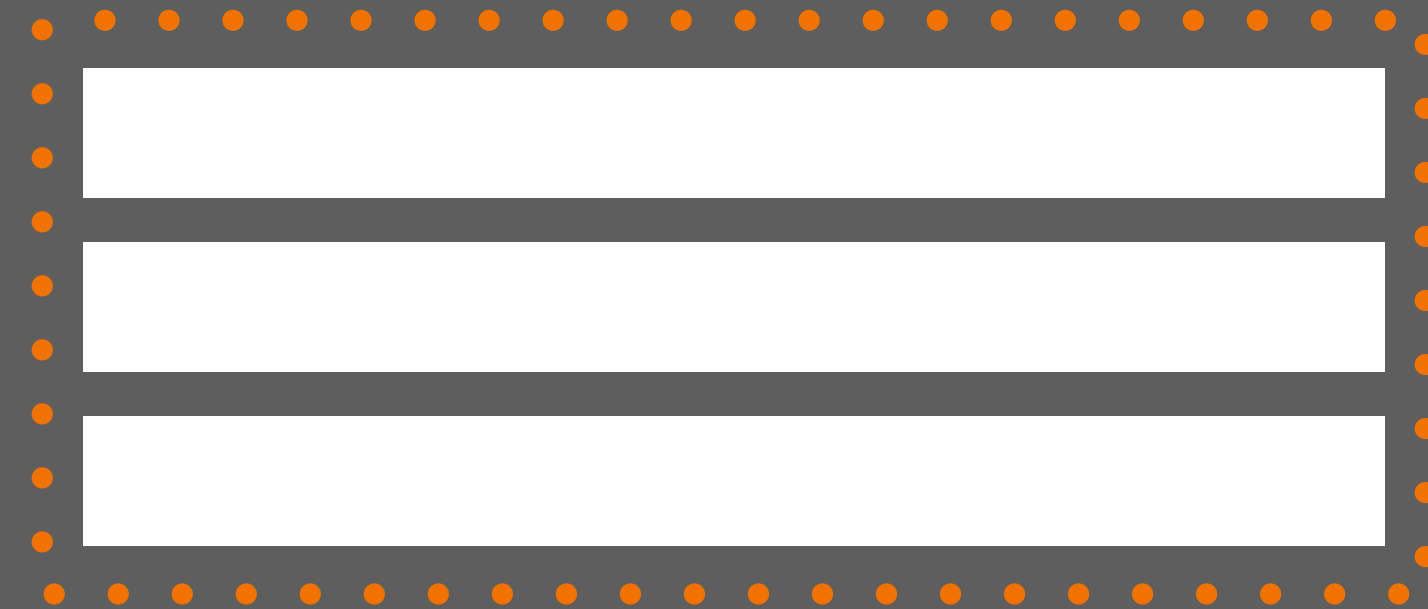
Hardware

Memory Partitions



Hardware

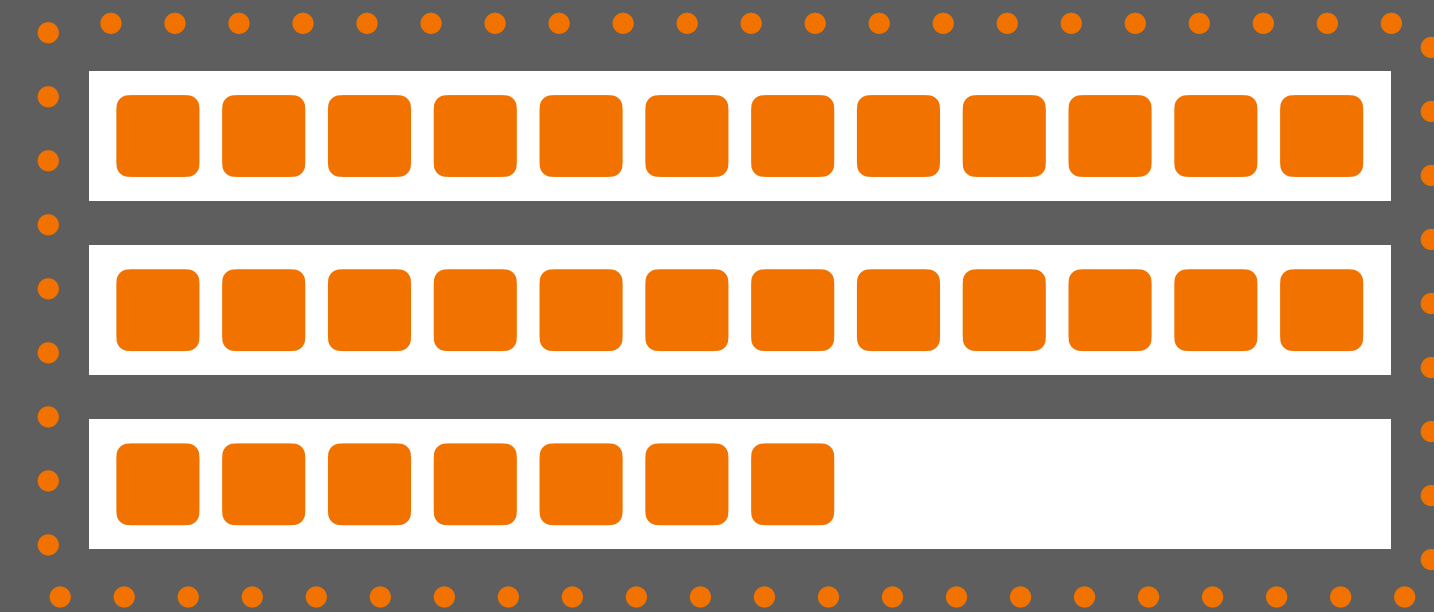
512 % 3 ≠ 0



Three horizontal white bars stacked vertically, enclosed in a dashed orange border.

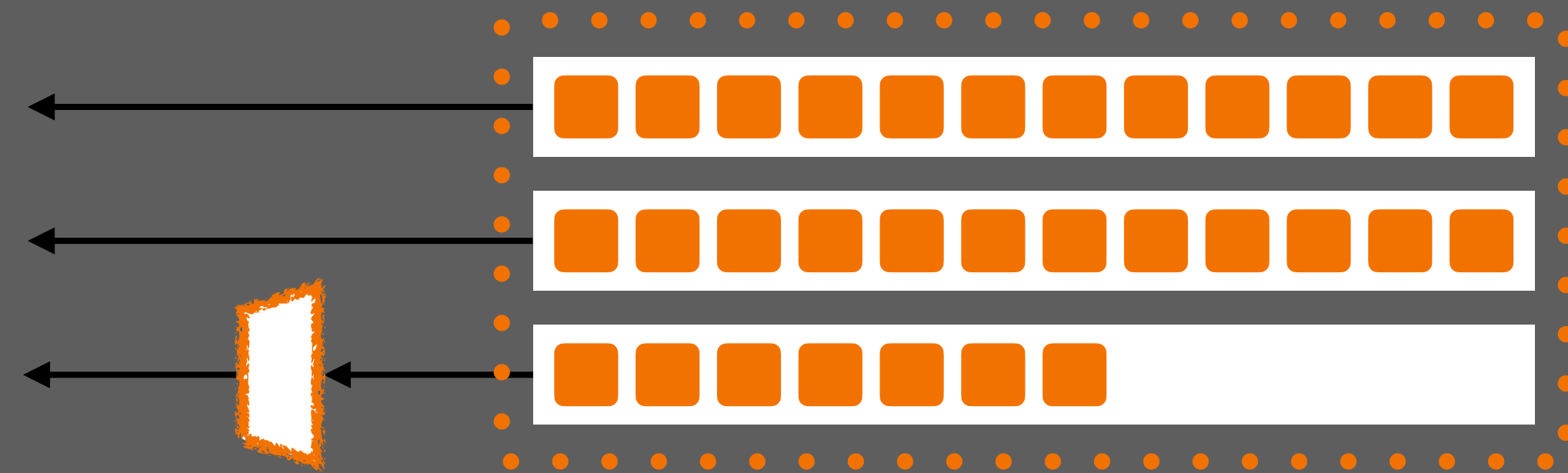
Hardware

512 % 3 ≠ 0

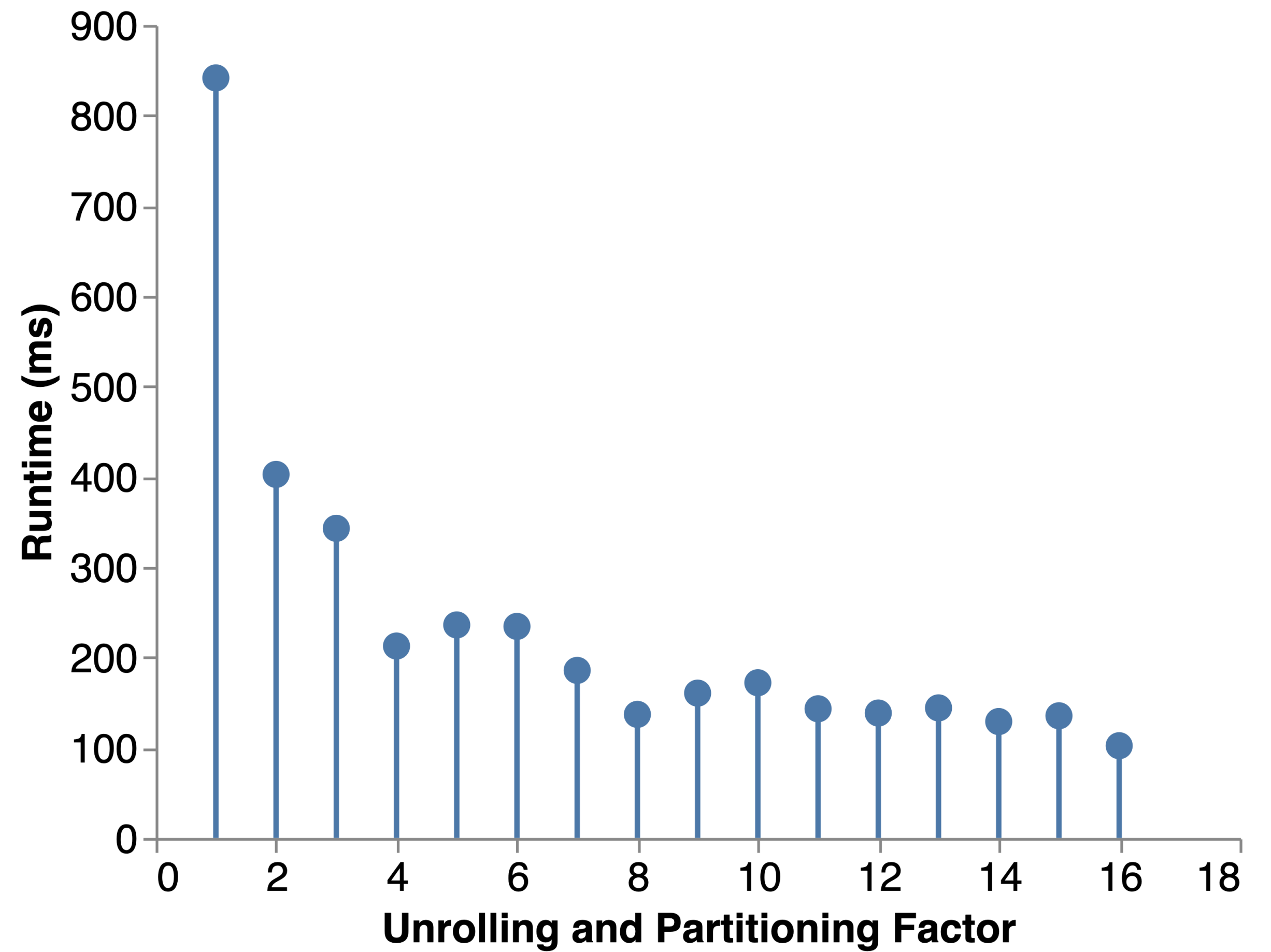
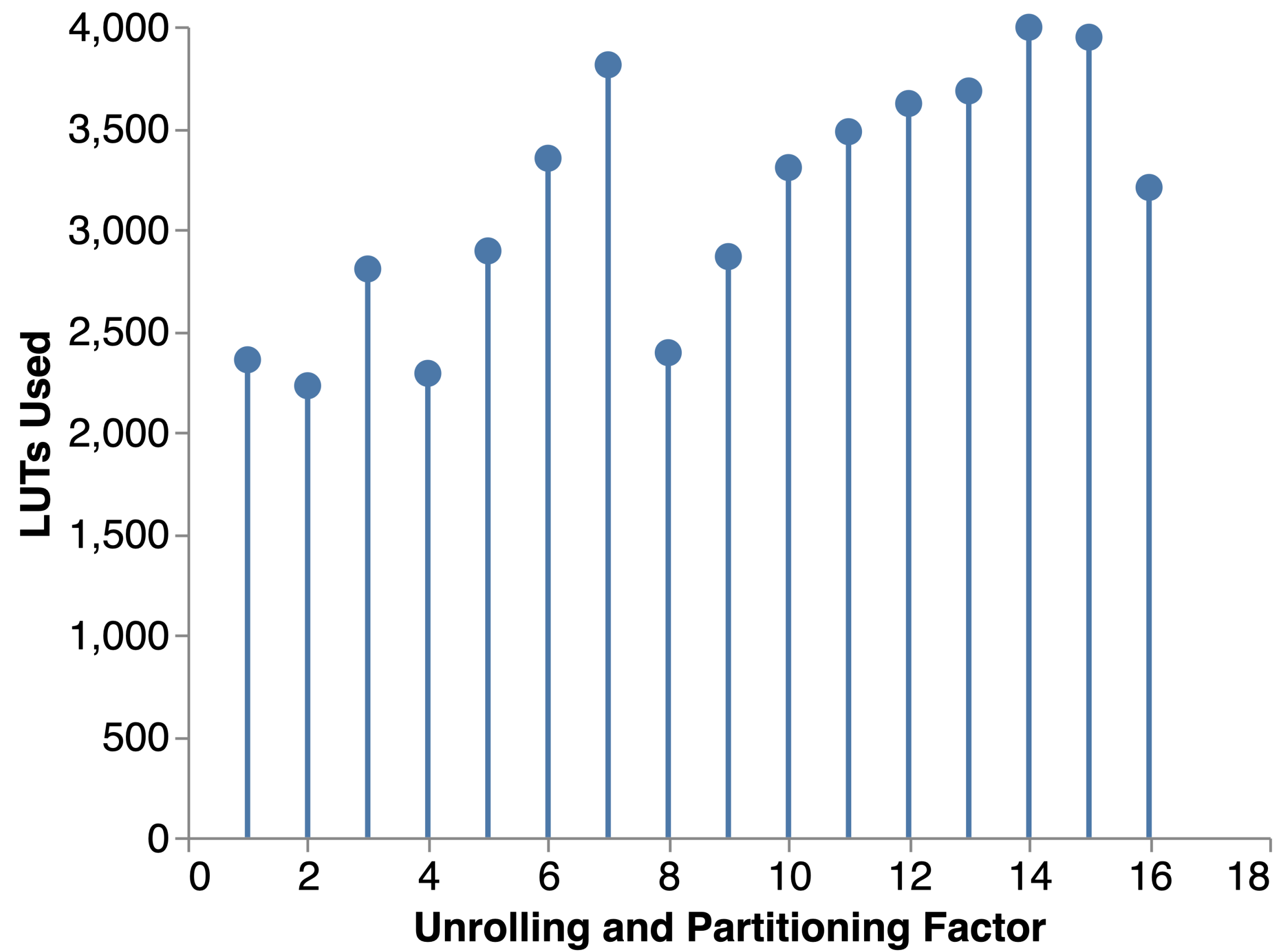


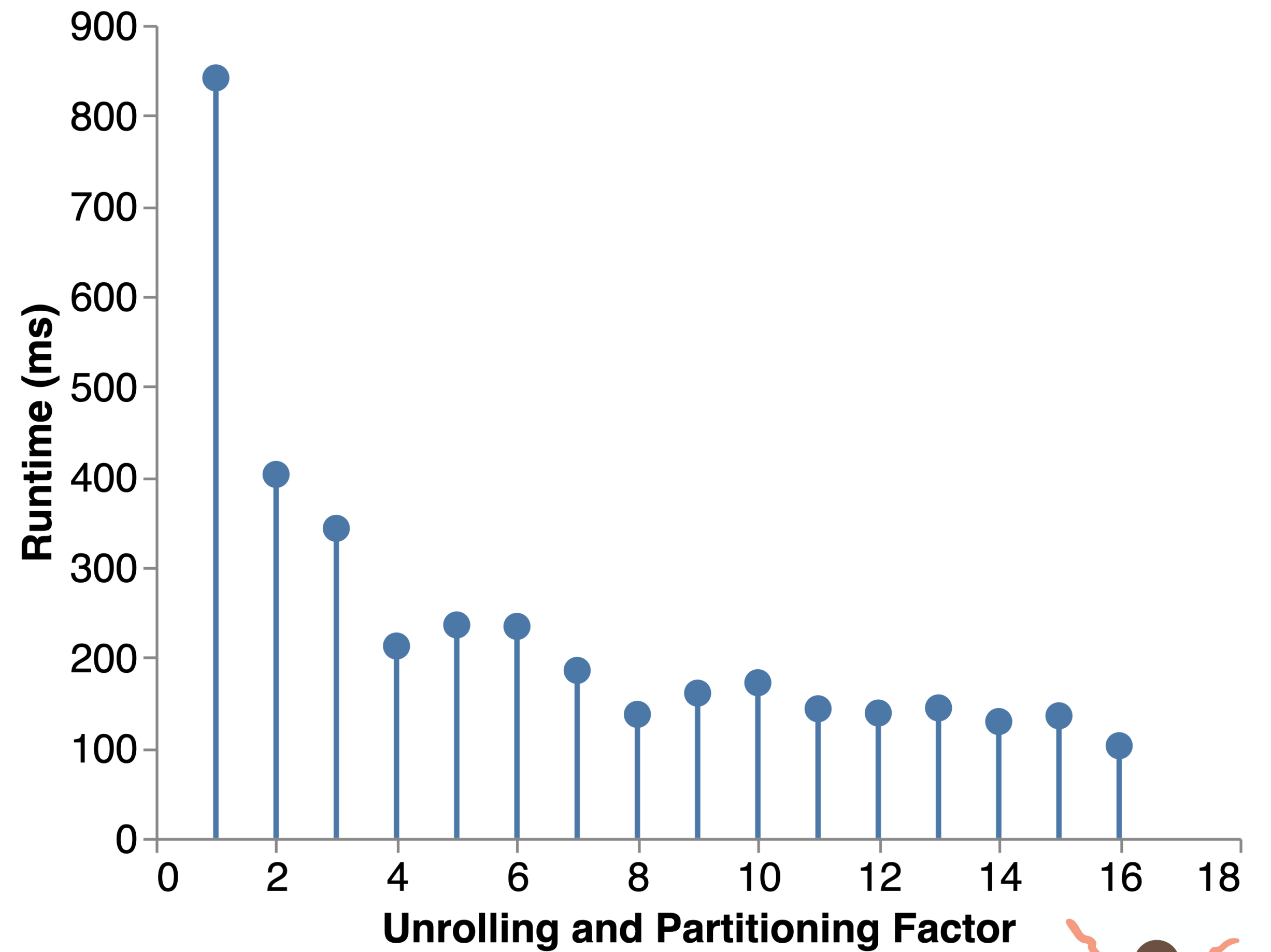
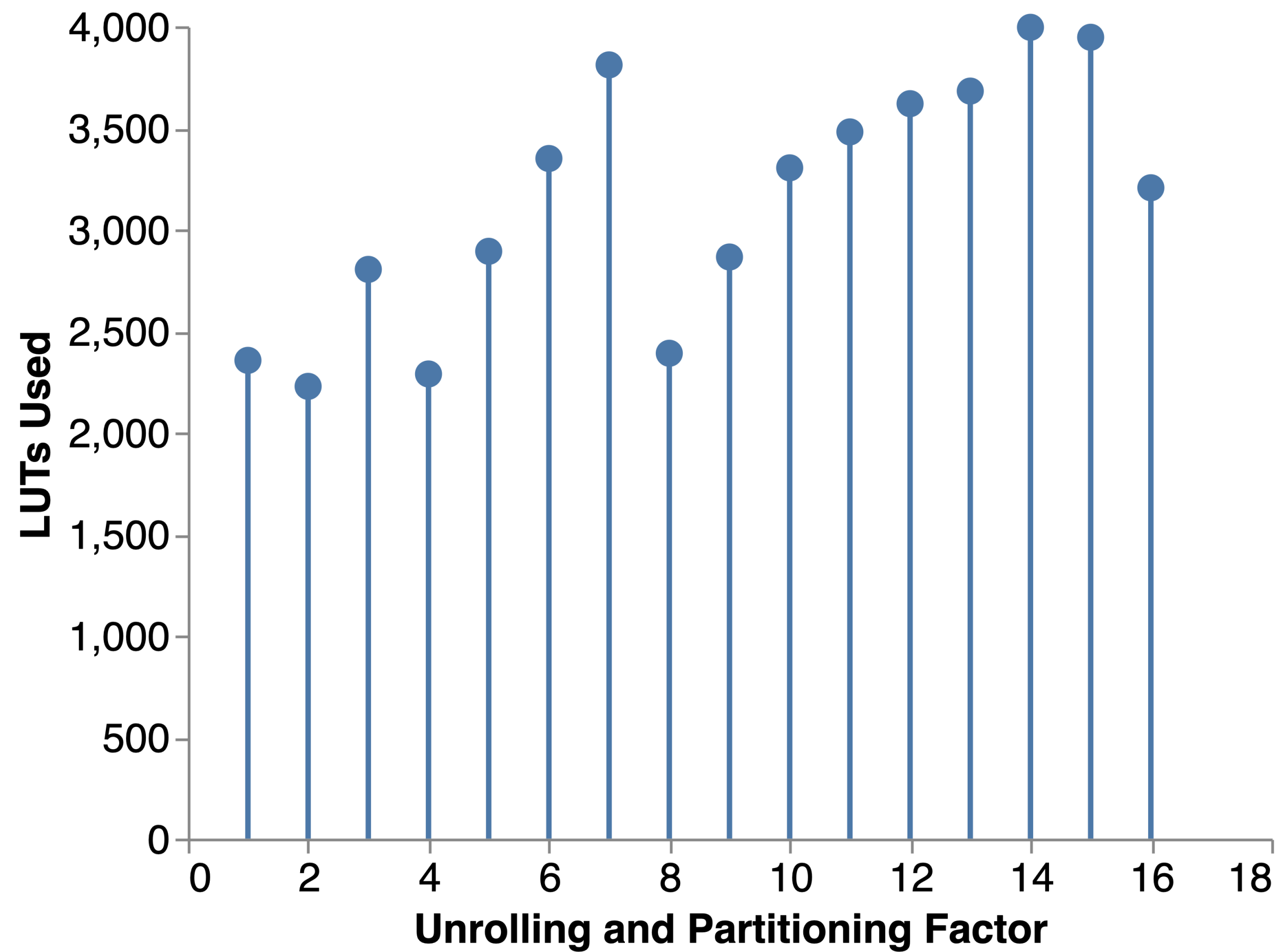
Mismatched
Partition Sizes

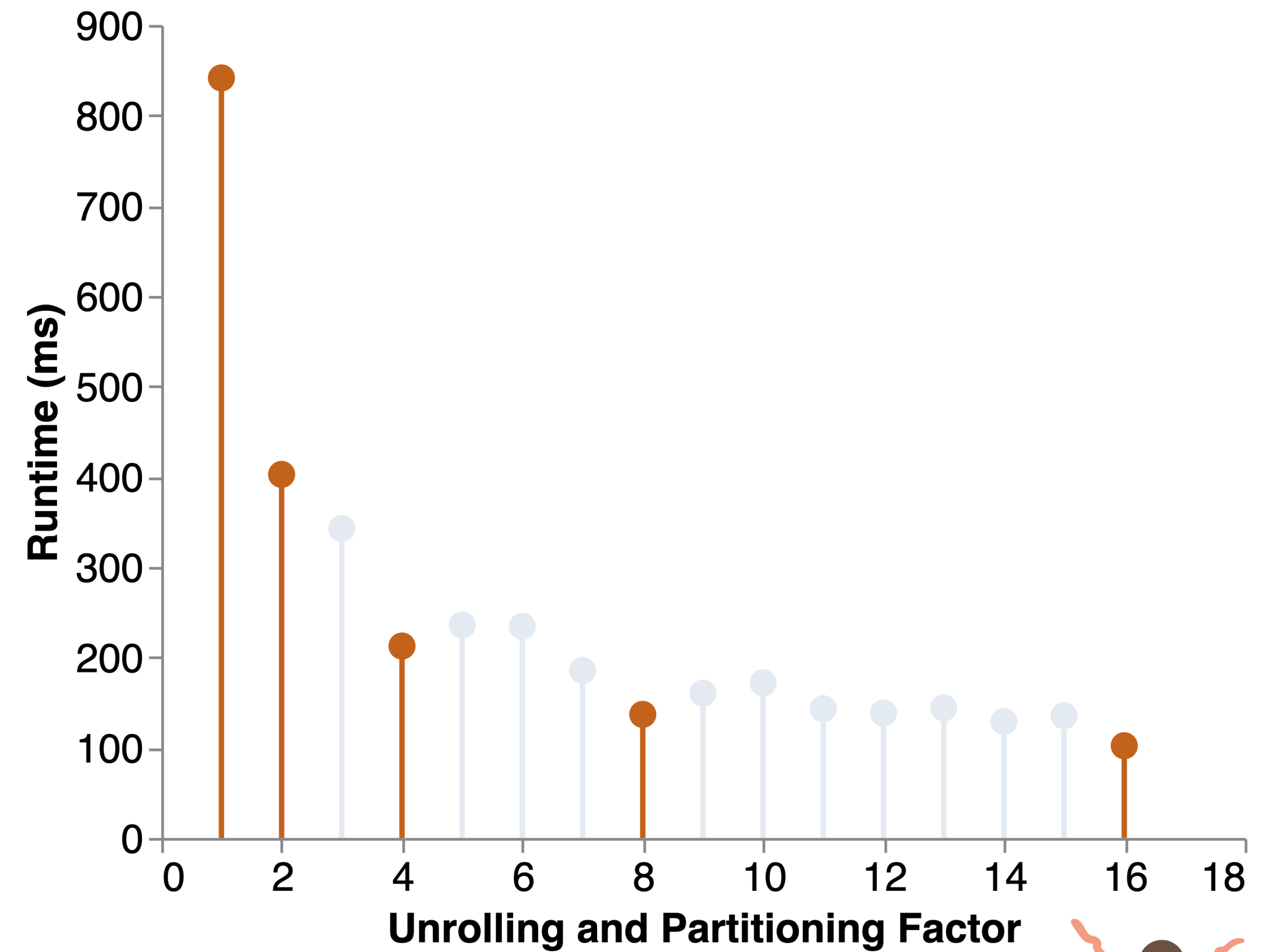
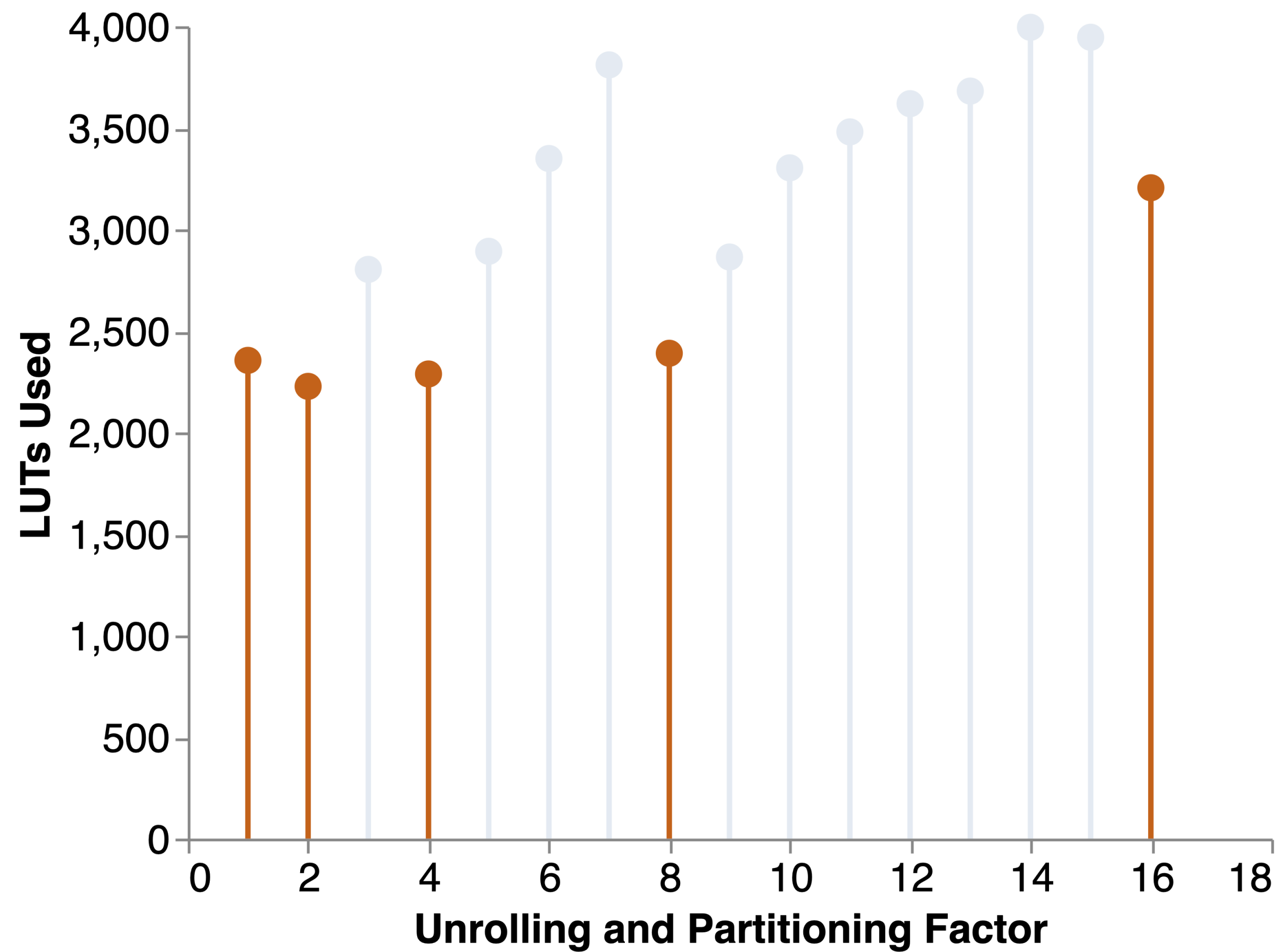
Hardware



Mismatched
Partition Sizes







HLS Really Works!

HLS Really Works! * * * * *

- * when you **unroll** designs
- * when **unrolling** and **partitioning** are aligned
- * when **partitioning** and **memory sizes** are aligned
- * when **ports times partitioning** is a factor of **unrolling**
- * when **memory accesses** are easily analyzable
- * when **reduction patterns** are easily analyzable

HLS Really Works! *

* * * * *

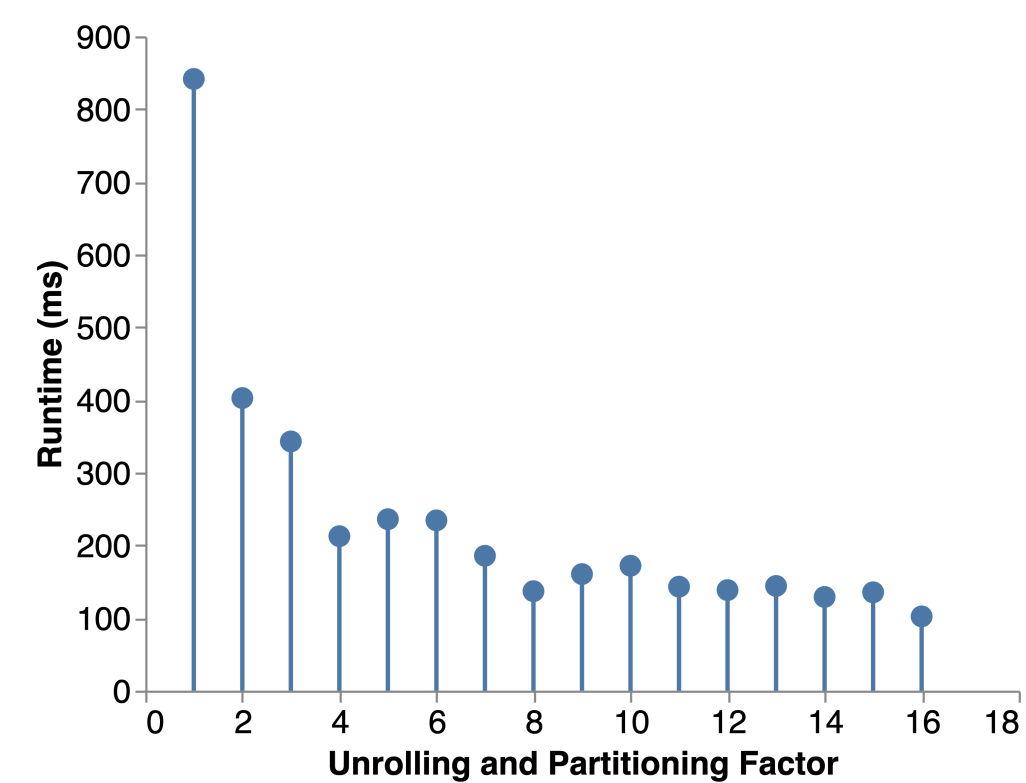
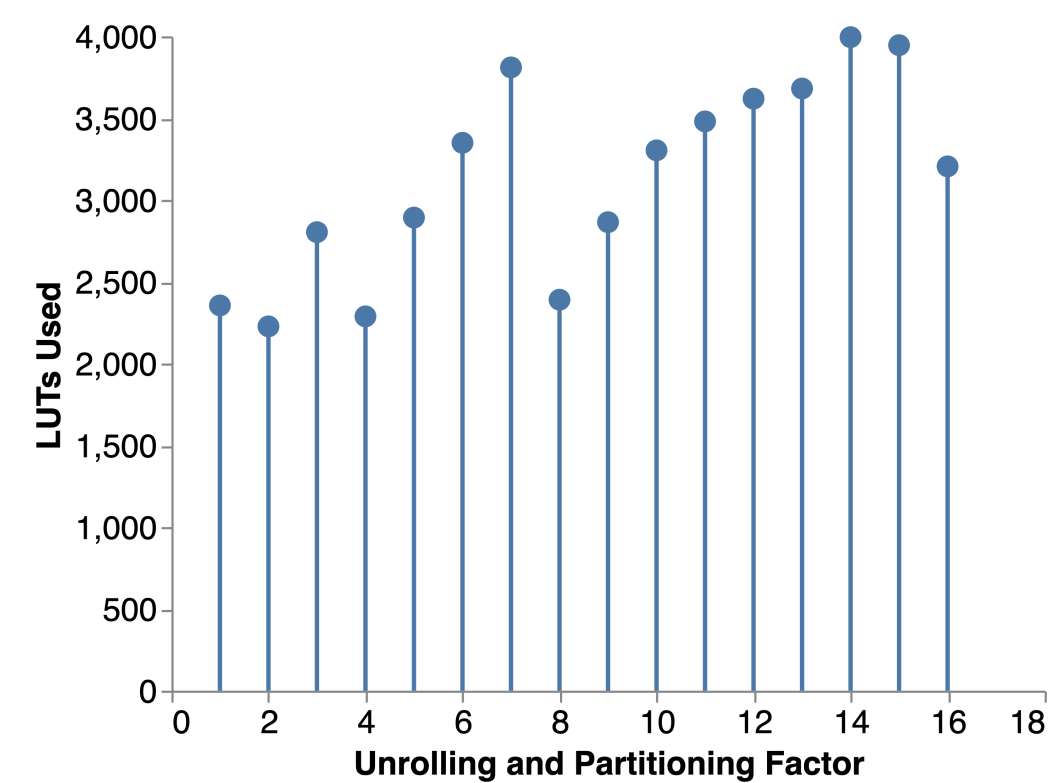


HLS Really Works! * * * * *

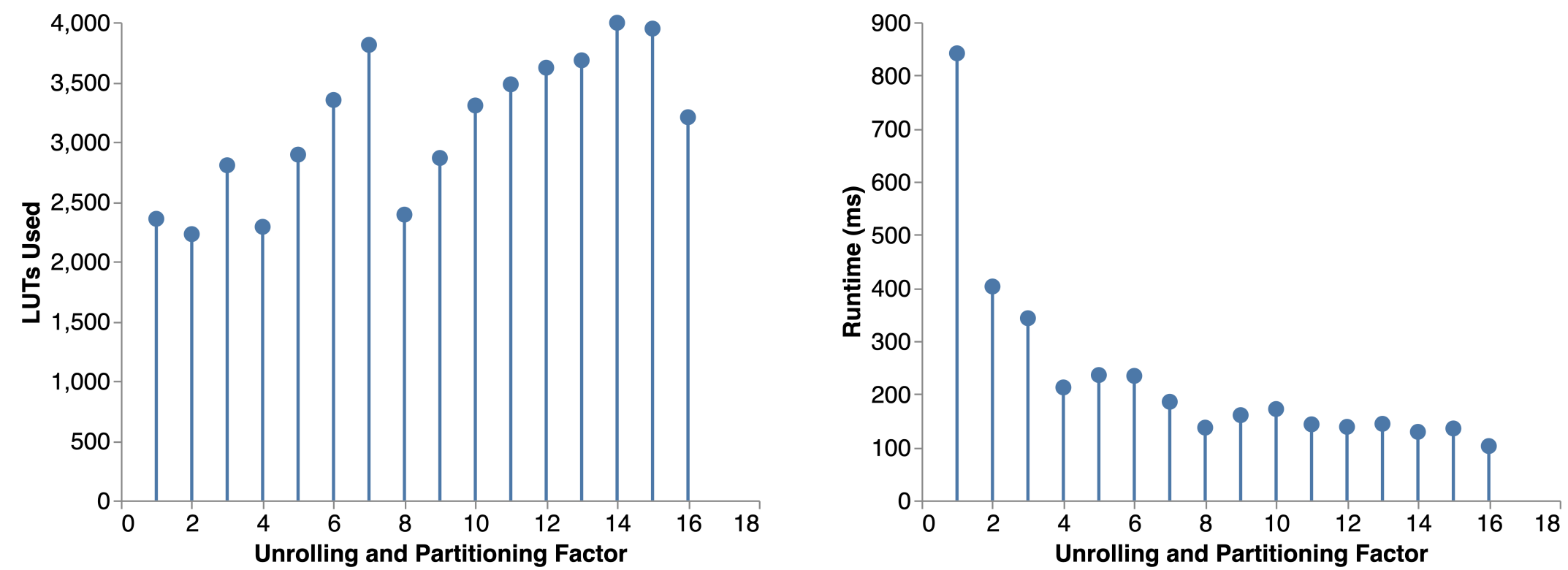
The language doesn't capture
timing and **resource constraints**.



Unpredictable

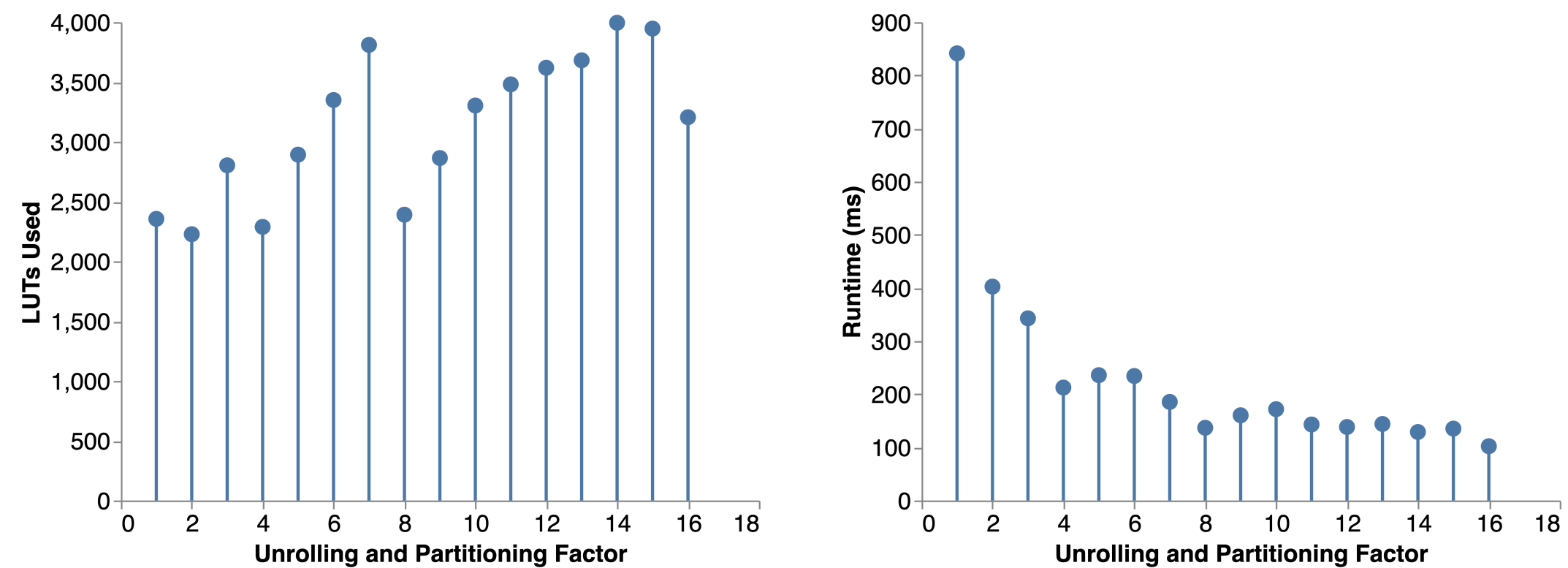


Unpredictable



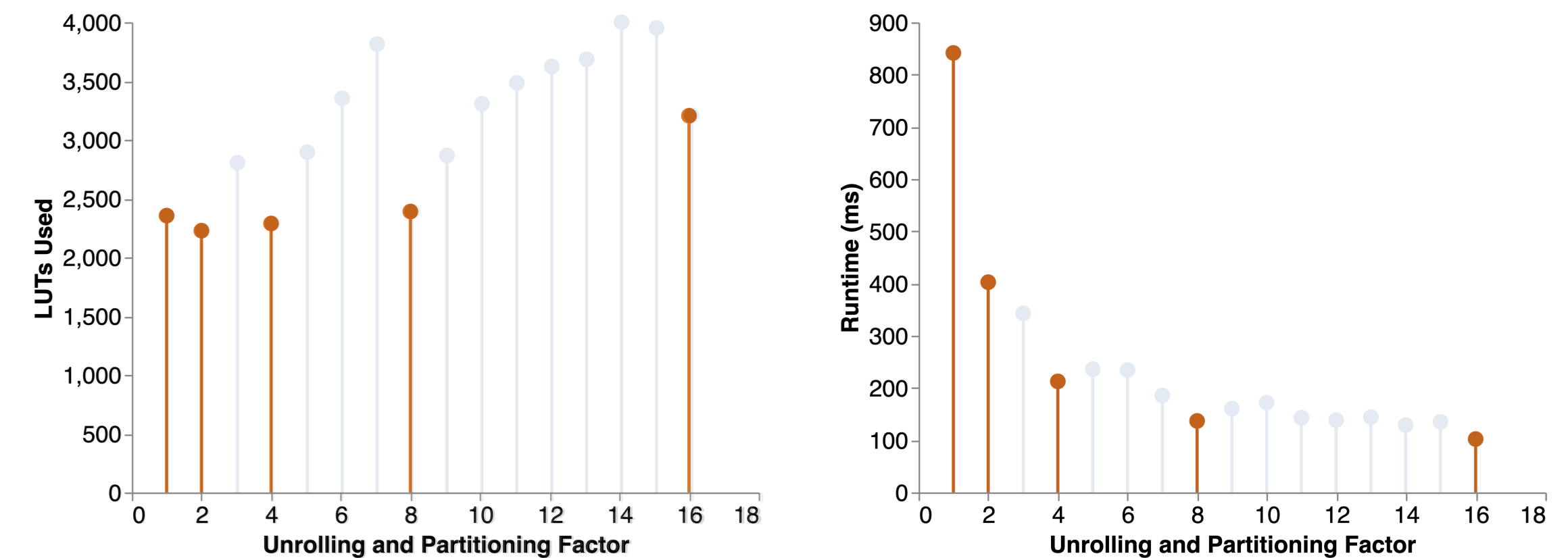
16 designs =
32 hours of
compilation

Unpredictable

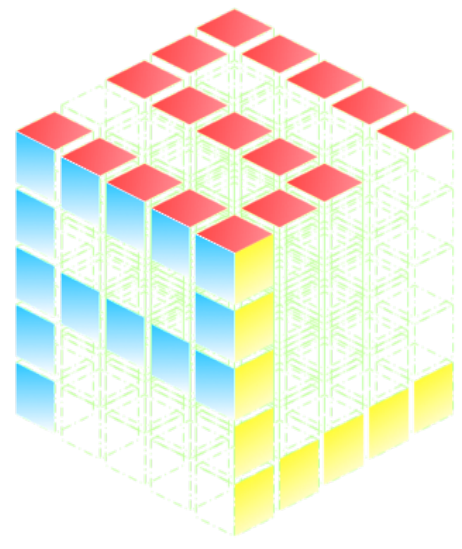


16 designs =
32 hours of
compilation

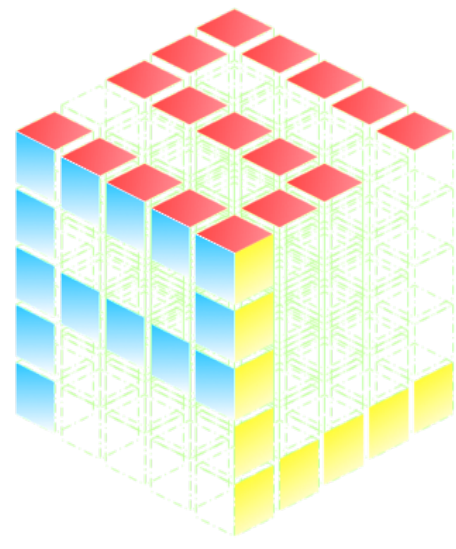
Predictable



4 designs =
8 hours of
compilation



Spatial: A Language and Compiler for Application Accelerators



Spatial: A Language and Compiler for Application Accelerators

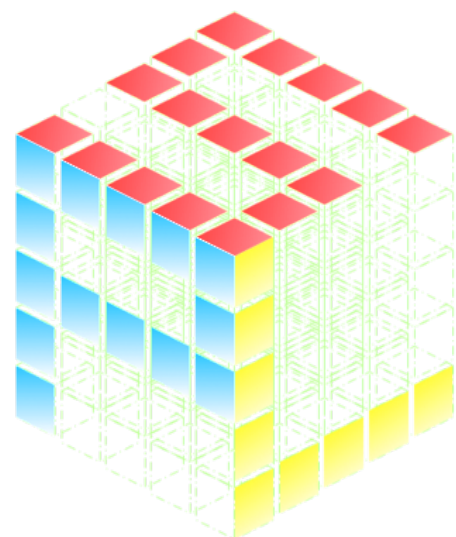
```

1  function GroupAccesses:
2    input:  $A \rightarrow$  set of reads or writes to  $m$ 
3     $G = \emptyset$  set of sets of compatible accesses
4    for all accesses  $a$  in  $A$ :
5      for all sets of accesses  $g$  in  $G$ :
6        if IComp( $a, a'$ ) for all  $a'$  in  $g$  then
7          add  $a$  to  $g$ 
8          break
9        else add  $\{a\}$  to  $G$ 
10   return  $G$ 
11 end function
12
13 function ConfigureMemory:
14   input:  $A_r \rightarrow$  set of reads of  $m$ 
15   input:  $A_w \rightarrow$  set of writes to  $m$ 
16    $G_r = \text{GroupAccesses}(A_r)$ 
17    $G_w = \text{GroupAccesses}(A_w)$ 
18    $I = \emptyset$  set of memory instances
19   for all read sets  $R$  in  $G_r$ :
20      $I_r = \{R\}$ 
21      $I_w = \text{ReachingWrites}(G_w, I_r)$ 
22      $i = \text{BankAndBuffer}(I_r, I_w)$ 
23     for each  $inst$  in  $I$ :
24        $I'_r = \text{ReadSets}[inst] + R$ 
25        $I'_w = \text{ReachingWrites}(G_w, I'_r)$ 
26       if OComp( $A_1, A_2$ )  $\forall A_1 \neq A_2 \in (G_w \cup I'_r)$  then:
27          $i' = \text{BankAndBuffer}(I'_r, I'_w)$ 
28         if Cost( $i'$ ) < Cost( $i$ ) + Cost( $inst$ ) then:
29           remove  $inst$  from  $I$ 
30           add  $i'$  to  $I$ 
31         break
32     if  $i$  has not been merged then add  $i$  to  $I$ 
33   return  $I$ 
34 end function

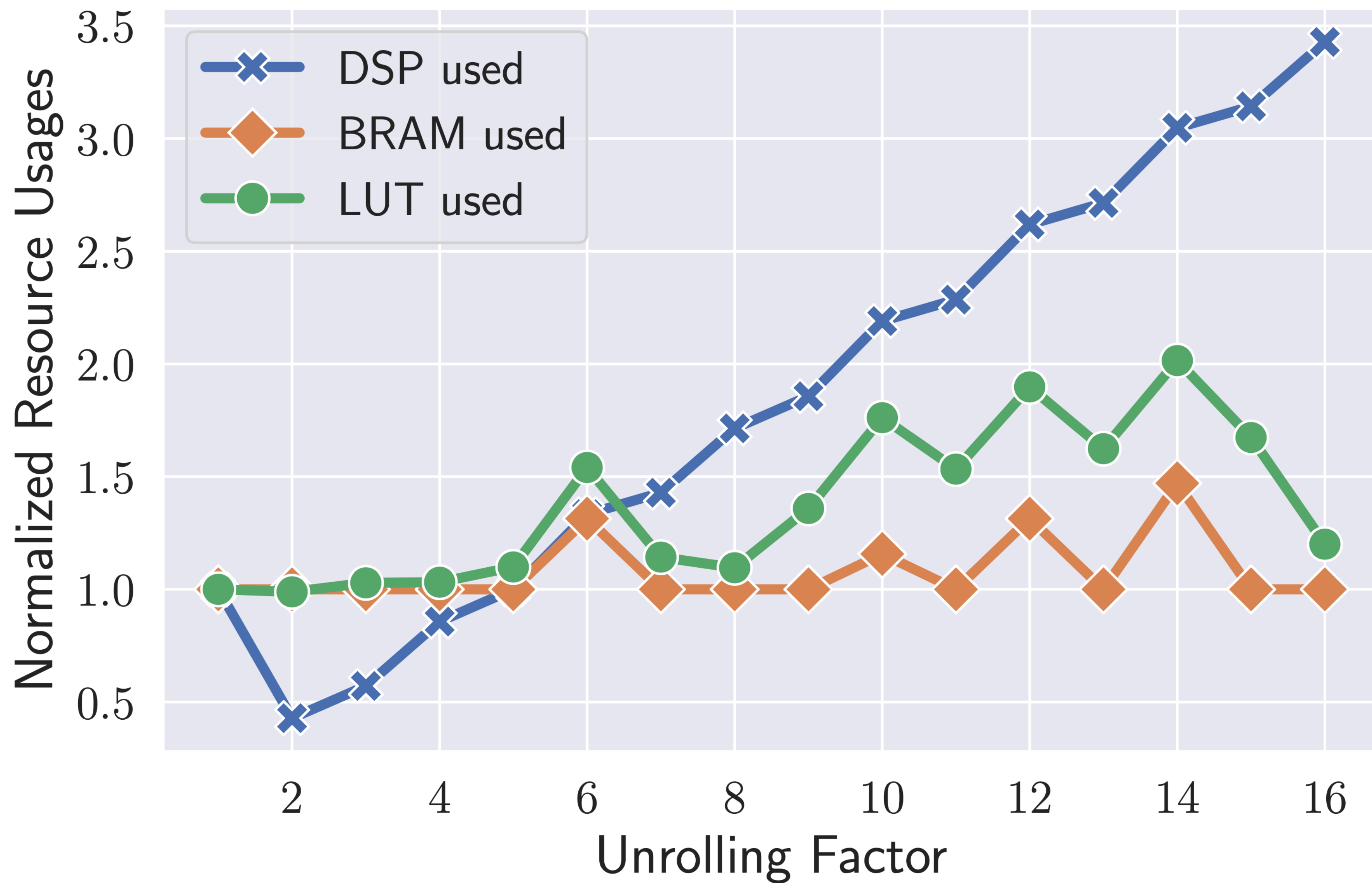
```

Figure 7. Banking and buffering algorithm for calculating instances of on-chip memory m .

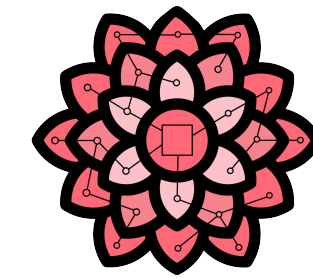
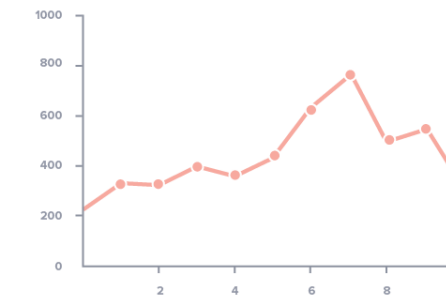
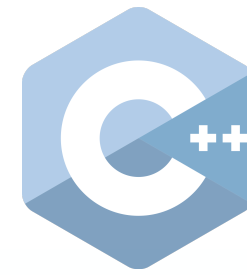
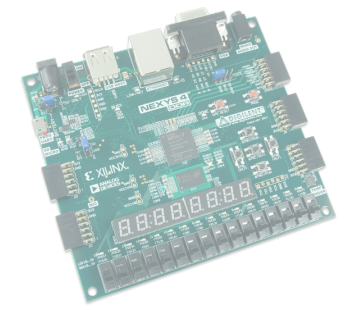
Automatic memory
partitioning



Spatial: A Language and Compiler for Application Accelerators

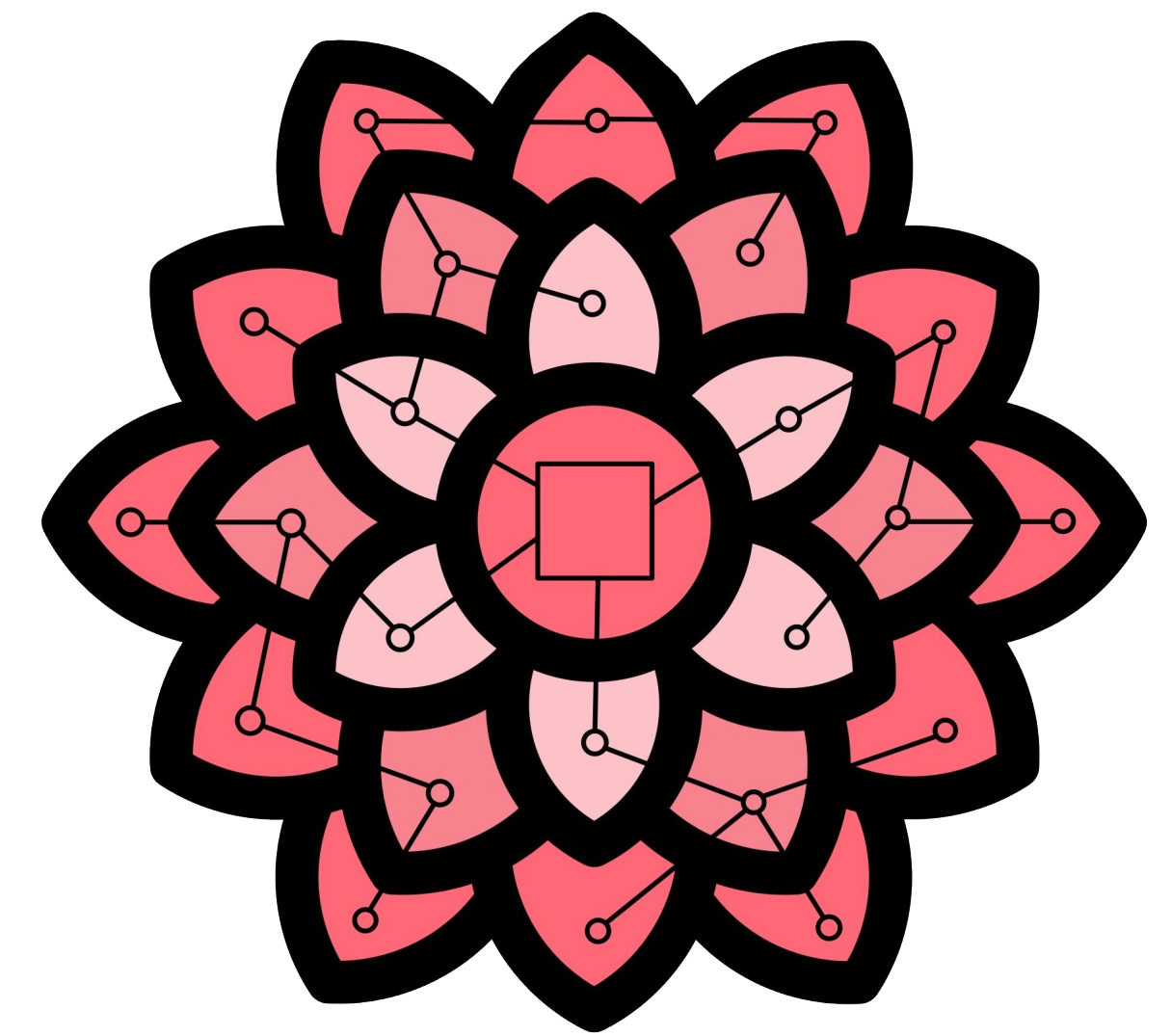


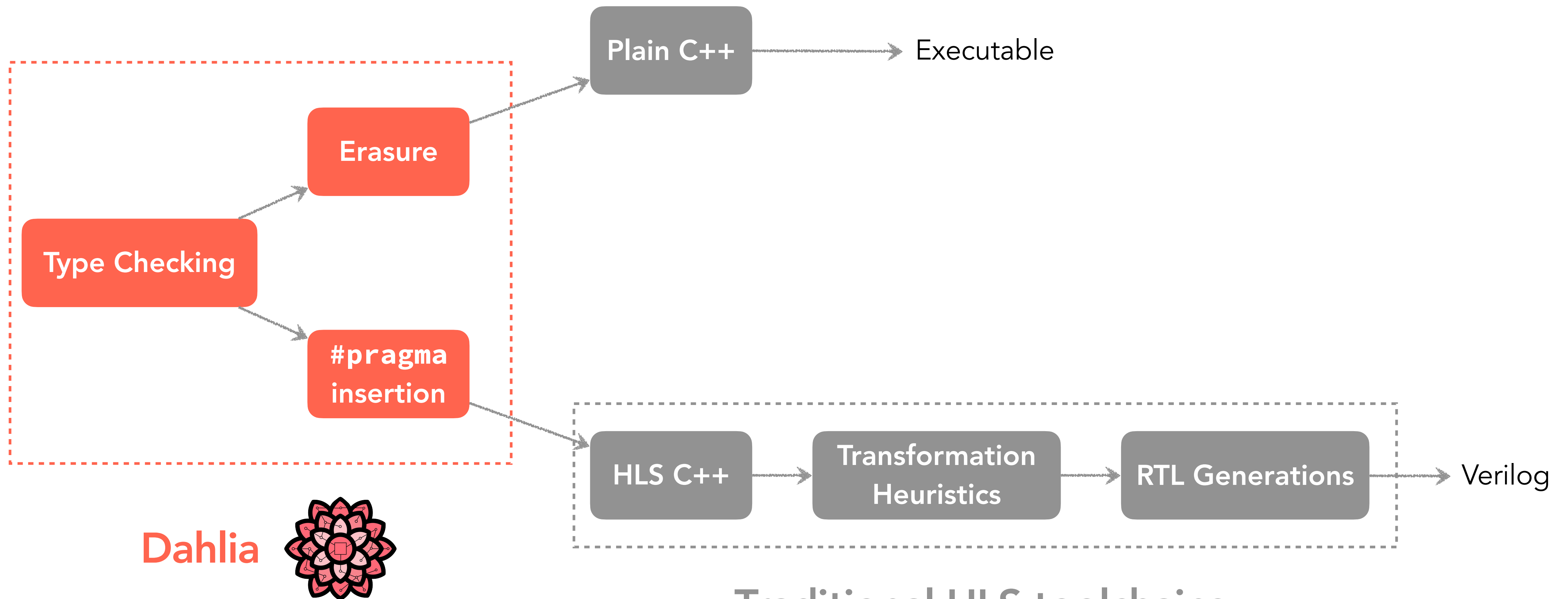
Ada's Journey



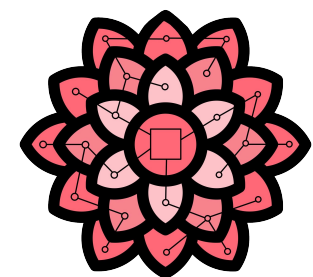
Dahlia

Predictable Accelerator Design



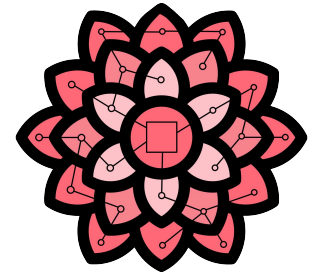


Traditional HLS toolchains



Dahlia

Hardware



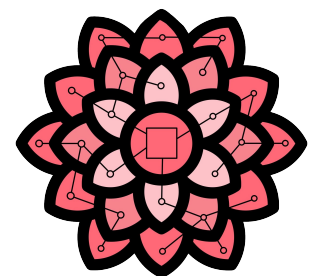
Dahlia

```
let m1: float[10];
```

```
let x = m1[0];
```

```
m1[1] := 1;
```

Hardware



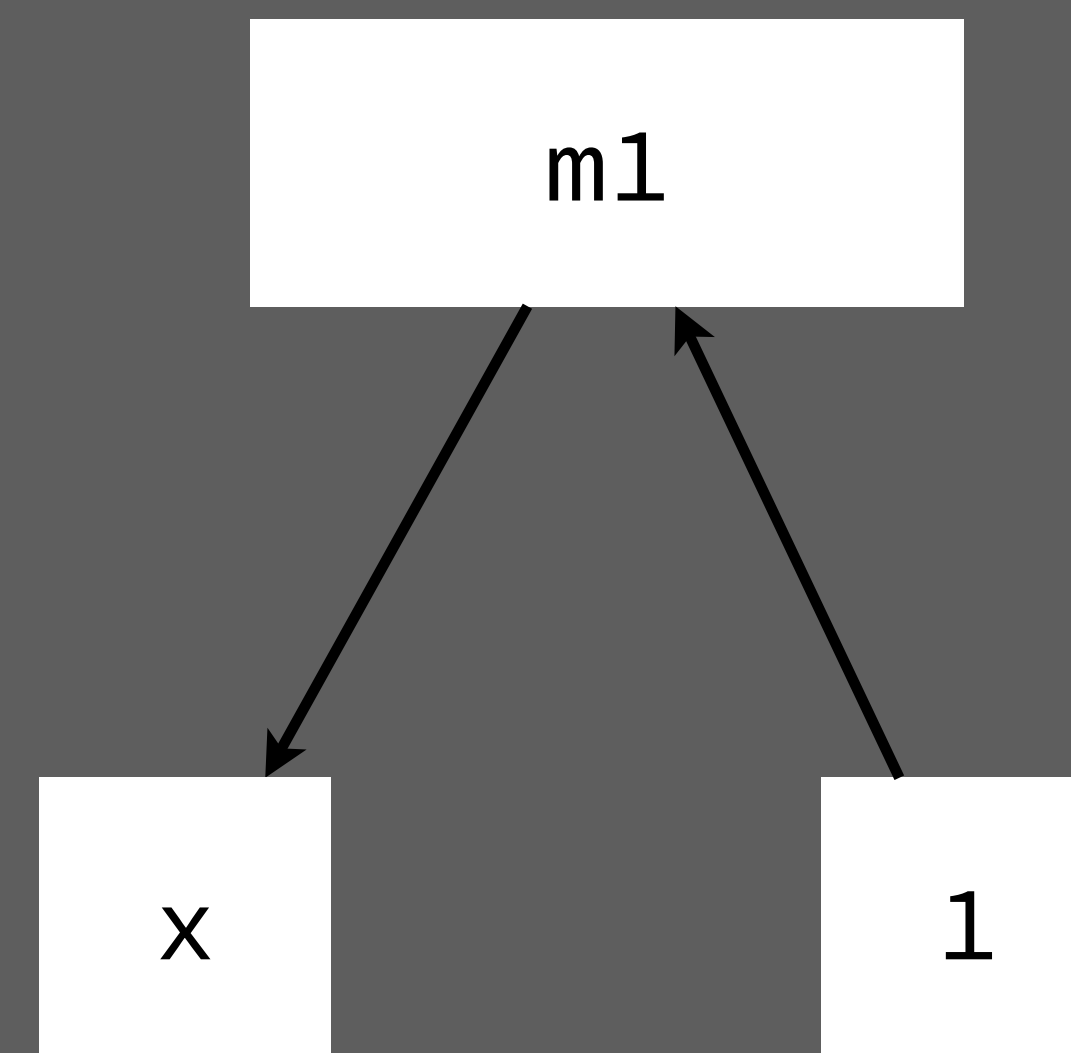
Dahlia

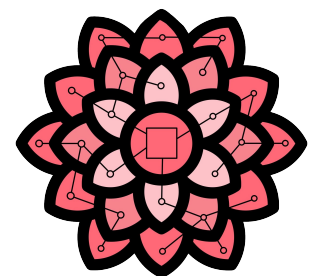
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Hardware





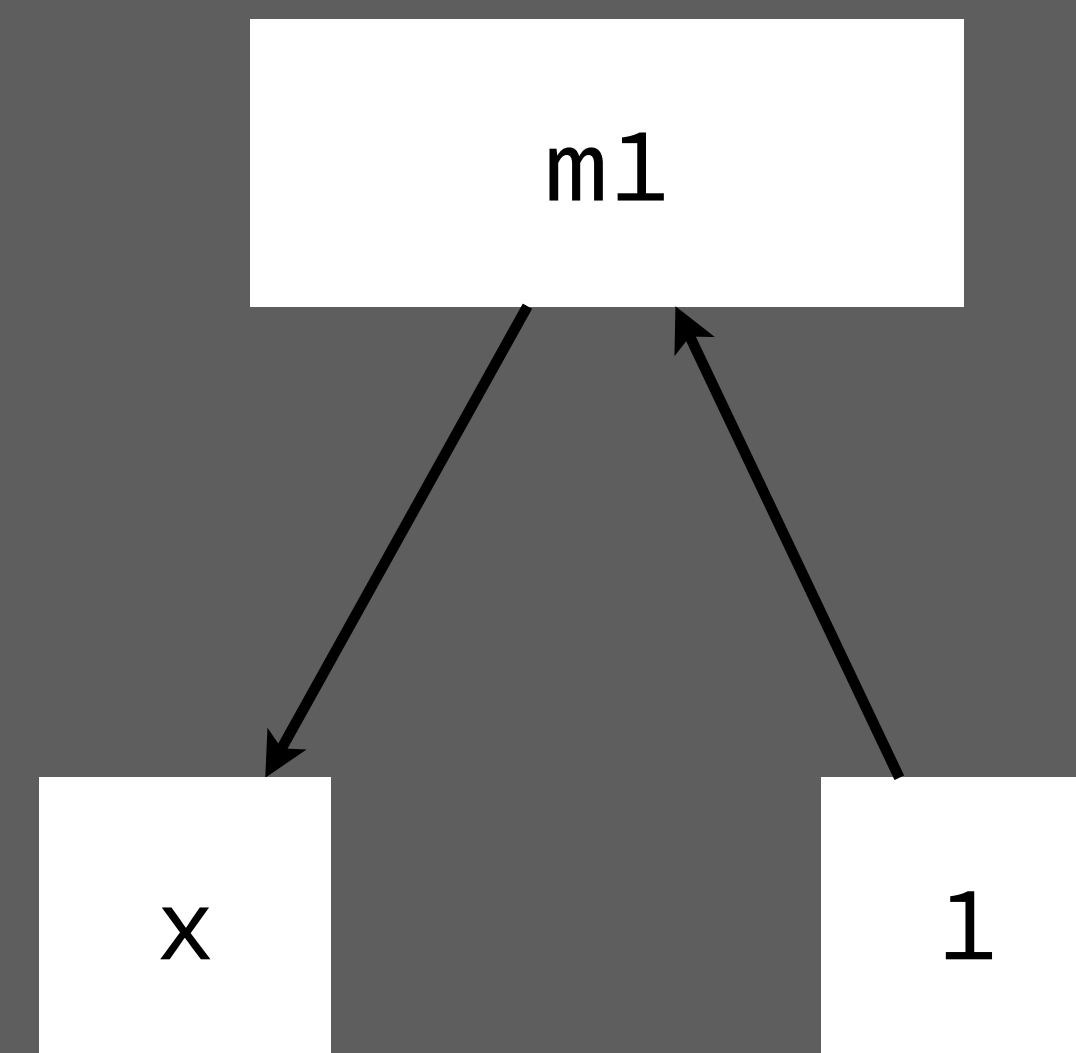
Dahlia

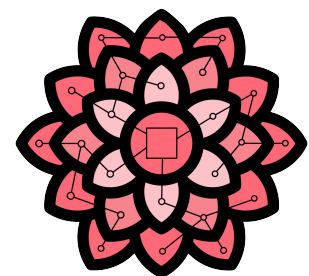
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```
let x = m1[0];
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```
m1[1] := 1;
```

Hardware





Dahlia

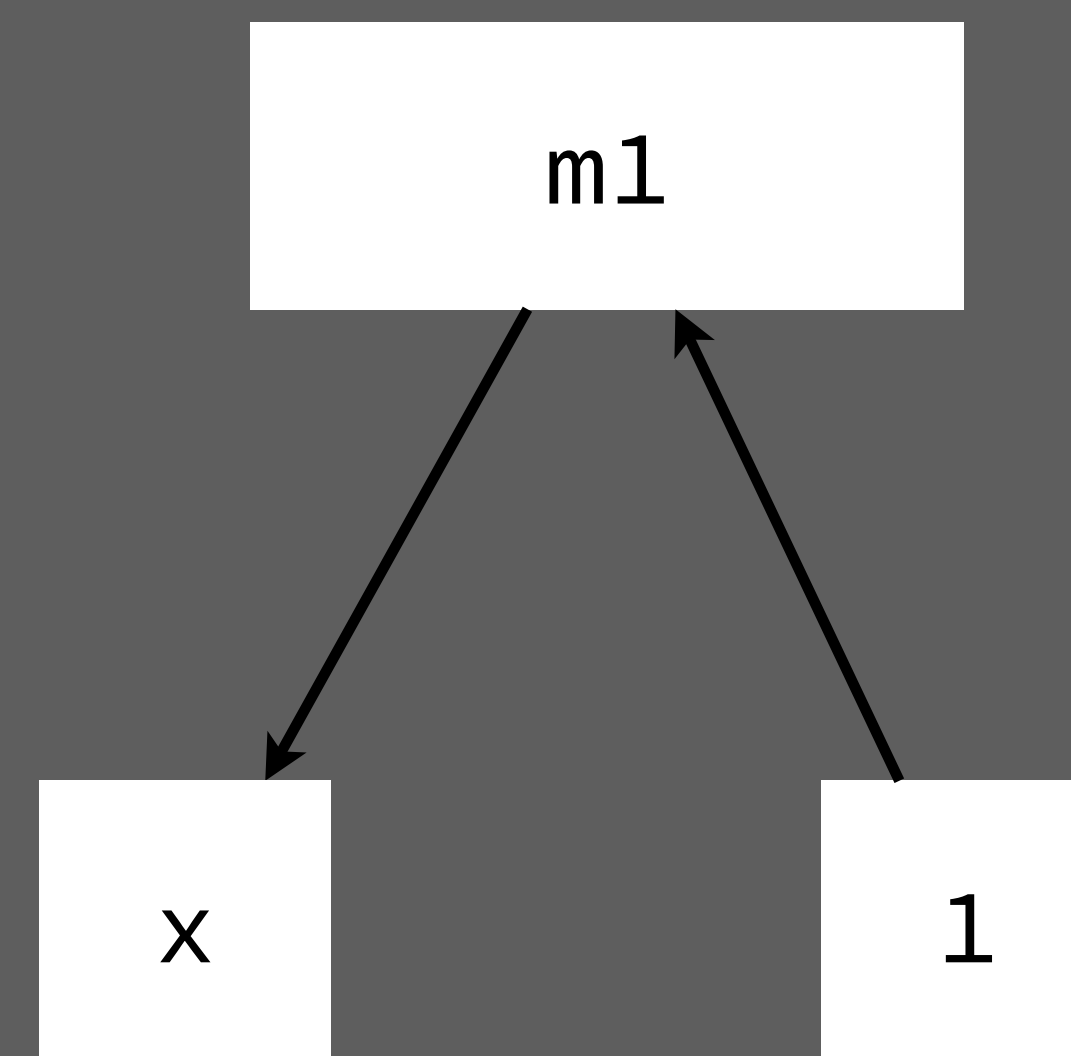
```
let m1: float[10];
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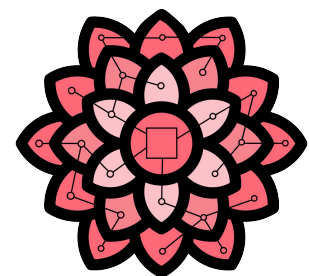
```
let x = m1[0];
```

```
m1[1] := 1;
```

Do these run
concurrently?

Hardware





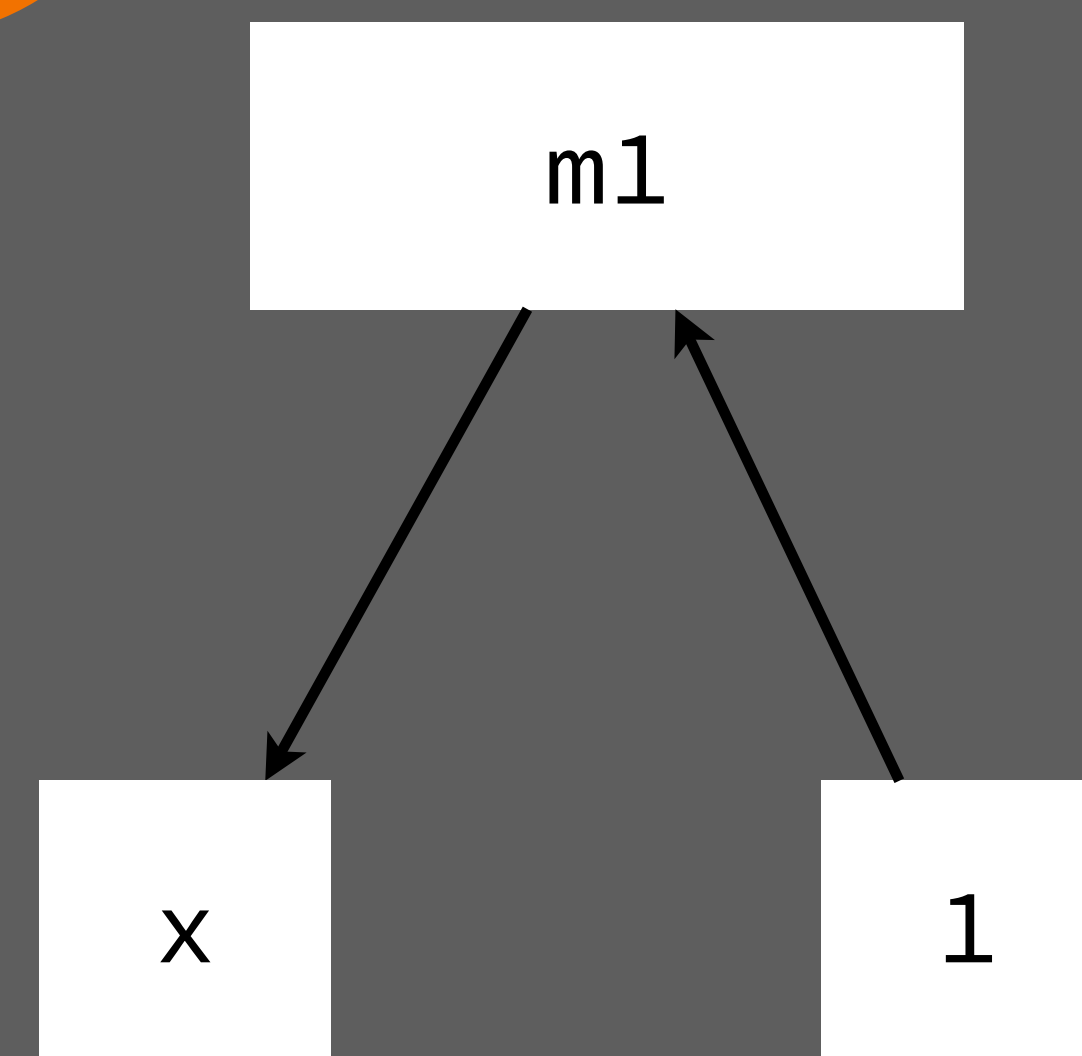
Dahlia

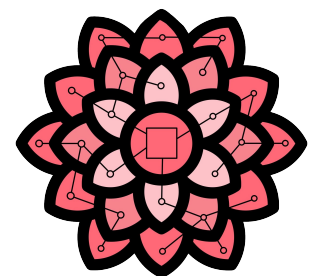
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let x = m1[0];  
m1[1] := 1;
```

Do these run
concurrently?

Hardware





Dahlia

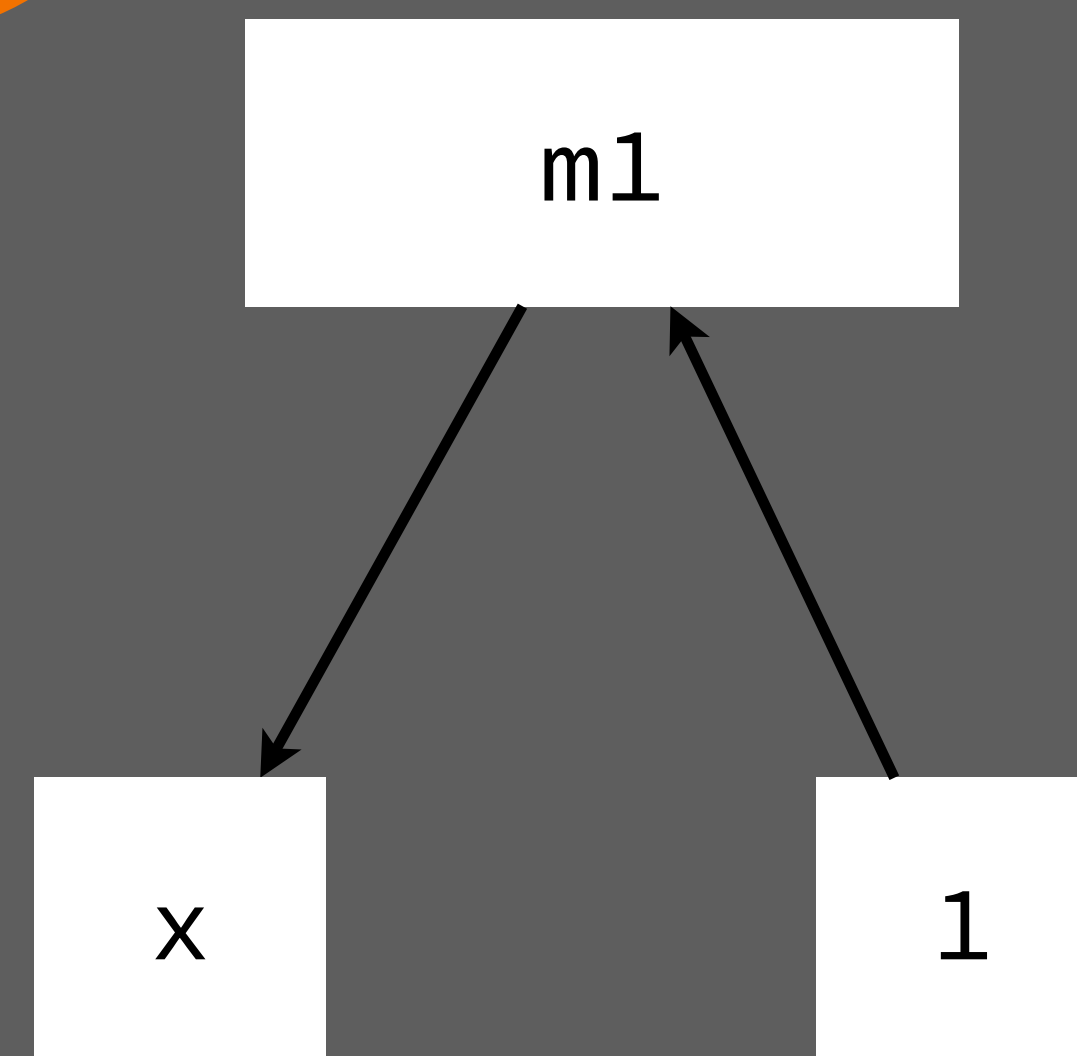
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let m1: float[10];
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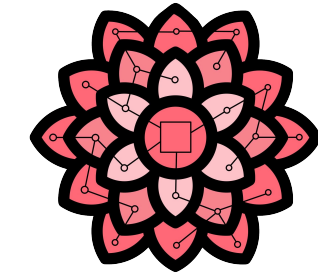
```
let x = m1[0];  
m1[1] := 1;
```

Do these run
concurrently?

Error: Affine resource 'm1'
already used in this context.

Hardware





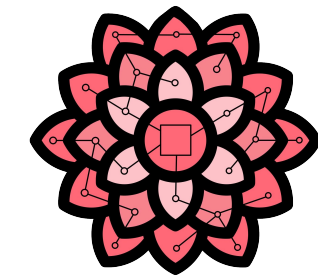
Dahlia

Affine Type System

```
let m1: float[10];
```

```
let x = m1[0];
```

```
m1[1] := 1;
```



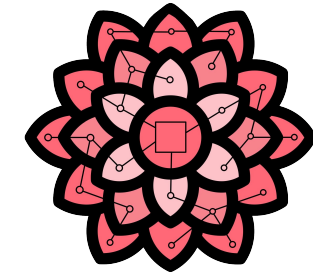
Dahlia

Affine Type System

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let m1: float[10];
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let x = m1[0];  
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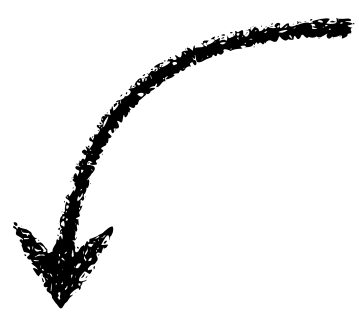
At most one **use** of a
variable



Dahlia

Affine Type System

Declaration

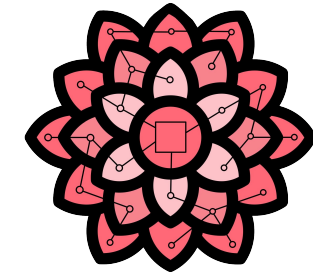


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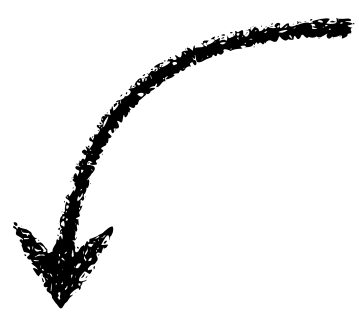
At most one **use** of a
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Dahlia

Affine Type System

Declaration



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let m1: float[10];
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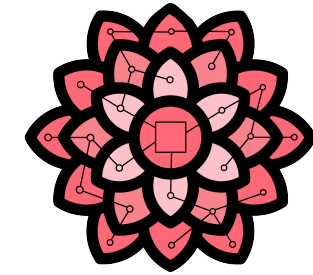
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let x = m1[0];
```

```
m1[1] := 1;
```



Use

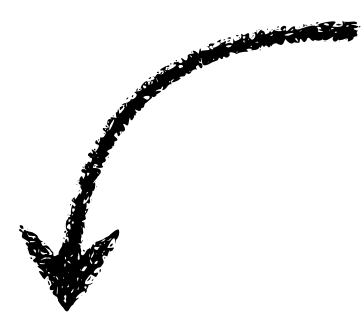
At most one **use** of a
variable



Dahlia

Affine Type System

Declaration

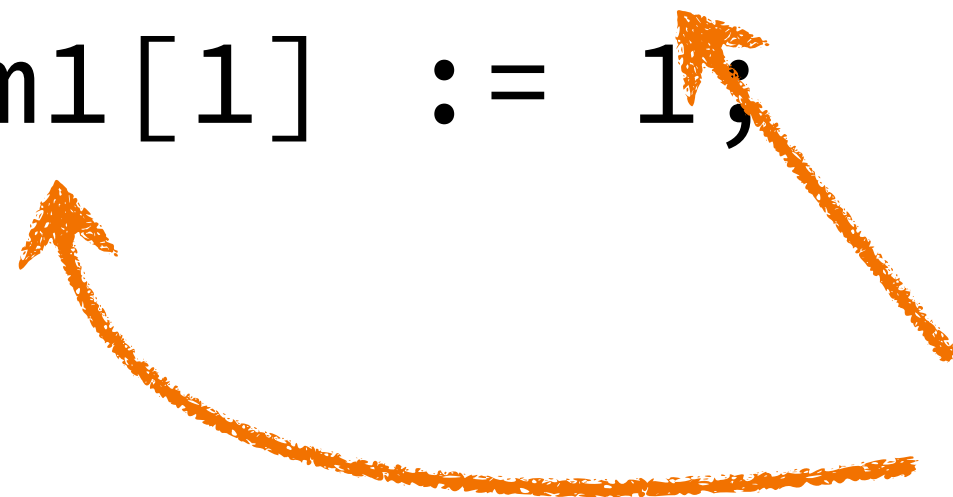


```
let m1: float[10];
```

```
let x = m1[0];
```

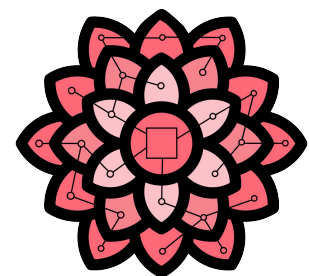
```
m1[1] := 1;
```

Use



At most one **use** of a variable

Error: Affine resource 'm1'
already used in this context.



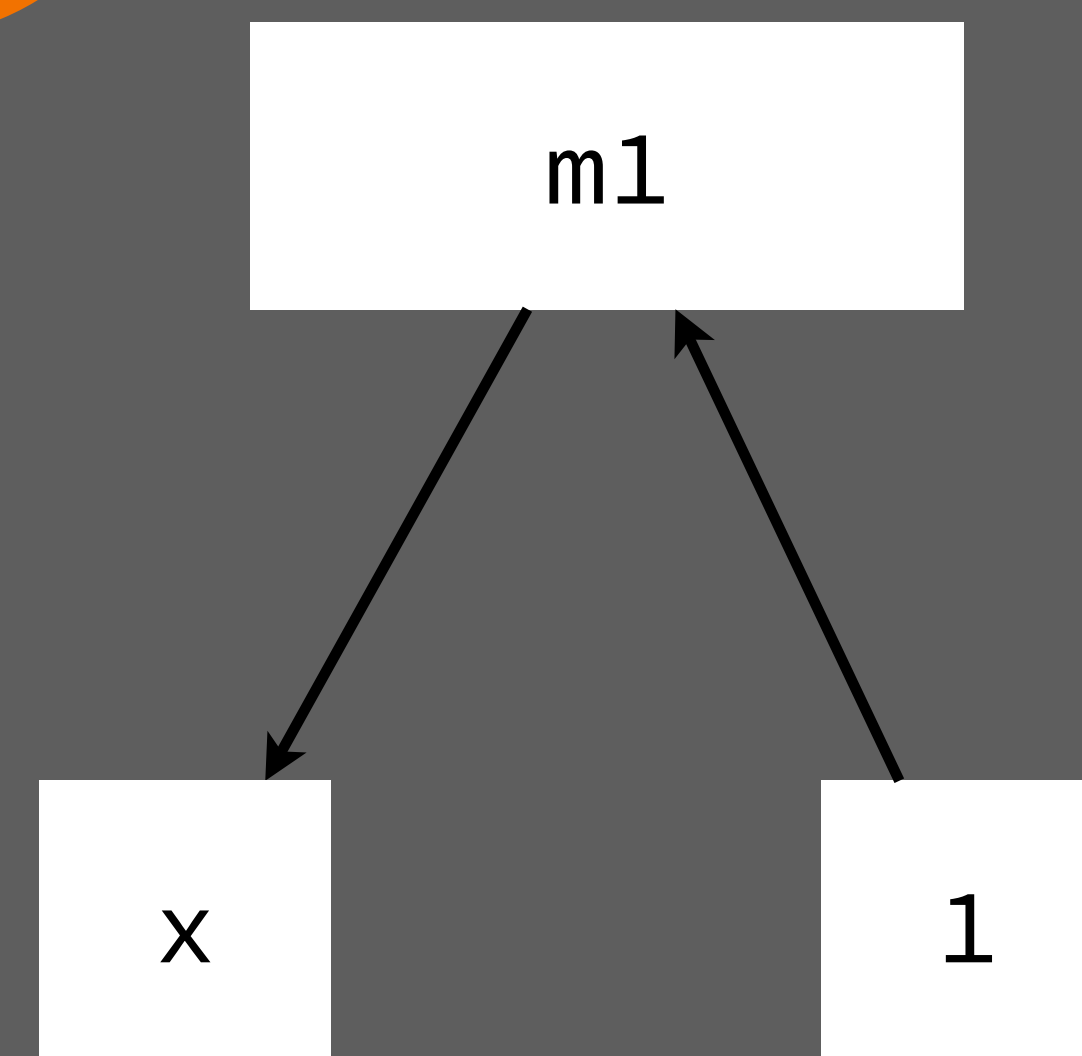
Dahlia

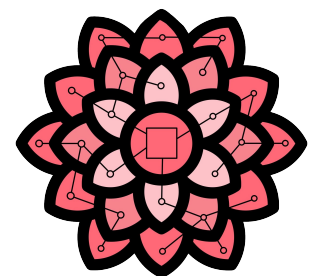
```
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let x = m1[0];  
m1[1] := 1;
```

Do these run
concurrently?

Hardware





Dahlia

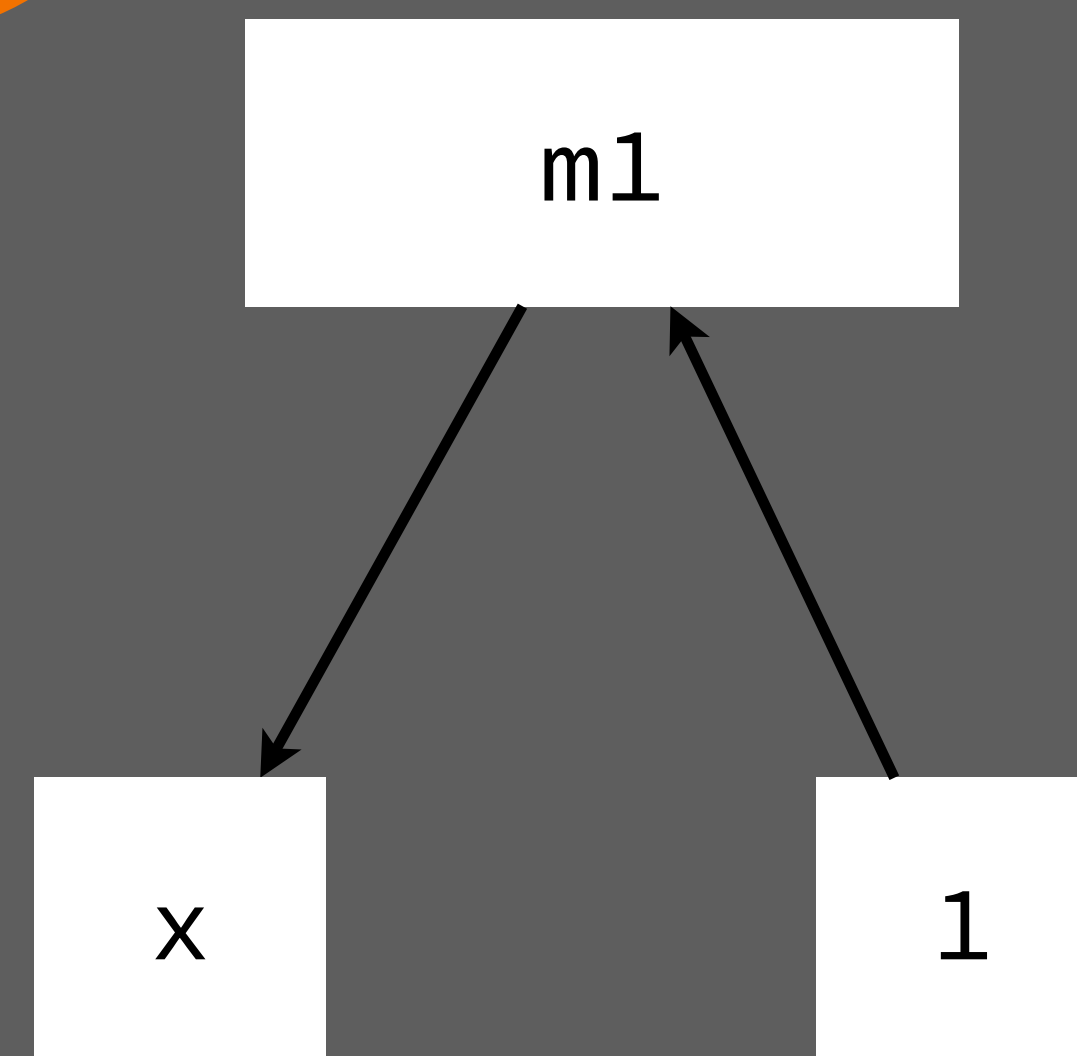
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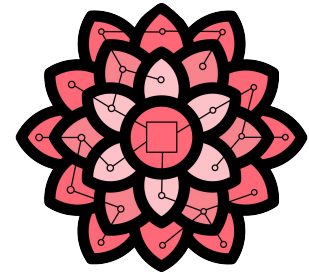
```
let x = m1[0];  
m1[1] := 1;
```

Do these run
concurrently?

Unordered Composition

Hardware





Dahlia

```
let m1: float[10];
```

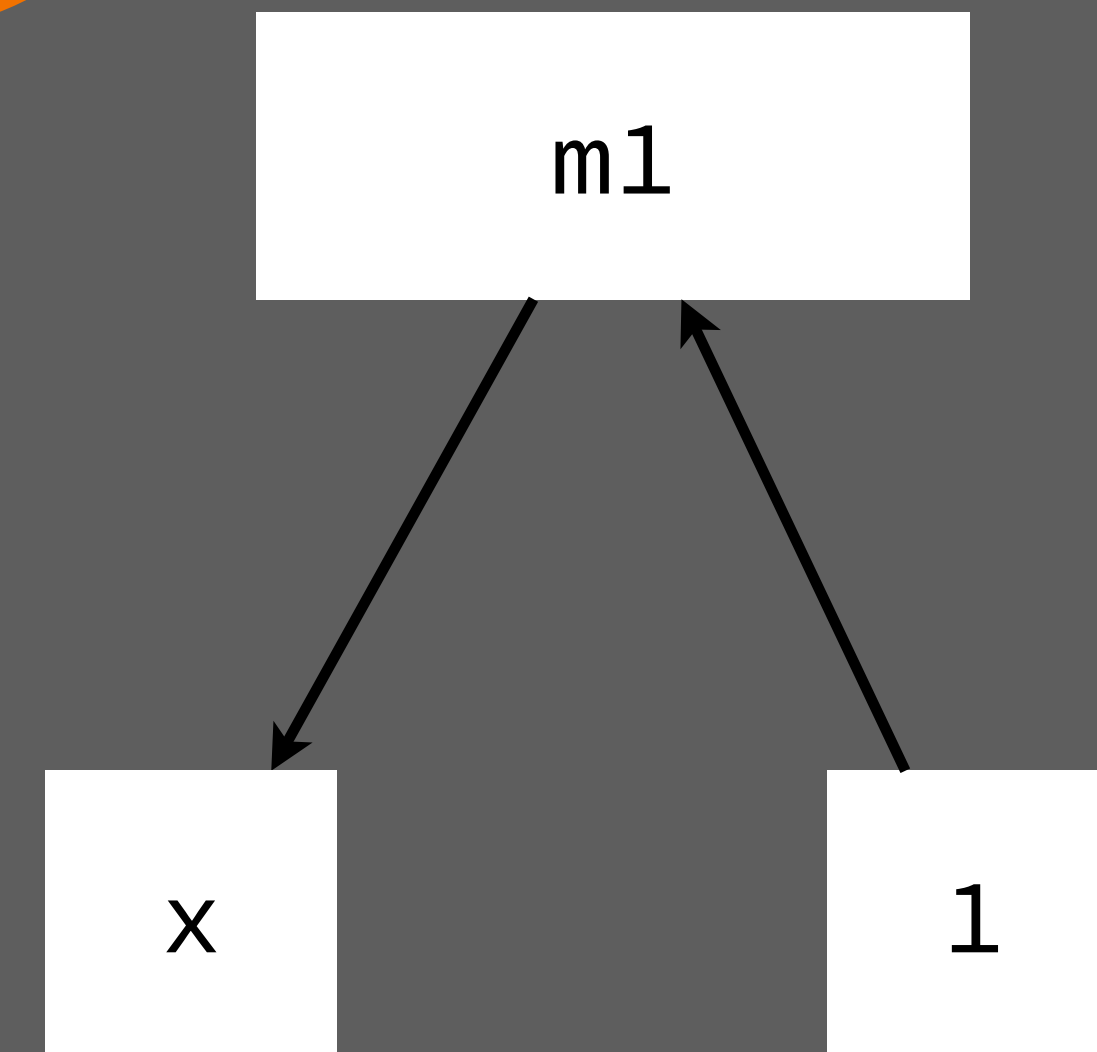
```
let x = m1[0];  
m1[1] := 1;
```

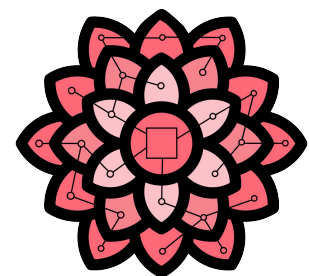
Do these run
concurrently?

Unordered Composition

- Run in **parallel** respecting data flow.
- Consume **affine** resources.

Hardware





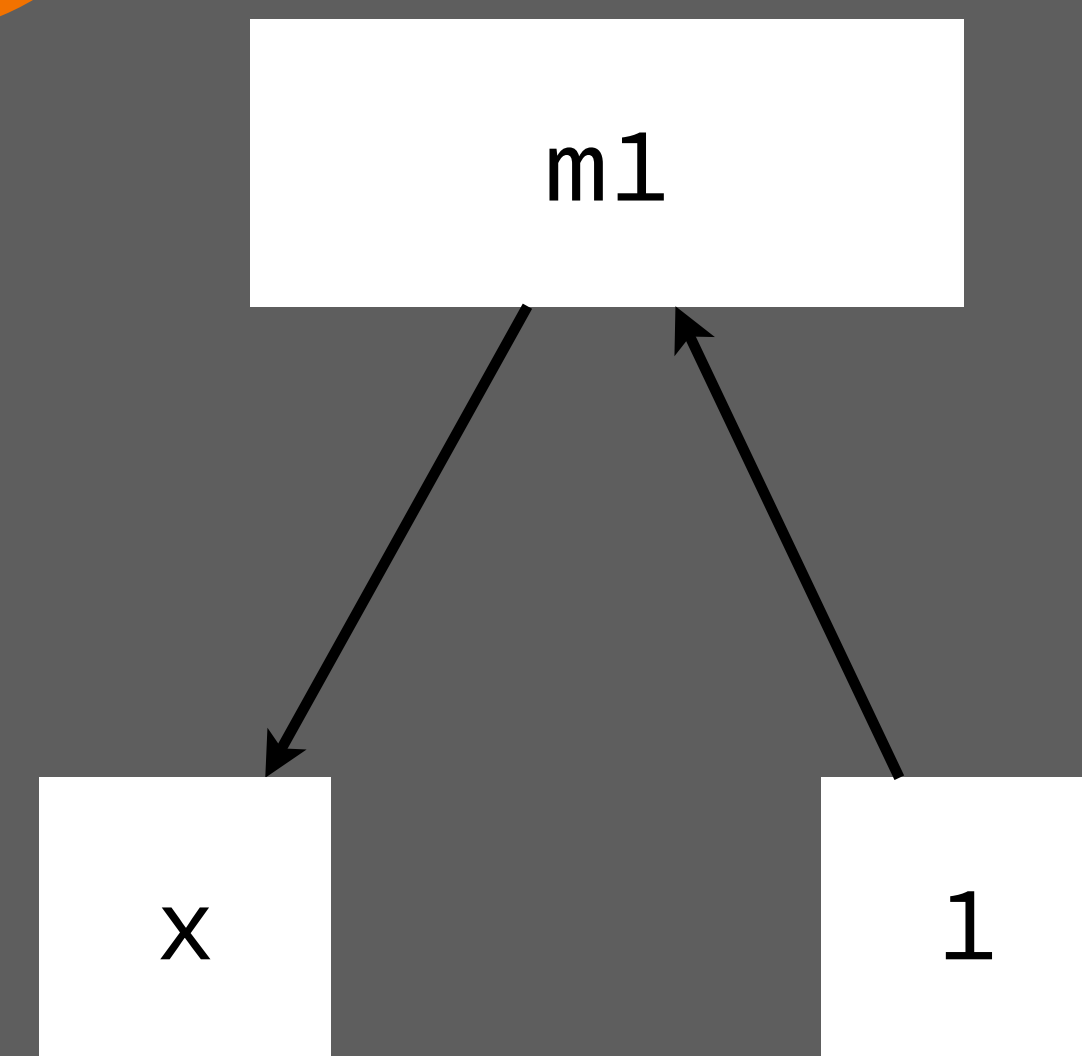
Dahlia

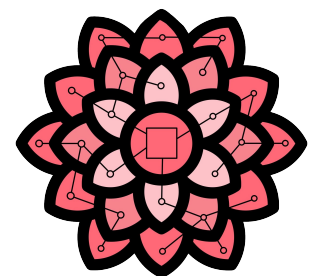
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let m1: float[10];
```

```
let x = m1[0];
```

```
m1[1] := 1;
```

Hardware





Dahlia

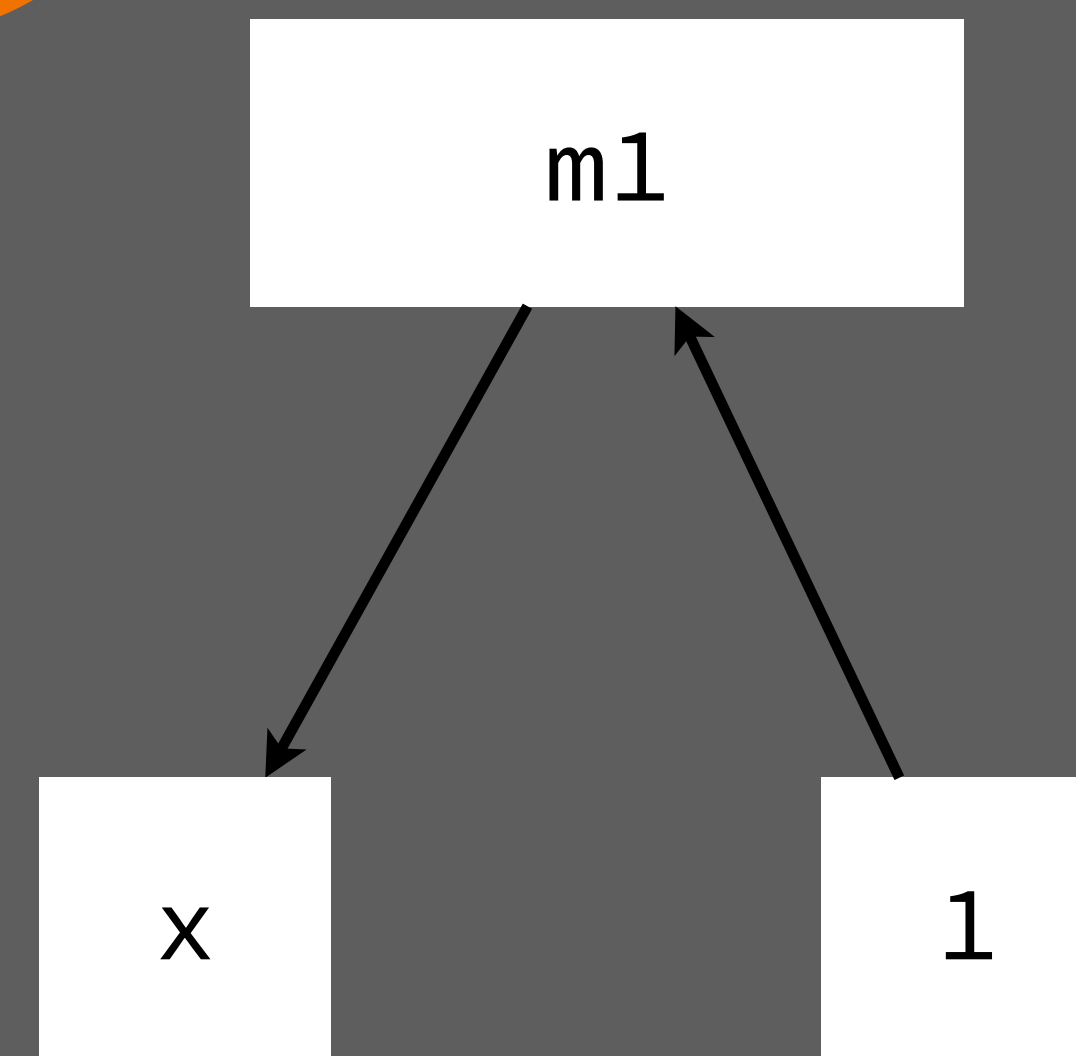
```
let m1: float[10];
```

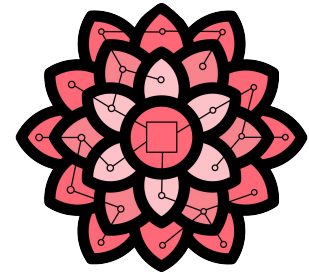
```
let x = m1[0];
```

```
m1[1] := 1;
```

Ordered Composition

Hardware





Dahlia

```
let m1: float[10];
```

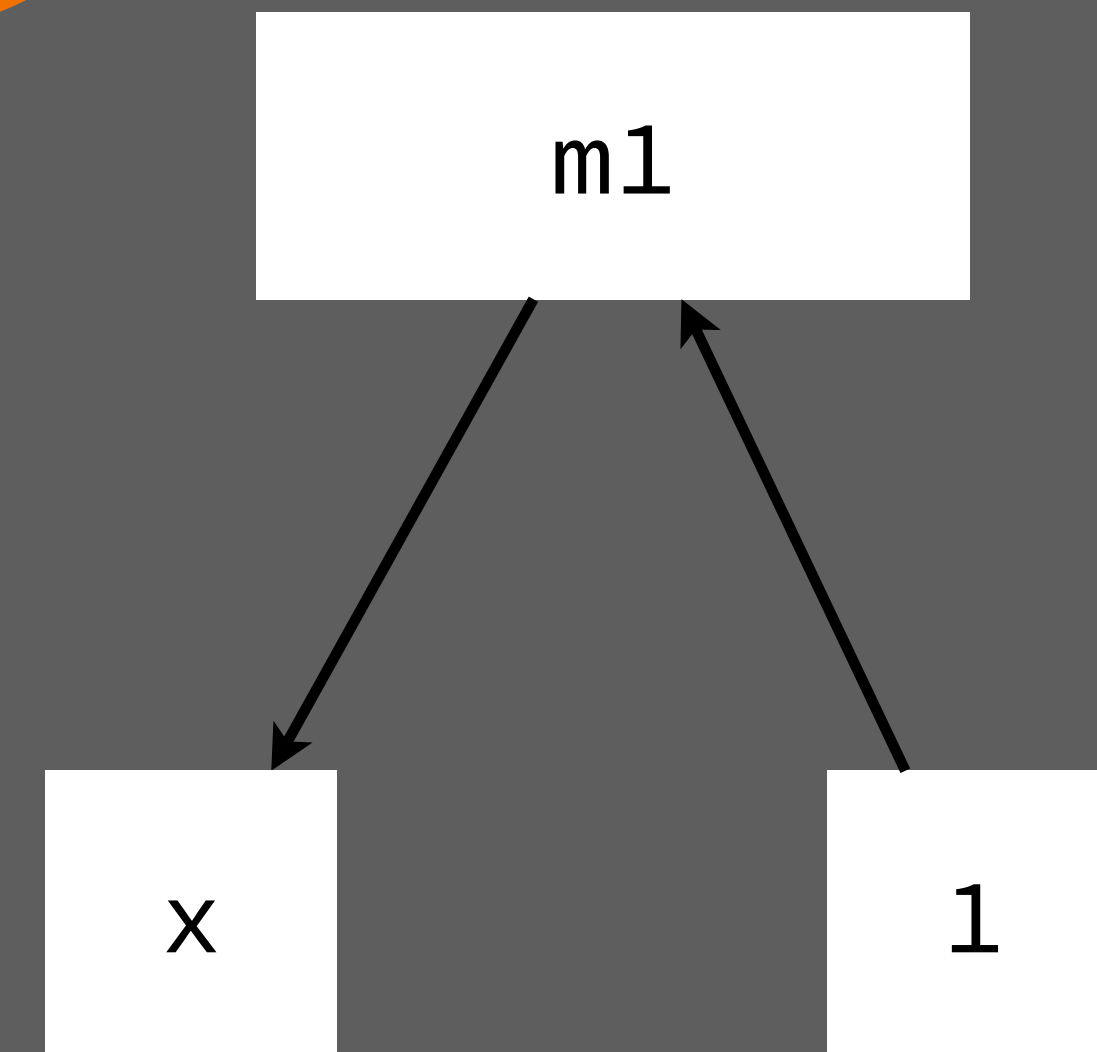
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m1[1] := 1;
```

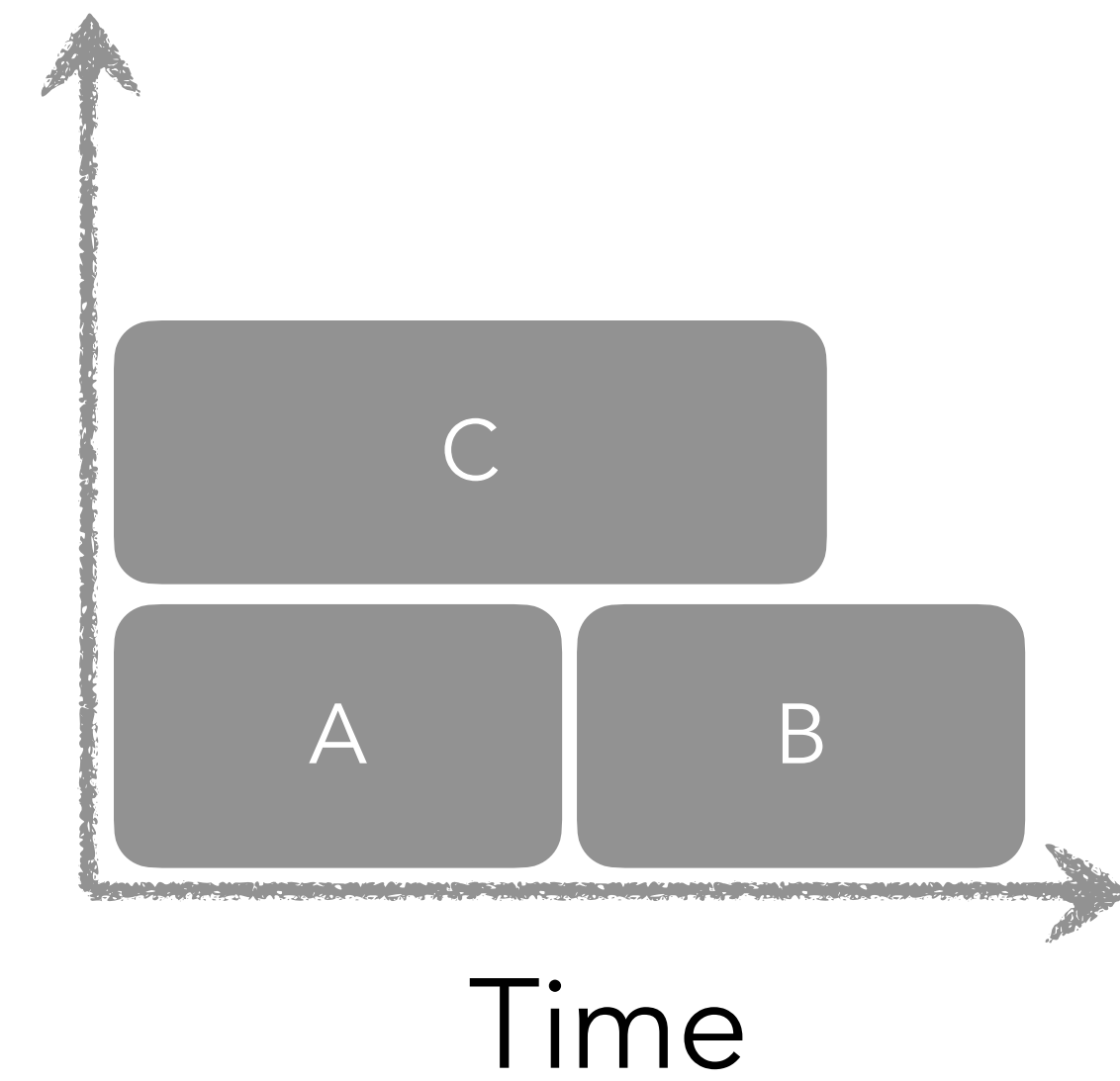
Ordered Composition

- Run **sequentially**.
- Renew **affine** resources.

Hardware

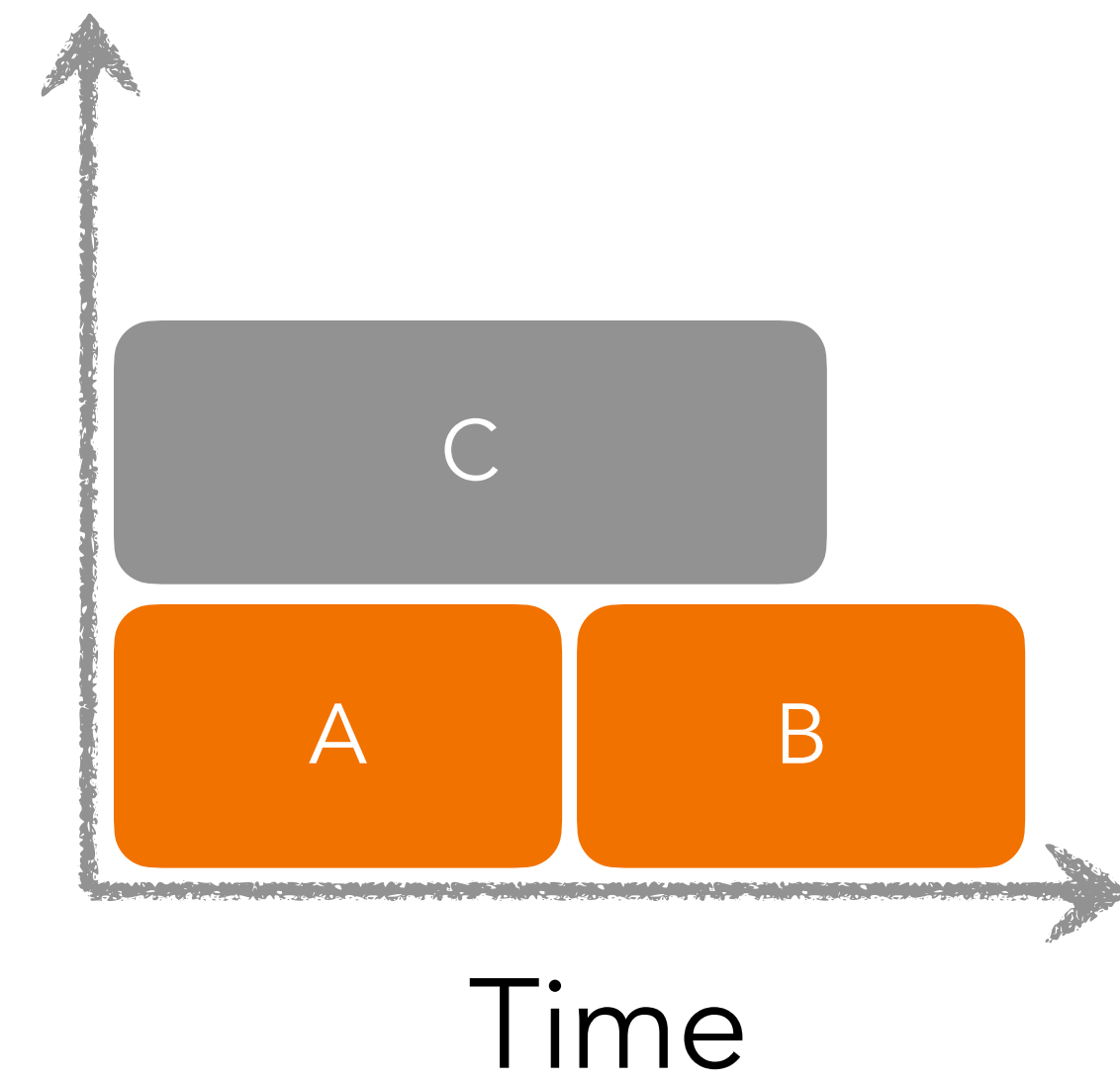


{ A --- B } ;
C



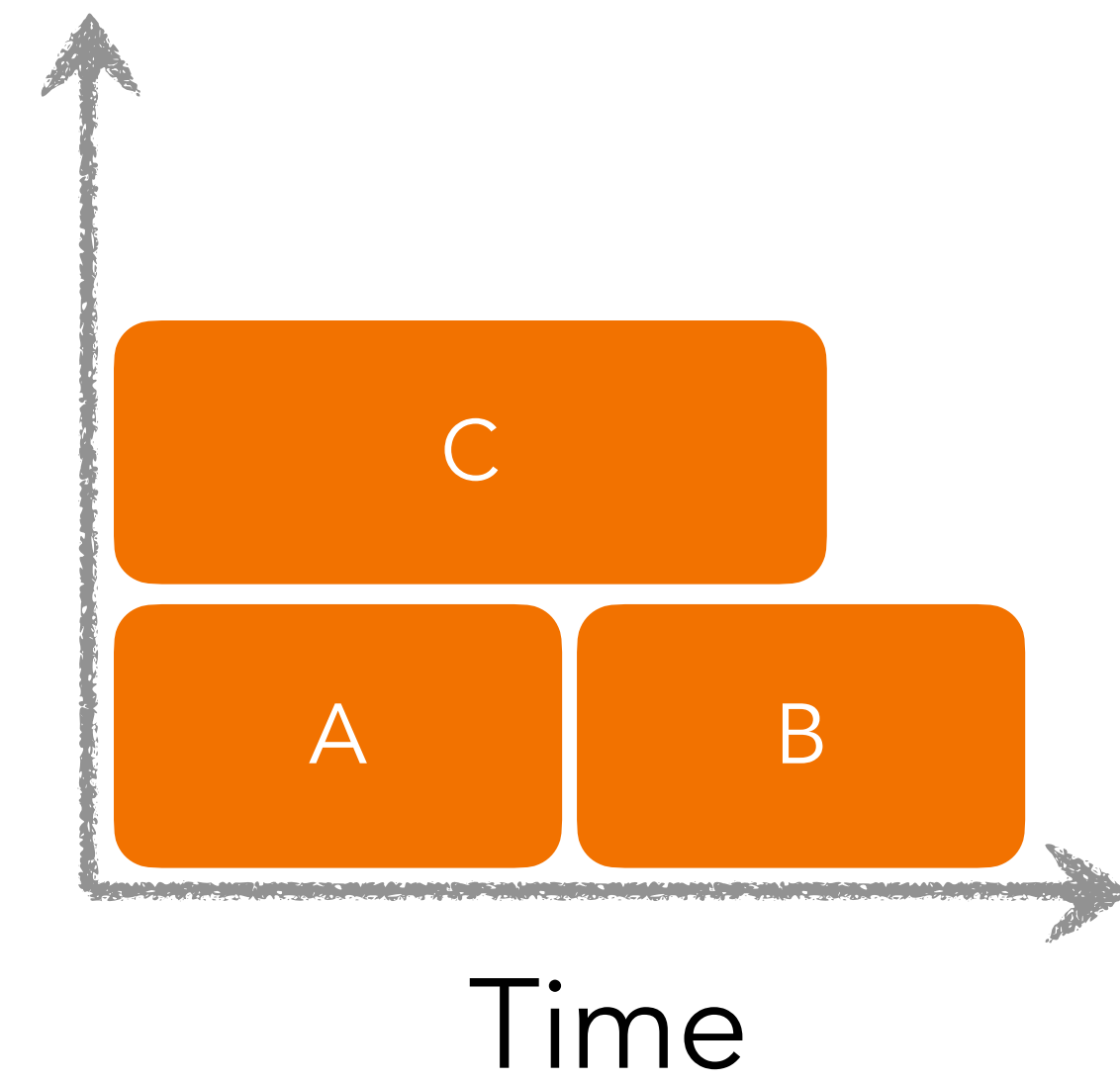
Temporally exclusive use of resources

$\{ A \text{ --- } B \};$
C

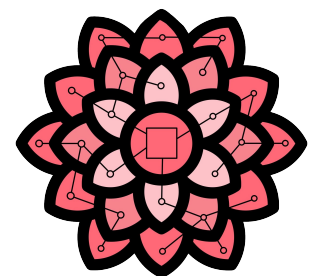


Temporally exclusive use of resources

{ A --- B } ;
C



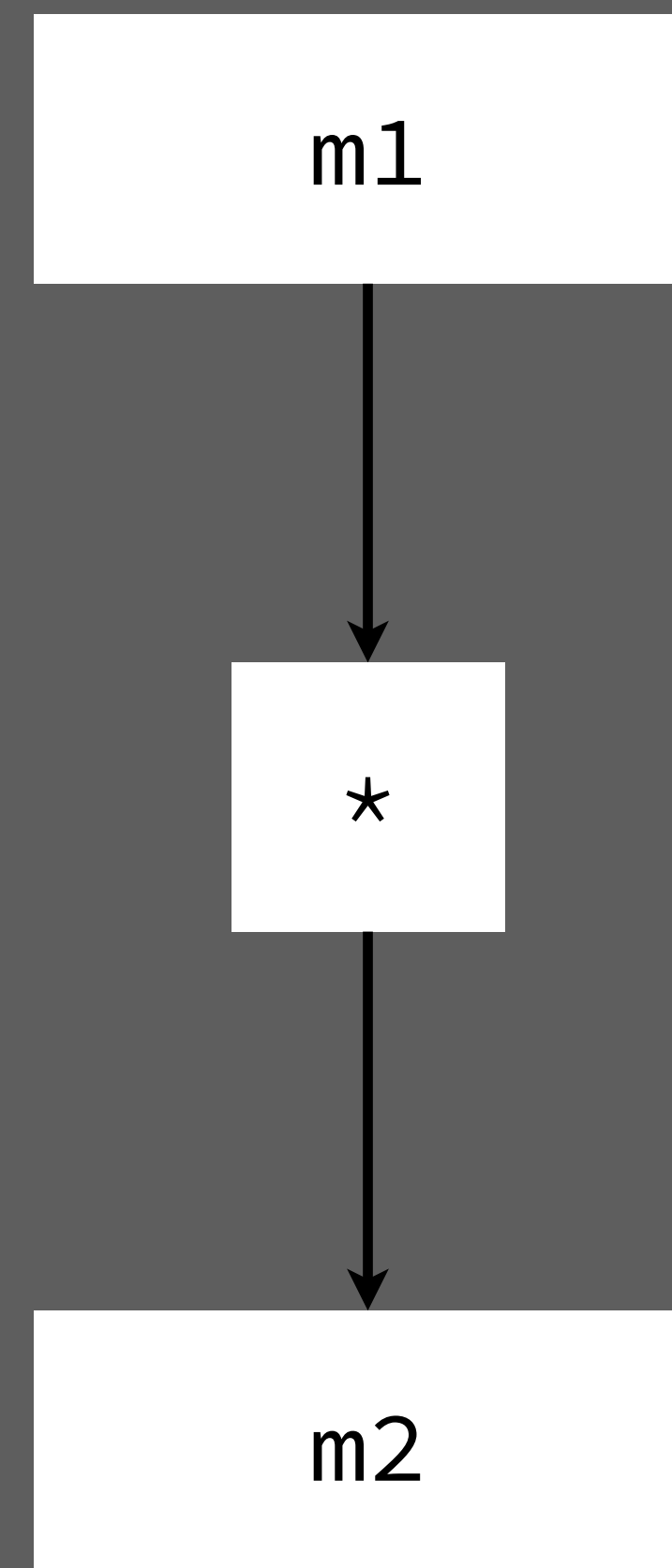
Temporally exclusive use of resources

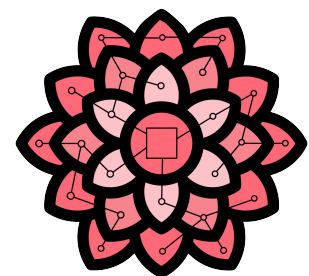


Dahlia

```
let m1: float[12];  
let m2: float[12];  
  
for (let i = 0 .. 12) {  
    m2[i] = m1[i] * 2;  
}
```

Hardware

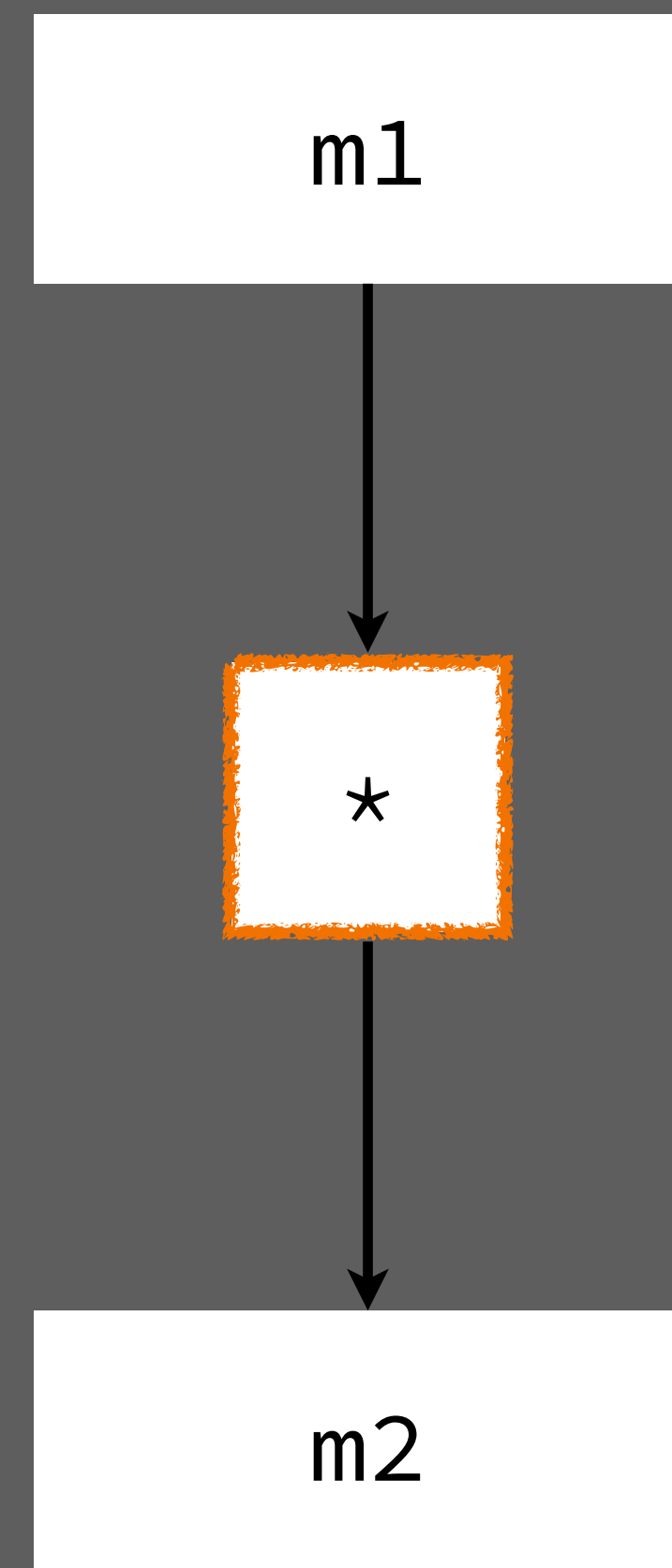


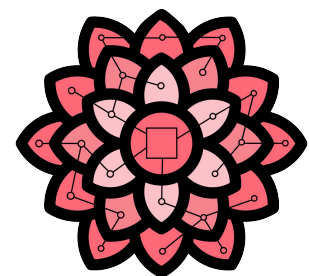


Dahlia

```
let m1: float[12];  
let m2: float[12];  
  
for (let i = 0 .. 12) unroll 3 {  
    m2[i] = m1[i] * 2;  
}
```

Hardware

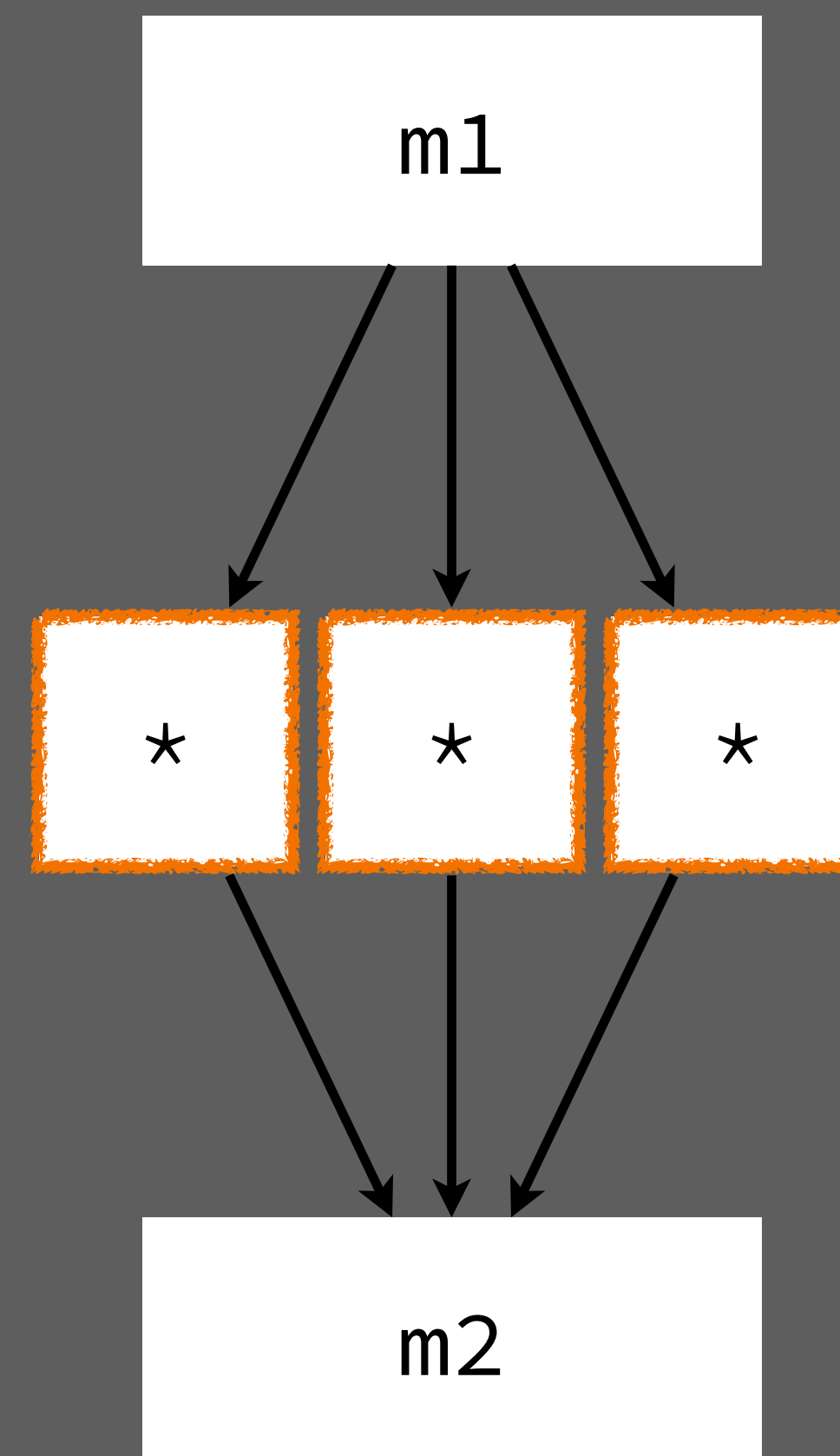


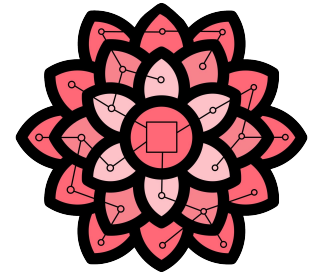


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Hardware



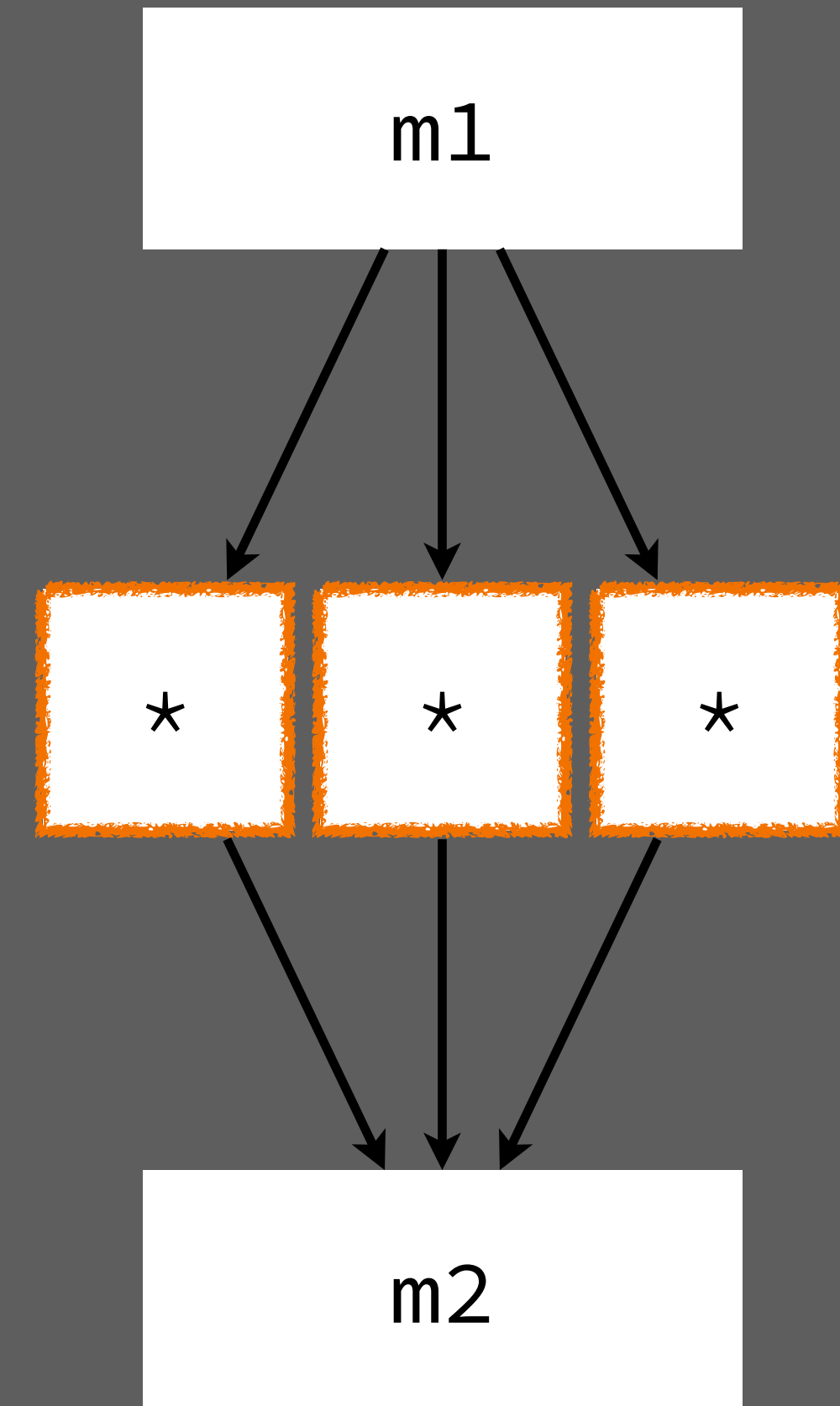


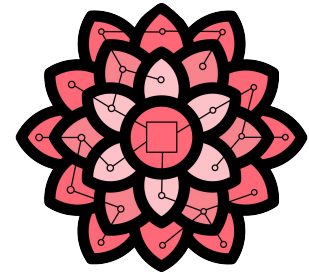
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}
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Error: Affine resource 'm1'
already used in this context.

Hardware



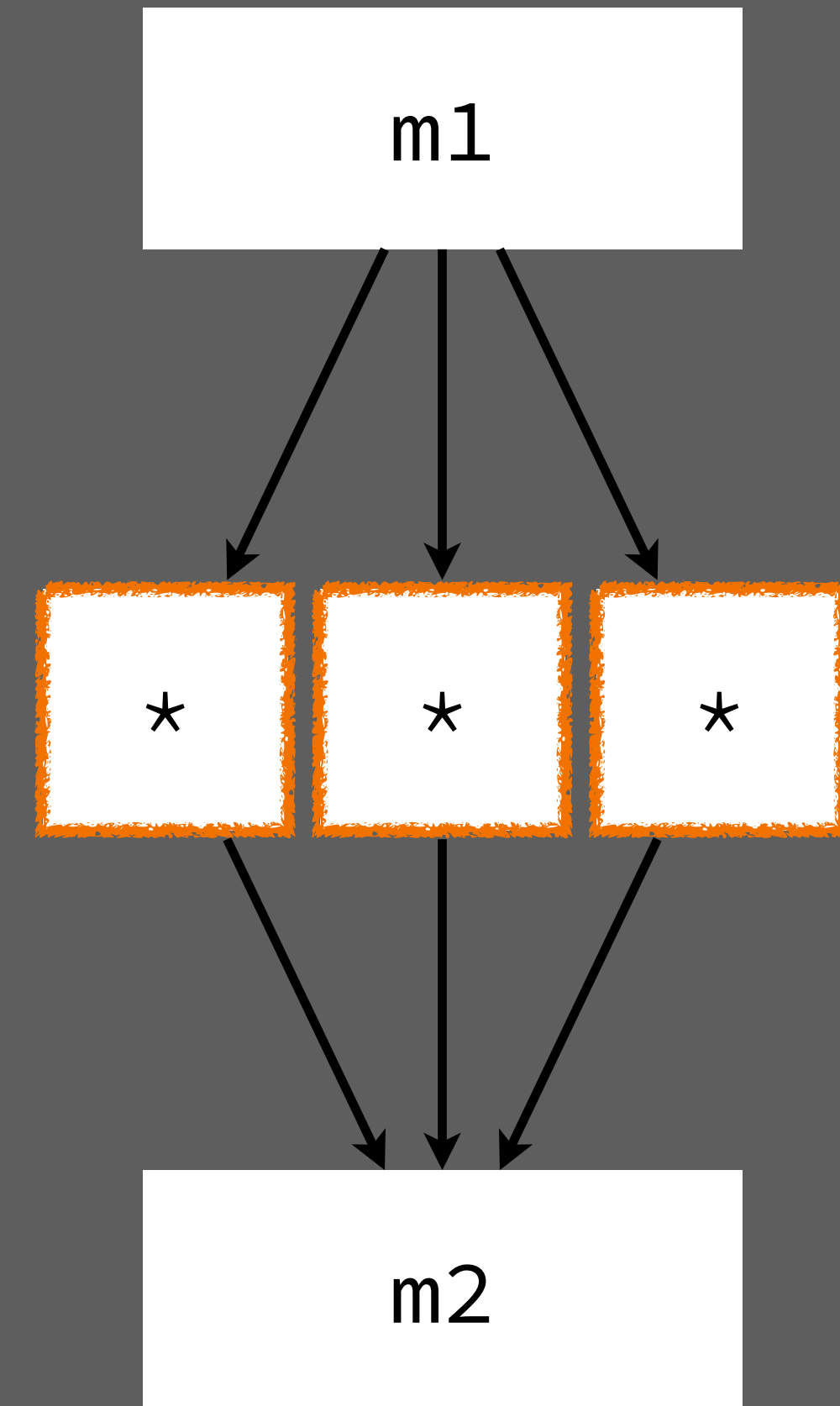


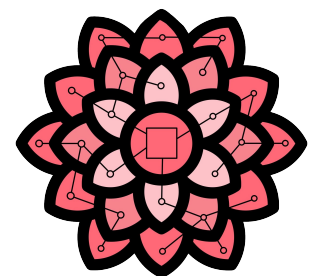
Dahlia

```
let m1: float[12];  
let m2: float[12];  
  
for (let i = 0 .. 4) {  
  m2[3*i+0] = m1[3*i+0] * 2;  
  m2[3*i+1] = m1[3*i+1] * 2;  
  m2[3*i+2] = m1[3*i+2] * 2;  
}
```

Error: Affine resource 'm1'
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Hardware

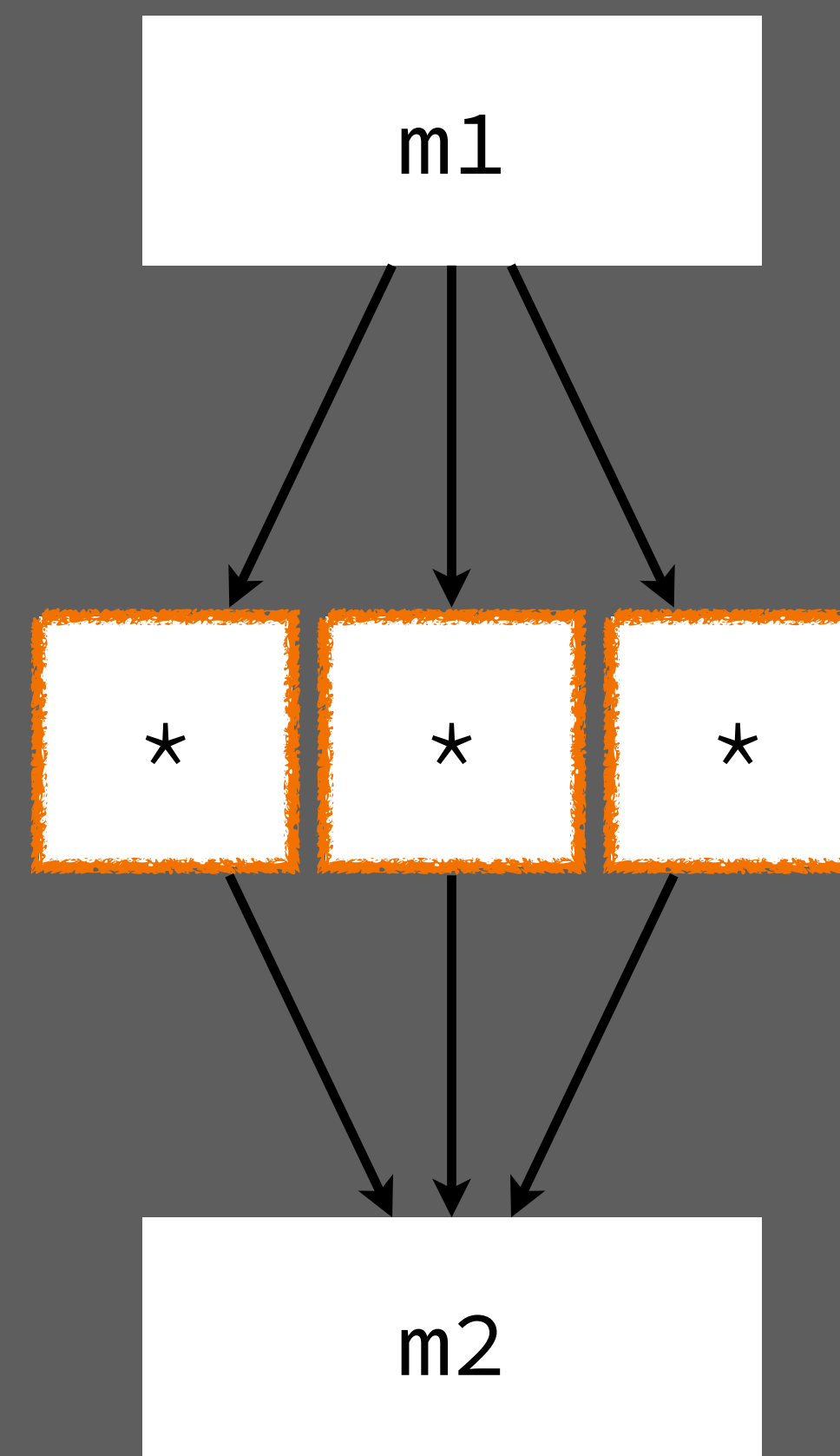


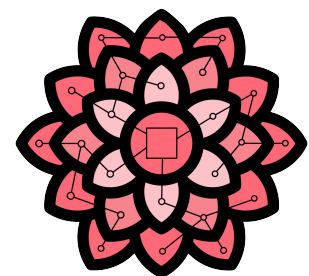


Dahlia

```
let m1: float[12 bank 3];  
let m2: float[12 bank 3];  
  
for (let i = 0 .. 12) unroll 3 {  
    m2[i] = m1[i] * 2;  
}
```

Hardware

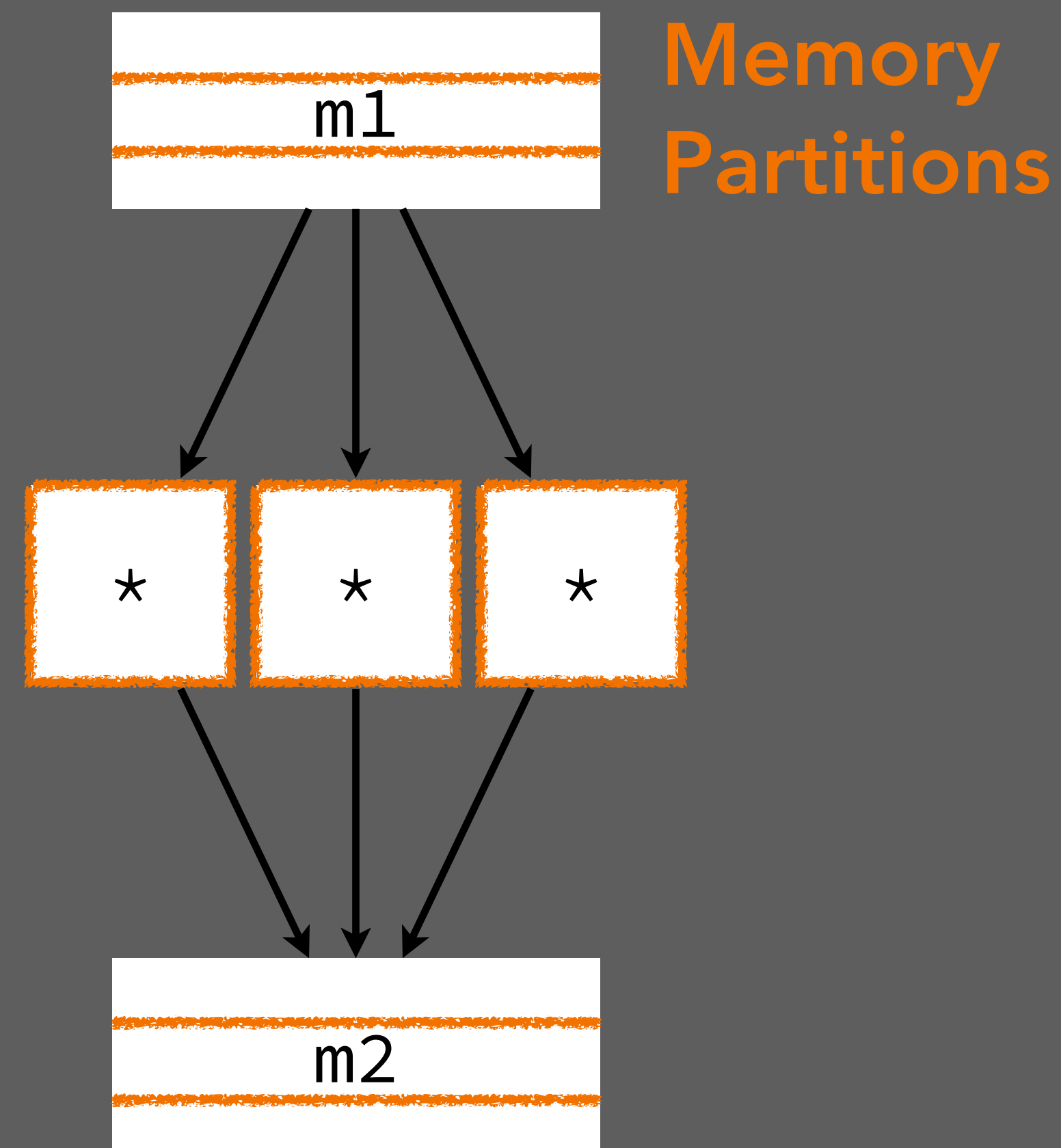


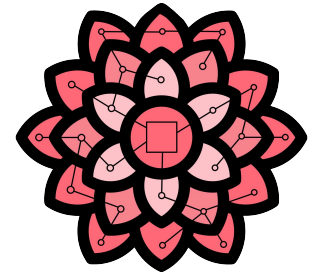


Dahlia

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let m1: float[12 bank 3];  
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Hardware



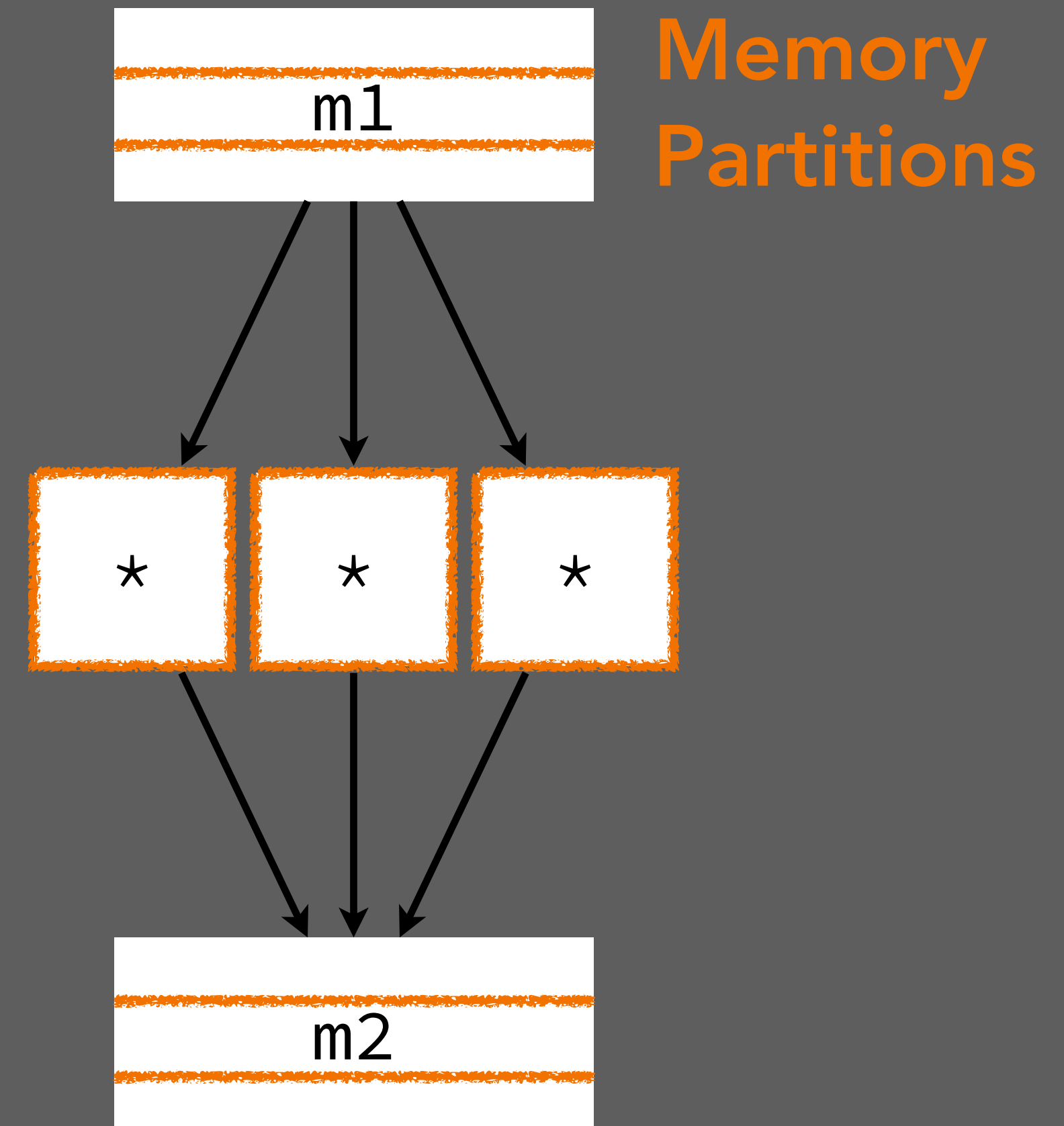


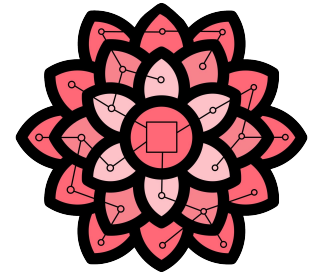
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for (let i = 0 .. 12) unroll 3 {  
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}
```

OK: Dahlia guarantees that parallel accesses use **disjoint partitions**.

Hardware



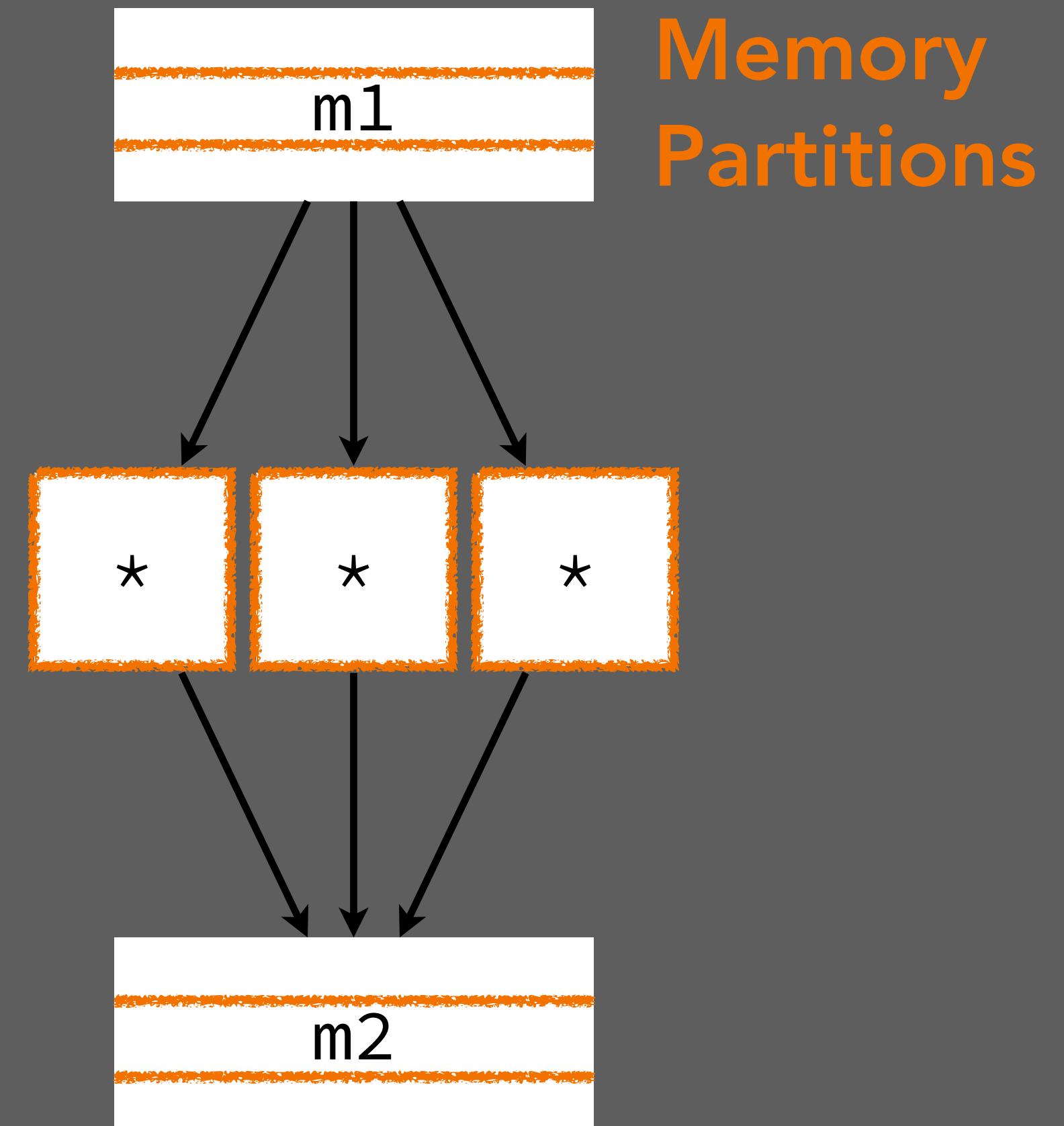


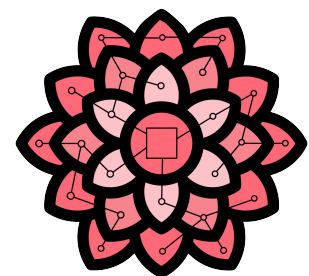
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  m2[3*i+1] = m1[3*i+1] * 2;  
  m2[3*i+2] = m1[3*i+2] * 2;  
}
```

OK: Dahlia guarantees that parallel accesses use **disjoint partitions**.

Hardware

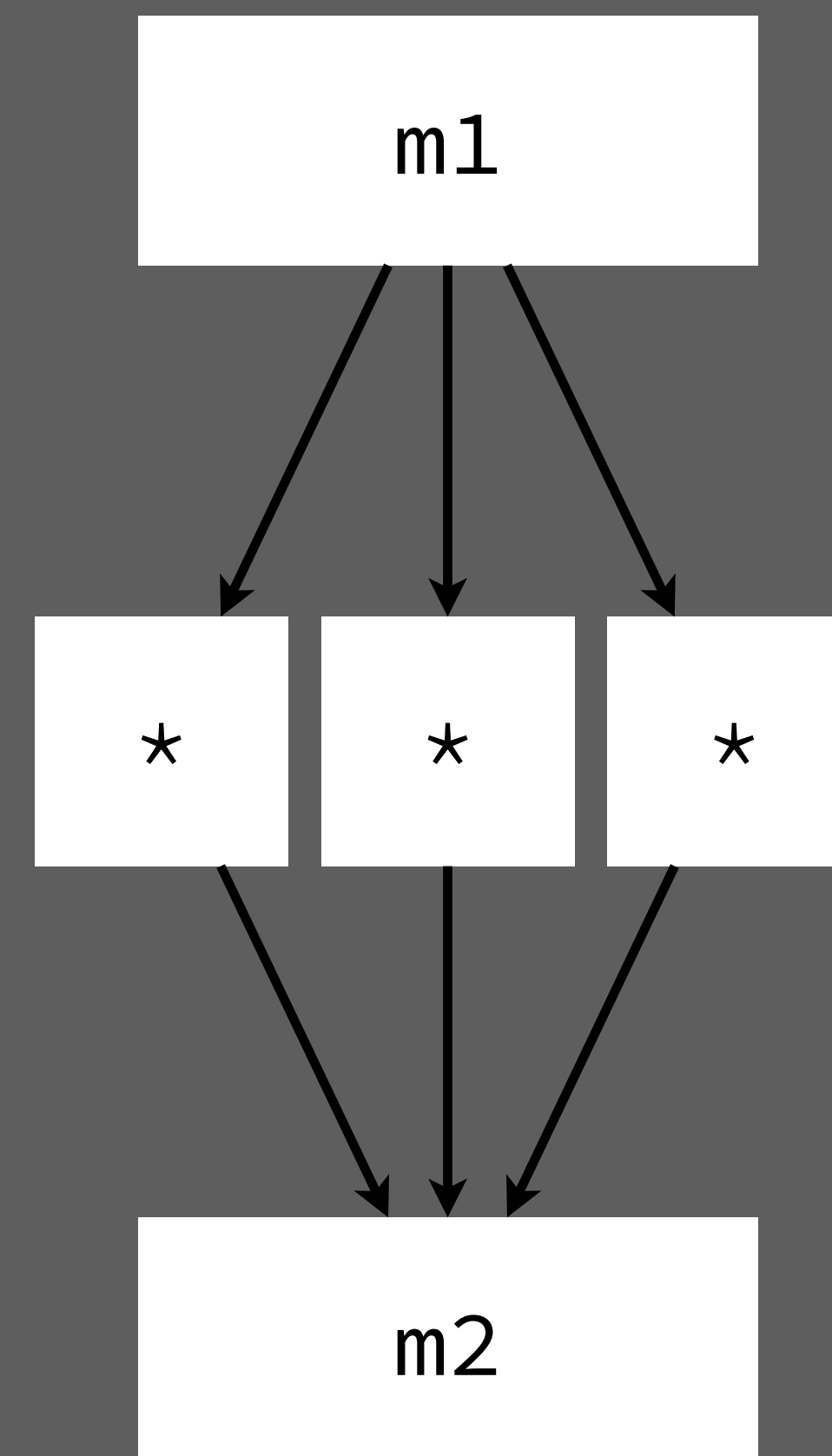


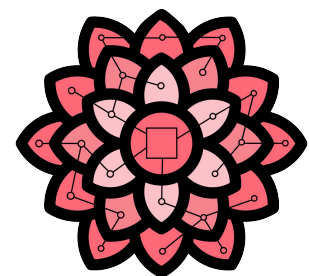


Dahlia

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let m1: float[12 bank 3];  
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for (let i = 0 .. 4) {  
  for (let j = 0 .. 3) {  
    m2[i] = m1[3*i+j] * 2;  
  }  
}
```

Hardware

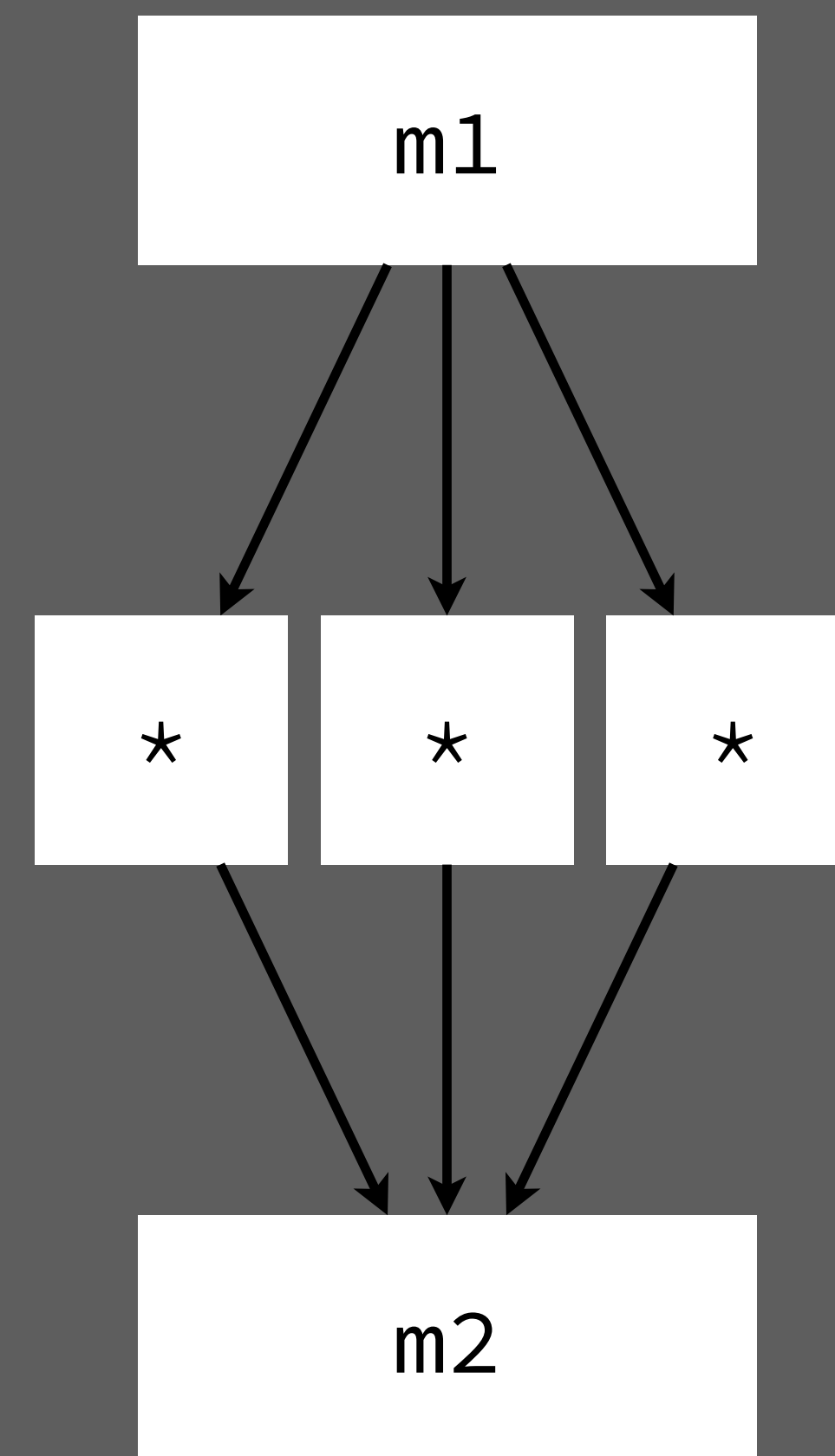


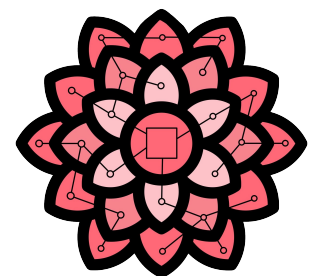


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Hardware

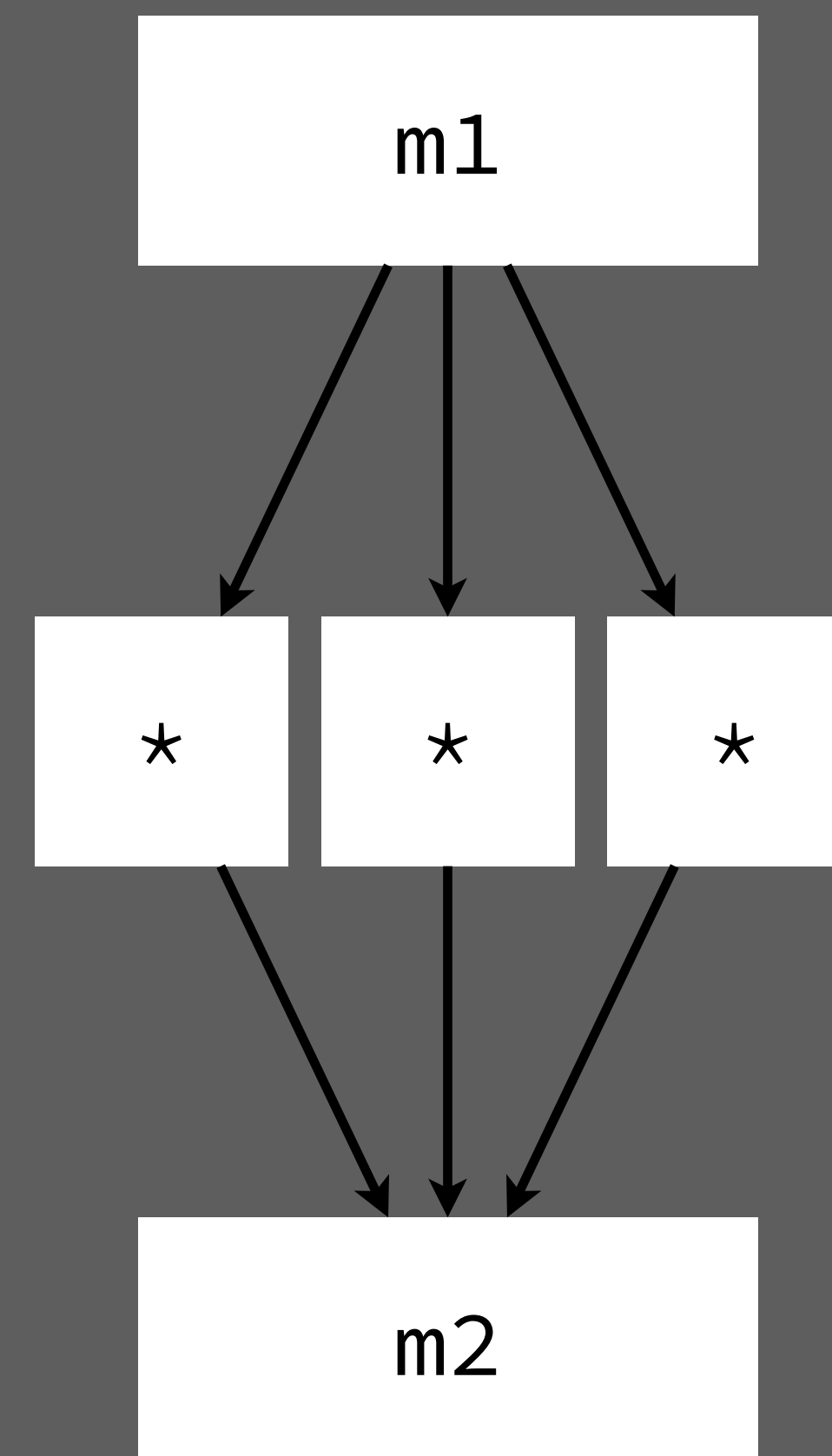


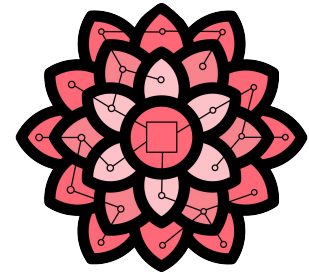


Dahlia

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for (let i = 0 .. 4) {  
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  }  
}
```

Hardware



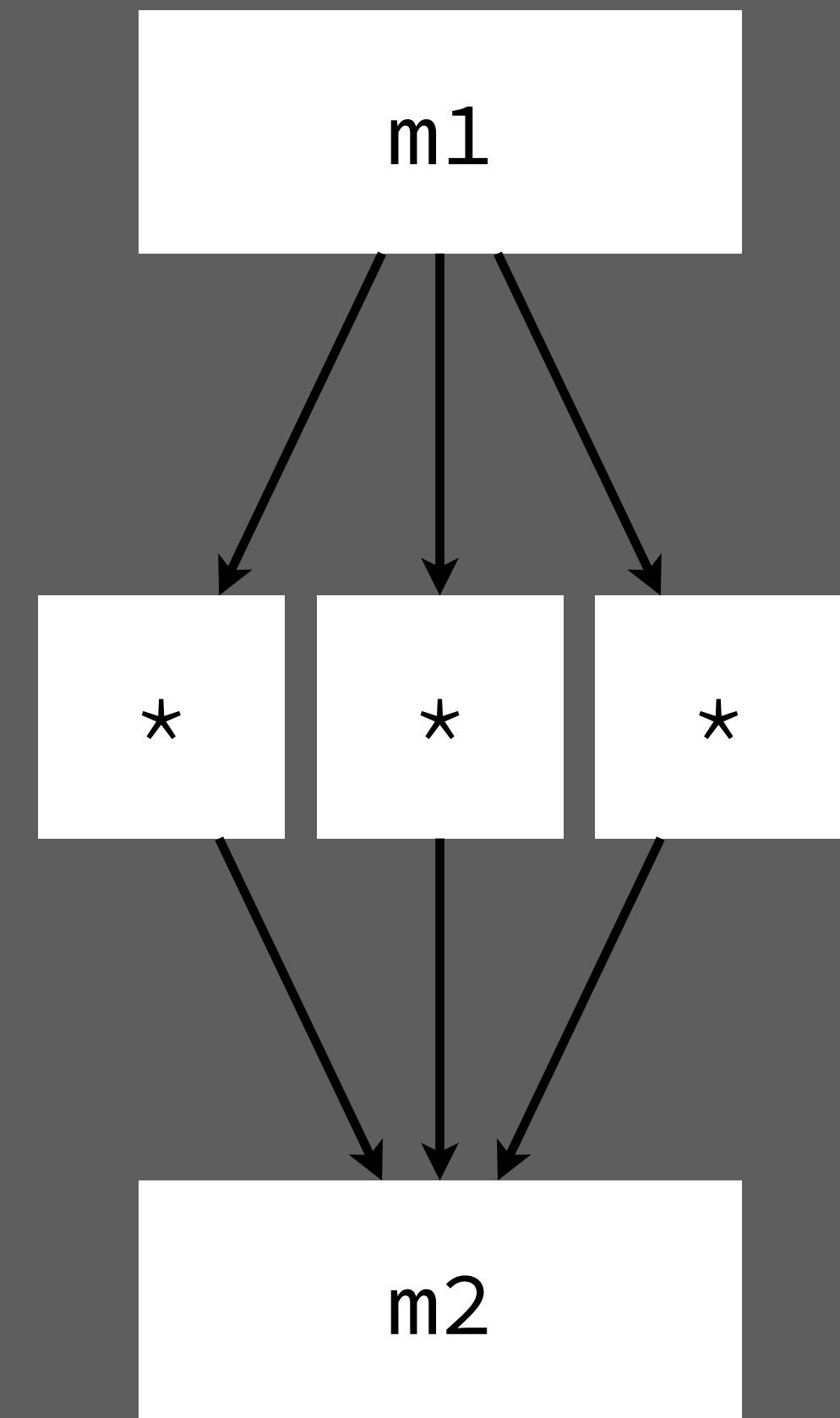


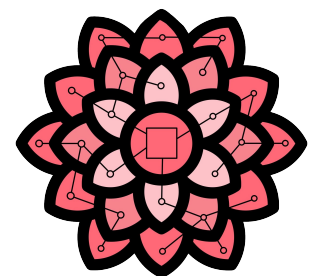
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}
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Parallelizing access patterns requires
unpredictable analyses

Hardware

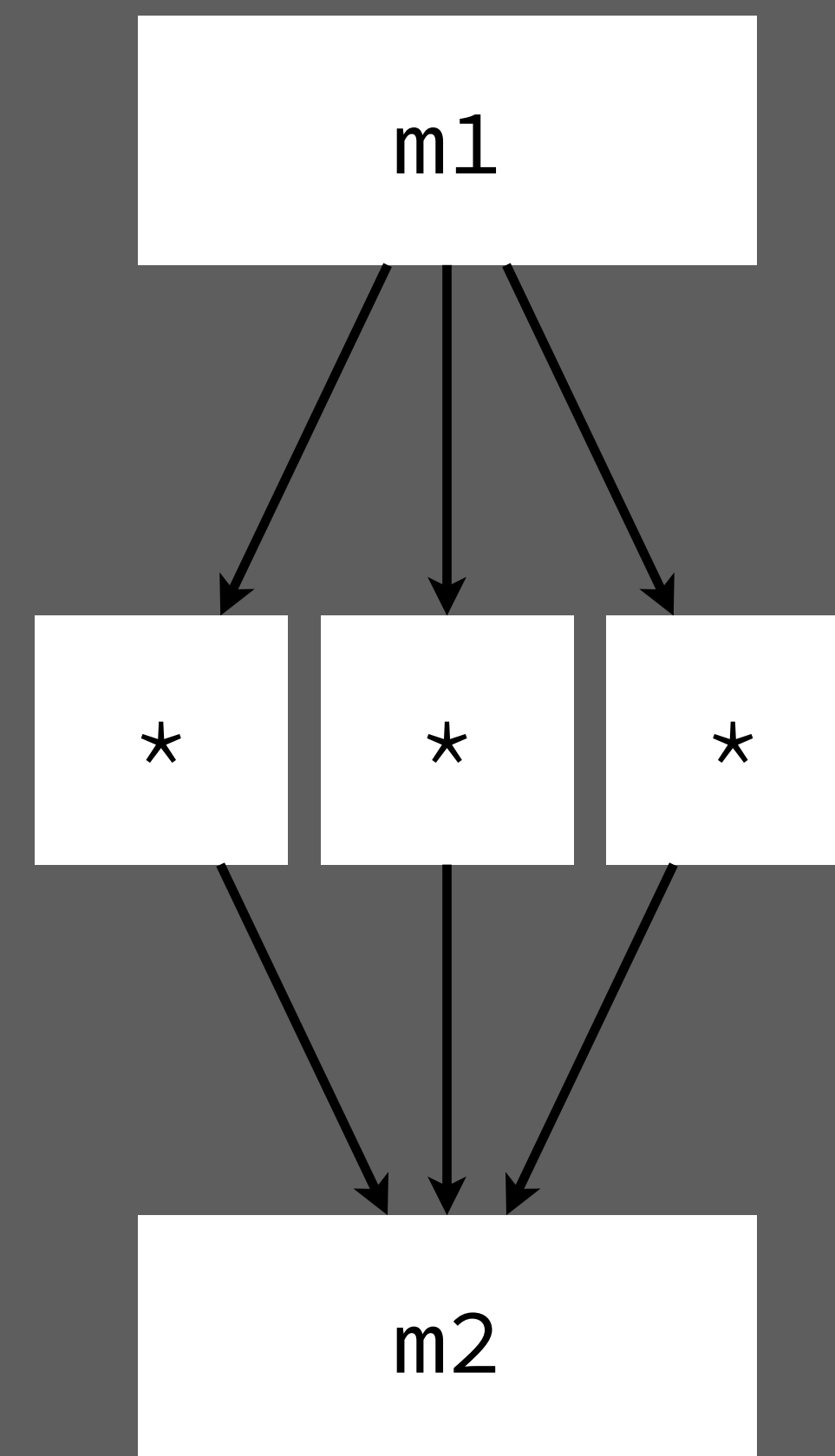


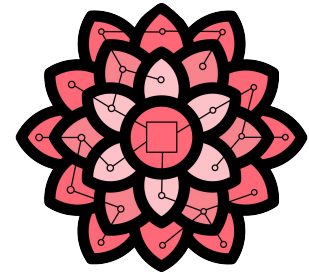


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  for (let j = 0 .. 3) unroll 3 {  
    m2[i] = m1[3*i+j] * 2;  
  }  
}
```

Hardware



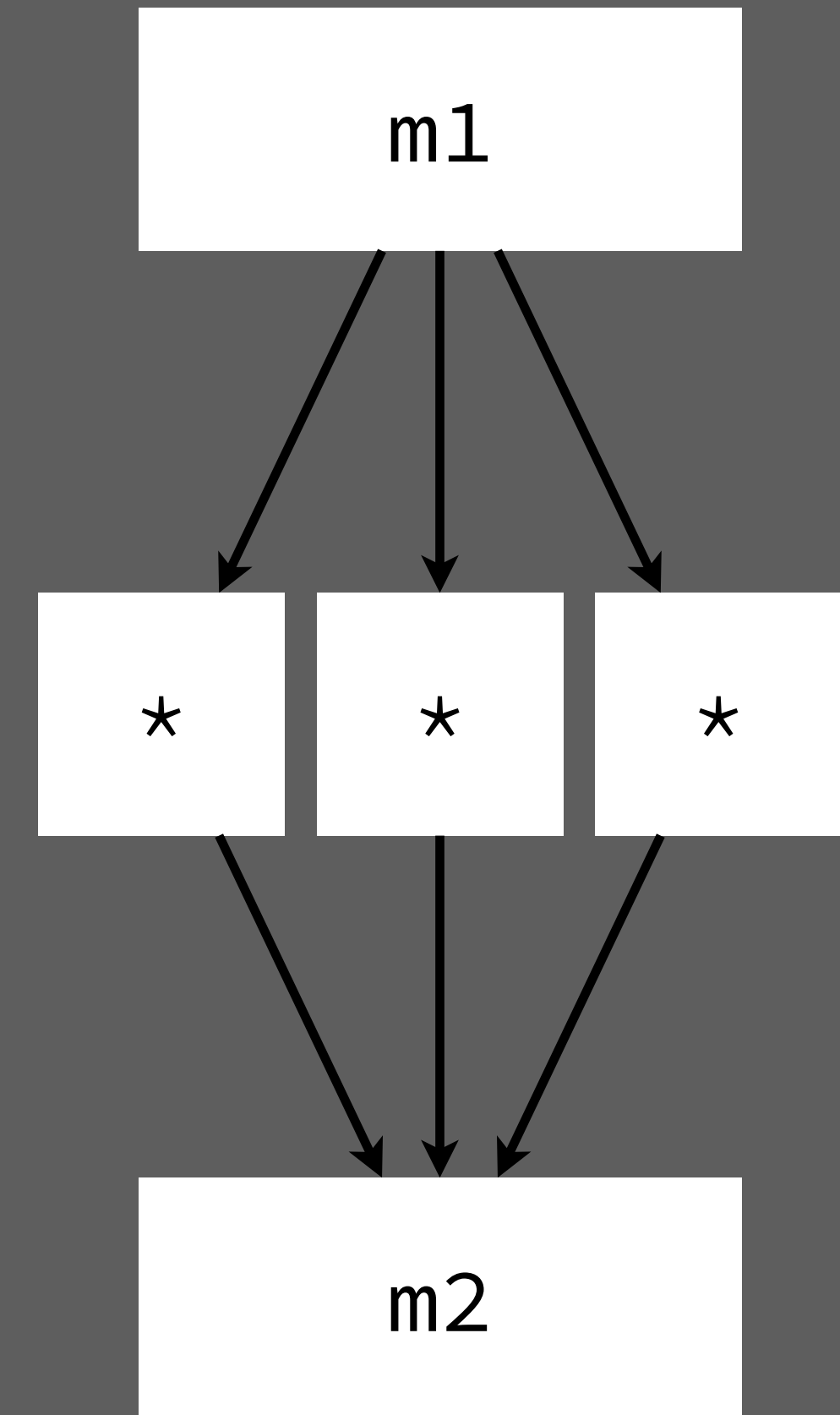


Dahlia

```
let m1: float[12 bank 3];  
let m2: float[12 bank 3];  
  
for (let i = 0 .. 4) {  
  for (let j = 0 .. 3) unroll 3 {  
    m2[i] = m1[3*i+j] * 2;  
  }  
}
```

Error: Cannot parallelize
dynamic access pattern.

Hardware



Memory Views

Memory Views

```
let m1: float[12 bank 3];
```

Memory **Views**

```
let m1: float[12 bank 3];
```

```
view v1 = suffix m1[by 2*i];
```

Memory **Views**

```
let m1: float[12 bank 3];
```

```
view v1 = suffix m1[by 2*i];
```

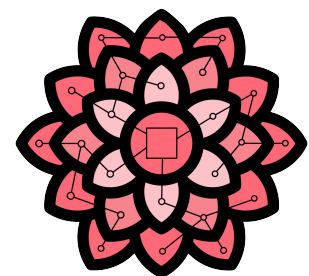
- Hardware cost of **indexing** logic

Memory **Views**

```
let m1: float[12 bank 3];
```

```
view v1 = suffix m1[by 2*i];
```

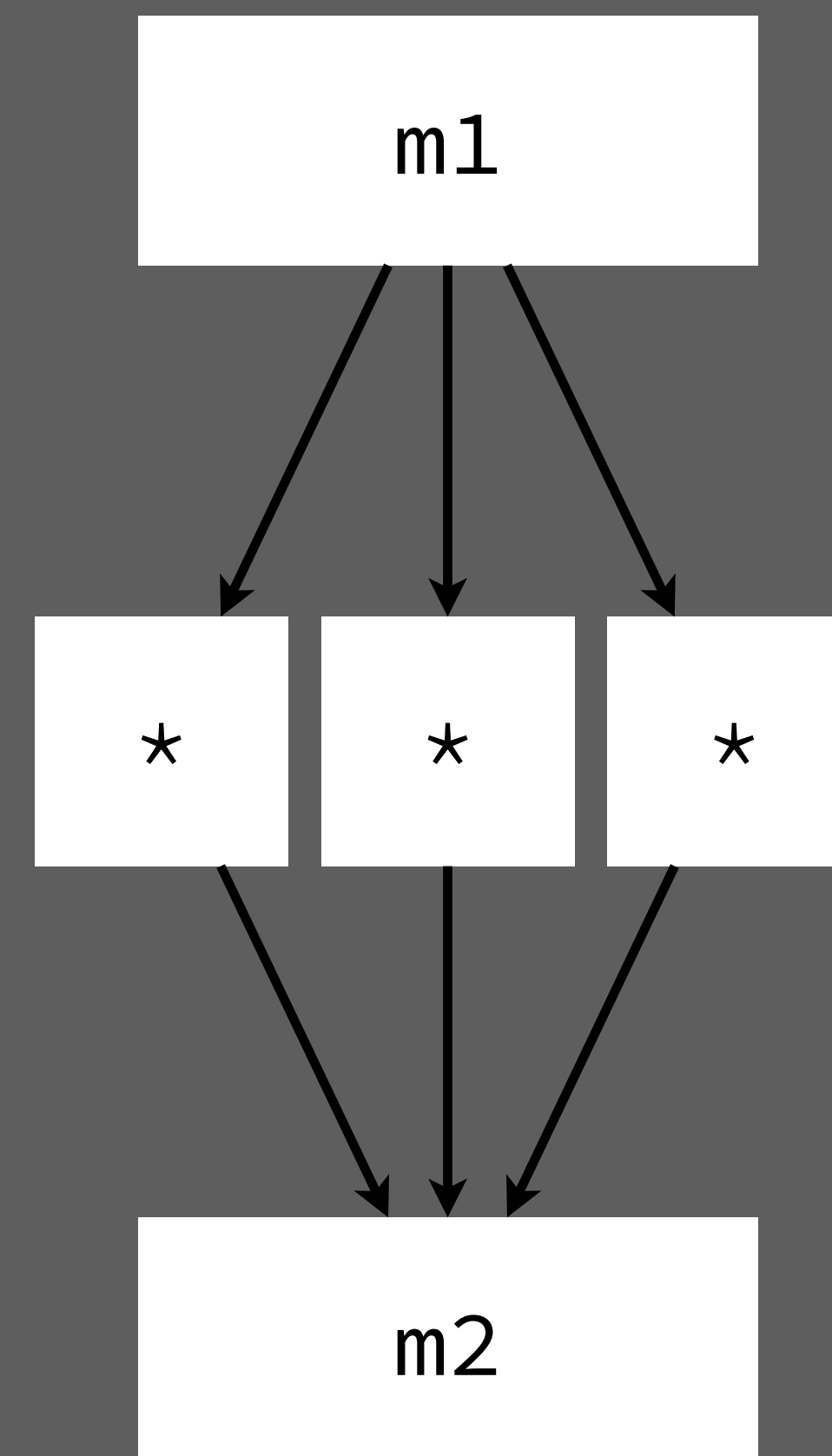
- Hardware cost of **indexing** logic
- **Proof** that accesses can be parallelized

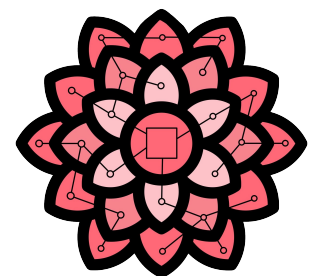


Dahlia

```
let m1: float[12 bank 3];  
let m2: float[12 bank 3];  
  
for (let i = 0 .. 4) {  
    view v1 = suffix m1[by 3*i];  
    for (let j = 0 .. 3) unroll 3 {  
        m2[i] = v1[j] * 2; // m1[3*i+j]  
    }  
}
```

Hardware



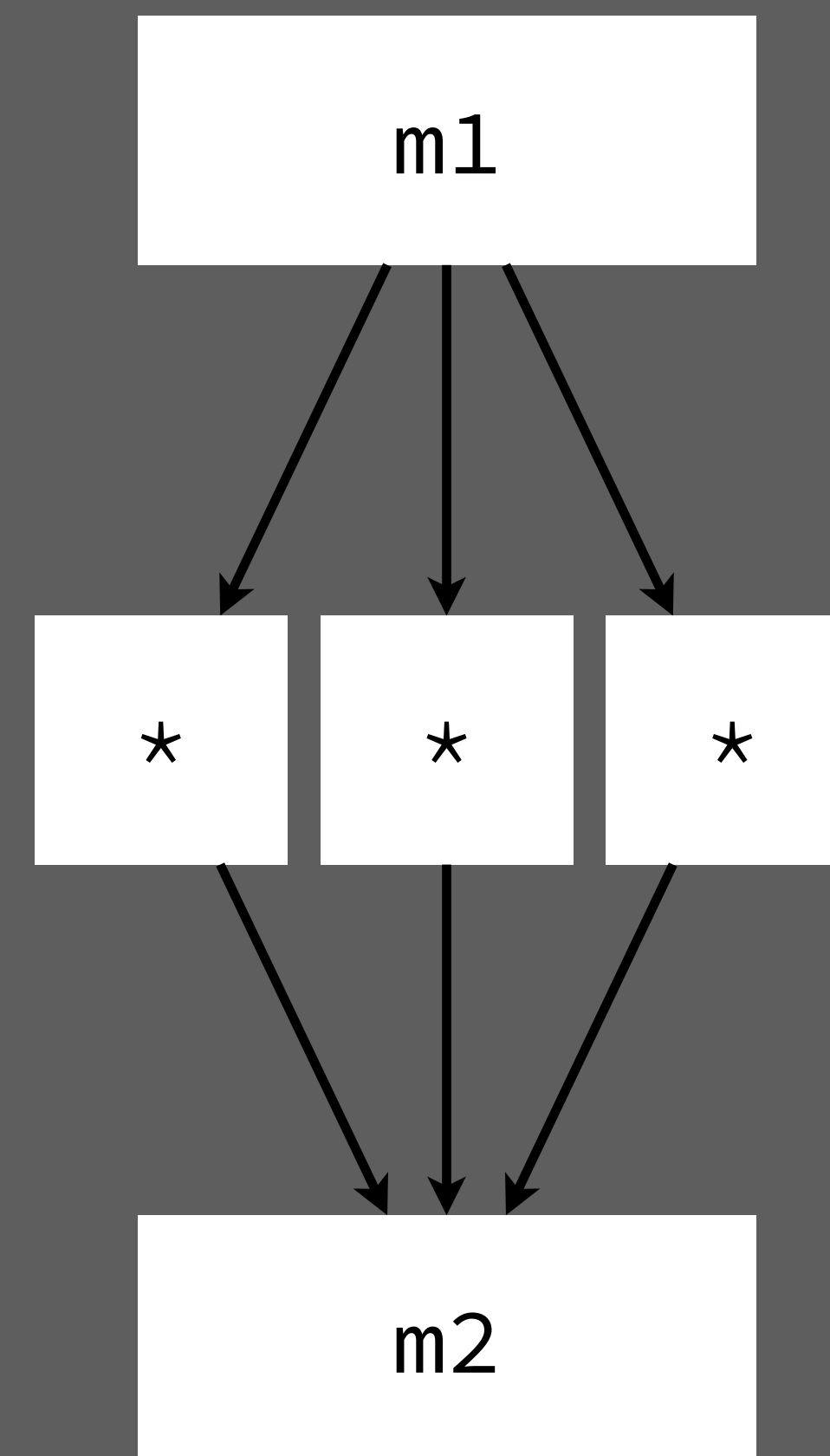


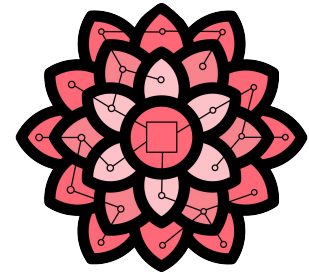
Dahlia

```
let m1: float[12 bank 3];  
let m2: float[12 bank 3];  
  
for (let i = 0 .. 4) {  
    view v1 = suffix m1[by 3*i];  
  
    for (let j = 0 .. 3) unroll 3 {  
        m2[i] = v1[j] * 2; // m1[3*i+j]  
    }  
}
```

Suffix View

Hardware





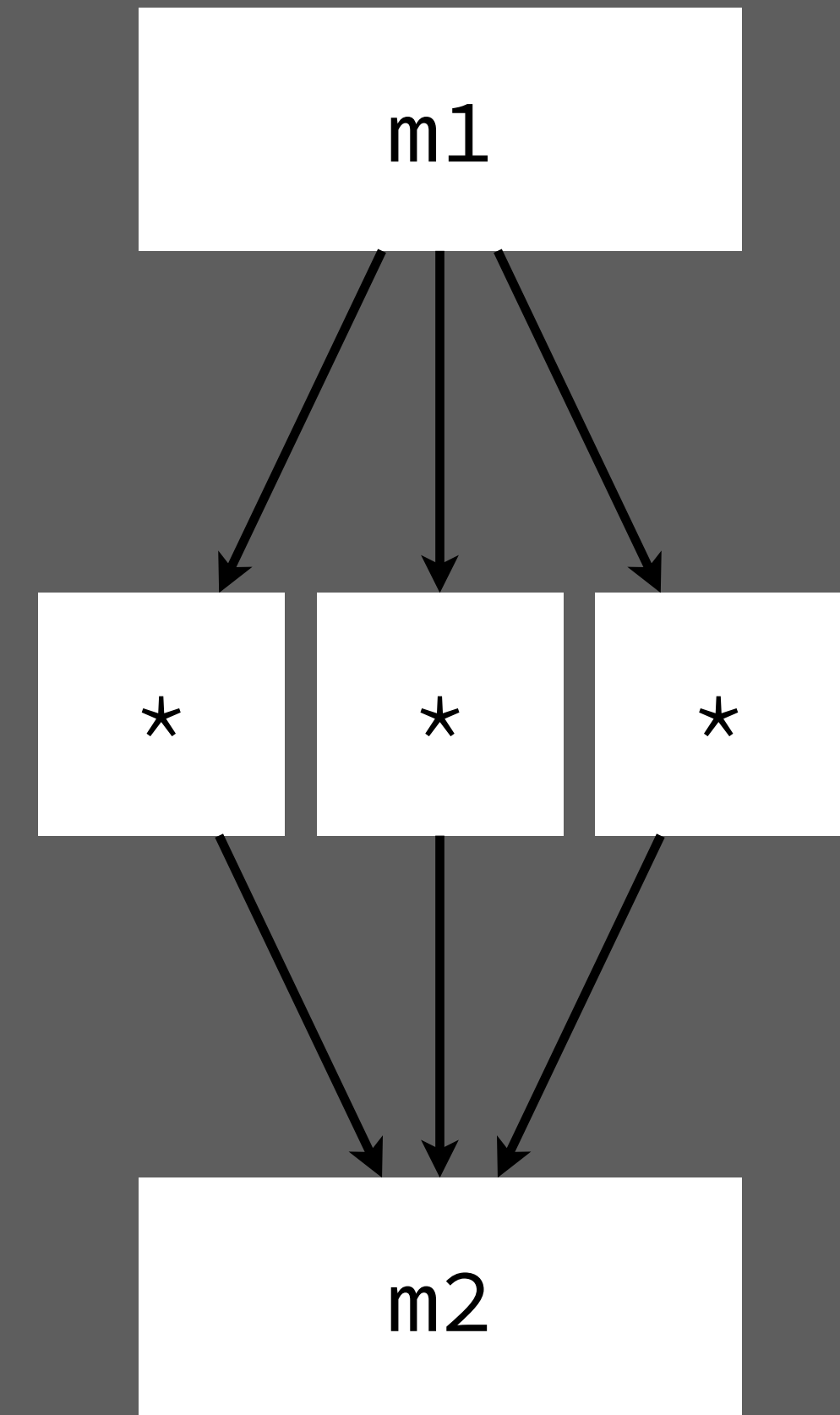
Dahlia

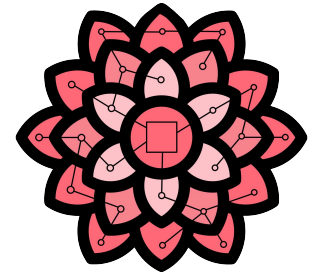
```
let m1: float[12 bank 3];  
let m2: float[12 bank 3];  
  
for (let i = 0 .. 4) {  
    view v1 = suffix m1[by 3*i];  
  
    for (let j = 0 .. 3) unroll 3 {  
        m2[i] = v1[j] * 2; // m1[3*i+j]  
    }  
}
```

Suffix View

- Offset by **banking factor**

Hardware

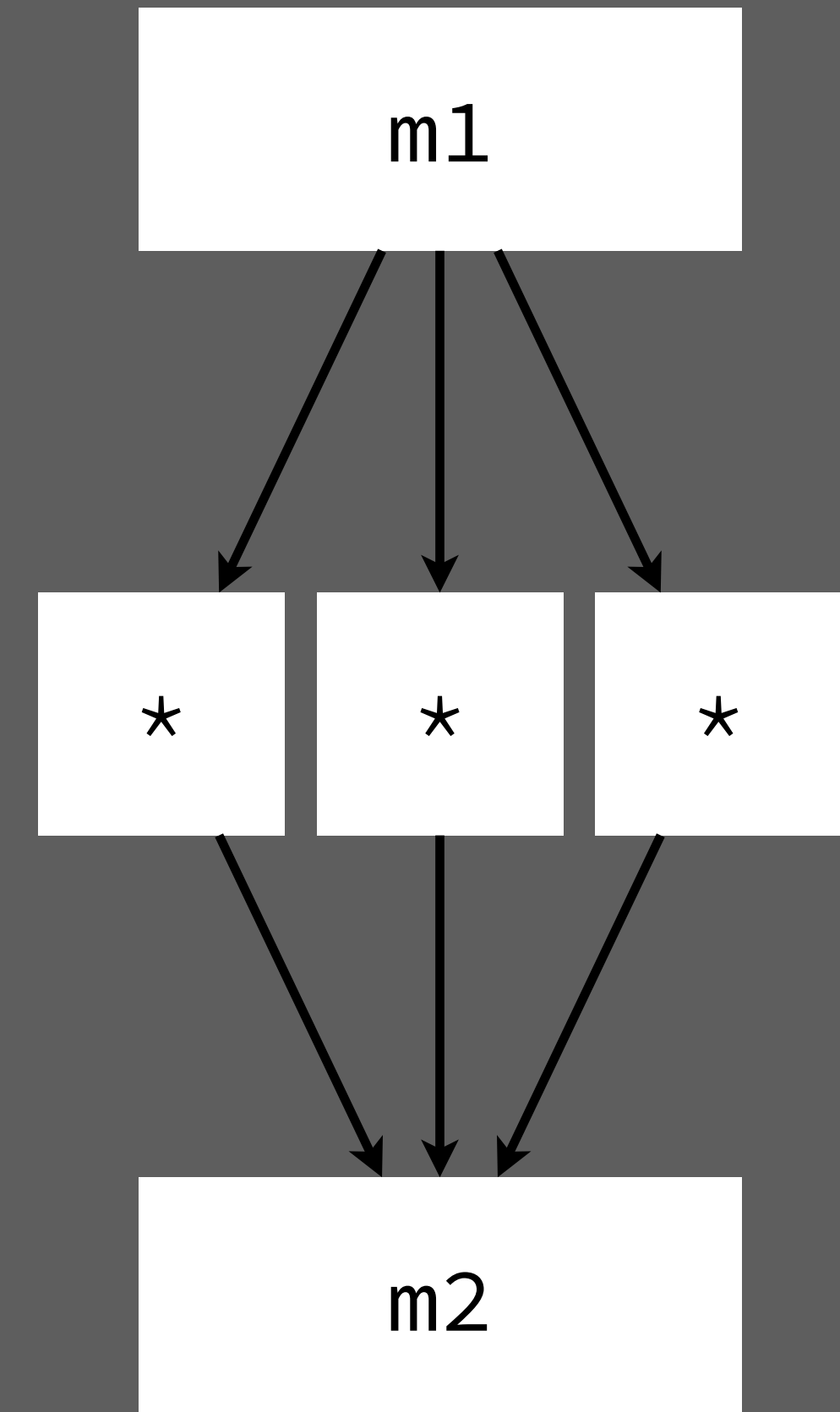


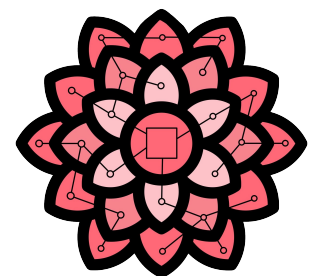


Dahlia

```
let m1: float[12 bank 3];  
let m2: float[12 bank 3];  
  
for (let i = 0 .. 4) {  
    view v1 = shift m1[by f(i)];  
  
    for (let j = 0 .. 3) unroll 3 {  
        m2[i] = v1[j] * 2; // m1[f(i)+j]  
    }  
}
```

Hardware



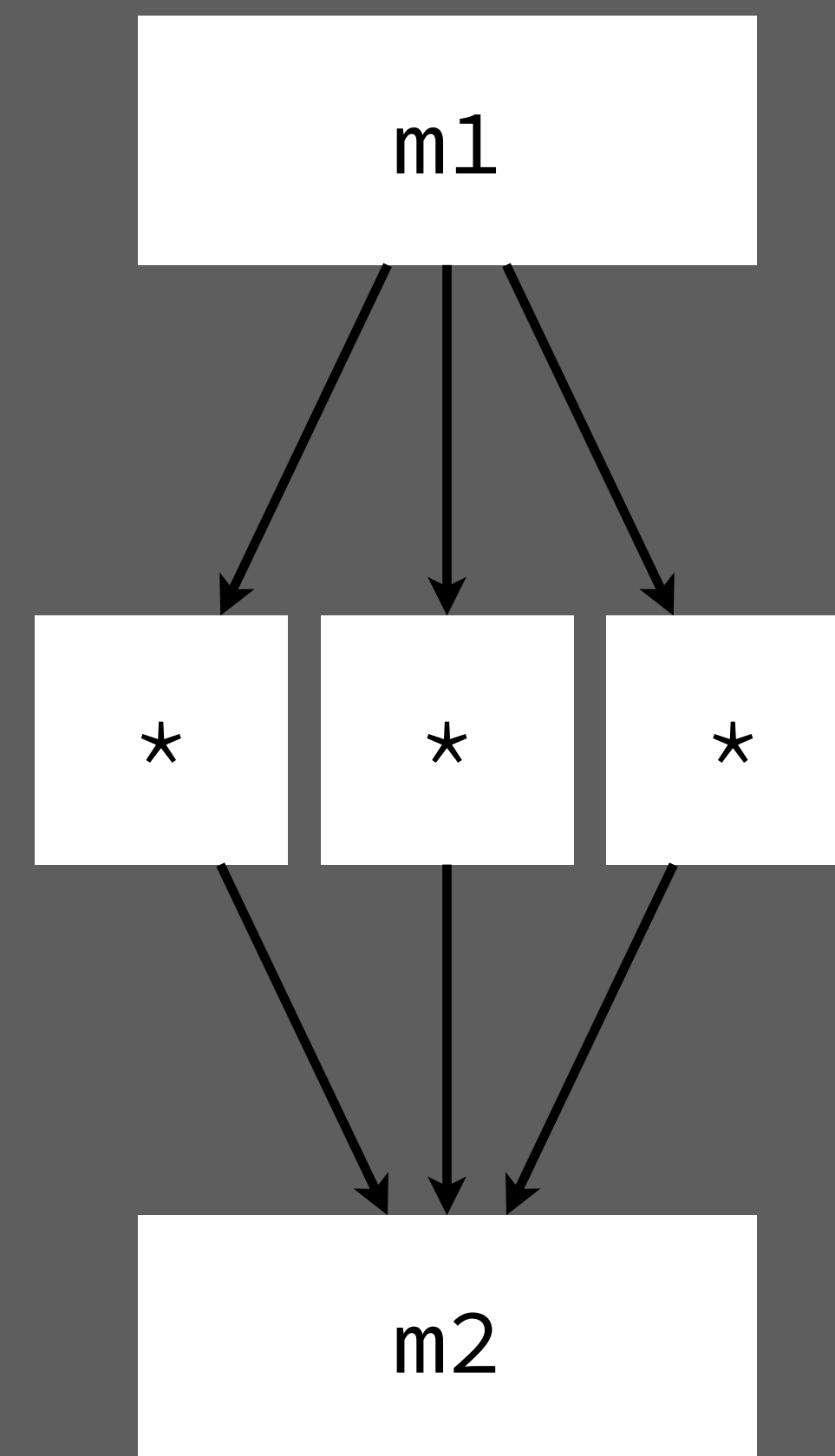


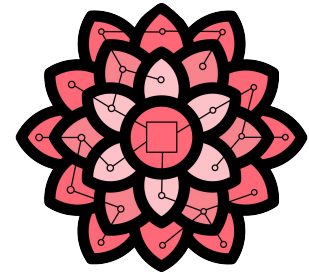
Dahlia

```
let m1: float[12 bank 3];  
let m2: float[12 bank 3];  
  
for (let i = 0 .. 4) {  
    view v1 = shift m1[by f(i)];  
  
    for (let j = 0 .. 3) unroll 3 {  
        m2[i] = v1[j] * 2; // m1[f(i)+j]  
    }  
}
```

Shift View

Hardware





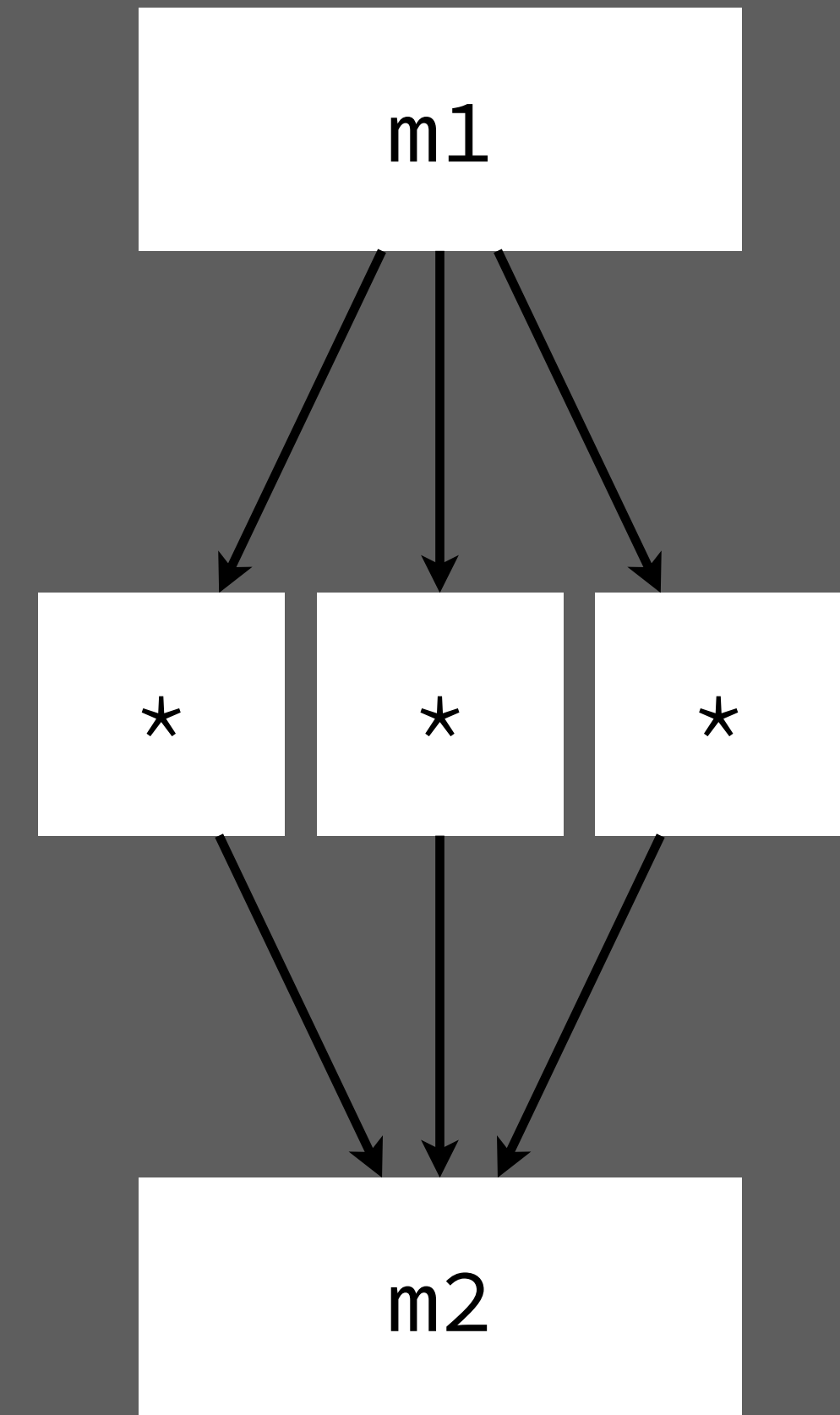
Dahlia

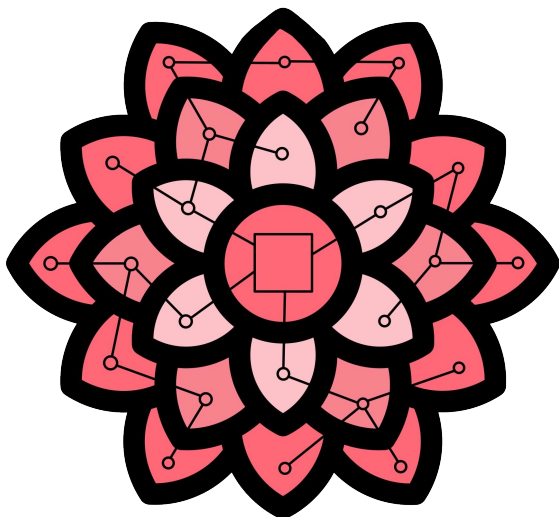
```
let m1: float[12 bank 3];  
let m2: float[12 bank 3];  
  
for (let i = 0 .. 4) {  
    view v1 = shift m1[by f(i)];  
  
    for (let j = 0 .. 3) unroll 3 {  
        m2[i] = v1[j] * 2; // m1[f(i)+j]  
    }  
}
```

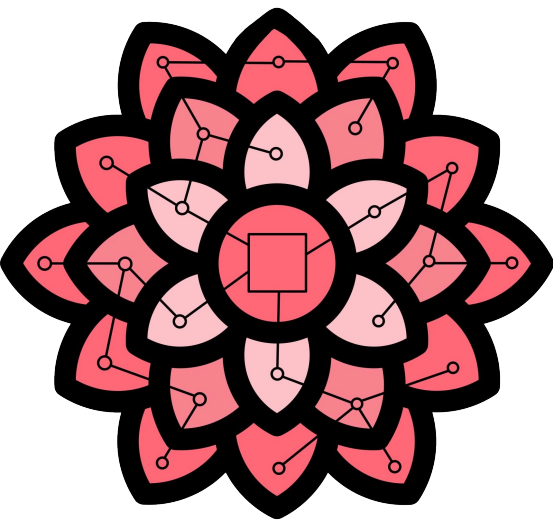
Shift View

- Arbitrary offset

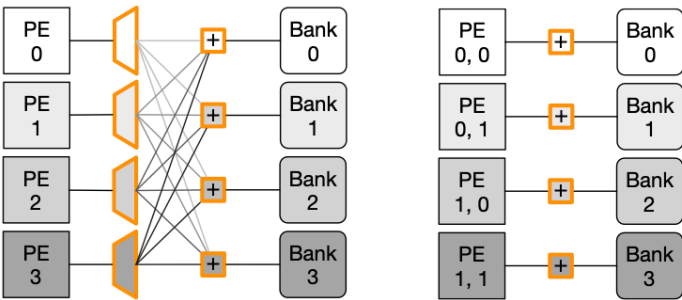
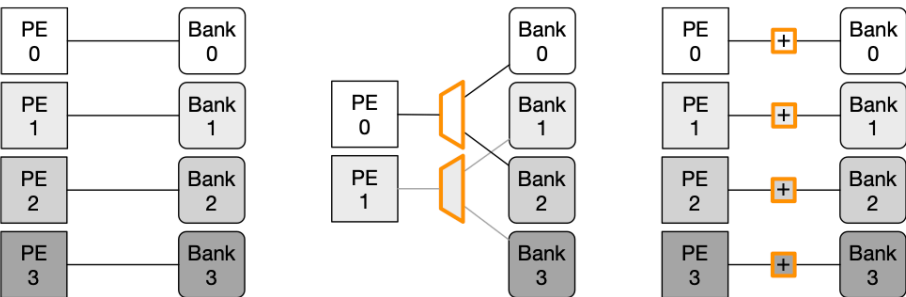
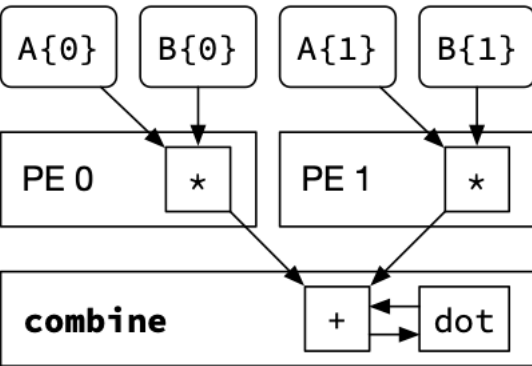
Hardware

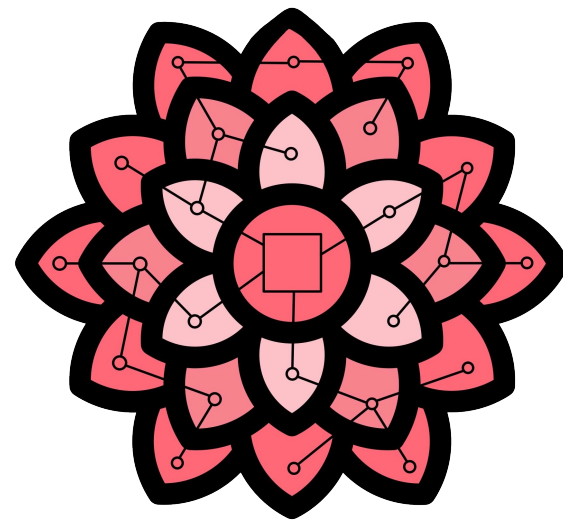






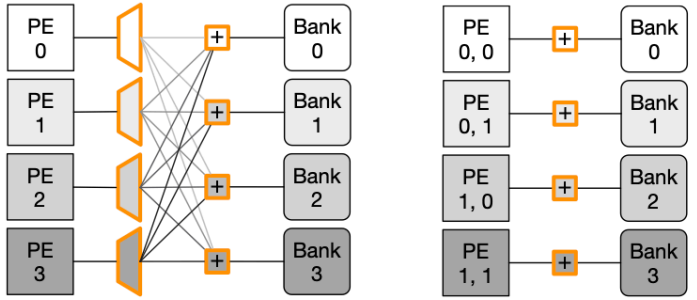
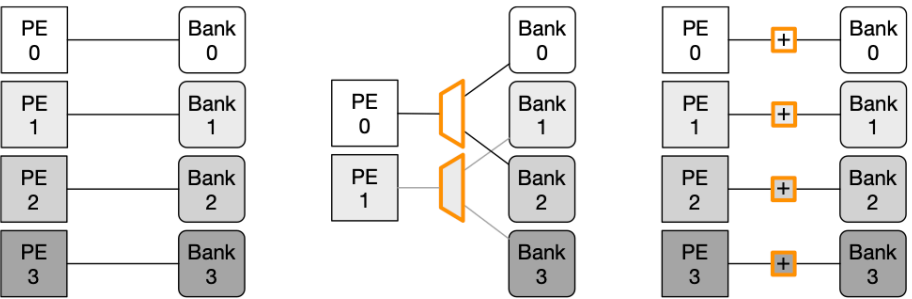
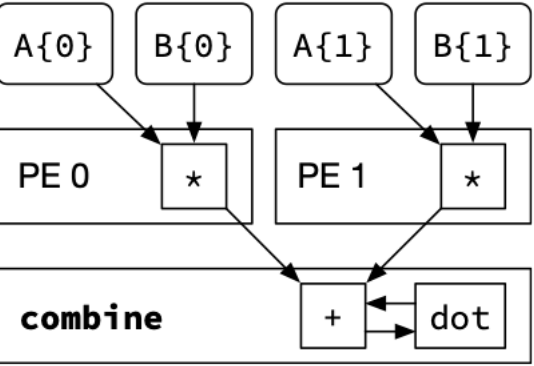
Language Features





Language Features

Formalism

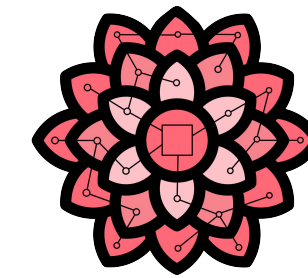
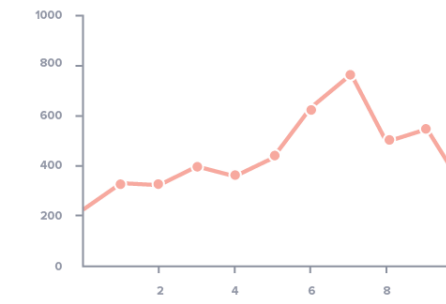
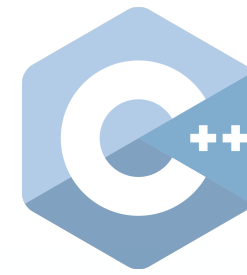
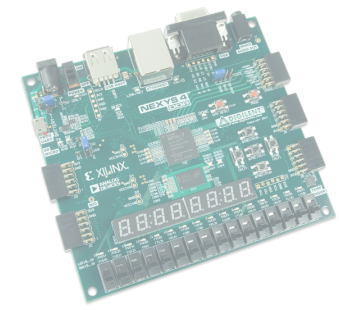


$x \in \text{variables} \quad a \in \text{memories} \quad n \in \text{numbers}$
 $b ::= \text{true} \mid \text{false} \quad v ::= n \mid b$
 $e ::= v \mid \text{bop } e_1 \ e_2 \mid x \mid a[e]$
 $c ::= e \mid \text{let } x = e \mid c_1 \longrightarrow c_2 \mid c_1 ; c_2 \mid \text{if } x \ c_1 \ c_2 \mid$
 $\quad \text{while } x \ c \mid x := e \mid a[e_1] := e_2 \mid \text{skip}$
 $\tau ::= \text{bit}\langle n \rangle \mid \text{float} \mid \text{bool} \mid \text{mem } \tau[n_1]$

$$\begin{array}{c}
 \boxed{\sigma_1, \rho_1, c \rightarrow \sigma', \rho', c'} \\
 \hline
 \frac{\sigma, \rho, e_1 \rightarrow \sigma', \rho', e'_1}{\sigma, \rho, a[e_1] := e_2 \rightarrow \sigma', \rho', a[e'_1] := e_2} \\
 \hline
 \frac{\sigma, \rho, e \rightarrow \sigma', \rho', e'}{\sigma, \rho, a[n] := e \rightarrow \sigma', \rho', a[n] := e'} \\
 \hline
 \frac{a \notin \rho}{\sigma, \rho, a[n] := v \rightarrow \sigma[a[n] \mapsto v], \rho \cup \{a\}, \text{skip}} \\
 \hline
 \frac{\sigma, \rho, e \rightarrow \sigma', \rho', e'}{\sigma, \rho, \text{let } x = e \rightarrow \sigma', \rho', \text{let } x = e'} \\
 \hline
 \sigma, \rho, \text{let } x = v \rightarrow \sigma[x \mapsto v], \rho, \text{skip} \\
 \hline
 \frac{\sigma, \rho, c_1 \rightarrow \sigma', \rho', c'_1}{\sigma, \rho, c_1 ; c_2 \rightarrow \sigma', \rho', c'_1 ; c_2} \qquad \frac{}{\sigma, \rho, \text{skip} ; c_2 \rightarrow \sigma, \rho, c_2} \\
 \hline
 \frac{}{\sigma, \rho, c_1 \longrightarrow c_2 \rightarrow \sigma, \rho, c_1 \overset{\rho}{\sim} c_2} \\
 \hline
 \frac{\sigma, \rho, c_1 \rightarrow \sigma', \rho', c'_1}{\sigma, \rho, c_1 \overset{\rho''}{\sim} c_2 \rightarrow \sigma', \rho', c'_1 \overset{\rho''}{\sim} c_2} \\
 \hline
 \frac{\sigma, \rho'', c_2 \rightarrow \sigma', \rho''', c'_2}{\sigma, \rho, \text{skip} \overset{\rho''}{\sim} c_2 \rightarrow \sigma', \rho, \text{skip} \overset{\rho'''}{\sim} c'_2} \\
 \hline
 \frac{}{\sigma, \rho, \text{skip} \overset{\rho''}{\sim} \text{skip} \rightarrow \sigma, \rho \cup \rho'', \text{skip}}
 \end{array}$$

$$\begin{array}{c}
 \boxed{\sigma_1, \rho_1, e \Downarrow \sigma_2, \rho_2, v} \\
 \hline
 \frac{\sigma_1, \rho_1, e_1 \Downarrow \sigma_2, \rho_2, v_1 \quad \sigma_2, \rho_2, e_2 \Downarrow \sigma_3, \rho_3, v_2 \quad v_3 = v_1 \text{ bop } v_2}{\sigma_1, \rho_1, \text{bop } e_1 \ e_2 \Downarrow \sigma_3, \rho_3, v_3} \\
 \hline
 \frac{\sigma(x) = v}{\sigma, \rho, x \Downarrow \sigma, \rho, v} \\
 \hline
 \frac{a \notin \rho_1 \quad \sigma_1, \rho_1, e \Downarrow \sigma_2, \rho_2, n \quad \sigma_2(a)(n) = v}{\sigma_1, \rho_1, a[e] \Downarrow \sigma_2, \rho_2 \cup \{a\}, v} \\
 \hline
 \boxed{\sigma_1, \rho_1, c \Downarrow \sigma_2, \rho_2} \\
 \hline
 \frac{\sigma_1, \rho_1, e \Downarrow \sigma_2, \rho_2, v}{\sigma_1, \rho_1, \text{let } x = e \Downarrow \sigma_2[x \mapsto v], \rho_2} \\
 \hline
 \frac{\sigma_1, \rho_1, c_1 \Downarrow \sigma_2, \rho_2 \quad \sigma_2, \rho_1, c_2 \Downarrow \sigma_3, \rho_3}{\sigma_1, \rho_1, c_1 \longrightarrow c_2 \Downarrow \sigma_3, \rho_2 \cup \rho_3} \\
 \hline
 \frac{\sigma_1, \rho_1, c_1 \Downarrow \sigma_2, \rho_2 \quad \sigma_2, \rho, c_2 \Downarrow \sigma_3, \rho_3}{\sigma_1, \rho_1, c_1 \overset{\rho}{\sim} c_2 \Downarrow \sigma_3, \rho_2 \cup \rho_3} \\
 \hline
 \frac{\sigma_1, \rho_1, c_1 \Downarrow \sigma_2, \rho_2 \quad \sigma_2, \rho_2, c_2 \Downarrow \sigma_3, \rho_3}{\sigma_1, \rho_1, c_1 ; c_2 \Downarrow \sigma_3, \rho_3} \\
 \hline
 \frac{\sigma_1, \rho_1, e_1 \Downarrow \sigma_2, \rho_2, \text{true} \quad \sigma_2, \rho_2, c_1 \Downarrow \sigma_3, \rho_3}{\sigma_1, \rho_1, \text{if } x \ c_1 \ c_2 \Downarrow \sigma_3, \rho_3} \\
 \hline
 \frac{\sigma_1, \rho_1, e_1 \Downarrow \sigma_2, \rho_2, \text{false} \quad \sigma_2, \rho_2, c_2 \Downarrow \sigma_3, \rho_3}{\sigma_1, \rho_1, \text{if } x \ c_1 \ c_2 \Downarrow \sigma_3, \rho_3} \\
 \hline
 \frac{\sigma_1, \rho_1, e_1 \Downarrow \sigma_2, \rho_2, \text{true} \quad \sigma_2, \rho_2, c \longrightarrow \text{while } x \ c \Downarrow \sigma_3, \rho_3}{\sigma_1, \rho_1, \text{while } x \ c \Downarrow \sigma_3, \rho_3} \qquad \frac{\sigma_1, \rho_1, e_1 \Downarrow \sigma_2, \rho_2, \text{false}}{\sigma_1, \rho_1, \text{while } x \ c \Downarrow \sigma_2, \rho_2}
 \end{array}$$

Ada's Journey



Type Systems

Well-typed programs
cannot **go wrong**

Type Systems

Well-typed programs
make **predictable trade-offs**

Design Space Exploration

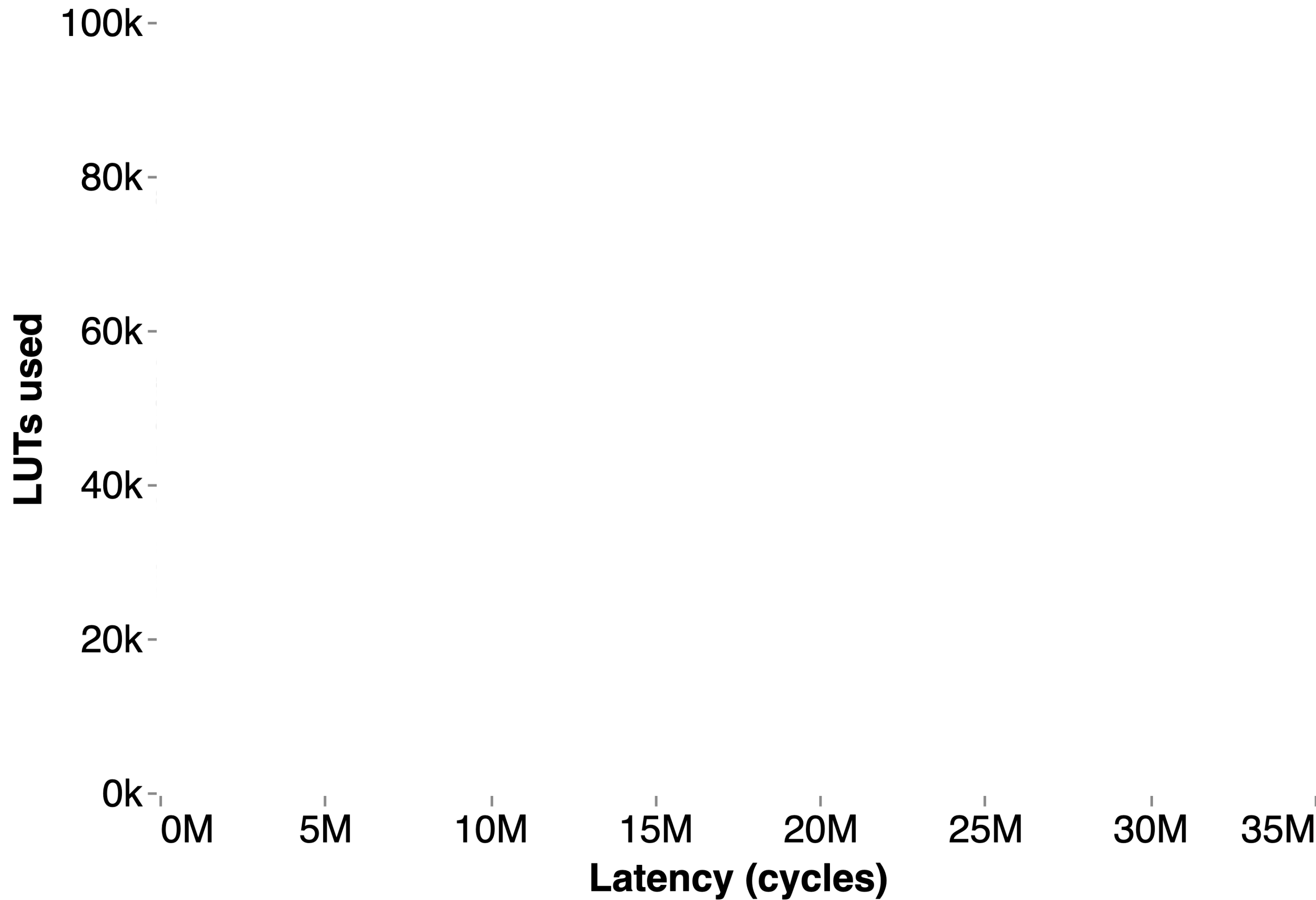
Design Space Exploration

Blocked matrix-
matrix multiply

32,000 designs

2,600 hours of
estimation

Design Space Exploration

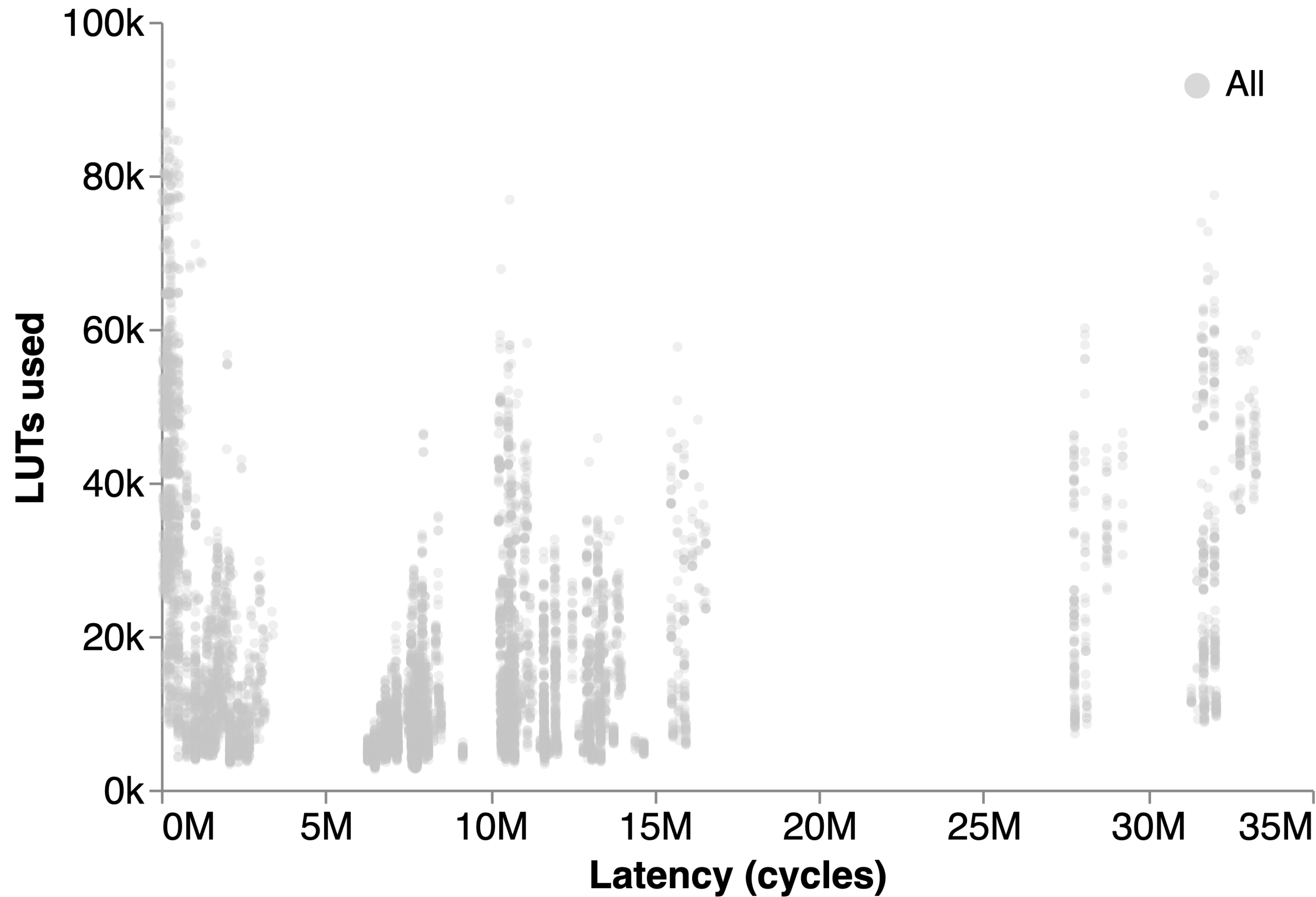


Blocked matrix-
matrix multiply

32,000 designs

2,600 hours of
estimation

Design Space Exploration

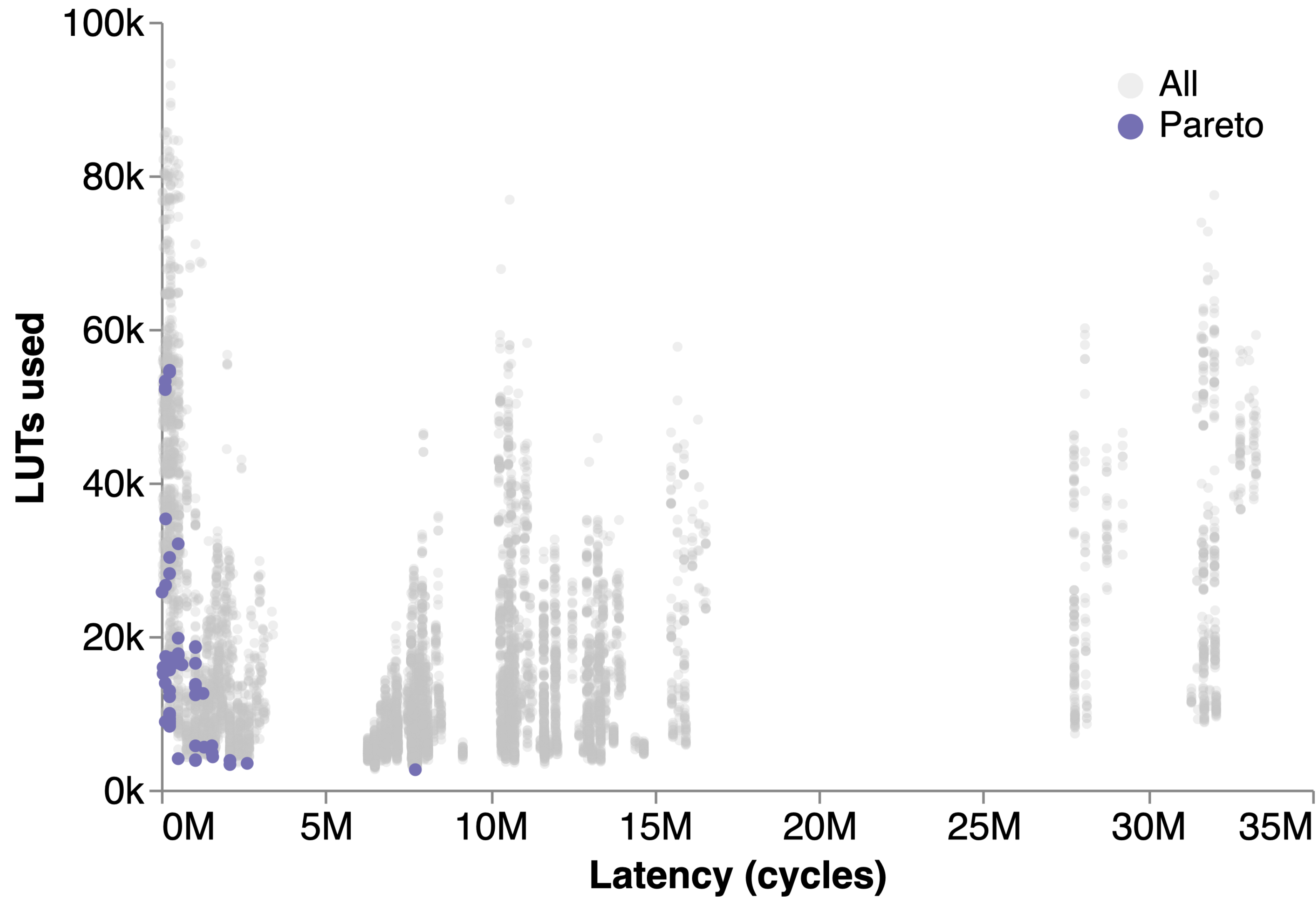


Blocked matrix-
matrix multiply

32,000 designs

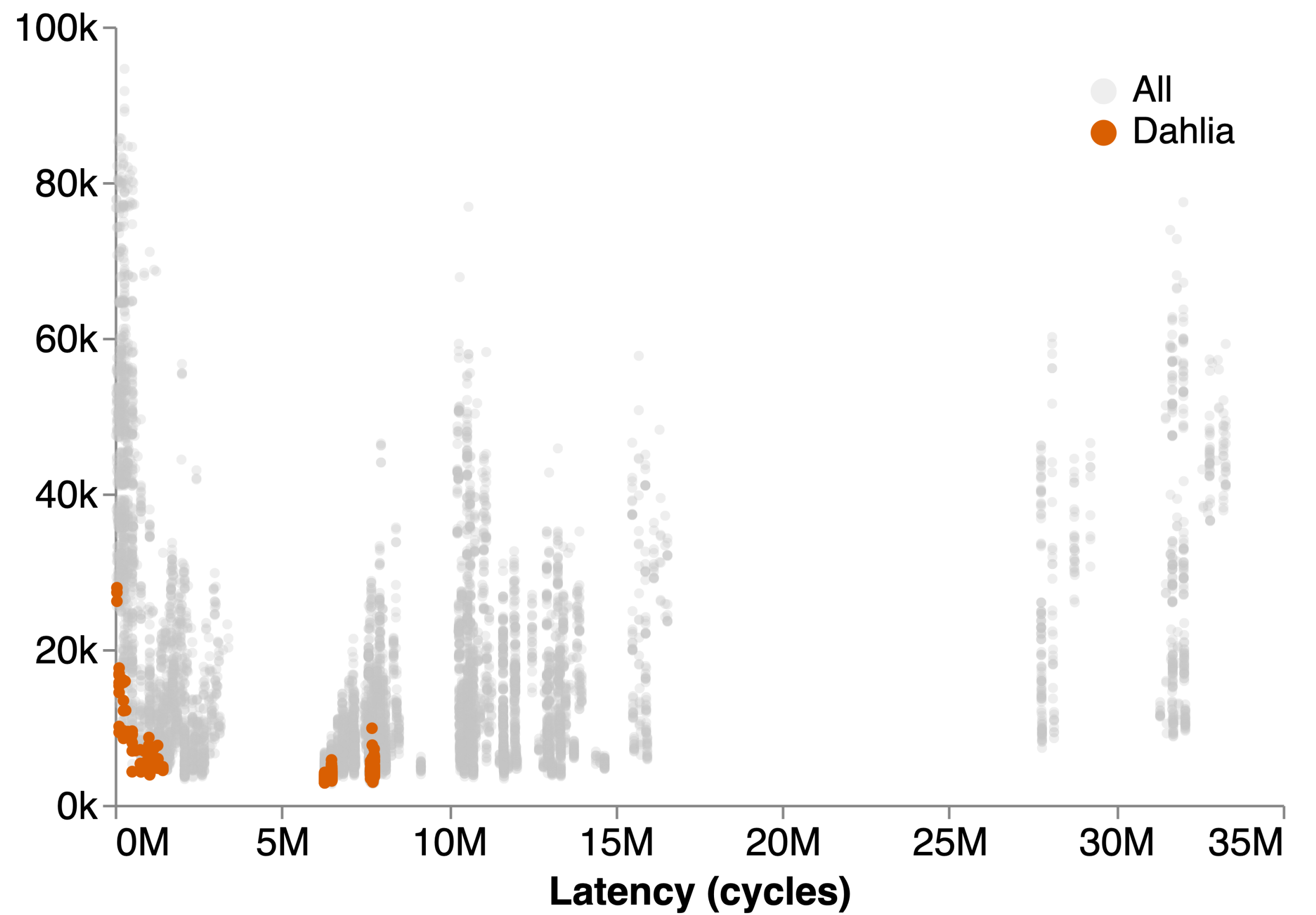
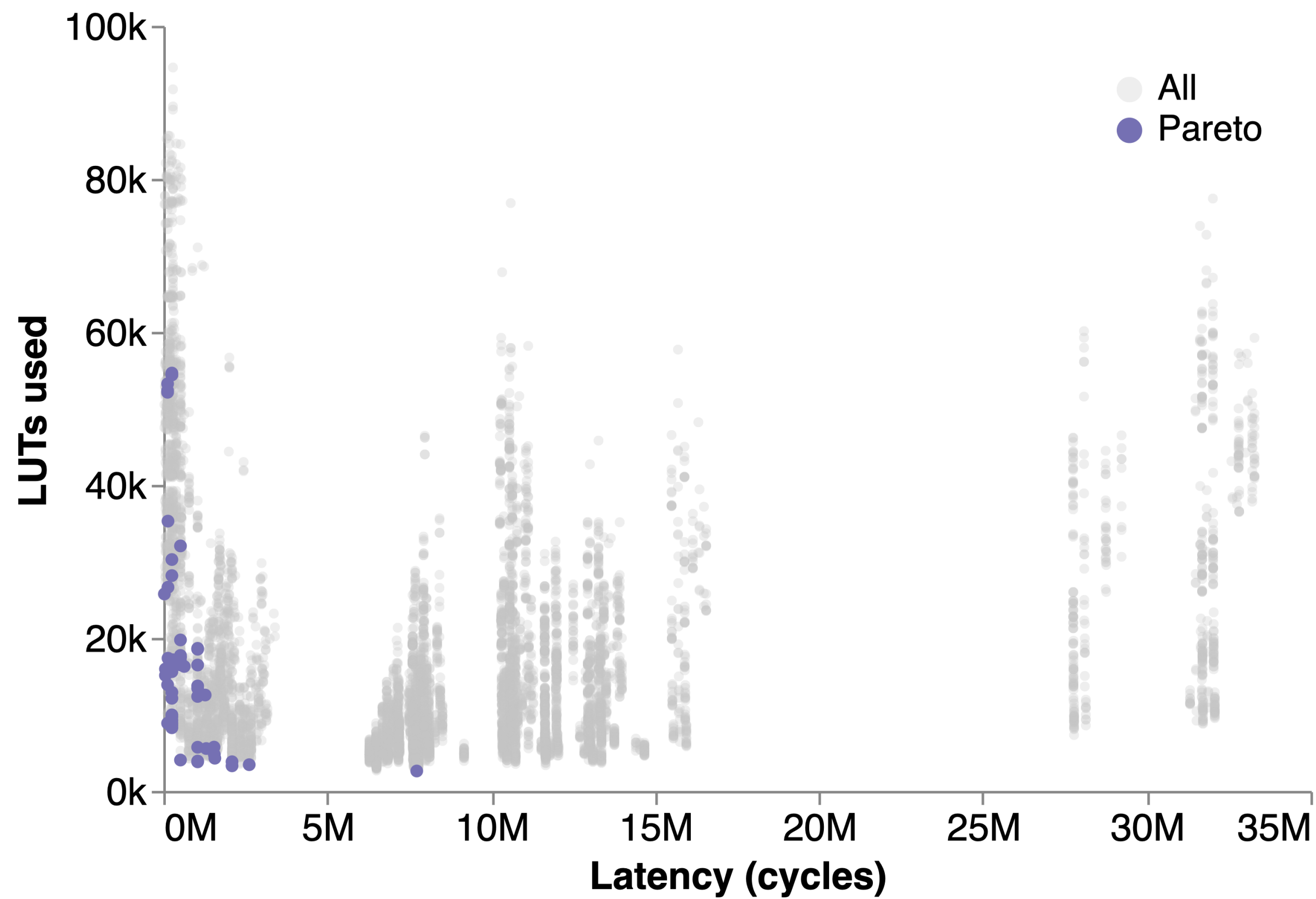
2,600 hours of
estimation

Design Space Exploration

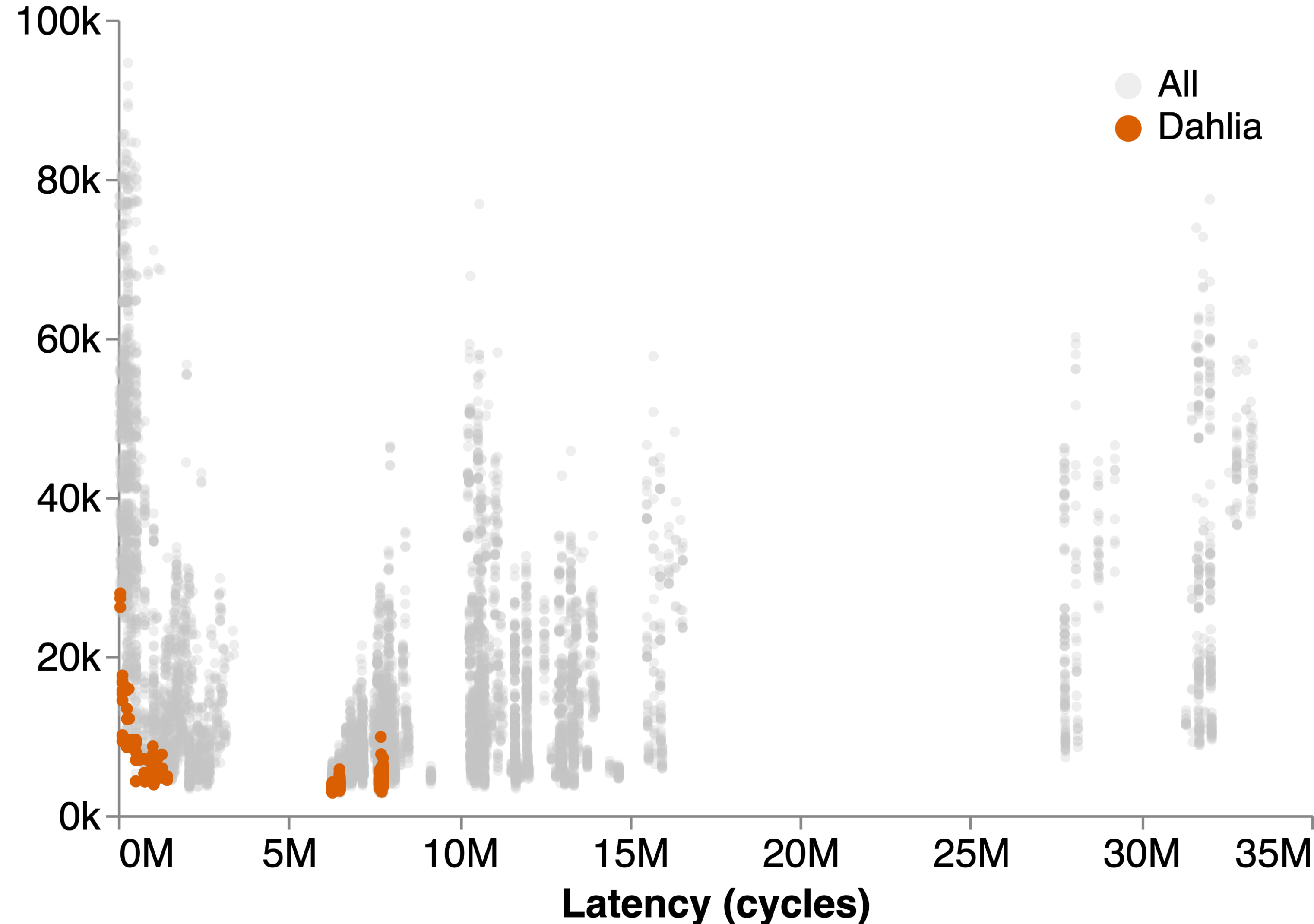


Pareto optimal w.r.t
FPGA resources and
runtime

Design Space Exploration

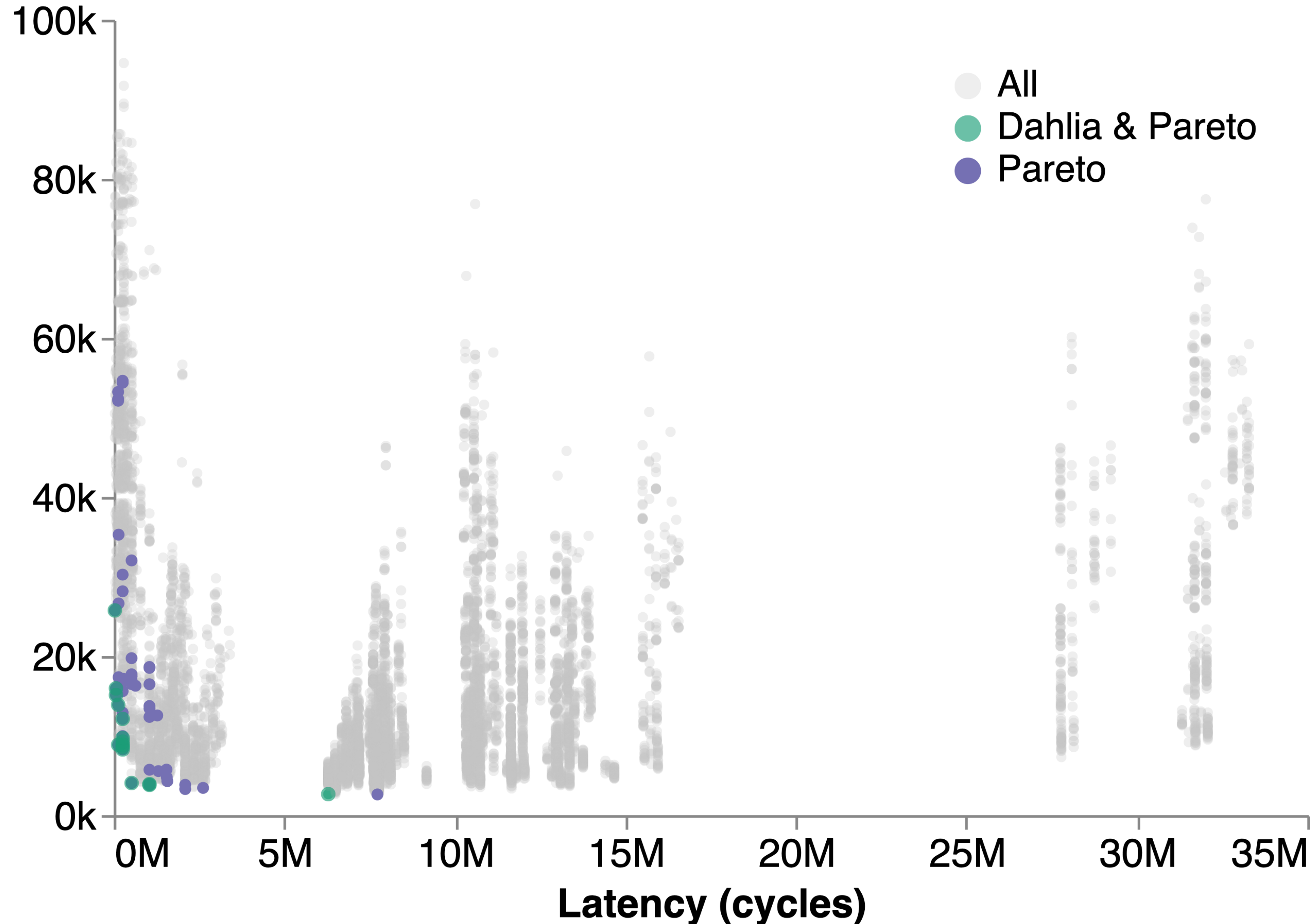


Design Space Exploration



354 out of 32,000
configurations
accepted

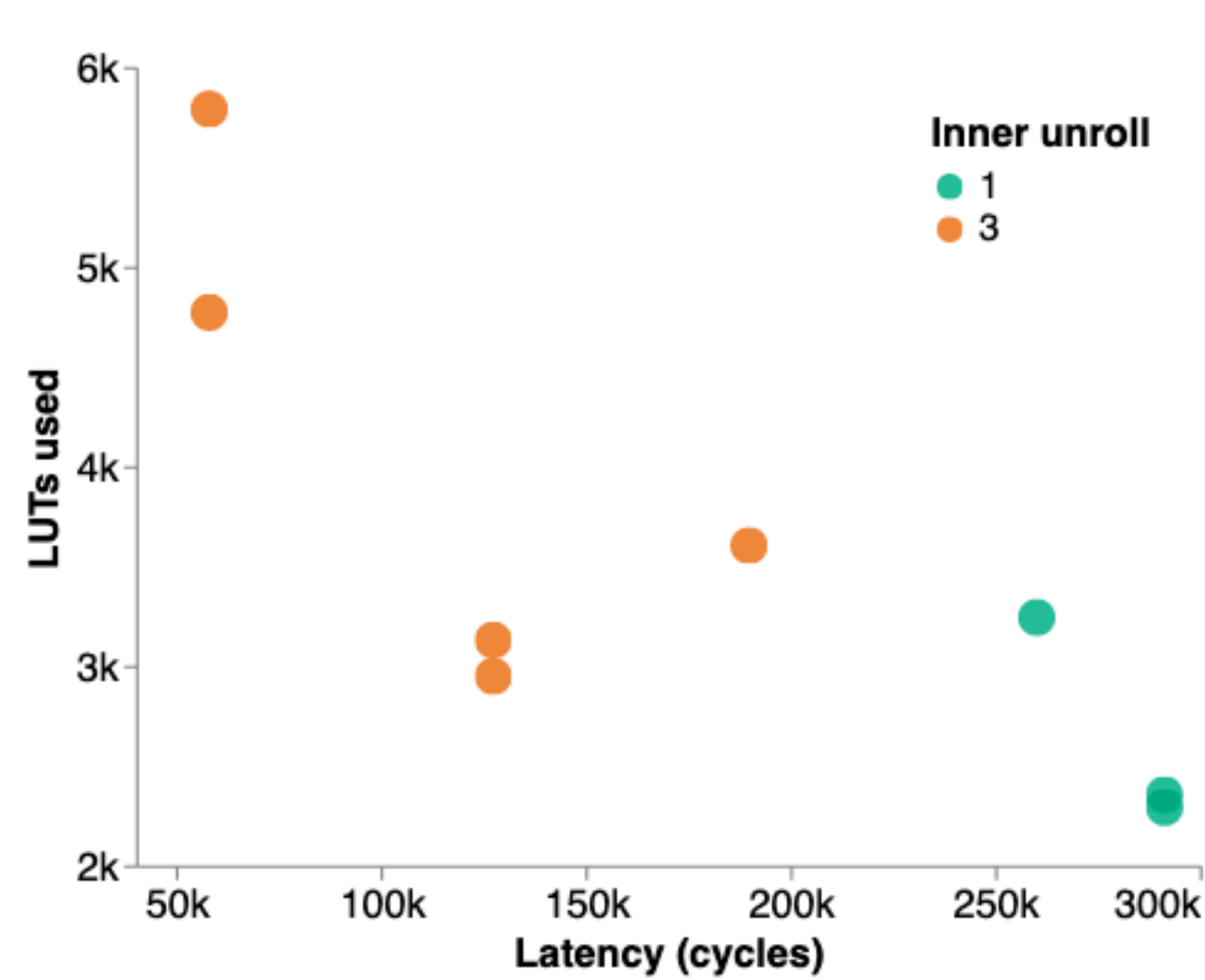
Design Space Exploration



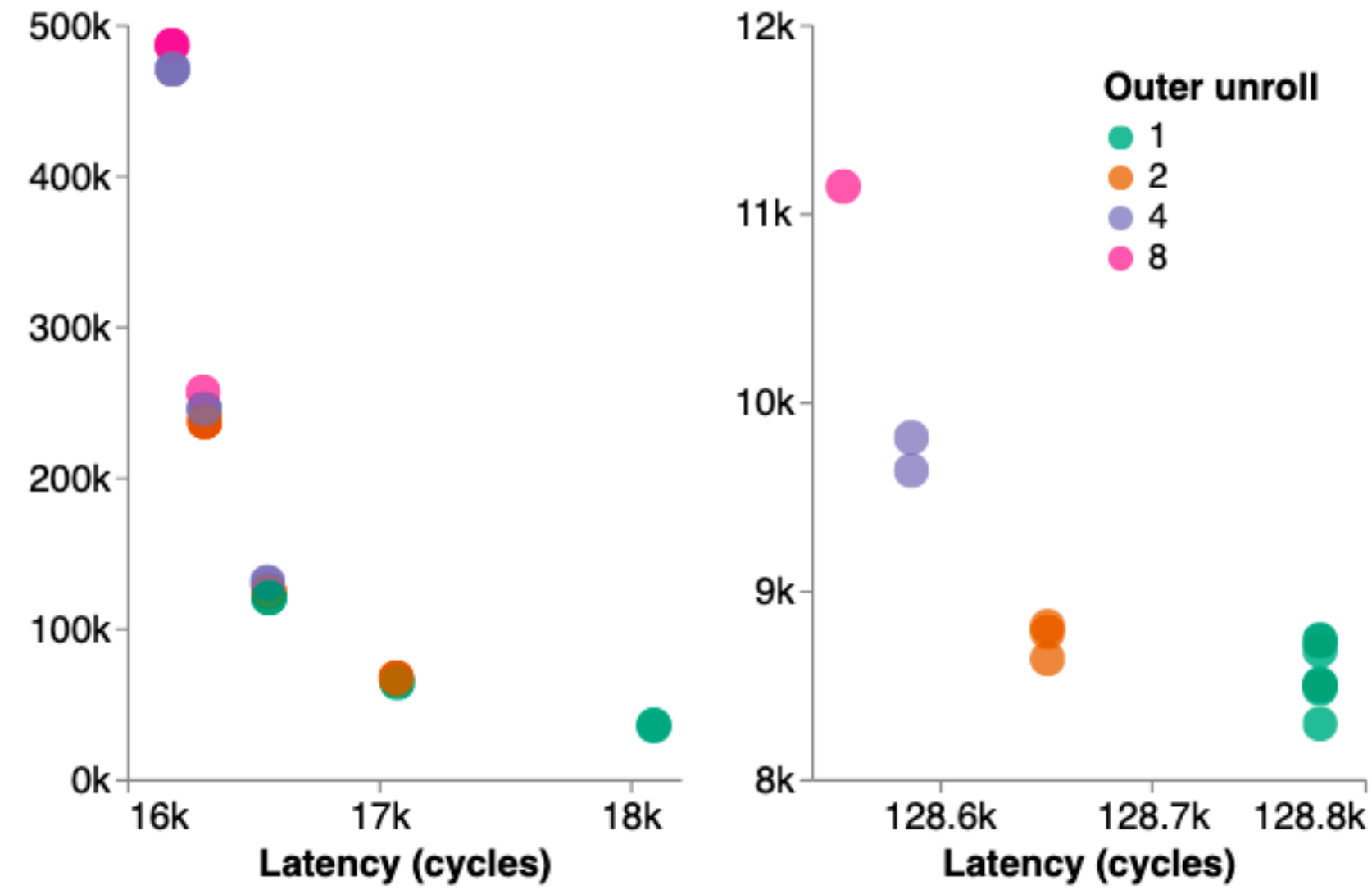
354 out of 32,000
configurations
accepted

15 out of **56 Pareto**
configurations
accepted

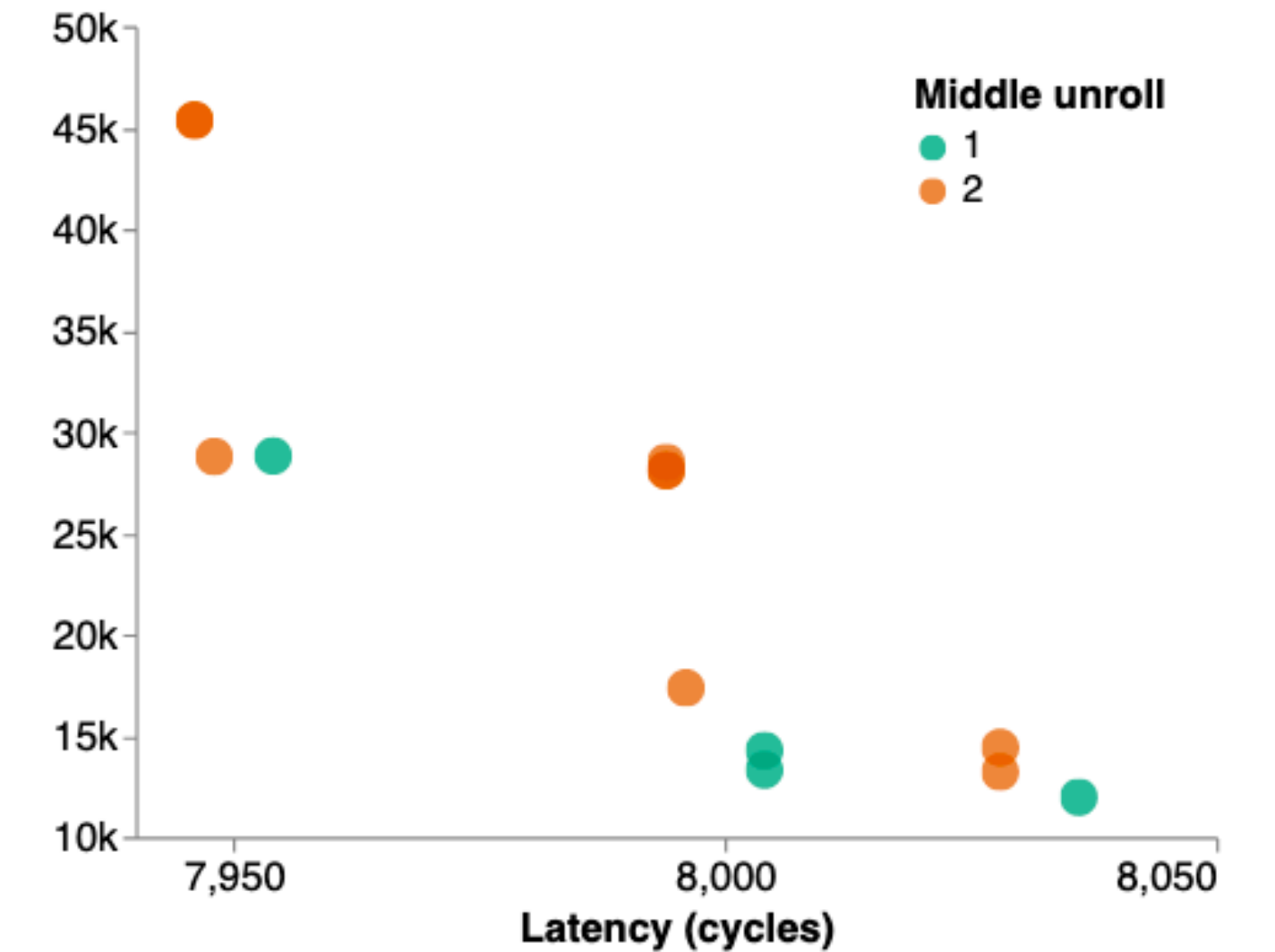
Qualitative Studies



(a) stencil2d with inner unroll.



(b) md-knn with outer unroll.



(c) md-grid with middle unroll.

Qualitative Studies

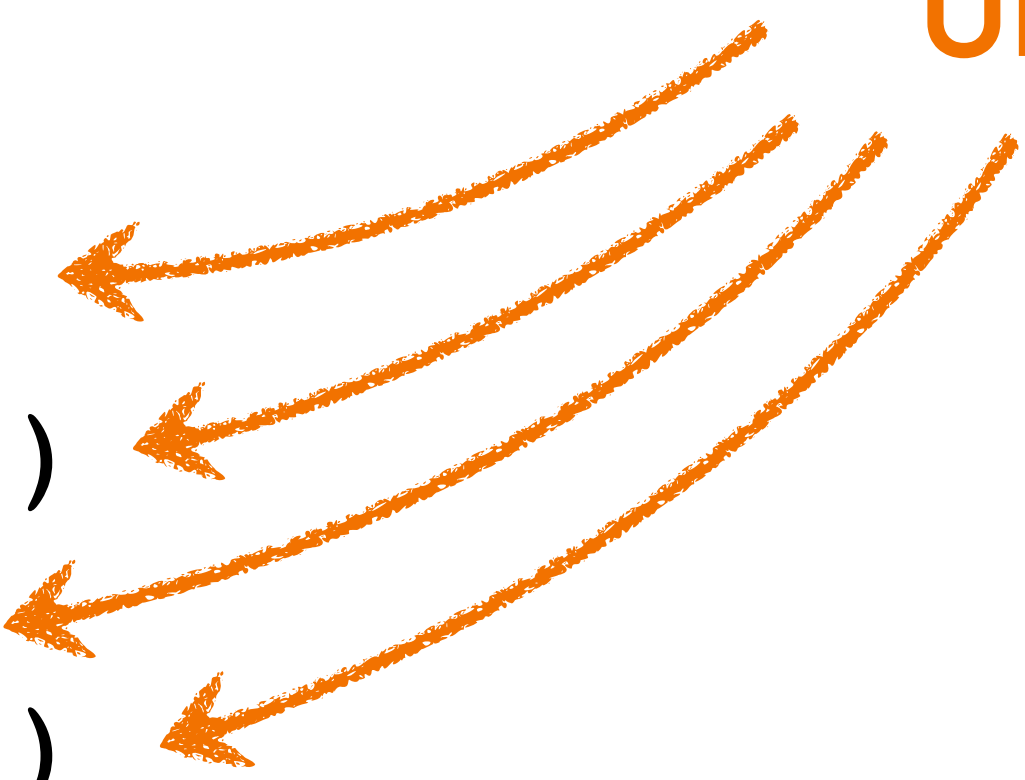
```
for (r=0; r<row_size; r++)  
    for (c=0; c<col_size; c++)  
        for (k1=0; k1<3; k1++)  
            for (k2=0; k2<3; k2++)  
                mul = filter[k1*3 + k2] *  
                    orig[(r+k1)*col_size + c+k2]
```

2D-Stencil

Qualitative Studies

```
for (r=0; r<row_size; r++)  
  for (c=0; c<col_size; c++)  
    for (k1=0; k1<3; k1++)  
      for (k2=0; k2<3; k2++)  
        mul = filter[k1*3 + k2] *  
              orig[(r+k1)*col_size + c+k2]
```

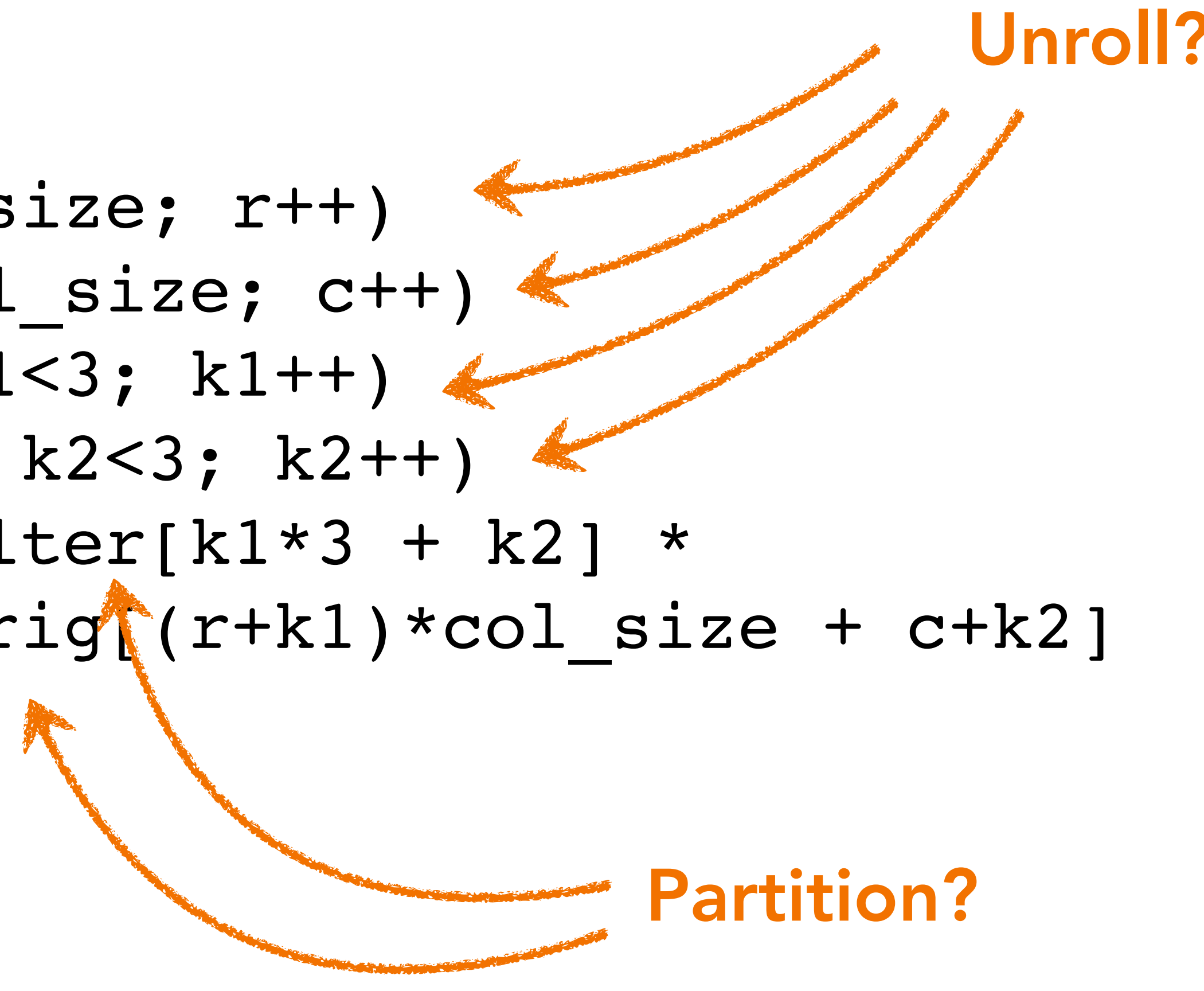
Unroll?



2D-Stencil

Qualitative Studies

```
for (r=0; r<row_size; r++)  
  for (c=0; c<col_size; c++)  
    for (k1=0; k1<3; k1++)  
      for (k2=0; k2<3; k2++)  
        mul = filter[k1*3 + k2] *  
              orig[(r+k1)*col_size + c+k2]
```



Unroll?

Partition?

2D-Stencil

Qualitative Studies

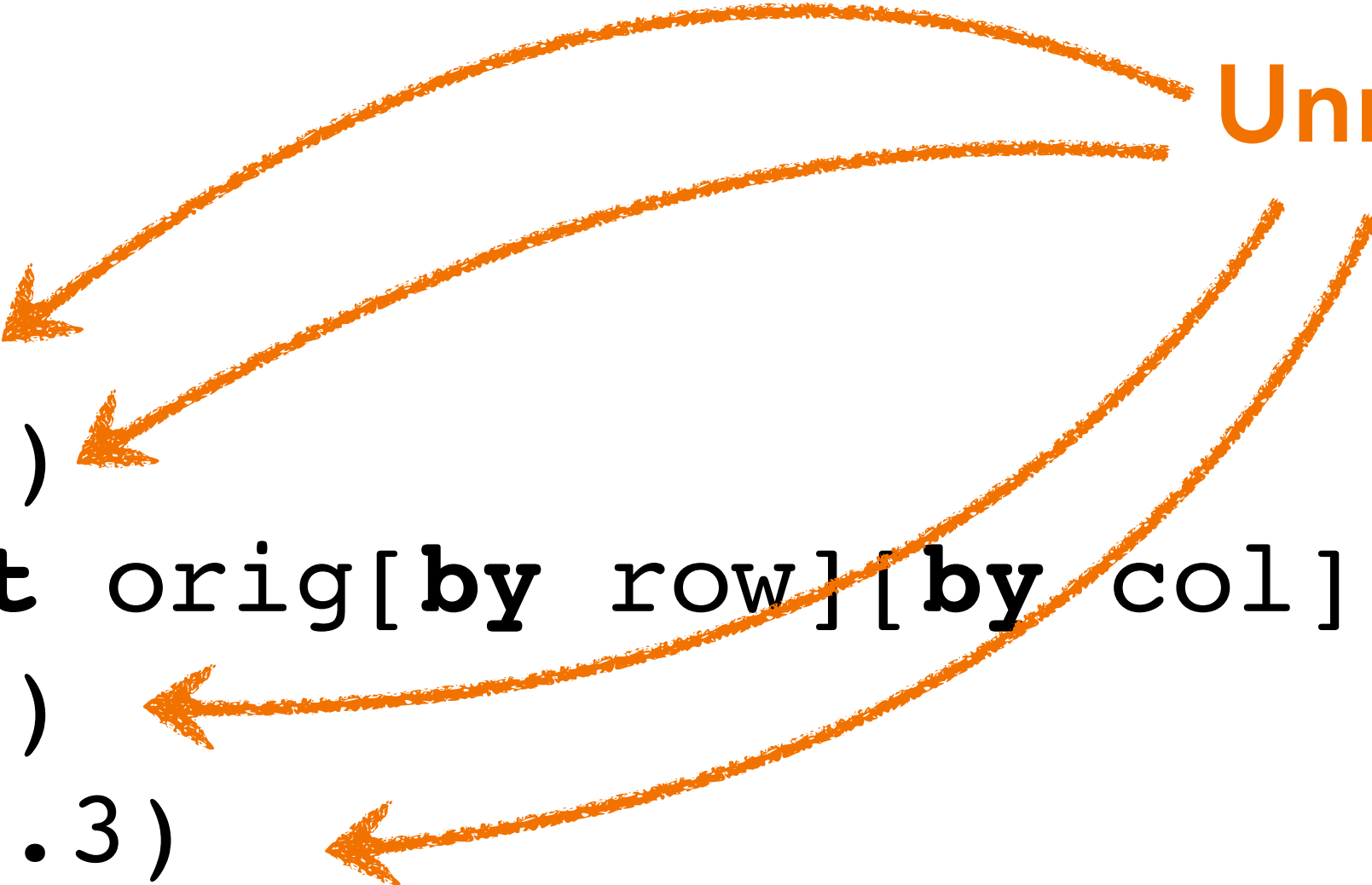
```
for (let row = 0..126)
  for (let col = 0..62)
    view window = shift orig[by row][by col];
    for (let k1 = 0..3)
      for (let k2 = 0..3)
        mul = filter[k1][k2] * window[k1][k2]
```

2D-Stencil

Qualitative Studies

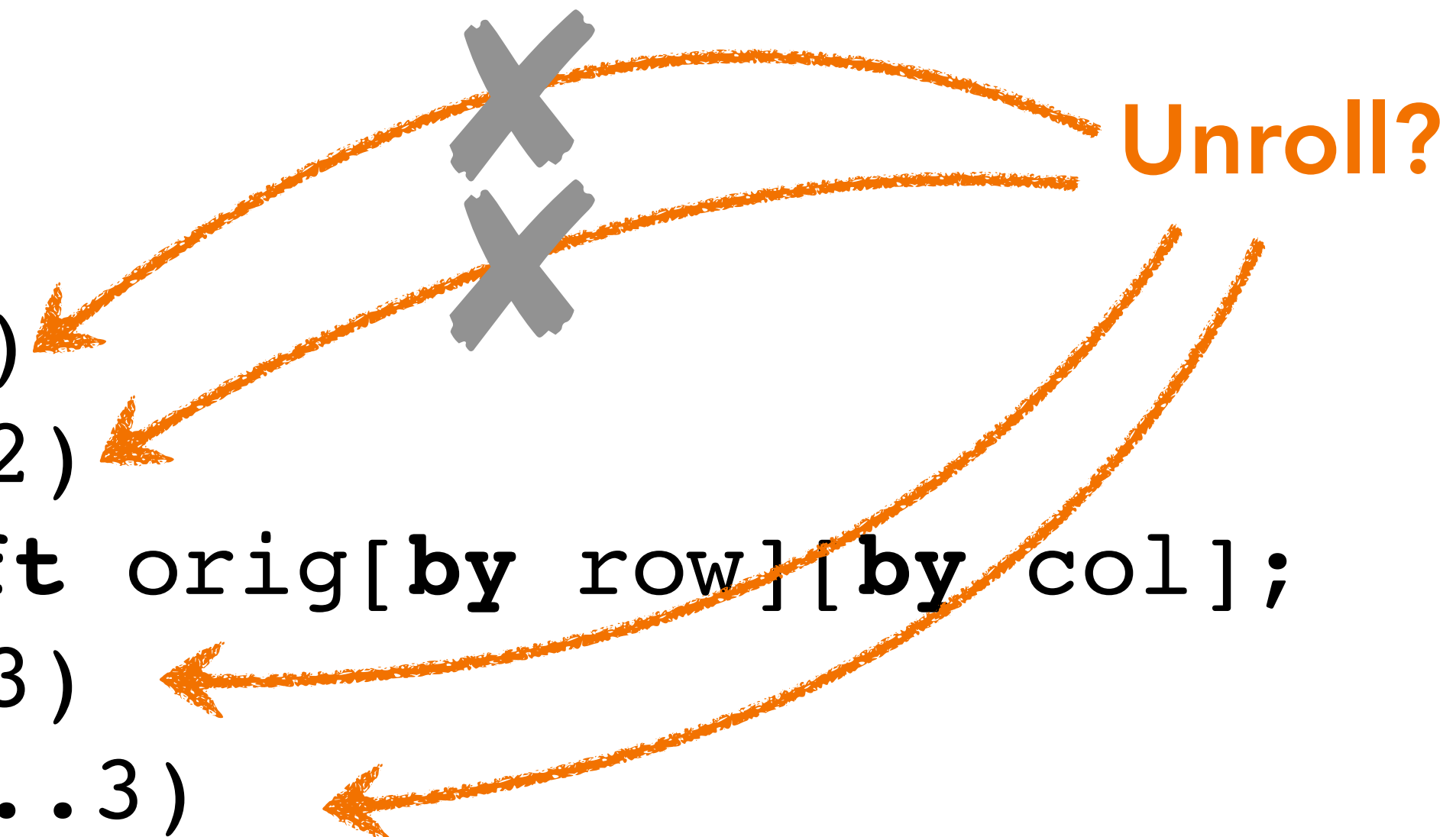
```
for (let row = 0..126)
  for (let col = 0..62)
    view window = shift orig[by row][by col];
    for (let k1 = 0..3)
      for (let k2 = 0..3)
        mul = filter[k1][k2] * window[k1][k2]
```

Unroll?



2D-Stencil

Qualitative Studies



Unroll?

```
for (let row = 0..126)
  for (let col = 0..62)
    view window = shift orig[by row][by col];
    for (let k1 = 0..3)
      for (let k2 = 0..3)
        mul = filter[k1][k2] * window[k1][k2]
```

The diagram illustrates the unrolling of nested loops. Four orange arrows originate from the text 'Unroll?' and point to the four nested loop headers: 'for (let row = 0..126)', 'for (let col = 0..62)', 'for (let k1 = 0..3)', and 'for (let k2 = 0..3)'. Two large grey 'X' marks are placed over the arrows pointing to the 'row' and 'col' loops, indicating that these loops are not to be unrolled.

2D-Stencil

Qualitative Studies

```
for (let row = 0..126)
  for (let col = 0..62)
    view window = shift orig[by row][by col];
    for (let k1 = 0..3) unroll 3
      for (let k2 = 0..3) unroll 3
        mul = filter[k1][k2] * window[k1][k2]
```

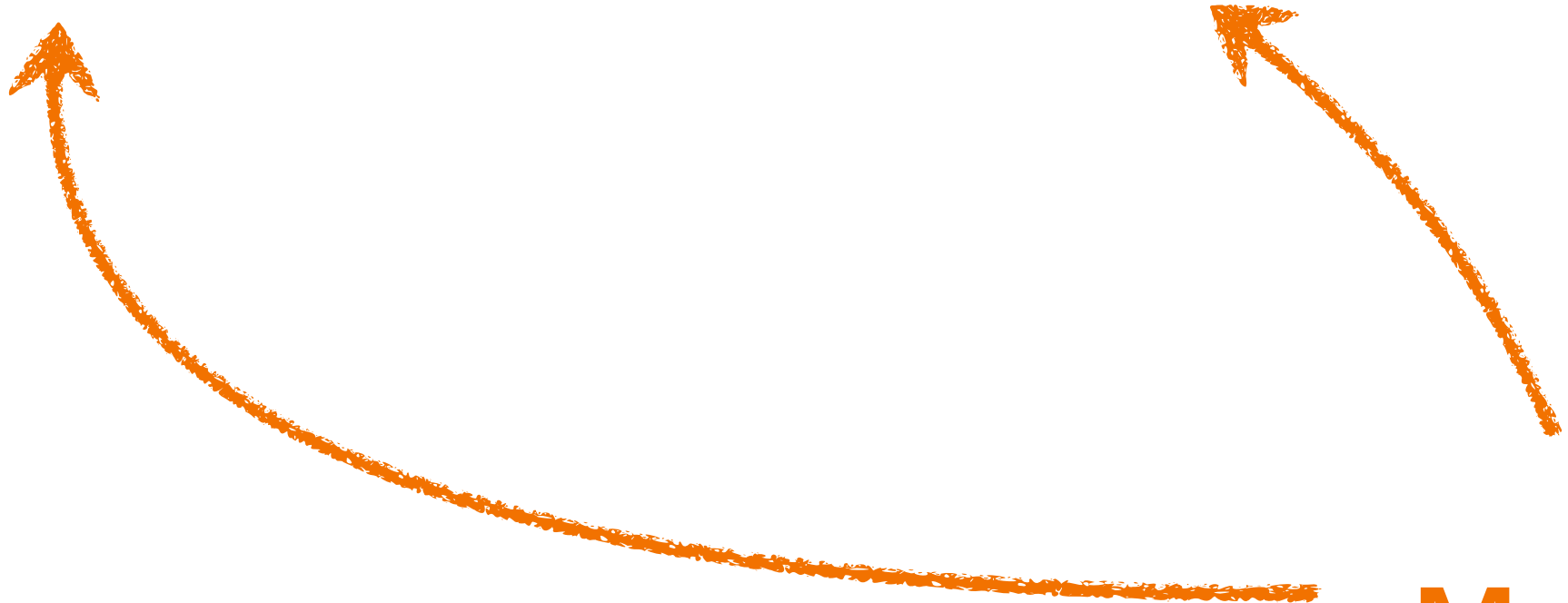
2D-Stencil

Qualitative Studies

```
for (let row = 0..126)
  for (let col = 0..62)
    view window = shift orig[by row][by col];
    for (let k1 = 0..3) unroll 3
      for (let k2 = 0..3) unroll 3
        mul = filter[k1][k2] * window[k1][k2]
```

2D-Stencil

Must partition



The Future

The Future

Resource Polymorphism

```
let m1: float[12 bank  $M_1$ ];  
let m2: float[12 bank  $M_2$ ];  
  
for (let i = 0 .. 12) unroll  $N$  {  
    m2[i] = m1[i] * 2;  
}  
  
for (let i = 0 .. 12) unroll  $K$  {  
    sum += m2[i];  
}
```


The Future

Resource Polymorphism

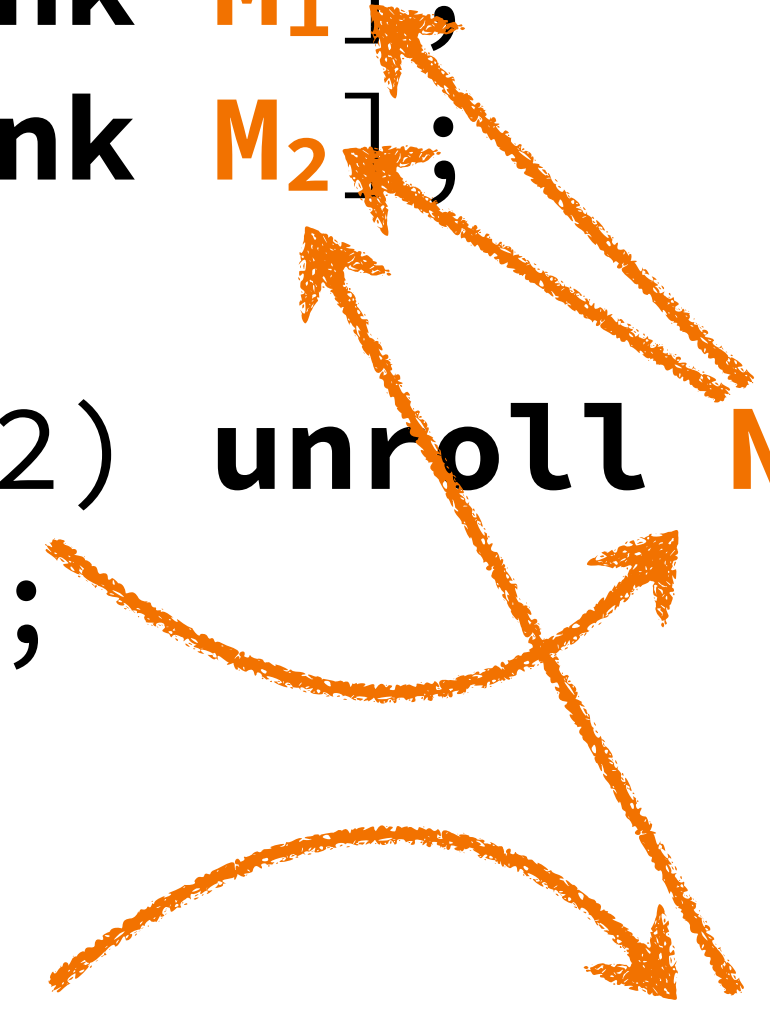
```
let m1: float[12 bank  $M_1$ ];  
let m2: float[12 bank  $M_2$ ];  
  
for (let i = 0 .. 12) unroll  $N$  {  
  m2[i] = m1[i] * 2;  
}  
  
for (let i = 0 .. 12) unroll  $K$  {  
  sum += m2[i];  
}
```

The diagram illustrates resource polymorphism. It shows two memory banks, M_1 and M_2 , which are used in two different loops. The first loop, labeled **unroll** N , uses M_1 to calculate $m2[i]$. The second loop, labeled **unroll** K , uses M_2 to calculate the sum. Arrows indicate that M_1 is associated with the first loop and M_2 is associated with the second loop, demonstrating how resources are mapped to different parts of the code.

The Future

Resource Polymorphism

```
let m1: float[12 bank  $M_1$ ];  
let m2: float[12 bank  $M_2$ ];  
  
for (let i = 0 .. 12) unroll  $N$  {  
  m2[i] = m1[i] * 2;  
}  
  
for (let i = 0 .. 12) unroll  $K$  {  
  sum += m2[i];  
}
```



The diagram illustrates resource polymorphism by showing how variables M_1 , M_2 , N , and K are used in the code and then defined on the right. Orange arrows point from each variable in the code to its definition:

- M_1 points from the `bank  $M_1$` in the first `let` statement to $M_1 = \{ \dots \}$.
- M_2 points from the `bank  $M_2$` in the second `let` statement to $M_2 = \{ \dots \}$.
- N points from the `unroll  $N$` in the first `for` loop to $N = \{ \dots \}$.
- K points from the `unroll  $K$` in the second `for` loop to $K = \{ \dots \}$.

$M_1 = \{ \dots \}$
 $M_2 = \{ \dots \}$
 $N = \{ \dots \}$
 $K = \{ \dots \}$

The Future

Modularity

The Future

Modularity

```
def dot(m1: float[10], m2: float[10]) {  
    let sum = 0;  
    for (let i = 0 .. 10) {  
        sum += m1[i] * m2[i];  
    }  
    return sum;  
}
```

```
dot(A, B);
```

The Future

Modularity

```
def dot(m1: float[10 bank 5], m2: float[10 bank 5]) {  
    let sum = 0;  
    for (let i = 0 .. 10) unroll 5 {  
        sum += m1[i] * m2[i];  
    }  
    return sum;  
}
```

```
dot(A, B); // A, B need exactly 5 banks
```

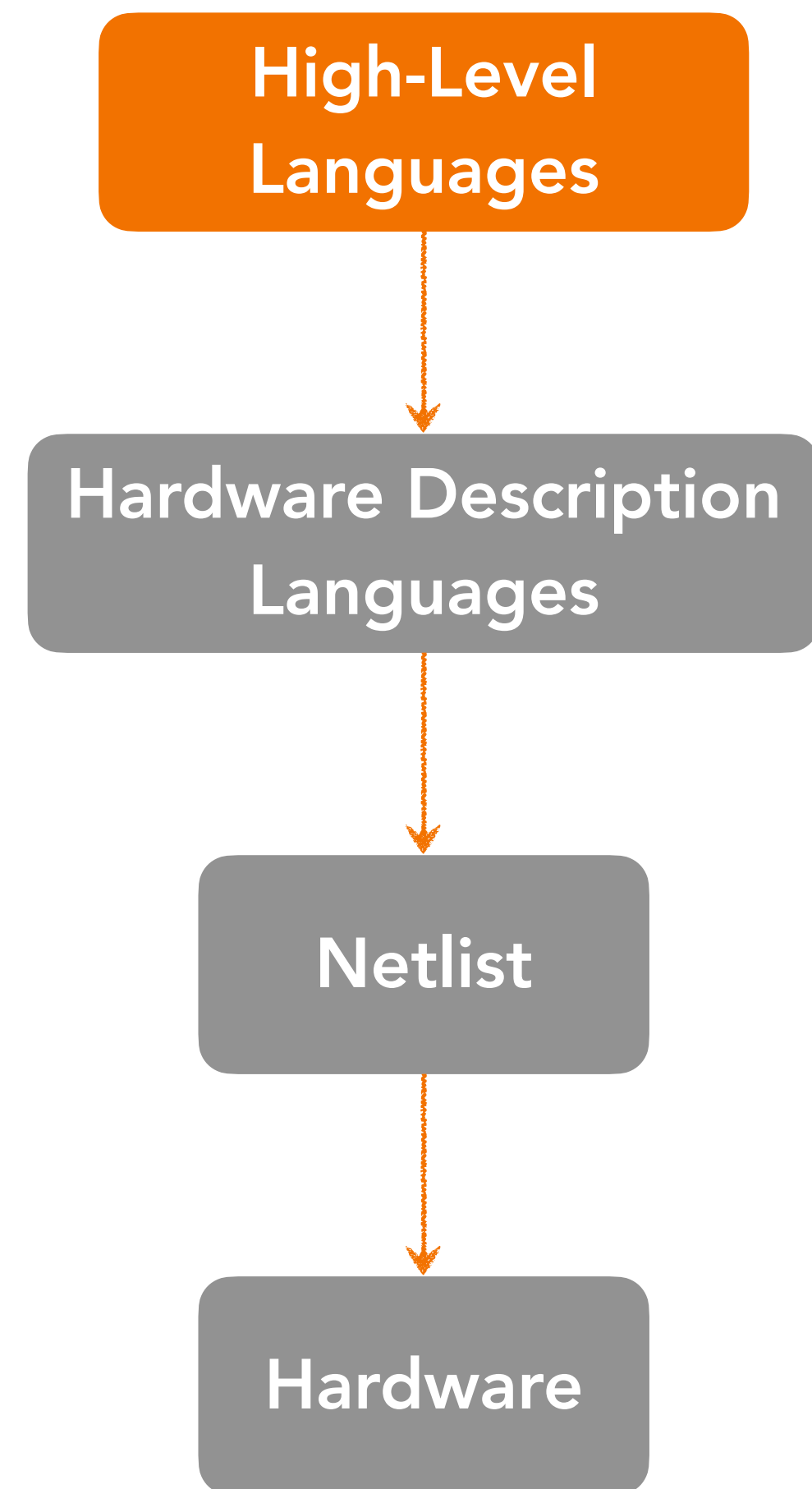

The Future

Modularity

```
def dot(m1: float[10 bank 5], m2: float[10 bank 5]) {  
    let sum = 0;  
    for (let i = 0 .. 10) unroll 5 {  
        sum += m1[i] * m2[i];  
    }  
    return sum;  
}  
  
for (let i = 0..2) unroll 2 {  
    dot(A, B); // How many banks? How many copies of dot?  
}
```

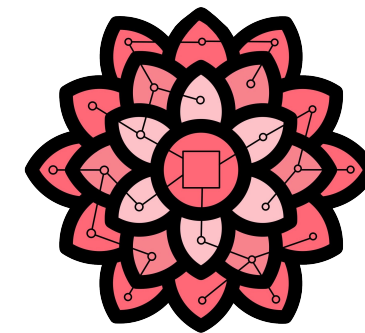
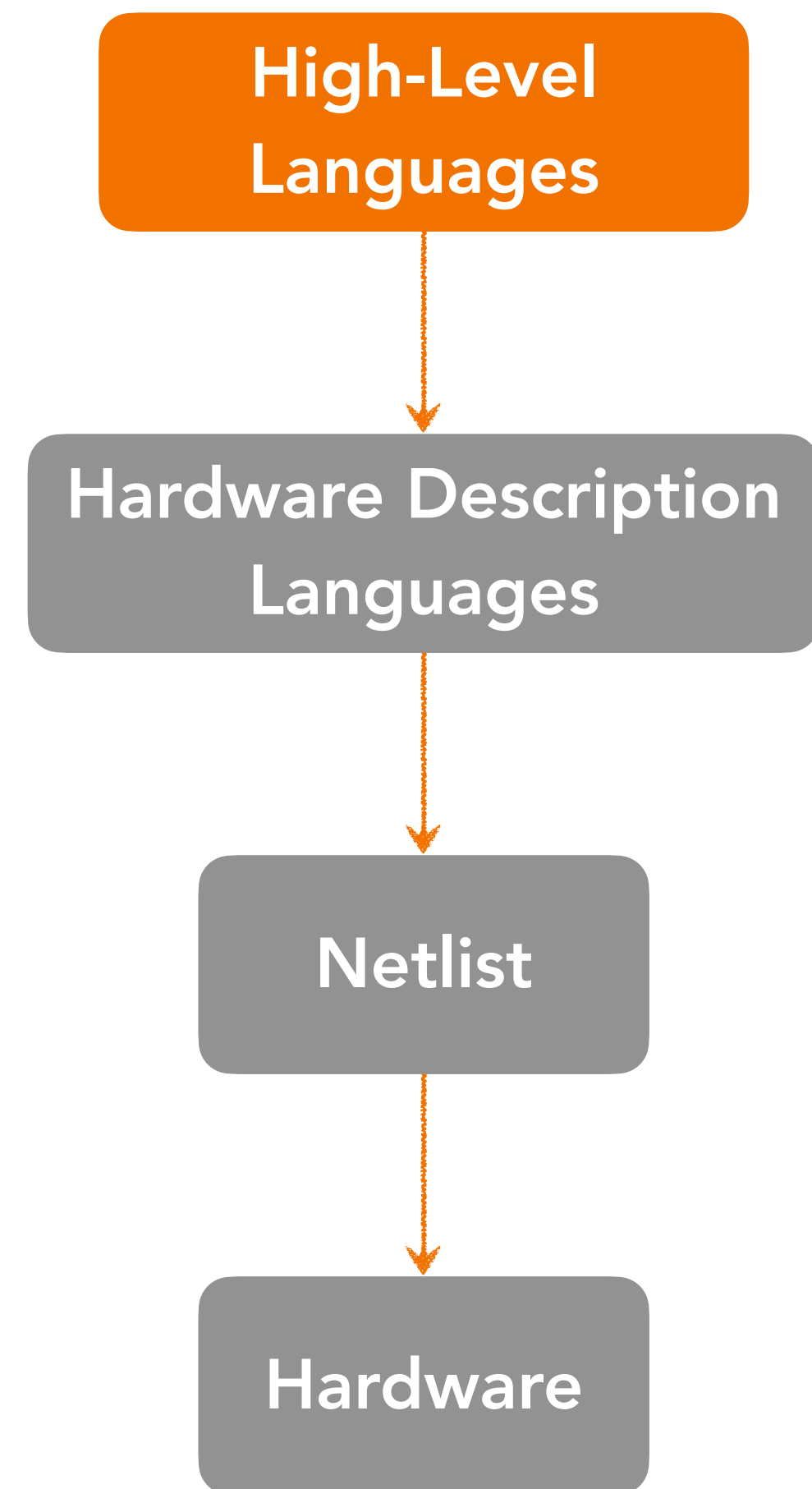
The Future

A Predictable Stack



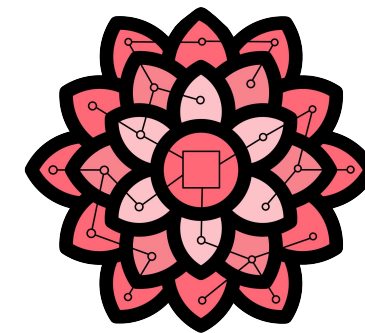
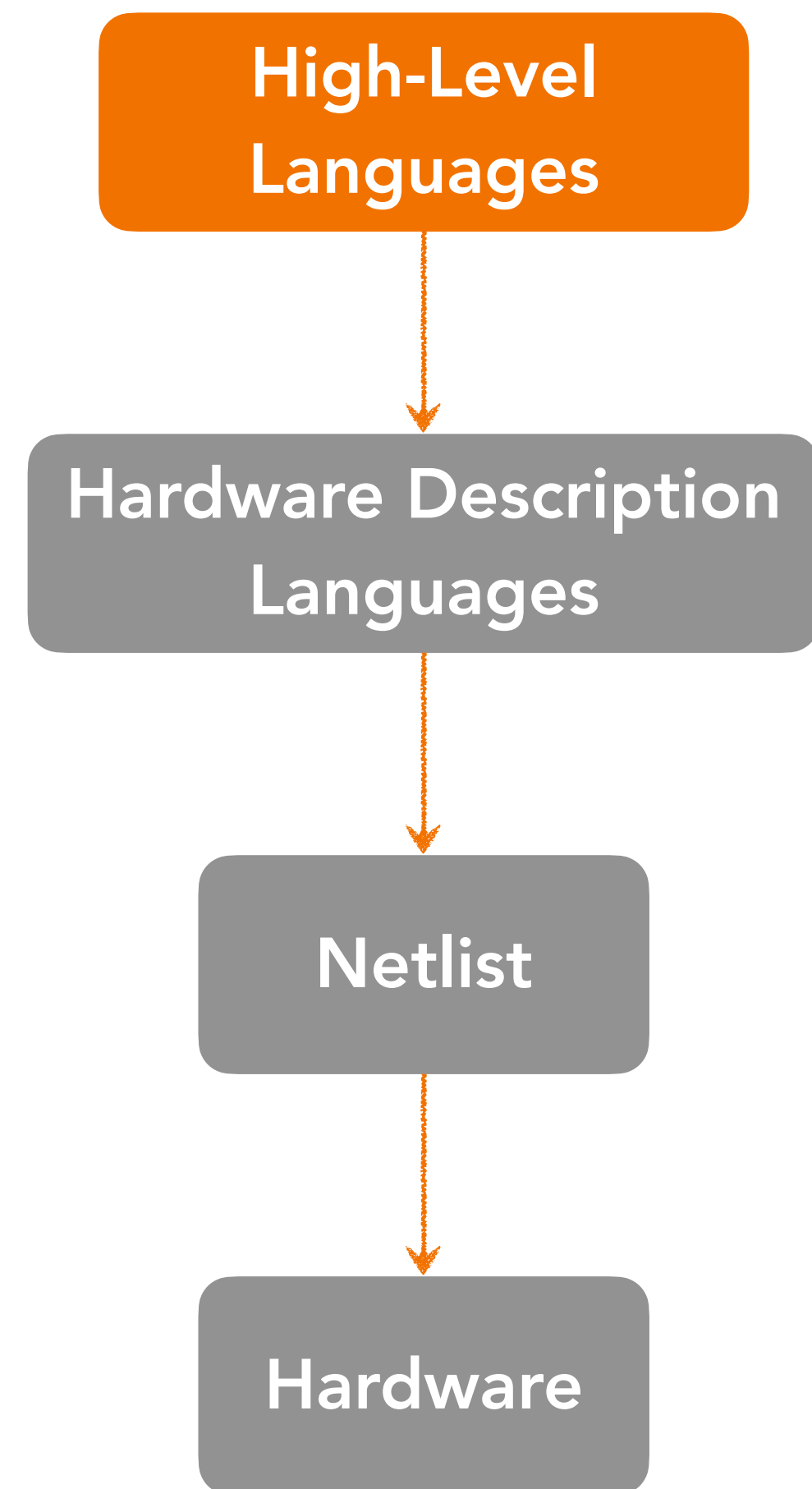
The Future

A Predictable Stack



The Future

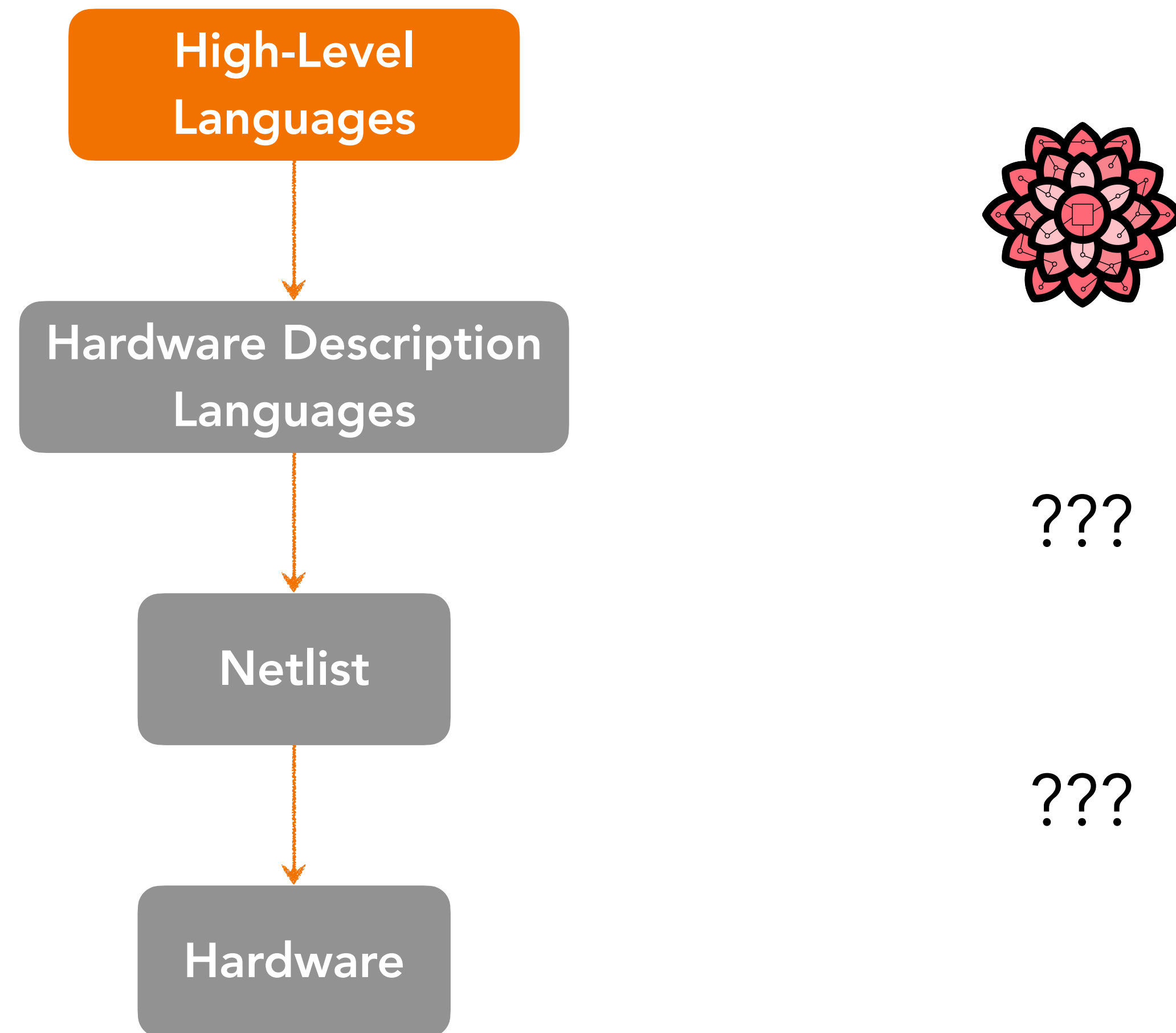
A Predictable Stack



???

The Future

A Predictable Stack

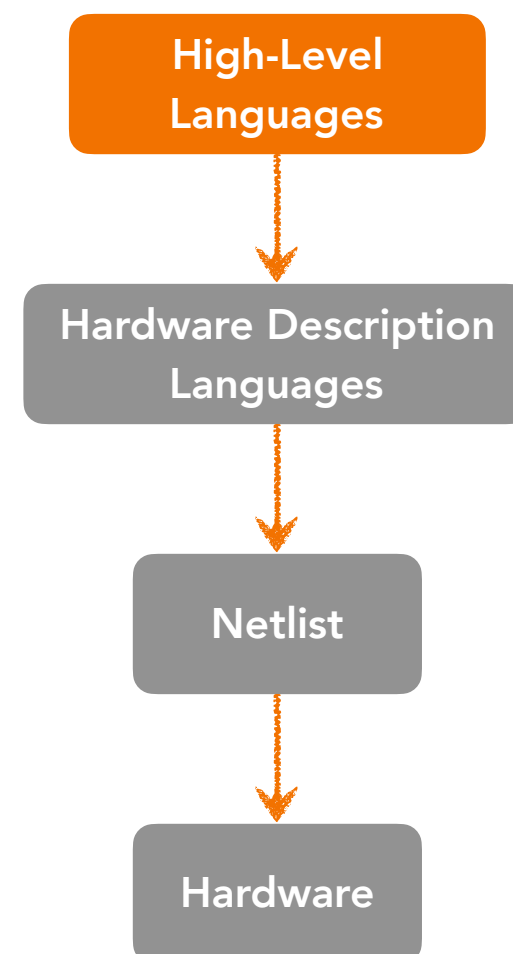


The Future

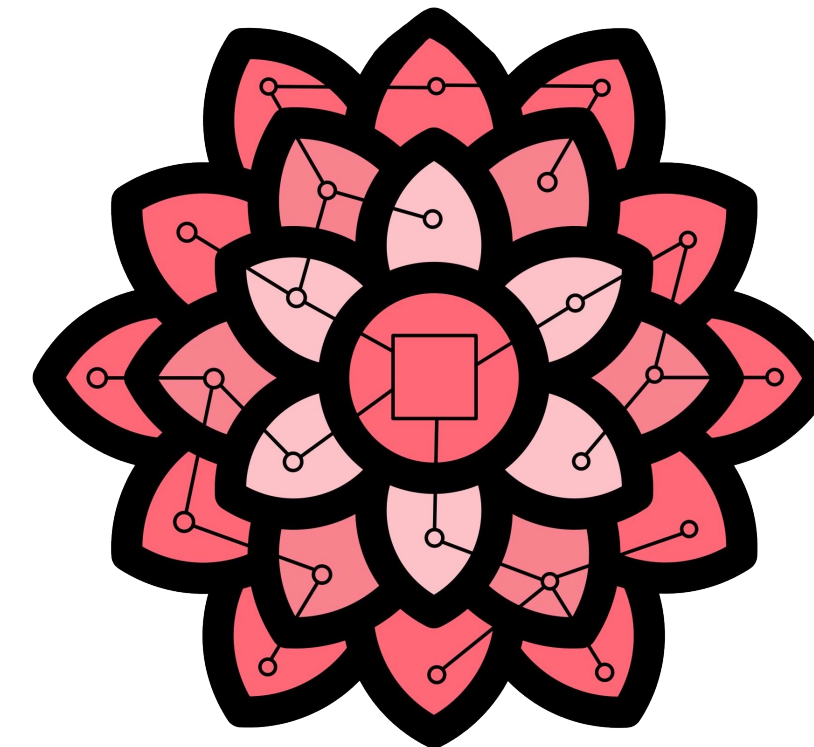
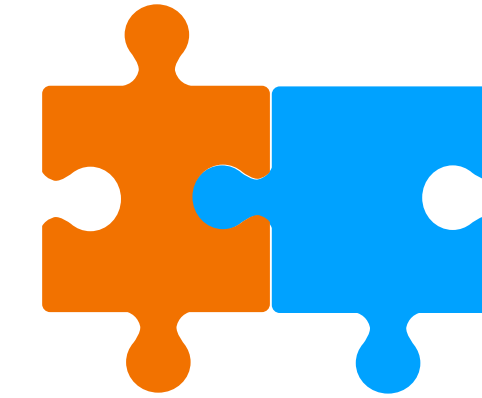
Resource Polymorphism

```
let m1: float[12 bank M];  
let m2: float[12 bank M];  
  
for (let i = 0 .. 12) unroll N {  
  m2[i] = m1[i] * 2;  
}
```

A Predictable Stack



Modularity



Predictability from
languages to LUTs

capra.cs.cornell.edu/dahlia