MASSACHVSETTS INSTITVTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science
6.001—Structure and Interpretation of Computer Programs
Spring Semester, 1999

Recitation – Wednesday, March 31

1. Environment WHAT?

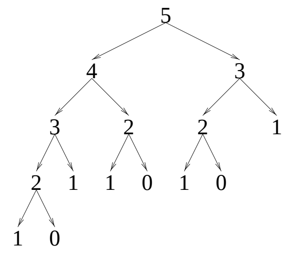
I barely remember the week before spring break. Let's start with an environment diagram warmup:

```
(define f
  (lambda (x)
    (lambda (y)
      (set! x (+ x y))
      x)))
(define g (f 4))
(g 3)
```

2. Counting Fibs

Recall the function `fib-1` that takes an integer `n` and computes the `n`'th Fibonacci number.

```
(define fib-1
  (lambda (n)
    (cond ((= n 0) 0)
          ((= n 1) 1)
          (else
           (+ (fib-1 (- n 1))
              (fib-1 (- n 2)))))))
```



What is the Order of Growth in Time for the procedure `fib-1`? This is a tough one to figure out. Maybe this tree will help. Consider the number of recursive calls to `fib-1` when the following is evaluated: `(fib-1 5)`

What if we want to see exactly how many times fib-1 is being called? Recall the function `make-count-proc` from last section. How can we use `make-count-proc` to define `fib-2` that will keep a count of the number of recursive calls?

```
(define make-count-proc
  (lambda (f)
    (let ((count 0))
      (lambda (x)
        (cond ((eq? x 'count) count)
              ((eq? x 'reset)
               (set! count 0)
               0)
              (else
               (set! count (+ count 1))
               (f x)))))))
```

```
(define fib-2
  (make-count-proc
   (lambda (n)
     (cond ((= n 0) 0)
           ((= n 1) 1)
           (else (+ (fib-2 (- n 1))
                    (fib-2 (- n 2)))))))
)

(fib-2 5)        ==>  5

(fib-2 'count)   ==>  15
```

Take a look at these calls to fib-2:

```
(fib-2 'reset)          ==> 0
(fib-2 30)              ==> 832040
(fib-2 'count)          ==> 2692537
```

That's pretty inefficient! We're recursively calling `fib-2` over and over again with the same argument and keep computing things we've already computed before. How can we fix this?

3. Memoizing

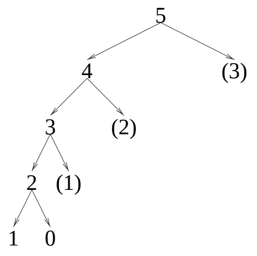
Recall that the procedure `make-count-proc` takes in a procedure and returns a very similar procedure (from the caller's point of view), but this new procedure keeps some local state around and does something else each time it is called.

Consider the procedure `memoize` that takes in a procedure of one argument and returns a procedure that keeps track of the all previously computed values. If a value passed in was passed in before, the procedure simply returns the saved value. Write the procedure `memoize`:

```
(define memoize
  (lambda (g)
    (let ((table '()))
      (lambda (y)
        (let ((result (assoc y table)))
          (if (pair? result)
              (cadr result)
              (let ((result (g y)))
                (set! table (cons (list y result) table))
                result))))))
)
```

Now define `fib-3` that uses memoization and a counter.

```
(define fib-3
  (make-count-proc
   (memoize
    (lambda (n)
      (cond ((= n 0) 0)
            ((= n 1) 1)
            (else
             (+ (fib-3 (- n 1))
                (fib-3 (- n 2))))))))) )
```



What is the Order of Growth in Time of `fib-3`?

```
(fib-3 'count) ==> 0
(fib-3 30) ==> 832040
(fib-3 'count) ==> 59
```

Challenge: Draw the Environment Diagram for the definition of `fib-3` (above) and then for the expression `(fib-3 3)`.