Sublinear Time Algorithms	September 1, 2020

Homework 1

Lecturer: Ronitt Rubinfeld — *Due Date: September 15, 2020*

Turn in your solution to each *problem on a separate sheet of paper, with your name on each one.*

1. Given a graph G of max degree d, and a parameter ϵ, give an algorithm which has the following behavior: if G is connected, then the algorithm should pass with probability 1, and if G is ϵ-far from connected (at least $\epsilon \cdot dn$ edges must be added to connect G), then the algorithm should fail with probability at least 3/4. Your algorithm should look at a number of edges that is independent of n, and polynomial in d, ϵ. For extra credit, try to make your algorithm as efficient as possible in terms of n, d, ϵ.

   For this homework set, when proving the correctness of your algorithm, it is ok to show that if the input graph G is likely to be passed, then it is ϵ-close to a graph G' which is connected, without requiring that G' has degree at most d.

2. In class we gave an MST approximation algorithm for graphs in which the weights on each edge were integers in the set $\{1..w\}$. Show that one can get an approximation algorithm when the weights can be any value in the range $[1..w]$ (it is ok to get a slightly worse running time in terms of $w, 1/\epsilon$).

3. The diameter of an unweighted graph is the maximum distance between any pair of nodes. Give a tester for graphs with degree at most d (where d is a constant and the graph is represented in the adjacency list model) that have low diameter. The tester should have the following specific behavior:

   (a) Graphs with diameter at most D are always accepted.

   (b) Graphs which are ϵ-far (that is, at least ϵdn edges must be added) from having diameter $4D + 2$ are failed with probability at least 2/3.

   (c) The query complexity of the tester should be $O(1/\epsilon^c)$ for some constant $1 \leq c \leq \infty$.

   For this homework, when proving the correctness of your algorithm, it is ok to show that if the input graph G is likely to be passed, then it is ϵ-close to a graph G' which has diameter $4D + 2$, without requiring that G' has degree at most d.

   Hint: Prove that every connected graph on n nodes can be transformed into a graph of diameter at most D by adding at most $O(n/D)$ edges.