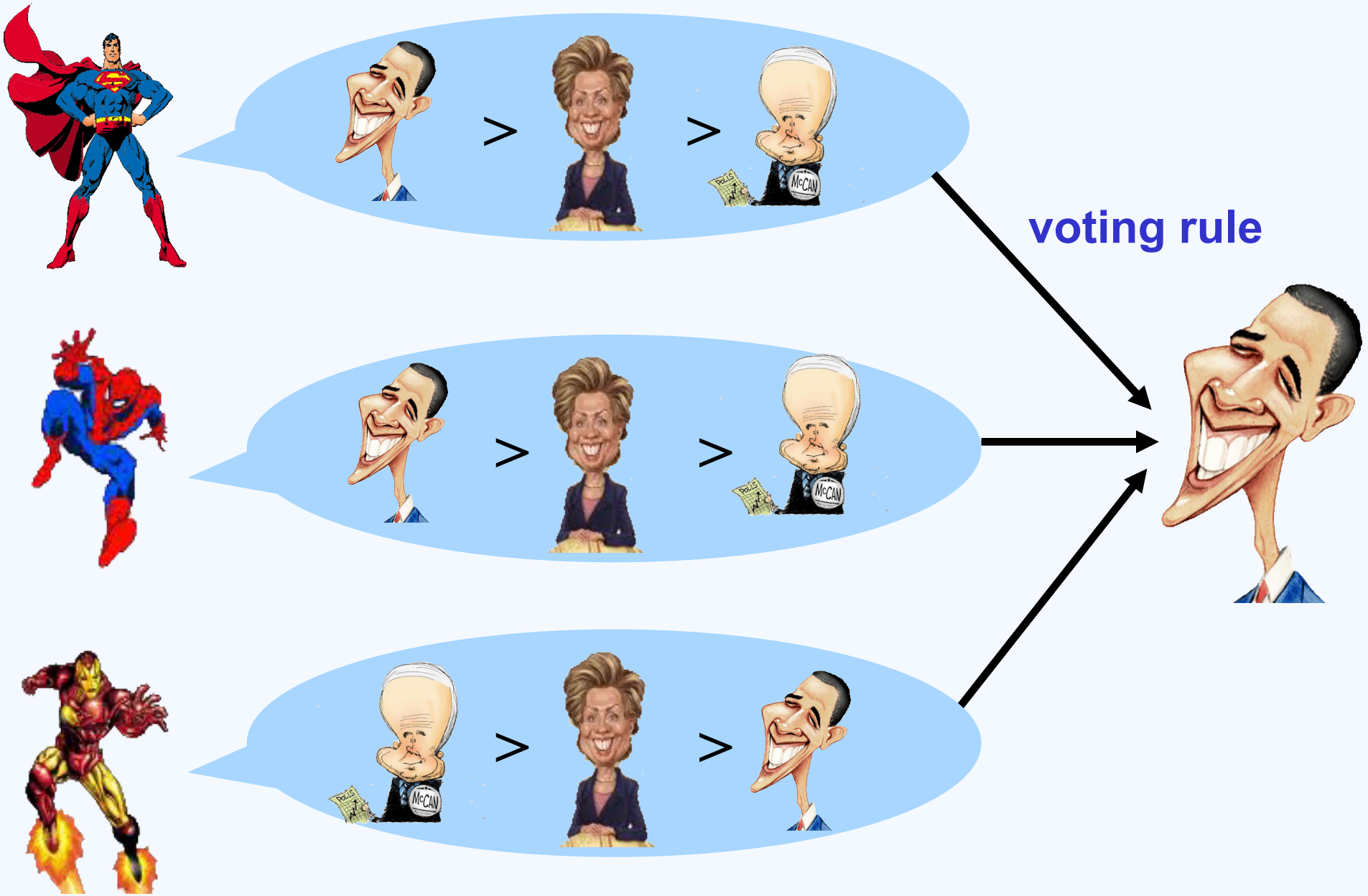
Complexity of Computing the Margin of Victory for Various Voting Rules

Ronald L. Rivest Emily Shen Lirong Xia

CAEC, Nov. 18, 2011

Voting



Criteria for voting rules

- Lots of voting rules (plurality, approval, instant runoff voting, etc.) – How to choose one?
- “Traditional” criteria: monotonicity, consistency, majority, etc.
- More recently: computational complexity of manipulation (strategic voting)
- We consider: efficient auditability – specifically, computational complexity of computing ***margin of victory*** (related to manipulation problems)

Margin of Victory (MoV)

- **Definition**: Given a profile of ballots, the ***margin of victory*** is the smallest number k such that k modified ballots could change the election winner
- Margin of victory is critical to ***efficient, effective post-election audits***
  - To provide a given level of statistical confidence, landslide election requires much less checking than a close election
- Margin of victory is a ***measure of closeness*** of election, suggests level of political mandate won by winner

Margin of Victory Examples

- Plurality
  - A:10 votes, B: 15 votes, C: 4 votes
  - Margin of victory = 3
- Instant-runoff voting (IRV)

$A>B>C$	$B>A>C$	$C>A>B$
10	15	4

  - Margin of victory = 1

The MoV computational problem

- Computational problem MoV: compute margin of victory of a profile of ballots
- Decision problem MoVk: Is the margin of victory at most k?
- MoV problem closely related to previously studied manipulation problems: UCM, bribery

Margin of Victory & Related Manipulation Problems

Problem	Objective	By	Desired Complexity
Margin of Victory	Change the winner	Changing votes	Low
Unweighted Coalitional Manipulation	Make a given candidate win	Adding votes	High
Bribery	Make a given candidate win	Changing votes	High

Our Results

Voting rule	Margin of Victory	Unweighted Coalitional Manipulation	
Positional scoring rules Including Borda	This work P	P (1 manipulator)	[BTT89]
		NPC (2 or more)	[XCP10] [DKNW11] [BNW11]
Plurality with runoff	P	P	[ZPR09]
Copeland	NPC and FPT	P (1 manipulator)	[BTT89]
		NPC (2 or more)	[FHS08,10]
Maximin	NPC and FPT	P (1 manipulator)	[BTT89]
		NPC (2 or more)	[XZP+09]
STV	NPC for MoV_1	NPC	[BO91]
Ranked pairs	NPC for MoV_1	NPC	[XZP+09]
Nanson's rule	?	NPC	[NWX11]
Baldwin's rule	?	NPC	[NWX11]

Poly-time margin algorithm for plurality with runoff

- Let d be the current winner
- For every k
  - Check whether there is a way to make d not in the runoff by changing k votes
  - Check for every adversarial c, every threshold l, whether there is a way to change k votes such that
    - c and d are ranked at the top for at least l times
    - Any other alternative is ranked at the top for no more than l times
    - c beats d in their pairwise election

IRV Margin of Victory = 1 is NP-Complete

- Proof by reduction from unweighted coalitional manipulation problem
- Tweak UCM1 profile P to get new profile P' by:
  - Adding a new candidate d
  - Ranking d just below c in P
  - Adding $|P|+1$ voters who all rank d as 1st choice
- Show: MoV of P' is 1 if and only if UCM1 has a solution

Summary and Future Work

- We studied complexity of computing the margin of victory for some common voting rules

Future work:

- Complexity of MoVk ($k > 1$) for IRV, ranked pairs
- Practical algorithms to compute/approximate margin of victory for IRV, ranked pairs
  - Heuristics, approximation algorithms