A Gentle Introduction to Bilateral Filtering and its Applications

Naïve Image Smoothing: Gaussian Blur

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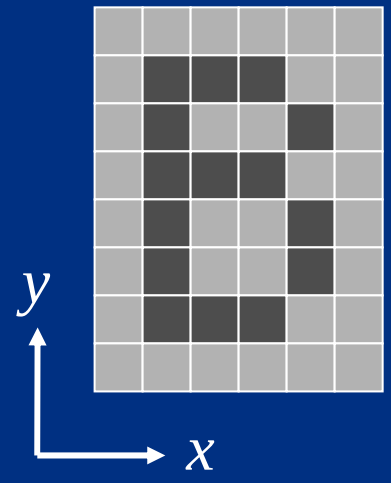
Notation and Definitions



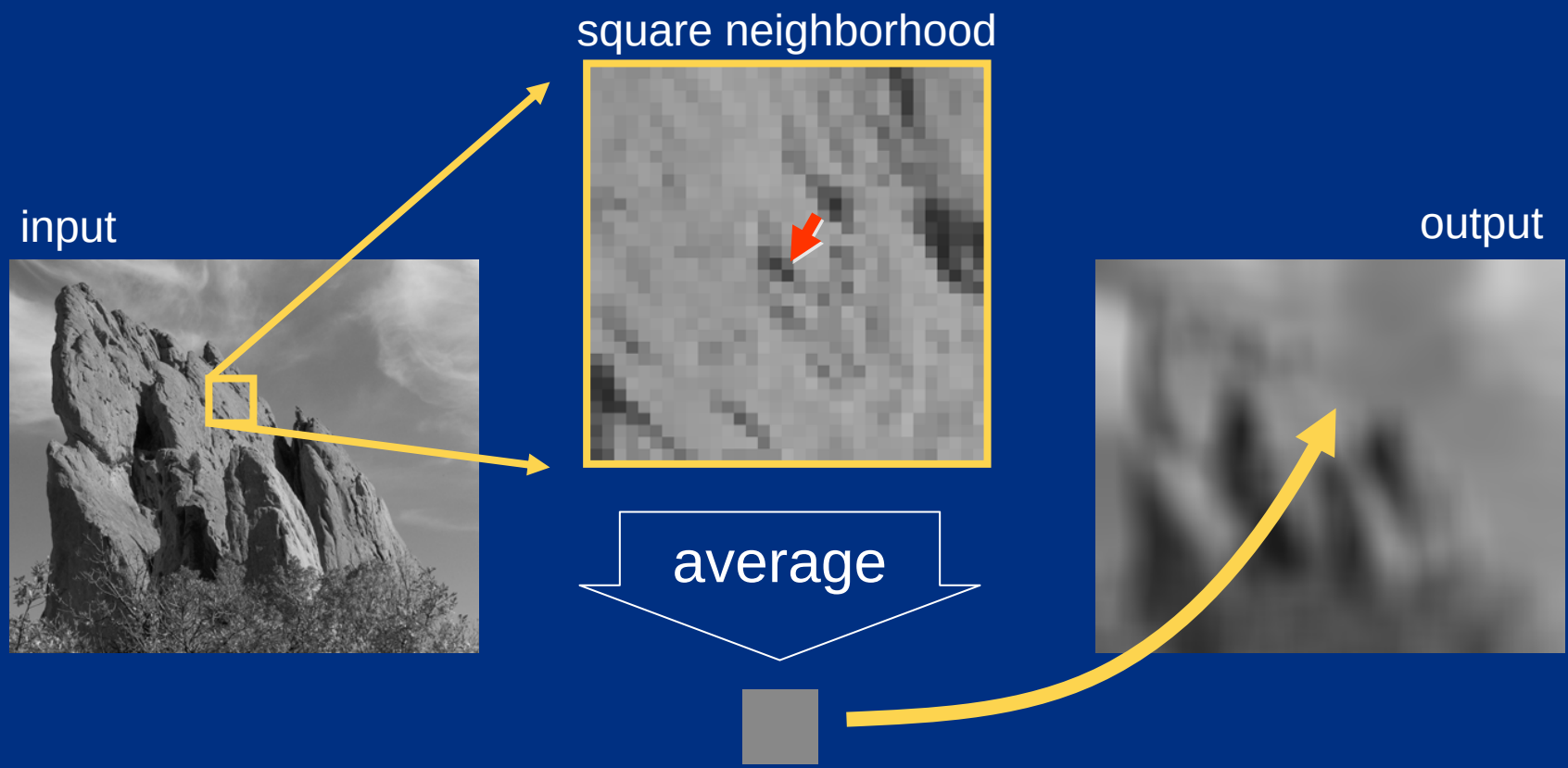
- Image = 2D array of pixels
- Pixel = intensity (scalar) or color (3D vector)
- $I_{\mathbf{p}}$ = value of image I at position: $\mathbf{p} = (p_x , p_y)$
- $F [I]$ = output of filter F applied to image I

Strategy for Smoothing Images

- Images are not smooth because adjacent pixels are different.
- Smoothing = making adjacent pixels look more similar.
- Smoothing strategy

  pixel $\rightarrow$ average of its neighbors

Box Average



Equation of Box Average

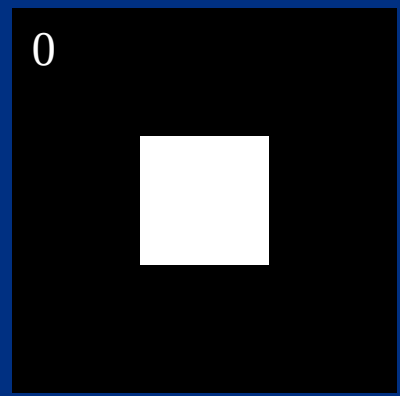
$$BA[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} B_{\sigma}(\mathbf{p} - \mathbf{q}) I_{\mathbf{q}}$$

result at pixel $\mathbf{p}$

sum over all pixels $\mathbf{q}$

normalized box function

intensity at pixel $\mathbf{q}$



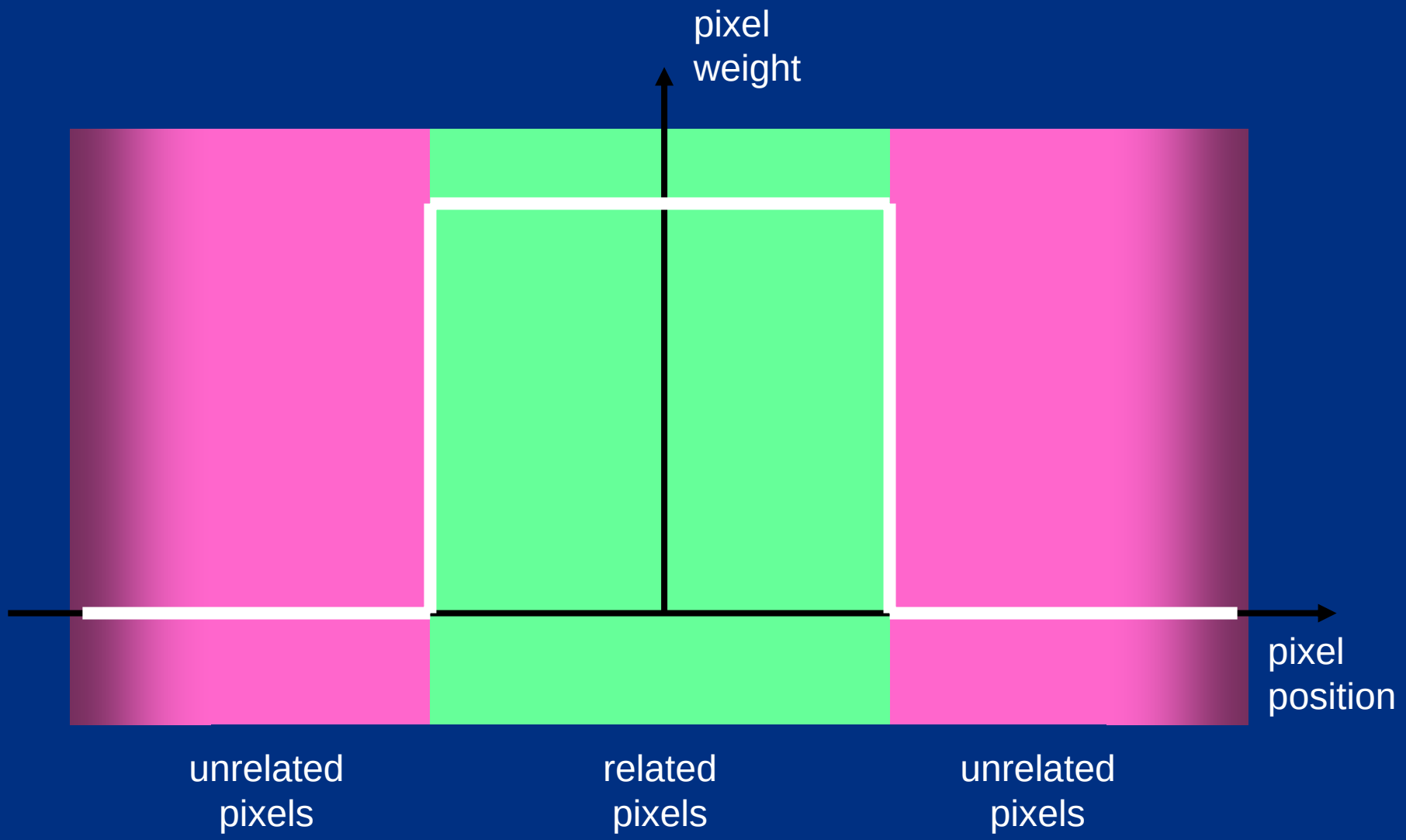
Square Box Generates Defects

- Axis-aligned streaks
- Blocky results

output

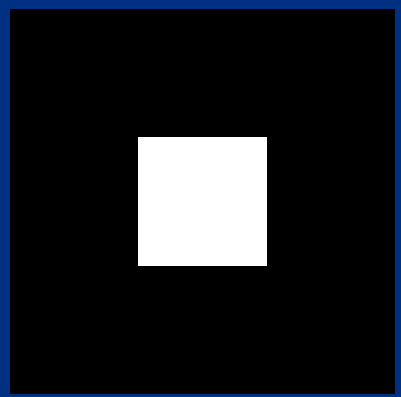
input

Box Profile

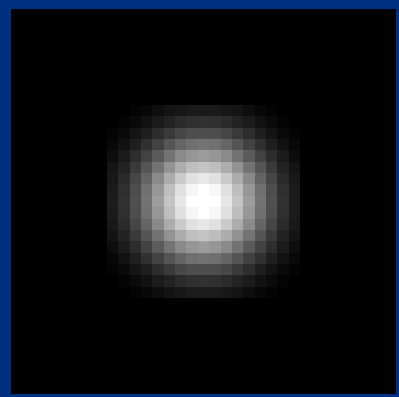

pixel
weight
pixel
position
unrelated
pixels
related
pixels
unrelated
pixels

Strategy to Solve these Problems

- Use an isotropic (*i.e.* circular) window.
- Use a window with a smooth falloff.

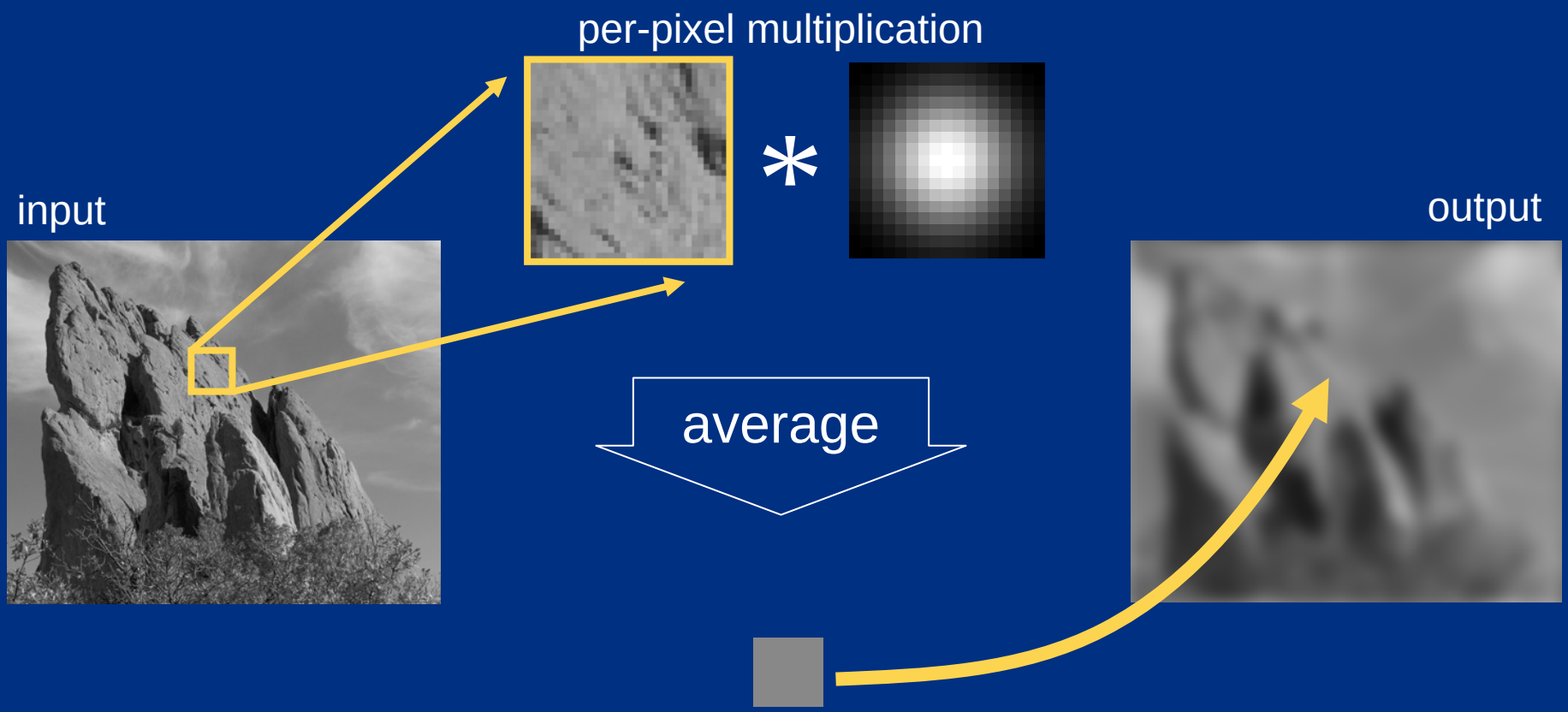


box window



Gaussian window

Gaussian Blur




input


box average

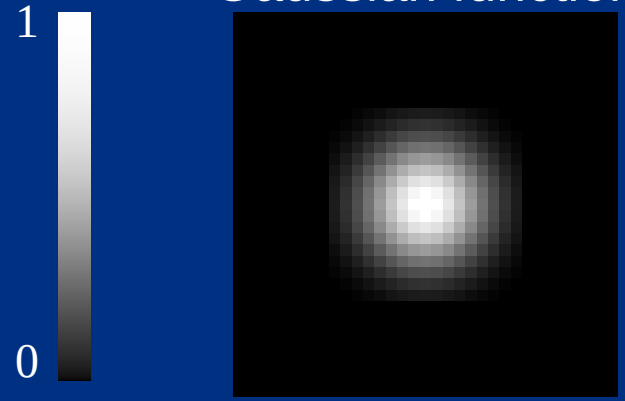

Gaussian blur

Equation of Gaussian Blur

Same idea: **weighted average of pixels**.

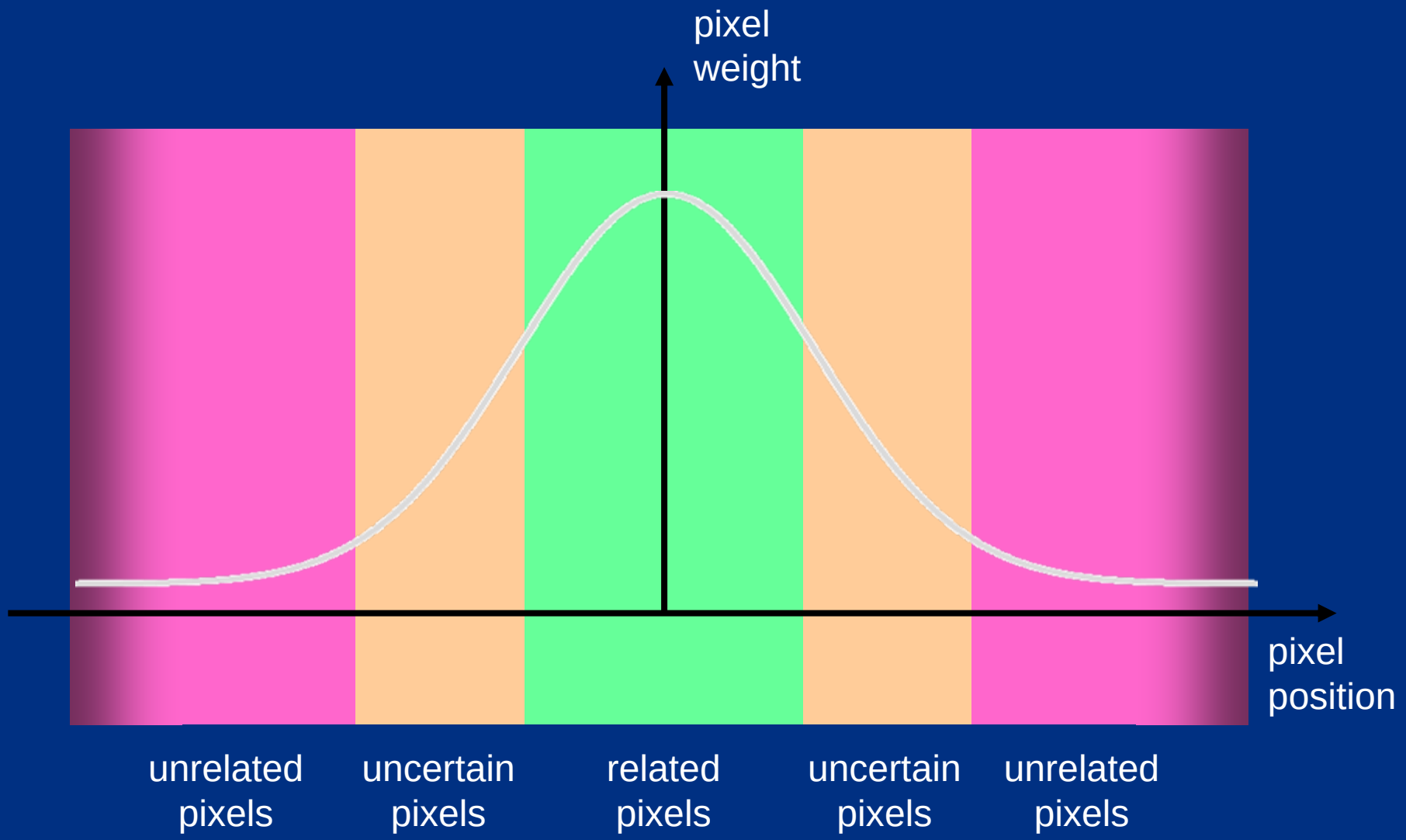
$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_{\sigma}\left(\| \mathbf{p} - \mathbf{q} \|\right) I_{\mathbf{q}}$$

normalized
Gaussian function



Gaussian Profile

$$G_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

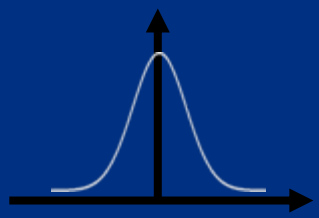


Spatial Parameter

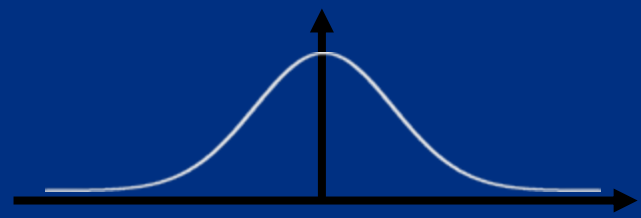
input

$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_{\sigma}\left(\| \mathbf{p} - \mathbf{q} \|\right) I_{\mathbf{q}}$$

size of the window



small σ



large σ

limited smoothing

strong smoothing

How to set σ

- Depends on the application.

- Common strategy: proportional to image size
  - e.g. 2% of the image diagonal
  - property: independent of image resolution

Properties of Gaussian Blur

- Weights independent of spatial location
  - linear convolution
  - well-known operation
  - efficient computation (recursive algorithm, FFT…)

Properties of Gaussian Blur

- Does smooth images
- But smoothes too much: **edges are blurred**.
  - Only spatial distance matters
  - No edge term

$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_{\sigma}\left(\|\mathbf{p}-\mathbf{q}\|\right) I_{\mathbf{q}}$$

space

input

output