

## Homework 0

*Lecturer: Ronitt Rubinfeld**Due Date: September 8, 2026*

**Homework guidelines:** The following problems are for your understanding. Do not turn in your solutions, but make sure you can solve it.

1. You are given an algorithm  $A$  for a decision problem (i.e., answer for each input is either 0 or 1), that runs in time  $T(n)$  on inputs of size  $n$ , with probability of error  $1/4$ . Show how to convert it into a new algorithm  $B$  that runs in time  $O(T(n) \log 1/\beta)$  with probability of error at most  $\beta$ . (Hint: run  $A$   $O(\log 1/\beta)$  times and take the “majority”, i.e., the most common, answer. Use Chernoff bounds to show that the correct answer is highly likely to be the output.)
2. Let  $f$  be a function which maps inputs of size  $n$  to a number. You are given an approximation scheme  $\mathcal{A}$  for  $f$  such that  $\Pr[\frac{f(x)}{1+\epsilon} \leq \mathcal{A}(x) \leq f(x)(1+\epsilon)] \geq 3/4$ , and  $\mathcal{A}$  runs in time polynomial in  $1/\epsilon, |x|$ . Construct an approximation scheme  $\mathcal{B}$  for  $f$  such that  $\Pr[\frac{f(x)}{1+\epsilon} \leq \mathcal{B}(x) \leq f(x)(1+\epsilon)] \geq 1 - \delta$ , and  $\mathcal{B}$  runs in time polynomial in  $\frac{1}{\epsilon}, |x|, \log \frac{1}{\delta}$ .
3. Let  $\Delta$  be a positive integer and  $G$  be a simple graph with  $n$  vertices and  $m$  edges. We assume that the degree of each vertex in  $G$  belongs to the interval  $[\Delta, 10\Delta]$ . Provide an algorithm that uses  $O(\epsilon^{-2} \ln 1/\delta)$  degree queries and output an estimate  $\tilde{d}$  of the average degree  $\bar{d} = 2m/n$  of the graph such that

$$\Pr[|\tilde{d} - \bar{d}| \leq \epsilon \bar{d}] \geq 1 - \delta.$$

In one degree query, you can get the degree of any vertex in  $G$ .