Homework guidelines: The following problems are for your understanding. Do not turn in your solutions, but make sure you can solve it.

1. You are given an algorithm $A$ for a decision problem (i.e., answer for each input is either 0 or 1), that runs in time $T(n)$ on inputs of size $n$, with probability of error $1/4$. Show how to convert it into a new algorithm $B$ that runs in time $O(T(n) \log 1/\beta)$ with probability of error at most $\beta$. (Hint: run $A$ $O(\log 1/\beta)$ times and take the “majority”, i.e., the most common, answer. Use Chernoff bounds to show that the correct answer is highly likely to be the output.)

2. Let $f$ be a function which maps inputs of size $n$ to a number. You are given an approximation scheme $A$ for $f$ such that $\Pr\left[ \frac{f(x)}{1+\epsilon} \leq A(x) \leq f(x)(1+\epsilon) \right] \geq 3/4$, and $A$ runs in time polynomial in $1/\epsilon, |x|$. Construct an approximation scheme $B$ for $f$ such that $\Pr\left[ \frac{f(x)}{1+\epsilon} \leq B(x) \leq f(x)(1+\epsilon) \right] \geq 1 - \delta$, and $B$ runs in time polynomial in $\frac{1}{\epsilon}, |x|, \log \frac{1}{\delta}$.

3. (Coupon Collector Problem). Given a die with $n$ sides. What is the expected number of times you need to roll the die in order to see each of the $n$ sides? (Hint: Given that you saw $i$ sides, how many times do you need to roll the die to see the $(i+1)^{st}$ side? Then use linearity of expectation.)