MASSACHVSETTS INSTITVTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science
6.001—Structure and Interpretation of Computer Programs
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Section 4
Final Review Handout [1]

Iterative vs. Recursive Processes and Order of Growth

Consider the two procedures below. Assume the arguments passed in are always positive integers.

```
(define (odd? x)
  (if (= x 1)
      #t
      (even? (- x 1))))

(define (even? x)
  (not (odd? x)))
```

Given the above definitions, when the expression `(odd?  24)` is evaluated, does that generate a recursive or iterative process? Why?

$\Longrightarrow$ **This generates a recursive process because the `not` operations are delayed.**

What is the order of growth of in time of the procedure `even?`?

$\Longrightarrow$ $\mathbf{\Theta(x)}$

What is the order of growth of in space of the procedure `odd?`?

$\Longrightarrow$ $\mathbf{\Theta(x)}$

Change one of the two functions above to make the process the opposite of what it currently is (eg. If it's a recursive process, make it iterative and if it's an iterative process, make it recursive).

$\Longrightarrow$
```
(define (even? x)
  (if (= x 0)
      #t
      (odd? (- x 1))))
```

Consider the function below:

```
(define (power-of-2? n)
  (cond ((= n 1) #t)
        ((< n 1) #f)
        (else
          (power-of-2? (/ n 2)))))
```

Does the function `power-of-2?` generate a recursive or iterative process? Why?

$\Longrightarrow$ **Iterative. No delayed operations.**

What is the order of growth of in time of the procedure `power-of-2?`?

$\Longrightarrow$ $\mathbf{\Theta(\log(x))}$

What is the order of growth of in space of the procedure `power-of-2?`?

$\Longrightarrow$ $\mathbf{\Theta(1)}$

[1]**Disclaimer: This handout was created as a study guide for the final. It is NOT necessarily complete. There could be other material not covered in this handout that is still fair game for the final. This is also NOT a practice final, in that I didn't time how long it should take. I just tried to come up with some problems that would help in reviewing for the final.**

Higher Order Procedures and Tree Structures

We'd like to write an evaluator for simple in-fix numerical expressions. For example, consider the trees generated by the following expressions. Careful: Notice that the operators are **not** quoted.

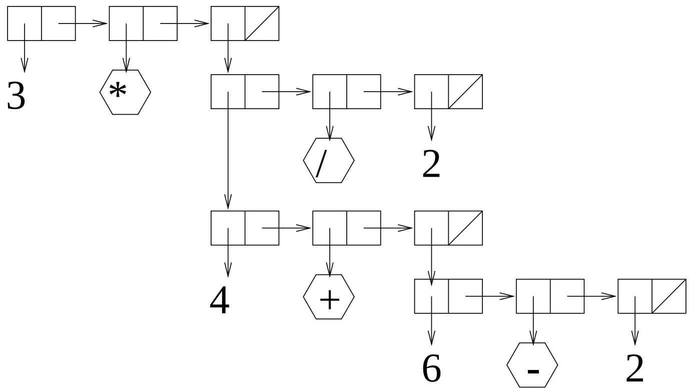
```
(define exp1 (list 3 + 5))
(define exp2 (list 3 * (list (list 4 + (list 6 - 2)) / 2)))
```

How does `exp1` print when evaluated?

⇒ `(3 #[arity-dispatched-procedure 16] 5)`

Draw the box and pointer diagram for the tree structure of `exp2`

⇒



Write a function `infix` that evaluates these types of expressions. For example,

```
(infix 4)     ==>  4
(infix exp1)  ==>  8
(infix exp2)  ==> 12
```

⇒
```
(define (infix exp)
  (if (not (pair? exp))
      exp
      ((cadr exp)
       (infix (car exp))
       (infix (caddr exp)))))
```

Consider a function `infix->prefix` that takes an infix expression like `exp1` and `exp2` above, and transforms the expression into prefix form (e.g. it swaps the order of the operator and first operand). For example, the expression `(infix->prefix (list (list 3 * 2) + 4))` will produce the same list structure as (i.e. will be `equal?` with) the value of following expression:
`(list + (list * 3 2) 4)`.

Write the function `infix->prefix`.

⇒
```
(define (infix->prefix exp)
  (if (not (pair? exp))
      exp
      (list (cadr exp)
            (infix (car exp))
            (infix (caddr exp)))))
```

Local State

Assume scheme has the following special form `time`, which takes one argument and times how long it takes to evaluate the argument. Time returns a list of two elements, the first is the amount of time it took to evaluate the expression, and the second is the value of the expression. For example,

```
(time (sqrt 4))     ==>  (20 2)
(time (sqr 5))      ==>  (10 25)
(time (list 1 2 3)) ==>  (7 (1 2 3))
(time (/ 1 0))      ==>  Error: divide by zero
```

Assume, for simplicity, that `(sqrt x)` always takes 20 time units to compute and `(sqr x)` always takes 10 time units to compute for all positive numbers.

Louis has a program that he wants to make run faster. He wants to see what procedures are eating up the most amount of time. He decides to write a procedure called `make-timed-procedure` which takes a procedure and returns a procedure that does the same thing as the original, but also keeps track of the total time spent in the procedure. For example,

```
(define tsqrt (make-timed-procedure sqrt))
(define tsqr (make-timed-procedure sqr))
```

```
(tsqrt 'time) ==> 0
(tsqrt 4)     ==> 2
(tsqrt 'time) ==> 20
(tsqrt 4)     ==> 2
(tsqrt 'time) ==> 40
```

```
(tsqr 'time)  ==> 0
(tsqr 10)     ==> 100
(tsqr 'time)  ==> 10
(tsqr 10)     ==> 100
(tsqr 'time)  ==> 20
```

Louis writes the following procedure:

```
(define make-timed-procedure-1
  (let ((total-time 0))
    (lambda (proc)
      (lambda (x)
        (if (eq? x 'time)
            total-time
            (let ((result (time (proc x))))
              (set! total-time (+ total-time (car result)))
              (cadr result)))))))
```

After defining the following two functions, Louis trys out his code. Show the output of each of the following expressions, assuming that each call to `sqrt` takes 20 and each call to `sqr` takes 10 time units.

```
(define tsqrt-1 (make-timed-procedure-1 sqrt))
(define tsqr-1 (make-timed-procedure-1 sqr))
```

⇒
```
(map tsqrt-1 (map tsqr-1 '(-1 2 -3 4 -5))) ==>  (1 2 3 4 5)
(tsqrt-1 'time)                             ==>  150
(tsqr-1 'time)                              ==>  150
```

This isn't quite the behavior we wanted. What change needs to be made to `make-timed-procedure` so that we get the correct behavior?

⇒
```
(define make-timed-procedure
  (lambda (proc)
    (let ((total-time 0))
      (lambda (x)
        (cond ((eq? x 'time) total-time)
              ((eq? x 'reset) (set! total-time 0) 0)
              (else (let ((result (time (proc x))))
                      (set! total-time (+ total-time (car result)))
                      (cadr result))))))))
```

Object Oriented Programming and Environment Diagrams

Below is the object oriented system from the March 19th Lecture Notes (just included for reference)

```
(define (get-method object message)
  (object message))

(define (no-method name)
  (list 'no-method name))

(define (no-method? x)
  (if (pair? x)
      (eq? (car x) 'no-method)
      false))

(define (method? x)
  (not (no-method? x)))

(define (ask object message . args)
  (let ((method (get-method object message)))
    (if (method? method)
        (apply method (cons object args))
        (error "No method" message (cadr method)))))
```

Consider the following expressions:

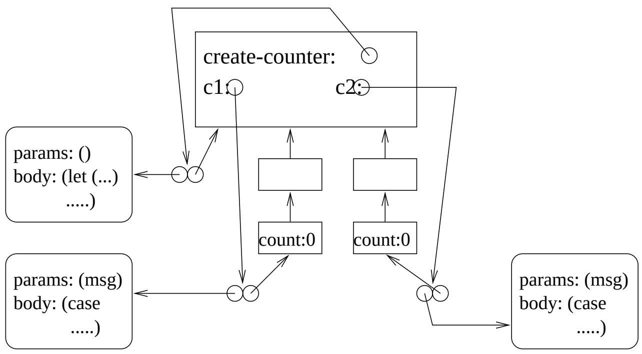
```
(define (create-counter)
  (let ((count 0))
     (lambda (message)
        (case message
          ((VALUE) (lambda (self) count))
          ((INCR) (lambda (self)
                    (set! count (+ 1 count))
                    count))
          ((DECR) (lambda (self)
                    (set! count (- count 1))
                    count))))))

(define c1 (create-counter))

(define c2 (create-counter))
```

Draw the environment diagram for the above three expressions.

⇒



Write the expression that you would use to get the value of counter `c1`.

⇒ **(ask c1 'value)**

Write the expression that you would use to increment counter `c1`.

⇒ **(ask c1 'incr)**

Streams and Higher Order Procedures

In this part, we are going to create an infinite stream of higher-order procedures. First, here are some simple functions that we are going to be using.

```
(define (compose f g) (lambda (x) (f (g x))))
(define (incr x) (+ x 1))
(define (sqr x) (* x x))
```

Now, Consider the function `compose-fstreams` that takes two streams-of-functions, `fs1` and `fs2`, and returns another stream-of-functions where each element is the result of composing the corresponding elements of `fs1` and `fs2`.

```
(define (compose-fstreams fs1 fs2)
  (cons-stream (compose (stream-car fs1) (stream-car fs2))
               (compose-fstreams (stream-cdr fs1) (stream-cdr fs2))))
```

We define the following two infinite streams of functions, one that increments and one that squares.

```
(define fs-incr (cons-stream incr fs-incr))
(define fs-sqr (cons-stream sqr fs-sqr))
```

We can now define another infinite stream-of-functions `fs` (Hint: this is similar to how we defined integers in terms of ones).

```
(define fs
  (cons-stream incr (compose-fstreams fs-incr fs)))
```

What is the first element of the stream `fs`?

$\Longrightarrow$ **The function that increments.**

What is the second element of the stream `fs`?

$\Longrightarrow$ **The function that adds 2 to a number.**

Consider the function `apply-fstream` and the definition of the stream `s`, below.

```
(define (apply-fstream fs x)
  (cons-stream
   ((stream-car fs) x)
   (apply-fstream (stream-cdr fs) x)))

(define s (apply-fstream fs 0))
```

What are the first 10 elements of the stream `s`?

$\Longrightarrow$ **(1 2 3 4 5 6 7 8 9 10 ...)**

Consider the following two streams that are defined.

```
(define t1 (apply-fstream (compose-fstreams fs fs-sqr) 0))
(define t2 (apply-fstream (compose-fstreams fs-sqr fs) 0))
```

What are the first 5 elements of the stream `t1`?

$\Longrightarrow$ **(1 2 3 4 5 ...)**

What are the first 5 elements of the stream `t2`?

$\Longrightarrow$ **(1 4 9 16 25 ...)**

Meta Circular Evaluator

We would like to introduce a new special form to our evaluator called `same?`. `Same?` always takes three arguments, and returns `#t` if all three arguments are `eq?`. `Same?`, however is smart in that if the first two arguments are different, then the third argument is **not** evaluated. Here are some examples of using `same?`.

```
(same? 'x 'x 'x)        ==>  #t
(same? 'x 'x 'y)        ==>  #f
(same? 'x 'y 'y)        ==>  #f
(same? 'x 'y (/ 1 0))   ==>  #f
(same? 'x 'x (/ 1 0))   ==>  Divide by Zero Error
```

To add this special form to the evaluator, we need to define some data abstraction.

Define the function `same??` that checks to see if an expression is a `same?` expression.

$\Longrightarrow$
```
(define (same?? exp) (tagged-list? exp 'same?))
```

Define the functions `same?-first` and `same?-second` that select out the first and second subexpressions (assume someone else defined `same?-third`).

$\Longrightarrow$
```
(define (same?-first exp) (cadr exp))
```
$\Longrightarrow$
```
(define (same?-second exp) (caddr exp))
```

Next, write the appropriate clause to add to the `cond` clause of `eval`, assuming that we have the function `eval-same?` that will evaluate a `same?` expression.

$\Longrightarrow$
```
((same?? exp) (eval-same? exp env))
```

Finally, write the `eval-same?` function that takes a `same?` expression and an environment and implements the special form as described above.

$\Longrightarrow$
```
(define (eval-same? exp env)
  (let ((val1 (eval (same?-first exp) env))
        (val2 (eval (same?-second exp) env)))
    (if (eq? val1 val2)
        (eq? val1 (eval (same?-third exp) env))
        #f)))
```

Fill in the blanks:

$\Longrightarrow$ We need to make `same?` a special form in our language because our language has **applicative order** evaluation. If, instead, our language had **normal order** evaluation, then we could simply define `same?` as a function.

Assuming our Scheme has the alternative method of evaluation (stated in the previous paragraph), define `same?` as a function.

$\Longrightarrow$
```
(define (same? a b c)
  (if (eq? a b)
      (eq? b c)
      #f))
```

Explicit Control Evaluator

The special form `same?` from the previous example was so helpful that we decided to add it to the explicit control evaluator. Assume that in addition to the registers we've used in the past, we've also got the register `TMP`.

Fill in the blanks in the following code that evaluates a `same?` expression.

⇒

```
ev-same?
  (assign unev (reg exp))
  (assign exp ((op same?-first) (reg exp)))
  (save continue)
  (save env)
  (save unev)
  (assign continue eval-after-first)
  (goto (label eval-dispatch))
eval-after-first
  (restore unev)
  (restore env)
  (assign tmp (reg val))
  (assign exp ((op same?-second) (reg unev)))
  (save tmp)
  (save env)
  (save unev)
  (assign continue eval-after-second)
  (goto (label eval-dispatch))
eval-after-second
  (restore unev)
  (restore env)
  (restore tmp)
  (test (op eq?) (reg tmp) (reg val))
  (branch (label eval-third-arg))
  (assign val #f)
  (restore continue)
  (goto (reg continue))
eval-third-arg
  (assign exp ((op same?-third) (reg unev)))
  (assign continue eval-after-third)
  (save tmp)
  (goto (label eval-dispatch))
eval-after-third
  (restore tmp)
  (restore continue)
  (test (op eq?) (reg tmp) (reg val))
  (branch (label all-same))
  (assign val #f)
  (goto (reg continue))
all-same
  (assign val #t)
  (goto (reg continue))
```

Concurrency

Consider the following definitions of `x` and `y`

```
(define x 2)
(define y 3)
```

In the table below, list all possible final values of `z` after the following expressions are evaluated.

```
(parallel-execute
  (lambda () (set! x (+ y y)))
  (lambda () (set! y (* x y))))

(define z (list x y))
```

$\Rightarrow$ **(6 18), (6 6), (9 6), (12 6)**

AMB Evaluator

Using the AMB evaluator, write an expression that returns the list `(x y z w)` such that

$$1 \leq x < y < z < w \leq 10$$

and

$$x^2 + y^2 + z^2 = w^2 \ .$$

You may find the following procedures helpful.

```
(define (require pred)
  (if (not pred) (amb) 'ok))

(define (ints-from-to a b)
  (if (> a b)
      (amb)
      (amb a (ints-from-to (+ a 1) b))))
```

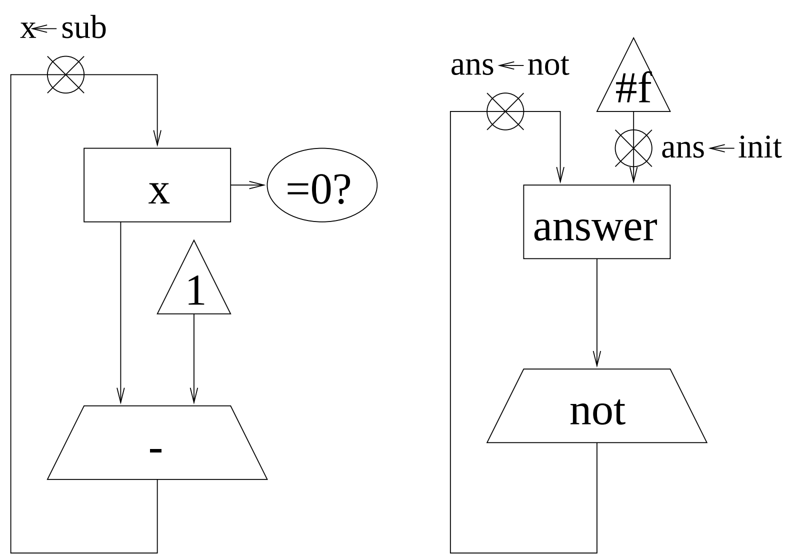
$\Rightarrow$

```
(let ((x (ints-from-to 1 10)))
  (let ((y (ints-from-to (+ x 1) 10)))
    (let ((z (ints-from-to (+ y 1) 10)))
      (let ((w (ints-from-to (+ z 1) 10)))
        (require (= (+ (* x x) (* y y) (* z z)) (* w w)))
        (list x y z w)))))
```

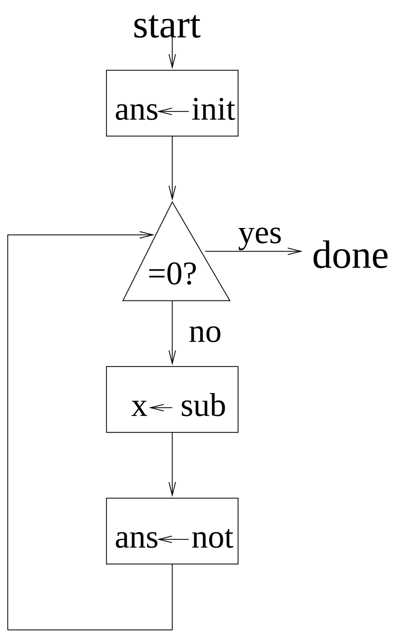
Register Machines

Draw the Data Paths AND Controller for a machine to compute whether the input `x` by successively subtracting 1 from the input. You can assume that the input will be a positive integer. Assume the only primitive operations you have are subtraction and testing equality. After your machine finishes running, the result in the `answer` register should be either `#t` or `#f`.

Data Path
⇒



Controller
⇒



Compilers

Consider the following compiled code:

```
1.    (assign val (op make-compiled-procedure) (label entry23) (reg env))
2.    (goto (label after-lambda22))
3.  entry23
4.    (assign env (op compiled-procedure-env) (reg proc))
5.    (assign env (op extend-environment) (const (f c)) (reg argl) (reg env))
6.    (save continue)
7.    (save env)
8.    (assign proc (op lookup-variable-value) (const c) (reg env))
9.    (assign val (op lookup-variable-value) (const f) (reg env))
10.   (assign argl (op list) (reg val))
11.   (test (op primitive-procedure?) (reg proc))
12.   (branch (label primitive-branch29))
13. compiled-branch28
14.   (assign continue (label proc-return30))
15.   (assign val (op compiled-procedure-entry) (reg proc))
16.   (goto (reg val))
17. proc-return30
18.   (assign proc (reg val))
19.   (goto (label after-call27))
20. primitive-branch29
21.   (assign proc (op apply-primitive-procedure) (reg proc) (reg argl))
22. after-call27
23.   (restore env)
24.   (restore continue)
25.   (assign val (op lookup-variable-value) (const f) (reg env))
26.   (test (op false?) (reg val))
27.   (branch (label false-branch25))
28. true-branch26
29.   (assign val (const 1))
30.   (goto (label after-if24))
31. false-branch25
32.   (assign val (const 2))
33. after-if24
34.   (assign argl (op list) (reg val))
35.   (assign val (op lookup-variable-value) (const a) (reg env))
36.   (assign argl (op cons) (reg val) (reg argl))
37.   (test (op primitive-procedure?) (reg proc))
38.   (branch (label primitive-branch33))
39. compiled-branch32
40.   (assign val (op compiled-procedure-entry) (reg proc))
41.   (goto (reg val))
42. primitive-branch33
43.   (assign val (op apply-primitive-procedure) (reg proc) (reg argl))
44.   (goto (reg continue))
45. after-call31
46. after-lambda22
47.   (perform (op define-variable!) (const a) (reg val) (reg env))
48.   (assign val (const ok))
```

Decompile the following groups of code. (Hint: Don't worry if the overall code looks a bit unusual.)

$\Longrightarrow$ Lines 8–22: `(c f)`

$\Longrightarrow$ Lines 25–33: `(if f 1 2)`

$\Longrightarrow$ Lines 8–45: `((c f) a (if f 1 2))`

$\Longrightarrow$ Lines 1–48:

```
(define a
        (lambda (f c)
          ((c f) a (if f 1 2))))
```