1. Given a graph $G$ of max degree $d$, and a parameter $\epsilon$, give an algorithm which has the following behavior: if $G$ is connected, then the algorithm should pass with probability 1, and if $G$ is $\epsilon$-far from connected (at least $\epsilon dn$ edges must be added to connect $G$), then the algorithm should fail with probability at least $3/4$. Your algorithm should look at a number of edges that is independent of $n$, and polynomial in $d, \epsilon$. For extra credit, try to make your algorithm as efficient as possible in terms of $n, d, 1/\epsilon$.

For this homework set, when proving the correctness of your algorithm, it is ok to show that if the input graph $G$ is likely to be passed, then it is $\epsilon$-close to a graph $G'$ which is connected, without requiring that $G'$ has degree at most $d$.

2. In class we gave an MST approximation algorithm for graphs in which the weights on each edge were integers in the set $\{1, \ldots, w\}$. Show that one can get an approximation algorithm when the weights can be any value in the range $[1, w]$ (it is ok to get a slightly worse running time in terms of $w, 1/\epsilon$).

3. The diameter of an unweighted graph is the maximum distance between any pair of nodes. Give a tester for graphs with degree at most $d$ (where $d$ is a constant and the graph is represented in the adjacency list model) that have low diameter. The tester should have the following specific behavior:

   (a) Graphs with diameter at most $D$ are always accepted.
   (b) Graphs which are $\epsilon$-far (that is, at least $\epsilon dn$ edges must be added) from having diameter $4D + 2$ are failed with probability at least $2/3$.
   (c) The query complexity of the tester should be $O(1/\epsilon^c)$ for some constant $1 \leq c \leq \infty$.

For this homework, when proving the correctness of your algorithm, it is ok to show that if the input graph $G$ is likely to be passed, then it is $\epsilon$-close to a graph $G'$ which has diameter $4D + 2$, without requiring that $G'$ has degree at most $d$.

**Hint:** Prove that every connected graph on $n$ nodes can be transformed into a graph of diameter at most $D$ by adding at most $O(n/D)$ edges.