Lecture 4b

Undirected s-t connectivity

random walks on graphs

Cover time:

$$C(G) \equiv \max_{v} E[\# \text{ steps to visit all nodes in } G \text{ when start at } v]$$

starting points

<u>Thm</u> $\forall G$, $C(G)$ is $O(n \cdot m)$

undirected graph

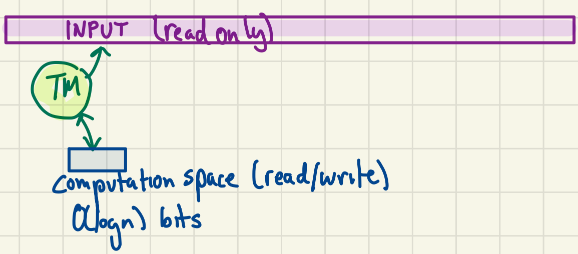
s-t Connectivity (UST-Conn)

Input undirected G
nodes s,t

Output if s,t in same conn. comp, "Yes"
else "NO"

many ways to solve in poly time
what about space?

RL = Class of problems solvable by randomized log-space computations
[no charge for input space (read-only), but can only store const # ptrs]



Thm UST-Conn $\in$ RL

Algorithm:

start at s

take a random walk for $c \cdot n^3$ steps

if ever see t output "Yes"

else output "NO"

Complexity:

time: $O(n^3) \times$ (time to pick random nbr)

space:

Keep track of:

step counter $O(\log n)$

space to pick random nbr:

e.g. scan nbrs & count d_u $O(\log n)$

toss d_u-sided die to get j

scan again to find j^{th} nbr

total $O(\log n)$

Behavior:

If s,t not connected, never output "Yes"

If s,t connected

$$h_{s,t} \le C_s(\underbrace{G_s}_{\text{connected component of } s \text{ (and } t)}) \le n^3$$

$$\Pr[\text{output "no"}] \le \underbrace{\Pr[\text{start at } s, \text{ walk } c \cdot C_s(G_s) \text{ steps \& don't see } t]}$$

$= \Pr[$ time to cover graph is $> c$ times its expectation$]$

$\le \frac{1}{c}$ (Markov's #)

∎

Comments

- Actually USTConn $\in L$!!! (Reingold)

- Open: is STConn for directed graphs $\in L$?

  ($\Rightarrow RL = L$)

  We know $RL \in L^{3/2}$

  recent improvement $RL \in \text{Space}\left(\frac{\log^{3/2} n}{\sqrt{\log\log n}}\right)$

  actually also BPspace(log n)

  due to William Hoza in Random '21