S-T connectivity (UST-Conn)

Input: Undirected $G$, nodes $s,t$
Output: "Yes" if $s,t$ connected
"No" o.w.

Can solve in poly time, in many ways.
What about small space?

RL = class of problems solvable by randomized log-space computations
(no change for input space (read only), but can only have
const #ptrs ...)

Thm: UST-Conn $\in$ RL

Algorithm:

- start at $s$
- take random walk for $2n^3$ steps
- if ever see $t$, output "Yes"
- o.w. output "No"

Complexity:
Keep track of $n$ steps so far
- edges at each node $x$: toss coin to pick one randomly
Behavior:

If \( s \) is not connected, never output "yes"

If \( s \) is connected

\[
 h_{st} \leq C_s(G_s) \leq n^3
\]

Pr \([\text{output "no"}]=\Pr[\text{start at } s, \text{ walk } \geq C \cdot E[C_s(G_s)] \text{ steps} \text{ and don't see } t] \]

\[
\leq \frac{1}{c} \quad \text{by Markov's } \#\]

Comments

- Actually \( \text{USTCONN } \neq \text{L} \) !!!

- Open is \( RL=L \) ?
  
  we know \( RL \leq L^{3/2} \)