MASSACHVSETTS INSTITVTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science
6.001—Structure and Interpretation of Computer Programs
Spring Semester, 1999

Recitation – Friday, April 16

1. Streams

In lecture yesterday, we looked at a simple implementation of streams which used two new special forms:

- `(delay x)` which returns a promise and is equivalent to `(lambda () x)`
- `(cons-stream a b)` which is equivalent to `(cons a (delay b))`

and a few data abstrations:

```
(define (force obj) (obj))
(define (stream-car stream) (car stream))
(define (stream-cdr stream) (force (cdr stream)))
```

Using these basic functions, we can build infinite streams. For example:

```
(define ones (cons-stream 1 ones))
(define twos (cons-stream 2 twos))
```

Here's another way we can define `twos` using `stream-map` a very useful function that works like `map`, except on streams:

```
(define twos
  (stream-map + ones ones))
```

2. Warm Up

Write a procedure `powers-of-2-from` which takes a power of 2 (`n`) and returns the stream n, n*2, (n*2)*2, ((n*2)*2)*2, ...

```
(define (powers-of-2-from n)

)
```

Define a stream of the whole numbers N = {0, 1, 2, 3 ...} using ones

```
(define whole

)
```

3. Taylor Series

We can represent an infinite Taylor Series as a stream. The series $f(x) = (a_0 x + a_1 x + a_2 x^2 + a_3 x^3 + \ldots)$ can simply be represented as the stream of numbers $(a_0\ a_1\ a_2\ a_3\ \ldots)$.

Recall that (for $-1 < x < 1$), $f(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$. What series could we use to represent this series?

For example, recall that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$. Using `stream-map`, `fact` (factorial), and `whole`, define the stream corresponding to this series.

```
(define e^x



)
```

4. Evaluating a Series

Now say we want to evaluate e^x for some x. Write a function `eval-series` that takes a series `s`, a value `x`, and the number of terms to use `n`, and evaluates the series.

```
(define (eval-series s x n)





)
```

Now we can evaluate our series: `(eval-series e^x -0.5 100)` `; value .6065306597126333`

5. More Stream Tools

Write the function `interleave` that takes two infinite streams and interleaves them. For example

```
(define all-ints
  (cons-stream 0 (interleave integers (stream-map - integers))))
```

This would be the infinite stream (0 1 -1 2 -2 3 -3 4 -4 ...)

```
(define (interleave s t)

)
```

6. Cosine Series

For example, recall that $cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$. How could we create this stream using the `e^x` stream we already created? (What stream could we create that we could multiply with `e^x`?)

$$\begin{array}{ccccccccccc} e^x & = & 1 & +x & +\frac{x^2}{2!} & +\frac{x^3}{3!} & +\frac{x^4}{4!} & +\frac{x^5}{5!} & +\frac{x^6}{6!} & +\ldots \\ cos(x) & = & 1 & +0 & -\frac{x^2}{2!} & +0 & +\frac{x^4}{4!} & +0 & -\frac{x^6}{6!} & +\ldots \\ \hline & & & & & & & & & \ldots \end{array}$$

How could we define the cosine stream in this way using `ones` and `zeros` and `interleave` twice?

```
(define cos-x

)
```

7. All Pairs

What about the set of all pairs of positive integers: $\{\langle x, y\rangle \mid x, y \in PositiveIntegers\}$? How can we capture this infinite-way infinite sequence into a stream? Let's define a procedure `pairs` that takes two infinite streams and returns a stream of all possible pairs of elements of the two streams.

```
(define (pairs s t)





))
```

	1	2	3	4	5	...
1	$\langle 1,1\rangle$	$\langle 1,2\rangle$	$\langle 1,3\rangle$	$\langle 1,4\rangle$	$\langle 1,5\rangle$	...
2	$\langle 2,1\rangle$	$\langle 2,2\rangle$	$\langle 2,3\rangle$	$\langle 2,4\rangle$	$\langle 2,5\rangle$	...
3	$\langle 3,1\rangle$	$\langle 3,2\rangle$	$\langle 3,3\rangle$	$\langle 3,4\rangle$	$\langle 3,5\rangle$	...
4	$\langle 4,1\rangle$	$\langle 4,2\rangle$	$\langle 4,3\rangle$	$\langle 4,4\rangle$	$\langle 4,5\rangle$	...
5	$\langle 5,1\rangle$	$\langle 5,2\rangle$	$\langle 5,3\rangle$	$\langle 5,4\rangle$	$\langle 5,5\rangle$	...
⋮	⋮	⋮	⋮	⋮	⋮	