MASSACHVSETTS INSTITVTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science
6.001—Structure and Interpretation of Computer Programs
Spring Semester, 1999

Recitation – Friday, April 23

1. Register Machines

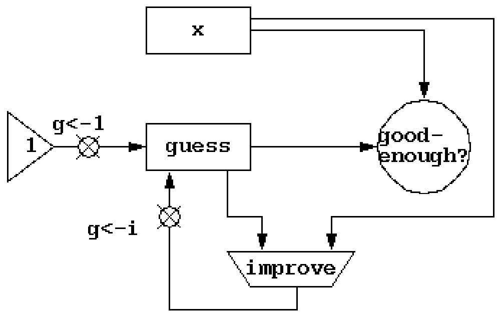
A register machine consists of (1) a finite set of **registers**, (2) a fixed set of **operators**, (3) a **controller** (set of instructions) and (4) a **stack**. We can represent register machines using **Data Path** diagrams and **Controller** diagrams or in a **language** that can be simulated in scheme.

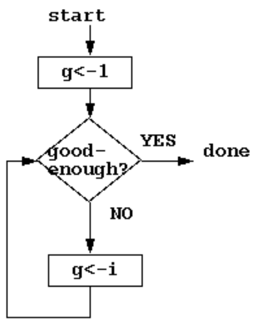
2. Register Machine Diagrams

Early in the semester, we saw some Scheme code to calculate square root using Newton's method:

```
(define (sqrt x)
  (define (good-enough? ...) ...)
  (define (improve guess) (average guess (/ x guess)))
  (define (sqrt-iter guess)
    (if (good-enough? guess)
        guess
        (sqrt-iter (improve guess))))
  (sqrt-iter 1))
```

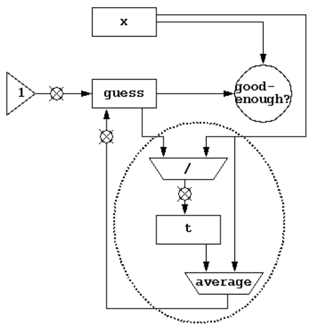
Unfortunately, Radio Shack^{TM} was out of square root chips, so we need to implement square root ourselves. Luckily they did have an `improve` chip and a `good-enough?` chip. Draw the data paths and control diagram for a square root machine.





When we connect it all up, we saw that the `improve` chip is broken, and Radio Shack doesn't have any others. However, they did have an `average` chip. Redraw the data paths and control diagram **as necessary** to replace the `improve` chip. Then, convert the diagrams into a register machine.

```
(registers x guess t )
(operations / average good-enough?  )
(controller
    (assign g (const 1))
  loop
    (test (op good-enough?) (reg guess) (reg x))
    (branch (label done))
    (assign t (op /) (reg guess) (reg x))
    (assign g (op average) (reg t) (reg guess))
    (goto (label loop))
  done)
```



3. Hand-Crafted Register Machines

What does the following register machine compute if we assume its input is in register `x` and its output is in register `y`. In other words, $y = f(x)$. What's `f`? $f(x) = 3x^2 + 7x + 5$

```
(registers x y t k sum )
(operations * + - = )
(controller
    (assign sum (const 0))
  loop
    (test (op =) (reg k) (const 0))
    (branch (label done))
    (assign x (reg k))
    (assign y (const 5))
    (assign t (op *) (reg x) (const 7))
    (assign y (op +) (reg t) (reg y))
    (assign t (op *) (reg x) (reg x))
    (assign t (op *) (reg t) (const 3))
    (assign y (op +) (reg t) (reg y))
    (assign k (op -) (reg k) (const 1))
    (assign sum (op +) (reg sum) (reg y))
    (goto (label loop))
  done
)
```

Modify the above code so that it computes $y = g(k) = f(1)+f(2)+\ldots+f(k)$, where `k` is an additional input register.

Below is the code to compute the same $f(x)$ as defined above. Given that code, write the code to compute $f(a)/f(b)$, where `a` and `b` are two input registers. You may use a `continue` register and a stack. (Don't duplicate the code to compute $f(x)$)

```
(registers x y t a b continue )
(operations * + / )
(controller
    (assign x (reg a))
    (assign continue (label f-of-b))
    (goto (label f-of-x))
  f-of-b
    (save y)
    (assign x (reg b))
    (assign continue (label divide-em))
    (goto (label f-of-x))

  ; This is the old f(x)
  ; Contract:  input in x, output in y,
  ;            when done, will jump to reg continue
  f-of-x
    (assign y (const 5))
    (assign t (op *) (reg x) (const 7))
    (assign y (op +) (reg t) (reg y))
    (assign t (op *) (reg x) (reg x))
    (assign t (op *) (reg t) (const 3))
    (assign y (op +) (reg t) (reg y))
    (goto (reg continue))

  divide-em
    (restore t)     ; f(a)
    (assign y (op /) (reg t) (reg y))
)
```