Your Name	November 23, 2024
6.5240 Problem Set 6	
Collaborators: *Firstname Lastname*	Due: December 4, 2024, 10pm

Homework guidelines: You may work with other students, as long as (1) they have not yet solved the problem, (2) you write down the names of all other students with which you discussed the problem, and (3) you write up the solution on your own. No points will be deducted, no matter how many people you talk to, as long as you are honest. If you already knew the answer to one of the problems (call these "famous" problems), then let us know that in your solution writeup – it will not affect your score, but will help us in the future. It's ok to look up famous sums and inequalities that help you to solve the problem, but don't look up an entire solution.

0. **(Matrix multiplication checker; not for credit — just a problem to think about before the lecture on Monday, Dec 2.)** You are given $n \times n$ matrices A, B, C whose elements are from $\mathbb{Z}_2$ (integers mod 2). Show a (randomized) algorithm running in $O(n^2)$ time which verifies $A \cdot B = C$. The algorithm should always output "pass" if $A \cdot B = C$ and should output "fail" with probability at least $3/4$ if $A \cdot B \neq C$. Assume the field operations $+, \times, -$ can be done in $O(1)$ steps.

1. **Removing adaptivity.**

   (a) Assume that your computational model is such that a query returns a single bit. In such a model, show that any algorithm making q queries can be made into a nonadaptive (i.e., where the queries do not depend on the results of any previous queries) tester that uses only 2^q queries.

   (b) We define a graph property to be a property that is preserved under graph isomorphism (i.e., if Π is a graph property, then for any isomorphic graphs G and G', G has property Π if and only if G' has property Π). Show that any adaptive algorithm in the adjancey matrix query model (i.e. given unit cost query to whether an edge (i, j) exists in the graph, for $i, j \in [n]$) for testing a given graph property which makes q queries can be made into a nonadaptive algorithm for testing the same graph property using only $O(q^2)$ queries in this model.

2. **Lower bound for monotone property testing via communication complexity.** *(Please note the lecture on November 25 will be helpful for this problem.)* Recall that $f : \{0,1\}^n \to \{0, \ldots, n\}$ is monotone if for all x, y such that $x_i \leq y_i$ for $i = 1, \ldots, n$, then $f(x) \leq f(y)$. Show that distinguishing whether f is monotone from the case that f is ε-far from monotone (i.e., there is no monotone g such that f and g differ on at most ε-fraction of the domain $\{0,1\}^n$) requires $\Omega(n)$ queries. *Hint: reduce from the communication complexity problem of disjointness.*