




Massachusetts Institute of Technology

# Part 1: Introduction to Path Planning

Lars Blackmore



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## Introductory Background

- Intro to Path Planning: AIMA Ch. 25.4
- Uninformed Search: AIMA Ch. 3
- Informed Search: AIMA Ch. 4

AIMA = Artificial Intelligence: A Modern Approach, by Russell and Norvig



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## Overview

- Problem Setup
  - From workspace to configuration space
- Path Planning in Continuous Configuration Spaces
  - Potential Field Methods
- Generating Roadmaps
  - Visibility graph
  - Voronoi diagrams
  - Cell decomposition
- Part 2: Path planning with Probabilistic Roadmaps
  - Brian



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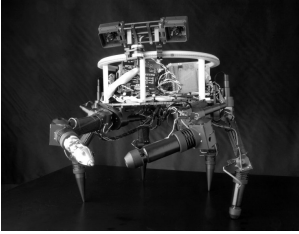
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### Introduction



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### Problem Definition

- Previous lectures:
  - Mapping the World from Sensor Information
  - Estimating Robot State
- This lecture:
  - Path planning
- Problem Statement: Compute a **collision-free path** for a rigid or articulated object (the robot) among **static** obstacles

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### Problem Definition

- Inputs:
  - Model of robot
  - Initial and goal robot configurations
- Output:
  - Continuous sequence of collision-free robot configurations connecting the initial and goal configurations

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### Path Planning and Modeling

- In order to make a planning problem tractable, typically abstract away complexities of robot model

```
graph TD; A[Path Planner] -- "Configurations in higher-level space" --> B[Low-level Controller]; B -- "Commands to actuators" --> C[Robot];
```

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### Path Planning and Modeling

- Example: Roomba
  - Goal defined in terms of location on 2-D floor
  - 2-D location is “controllable”
  - Plan at level of 2-D location
  - Low-level controller issues motor commands

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### Path Planning and Modeling

- Example: UAV
  - 2-D location is not “controllable”
  - Approach A:
    - Plan a path as if 2-D location controllable
    - Try to follow this path as best possible
  - Approach B:
    - Plan while taking into account lowerlevel dynamics model

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### Initial Assumptions for this Lecture

- Path planning in controllable spaces
  - Later in this lecture we will talk about planning with dynamics models
- Robot state known exactly
  - Estimation algorithms (e.g. SLAM) give distributions over robot state
  - Typical approach: use **mean** of distribution
  - Later lectures will talk about path planning with uncertainty



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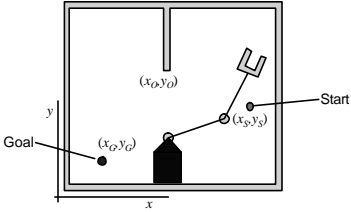

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### Workspace to Configuration Space

- Path planning problem usually defined in terms of workspace coordinates
  - Task: Go from start position to goal position
  - Constraint: Do not collide with any obstacles

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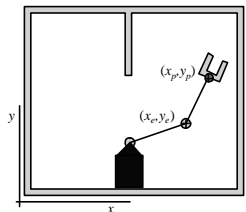

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### Workspace to Configuration Space

- In workspace coordinates, configuration of manipulator described by position of elbow and endpoint

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### Workspace to Configuration Space

- Problem: not all elbow/endpoint configurations possible

- To reach goal, must find path that has **feasible** configurations along **entire path**

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### Workspace to Configuration Space

- Problem can also be represented in **configuration space**

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### Workspace to Configuration Space

- Solution: plan in **configuration space**
  - Transform start and goal position into configuration space
  - Find a path in the configuration space

- Without obstacles, this is easy...

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### Workspace to Configuration Space

- With obstacles:
  - Transform obstacles into configuration space
- Plan a path in the configuration space

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### Workspace to Configuration Space

- With obstacles:
  - Transform obstacles into configuration space
- Plan a path in the configuration space

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### Workspace to Configuration Space

- With obstacles:
  - Transform obstacles into configuration space
- Plan a path in the configuration space

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### Workspace to Configuration Space

- With obstacles:
  - Transform obstacles into configuration space
- Plan a path in the configuration space

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### Workspace to Configuration Space

- With obstacles:
  - Transform obstacles into configuration space
- Plan a path in the configuration space

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### Workspace to Configuration Space

- With obstacles:
  - Transform obstacles into configuration space
- Plan a path in the configuration space

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### Workspace to Configuration Space

- Configuration space represents all feasible assignments to robot state, start and goal states
  - Now we can think about how to plan in this space
- Configuration space has same dimensionality as robot state
  - Configuration space can have higher or lower dimensionality than workspace
- Other pictures of configuration spaces...

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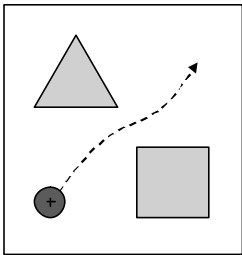
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### Workspace to Configuration Space

- Circular mobile robot in 2-D space



The diagram shows a square workspace containing a triangular obstacle at the top left, a square obstacle at the bottom right, and a circular robot with a '+' sign at the bottom left. A dashed arrow indicates a path from the robot to the top right corner of the workspace.

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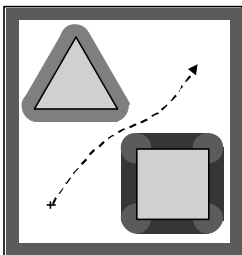
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### Workspace to Configuration Space

- Circular mobile robot in 2-D space



The diagram shows a square workspace containing a triangular obstacle at the top left and a square obstacle at the bottom right. A circular robot with a '+' sign is at the bottom left. A dashed arrow indicates a path from the robot to the top right corner. The robot and obstacles are shown with thick outlines, representing their footprint in the workspace.

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### Workspace to Configuration Space

- Polygonal Robot in 2-D Space

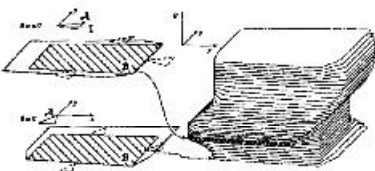


Figure 8.  $A$  and  $B$  are convex polygons.  $A$  is a free-flying robot.  $C$  is represented by parameterizing each configuration  $q$  by  $(x, \theta) \in \mathbb{R}^2 \times [0, 2\pi]$ . The represented Configuration is a volume bounded by patches of robot configs (see Chapter 1).

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### Overview

- Problem Setup
  - From workspace to configuration space
- Path Planning in Continuous Configuration Spaces
  - Potential Field Methods
- Generating RoadMaps
  - Visibility graph
  - Voronoi diagrams
  - Cell decomposition
- Part 2: Path planning with Probabilistic Roadmaps
  - Brian

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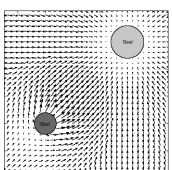
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### Potential Field Methods

- Works for continuous spaces
- Key idea:
  - Robot follows suitably defined function downhill until reaches goal
  - Function defined so that robot pushed towards goal and away from obstacles



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### Potential Field Methods

- Define potential function  $f$ 

$$f = F + ?$$
- Define attractive potential  $F$  for goal:
  - Example:
 
$$\Phi = k_1 \|\mathbf{x} - \mathbf{x}_{goal}\|^2$$

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### Potential Field Methods

- Example ? to guarantee no collisions?
 
$$\Psi = \frac{k_2}{\|\mathbf{x} - \mathbf{x}_{obs}\|^2}$$

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### Potential Field Methods

- Overall  $f = F + ?$

- Robot follows gradient downhill so that  $\dot{\mathbf{x}} = -\nabla f$

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### Potential Field Methods

- Are we guaranteed to stop at the goal?
  - Not exactly with the functions defined so far

$$\Phi = k_1 \|\mathbf{x} - \mathbf{x}_{goal}\|^2 \quad \Psi = \frac{k_2}{\|\mathbf{x} - \mathbf{x}_{obs}\|^2}$$

- Solutions?
  - Small  $k_2$  (approximate solution)
  - Logarithmic attractor function
  - Discontinuous obstacle function

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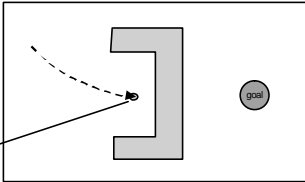
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### Potential Field Methods

- Drawbacks?
  - Not optimal
  - Complete?
- Potential field methods are not complete:
  - Local minima can exist in field



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### Potential Field Methods

- Main drawback:
  - No principled way of generating functions to guarantee completeness or optimality

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### Overview

- Problem Setup
  - From workspace to configuration space
- Path Planning in Continuous Configuration Spaces
  - Potential Field Methods
- **Generating Roadmaps**
  - Visibility graph
  - Voronoi diagrams
  - Cell decomposition
- Path planning with Probabilistic Roadmaps
  - Brian

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### From Continuous Maps to Roadmaps

- Want to plan a path in the configuration space
- Two classes of approach
  - Plan directly in the configuration space
    - Infinite space (continuous)
  - Represent configuration space as a roadmap
    - Plan in the roadmap
    - Assume that low-level controller takes care of conversion from discrete map to continuous space

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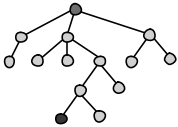
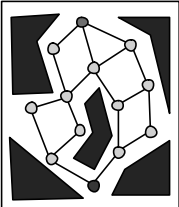
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### From Continuous Maps to Roadmaps

- Planning with roadmaps:
  - Reminder of how to search graphs



Example search techniques: BFS, DFS, A\*

See Russell & Norvig Ch. 3 & 4

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### From Continuous Maps to Roadmaps

- Techniques for roadmap generation
  - Visibility graph
  - Voronoi diagram
  - Cell decomposition

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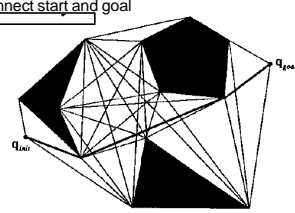
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### From Continuous Maps to Roadmaps

- Visibility Graph
  - Connect each obstacle corner to every other visible corner
  - Also connect start and goal



- Advantages:
  - Shortest path also shortest path in original space (in 2-D only)
- Disadvantages:
  - Not robust

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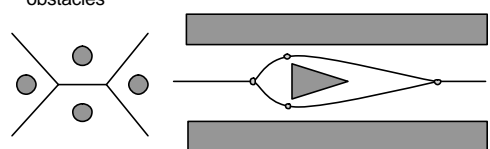
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### From Continuous Maps to Roadmaps

- Voronoi Graph
  - **Definition:** set of all points equidistant to two or more obstacles



- Disadvantages?
  - Hard to calculate (esp. in higher dimensional spaces)
  - Excessive clearance in wide-open spaces

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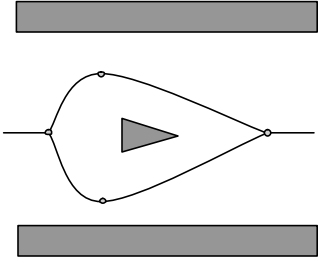
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### From Continuous Maps to Roadmaps

- Unnecessary clearance in wide open spaces



The diagram shows two horizontal gray bars, one above and one below. A path is drawn between them, starting from a point on the left, curving around the top bar, then around the bottom bar, and meeting at a point on the right. Inside this path is a gray triangle pointing to the right. This illustrates a path that maintains a large clearance from the obstacles, which is unnecessary.

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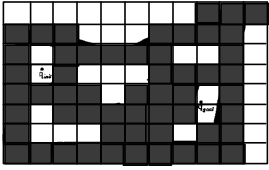
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### From Continuous Maps to Roadmaps

- Cell decomposition
  - Break up space into cells that are easily traversable
- Simplest approach: grid decomposition



The diagram shows a 10x10 grid of cells. Some cells are shaded black, representing obstacles. A path is shown as a sequence of white cells starting from a small square on the left and ending at another small square on the right. The path winds through the grid, avoiding the black obstacles.

- Can lose completeness

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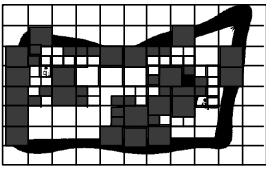
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### From Continuous Maps to Roadmaps

- Variable-sized cell decomposition



The diagram shows a grid of cells of varying sizes. Some cells are shaded black, representing obstacles. A path is shown as a thick black line that starts on the left, goes around the obstacles, and ends on the right. The cells along the path are larger and more uniform in size, while the cells in the corners and between obstacles are smaller.

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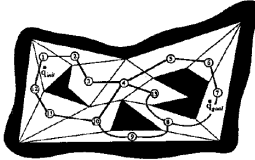
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### From Continuous Maps to Roadmaps

- Exact cell decomposition



- Cell decomposition drawbacks:
  - Computation can be difficult
  - Does not scale well to larger-dimensional spaces

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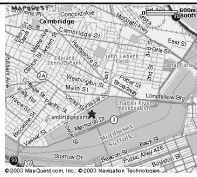

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### Exploring Roadmaps

- Shortest path
  - Dijkstra's algorithm
  - Bellman-Ford algorithm
  - Floyd-Warshall algorithm
  - Johnson's algorithm
- Informed search
  - Uniform cost search
  - Greedy search
  - A\* search
  - Beam search
  - Hill climbing

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