The Power of Gradients in Inverse Dynamics Problems

Tao Du
MIT CSAIL
What is a dynamic system?
What is a dynamic system?

“A dynamical system is particle or ensemble of particles whose state varies over time and thus obeys differential equations involving time derivatives.”

---Nature Portfolio
What is a dynamic system?

<table>
<thead>
<tr>
<th>States</th>
<th>Time derivatives</th>
</tr>
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<td>$s$,</td>
<td>$\frac{ds}{dt}$, $\frac{d^2s}{dt^2}$, ...</td>
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What is a dynamic system?

Dynamic model

\[ F( s, \frac{ds}{dt}, \frac{d^2s}{dt^2}, \ldots ) = 0 \]
What is a dynamic system?

Dynamic model

\[ F(s, \frac{ds}{dt}, \frac{d^2s}{dt^2}, \ldots) = 0 \]

Example

Rigid-body systems: Euler-Lagrange equation

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} - Q = 0 \]

Deformable objects: continuum mechanics

\[ \nabla \cdot \sigma + f = 0 \]

Fluid systems: Navier-Stokes equation

\[ \frac{du}{dt} + (u \cdot \nabla)u - \nu \nabla^2 u = -\frac{1}{\rho} \nabla p + g \]
The input and the output

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<td>( F(\ s, \ \frac{ds}{dt}, \frac{d^2s}{dt^2}, \ldots ) = 0 )</td>
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**Example**

Rigid-body systems: Euler-Lagrange equation

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Deformable objects: continuum mechanics

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The input and the output

**Input**
- Parameters

**Example**
- Intrinsic parameters
- Extrinsic parameters

**Dynamic model**

\[ F(s, \frac{ds}{dt}, \frac{d^2s}{dt^2}, \ldots) = 0 \]

**Example**

- Rigid-body systems: Euler-Lagrange equation
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<td>State sequences</td>
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Example

- **Intrinsic parameters**
  - Rigid-body systems: Euler-Lagrange equation
    \[
    \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} - Q = 0
    \]
  - Deformable objects: continuum mechanics
    \[
    \nabla \cdot \sigma + f = 0
    \]
  - Fluid systems: Navier-Stokes equation
    \[
    \frac{du}{dt} + (u \cdot \nabla)u - \nu \nabla^2 u = -\frac{1}{\rho} \nabla p + g
    \]

- **Extrinsic parameters**
  - States from simulation
  - States from experiments

Example
The forward dynamics problem

Given the model and parameters of a dynamic system, compute its state sequence.
The forward dynamics problem

Given the model and parameters of a dynamic system, compute its state sequence.

Parametrization
Initializing parameters
The forward dynamics problem

Given the model and parameters of a dynamic system, compute its state sequence.

**Parametrization**
Initializing parameters

**Modeling**
Deriving governing equations
The forward dynamics problem

Given the model and parameters of a dynamic system, compute its state sequence.

- **Parametrization**: Initializing parameters
- **Modeling**: Deriving governing equations
- **Evaluation**: Computing performance metrics
The forward dynamics problem

Airfoil
Navier-Stokes equations in COMSOL
Learning mesh-based simulation with graph networks
Pfaff et al. ICLR 2021

Dexterous hand
Euler-Lagrange equations in Issac Gym
A system for general in-hand object re-orientation
Chen et al. CoRL 2021
The inverse dynamics problem

Given the state sequence of a dynamic system, infer its model and parameters.
The inverse dynamics problem

Given the state sequence of a dynamic system, infer its model and parameters.

- **Parametrization**: Initializing parameters
- **Modeling**: Deriving governing equations
- **Evaluation**: Computing performance metrics
The inverse dynamics problem

Inferring intrinsic parameters
Real2Sim
[Hahn et al. SIGGRAPH 19]

Inferring extrinsic parameters
DeepMimic
[Peng et al. SIGGRAPH 18]
Our topic today: the gradient methodology

- **Parametrization**: Initializing parameters
- **Modeling**: Deriving governing equations
- **Evaluation**: Computing performance metrics
Our topic today: the gradient methodology

- **Parametrization**: Initializing parameters
- **Modeling**: Deriving governing equations
- **Evaluation**: Computing performance metrics

SIGGRAPH 2021

SIGGRAPH Asia 2016
Our topic today: the gradient methodology

- **Parametrization**: Initializing parameters
- **Modeling**: Deriving governing equations
- **Evaluation**: Computing performance metrics
Our topic today: the gradient methodology

- **Parametrization**
  Initializing parameters

- **Modeling**
  Deriving governing equations

- **Evaluation**
  Computing performance metrics

SIGGRAPH 2021

SIGGRAPH 2022

SIGGRAPH Asia 2016

SIGGRAPH 2022

ICLR 2022

RA-L 2022
Our topic today: the gradient methodology

- **Parametrization**: Initializing parameters
- **Modeling**: Deriving governing equations
- **Evaluation**: Computing performance metrics
DiffAqua: A Differentiable Computational Design Pipeline for Soft Underwater Swimmers with Shape Interpolation

Pingchuan Ma, Tao Du, John Z. Zhang, Kui Wu, Andrew Spielberg, Robert K. Katzschmann, Wojciech Matusik

SIGGRAPH 2021
Problem statement

Find the optimal *shape* and *control* of soft robotic fishes to achieve *extremal* performance for underwater tasks.
Applications of soft robotic fish

SoFl
[Katzschmann et al. Science Robotics 18]

Project CETI
Why is it an inverse dynamics problem

- Parametrization
- Modeling
- Evaluation

Desired performance
- e.g. flow-resistance
Why is it an inverse dynamics problem

- **Parametrization**
- **Modeling**
  - Known dynamic model
    - Continuum mechanics
    - $\nabla \cdot \sigma + f = 0$
- **Evaluation**
  - Desired performance
    - e.g. flow-resistance
Why is it an inverse dynamics problem

**Parametrization**  
Shape and control to be determined

**Modeling**  
Known dynamic model  
Continuum mechanics

\[ \nabla \cdot \sigma + f = 0 \]

**Evaluation**  
Desired performance  
e.g. flow-resistance
The challenges

Fishes are **soft**: many degrees of freedom are needed.
Fishes are **diverse**: it’s difficult to find one compact representation for all.
Parametrization is the key

40k DoFs
3 fins

40k DoFs
1 fin

40k DoFs
5 fin
Our approach: Wasserstein gradients

- 40k DoFs
- 3 fins

- 40k DoFs
- 1 fin

- 40k DoFs
- 5 fin
Our approach: Wasserstein gradients

- 40k DoFs, 3 fins
- 3 DoFs, 1-5 fins
- 40k DoFs, 1 fin
- 40k DoFs, 5 fin
Our approach: Wasserstein gradients

- 40k DoFs, 3 fins
- 40k DoFs, 1 fin
- 40k DoFs, 5 fin
- 3 DoFs, 1-5 fins
- Score
Our approach: Wasserstein gradients

40k DoFs
3 fins

3 DoFs
1-5 fins

40k DoFs
1 fin

40k DoFs
5 fin

Score
Example: speedy fish

- Optimized control only
- Optimized control only
- Optimized control only
- Optimized shape and control
Example: flow-resistant fish

Unoptimized

Optimized

Wasserstein weights
Underwater Soft Robot Modeling and Control with Differentiable Simulation

Tao Du*, Josie Hughes*, Sebastien Wah, Wojciech Matusik, Daniela Rus
IEEE RA-L/RoboSoft 2021
Summary

- **Parametrization**
  - Initializing parameters

- **Modeling**
  - Deriving governing equations

- **Evaluation**
  - Computing performance metrics

SIGGRAPH 2021

SIGGRAPH 2022

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ICLR 2022

RA-L 2022
Summary

- **Parametrization**
  Initializing parameters

- **Modeling**
  Deriving governing equations

- **Evaluation**
  Computing performance metrics
DiffPD: Differentiable Projective Dynamics

Tao Du, Kui Wu, Pingchuan Ma, Sebastien Wah, Andrew Spielberg, Daniela Rus, Wojciech Matusik

ACM Transactions on Graphics (SIGGRAPH 2022)
Feature highlights

- Gradients
- Speed
- Robustness
Feature highlights

Gradients

Speed

Robustness

ChainQueen
Hu et al. ICRA 19

Projective dynamics
Bouaziz et al. SIGGRAPH 14

ADD
Geilinger et al. SIGGRAPH Asia 2020
Applications: system identification
Applications: trajectory optimization

Initial guess

Optimized parameters

Contraction - Expansion
Applications: real-to-sim transfer

Initial guess

Optimized parameters
Summary

- **Parametrization**: Initializing parameters
- **Modeling**: Deriving governing equations
- **Evaluation**: Computing performance metrics

SIGGRAPH 2021

SIGGRAPH 2022

ICLR 2022

SIGGRAPH Asia 2016

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RA-L 2022
RISP: Rendering-Invariant State Predictor with Differentiable Simulation and Rendering for Cross-Domain Parameter Estimation

Pingchuan Ma*, Tao Du*, Joshua B. Tenenbaum, Wojciech Matusik, Chuang Gan
ICLR 2022 (oral)
Problem statement

Build a digital twin of a robot from its video of motion sequences.
Why is it an inverse dynamics problem

Parametrization  Modeling  Evaluation

Desired performance
Match motions from videos
Why is it an inverse dynamics problem

**Parametrization**

**Modeling**

Known dynamic model
Euler-Lagrange dynamics

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} - Q = 0
\]

**Evaluation**

Desired performance
Match motions from videos
Why is it an inverse dynamics problem

Parametrization
Control sequence to be determined

Modeling
Known dynamic model
Euler-Lagrange dynamics

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} - Q = 0 \]

Evaluation
Desired performance
Match motions from videos
The challenge

The unknown visual appearance parameters shadows the dynamics information.
The first idea: a state-prediction network
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The first idea: a state-prediction network
The state-of-the-art approach

Train the network using **domain randomization.**
Domain randomization failed here...

Reconstructed motion

Input video
Why did domain randomization fail?
Why did domain randomization fail?

- Dynamic model
  - states
  - actions

- Visual appearance
  - lighting
  - material
  - texture

- Video input

- Neural network

- Prediction states
Why did domain randomization fail?

The network needs to maintain invariance under different visual appearances.
The second idea: rendering-invariance

The network needs to maintain invariance under different visual appearances.

\[ \nabla = 0 \]
Results: quadrotors

Note that the rendering configuration is intentionally made different.
Results: dexterous hand

Note that the rendering configuration is intentionally made different.
Summary

Parametrization
Initializing parameters

Modeling
Deriving governing equations

Evaluation
Computing performance metrics
What is next?

Let me end the talk with what I consider one of the most inspiring inverse dynamics problems in history.

Tycho Brahe (1546-1601)  
Johannes Kepler (1571-1630)  
Sir Isaac Newton (1643-1727)
What is next?

The most rewarding inverse problem is to discover scientific laws.
Acknowledgment

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Acknowledgment


Acknowledgment

Page 29: Video credit to Jie Xu.


Thank you!

Papers, code, and data are available at https://people.csail.mit.edu/taodu
Email: taodu@csail.mit.edu