The Aperture Problem for Refractive Motion

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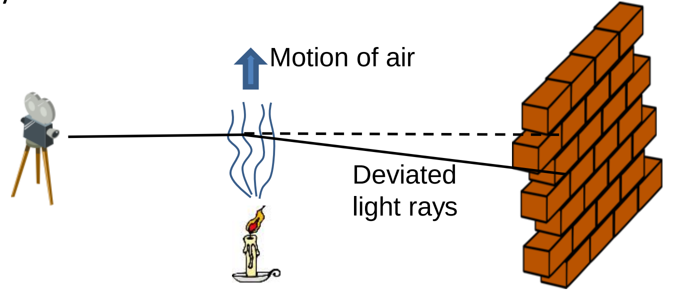
CVPR 2015 BOSTON JUNE 7-12

Abstract

Estimating Refractive Motion from a Sequence:

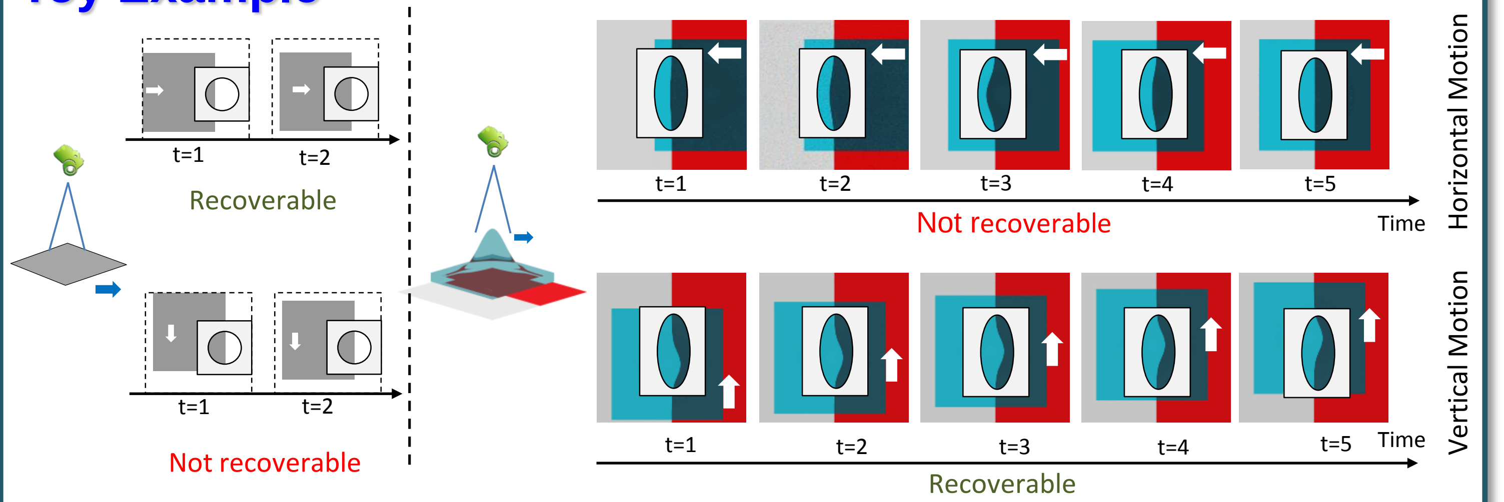
- **Objective**: tracking the movement of refractive objects (like hot air or a glass) in a single video.
- **Existing techniques**: Background Oriented Schlieren [1-3], Refractive flow [4].
- **Limitation**: no rigorous analysis about when refractive motion is recoverable.



Our Contribution:

We offer a theory for **the aperture problem** when a **refractive object** move between a camera and a static opaque background.

Toy Example



The traditional aperture problem for opaque objects

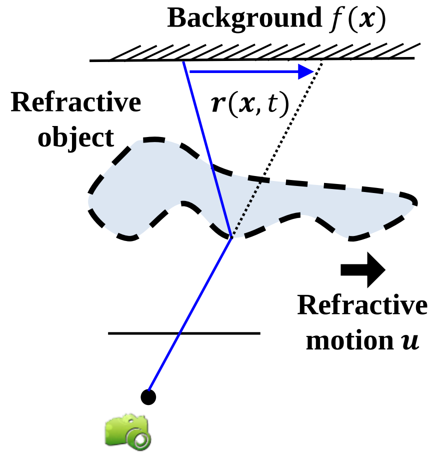
The aperture problem for refractive objects

Problem Setup

Image Formation:

$$g(x,t) = f(x - r(x - ut))$$

Observed sequence — Background image — Distortion due to refraction — Motion of refractive object



Motion Estimation:

Observation: $g(x,t)$ **Unknown:** u, r, f

Task: given the image sequence $g(x,t)$, what is the solution space of refractive motion u (existence, uniqueness)?

Aperture Theory

The observed image through the aperture	Constant	First order structure (a straight edge)	Second order structure (a conic curve)
The traditional aperture problem	$\mathcal{U} = \mathbb{R}^2$ (No information)	$\mathcal{U}$ is a line in $\mathbb{R}^2$	$\mathcal{U} = \mathbf{u}^*$
The refractive aperture problem	$\mathcal{U} = \mathbb{R}^2$ (No information)	$\mathcal{U} = \mathbb{R}^2$ (No information)	$\mathcal{U}$ is a line in $\mathbb{R}^2$

$\mathcal{U}$ is the solution space, and $\mathbf{u}^*$ is the ground truth motion.

Aperture Theory for Refractive Objects:

- Observing a plain pattern does not reveal any information about the motion of the object.
- The movement of a first order structure in the observed sequence still does not reveal any information about the motion of the object.
- The movement of a 2D structure in the observed sequence reveals the motion in only one direction.

Second Order Structure:

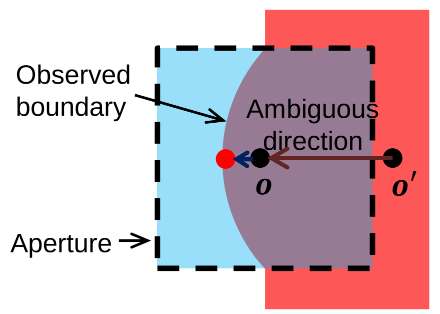
At each time point t, we fit a conic curve $\mathbf{x}^T A\mathbf{x} + \mathbf{q}^T\mathbf{x} + 1 = 0$ to the observed boundary within the aperture.

1. When A is a scalar a, we get **the simplified refractive flow equation ($\mathbf{n}_\perp$ is the direction of background structure):**

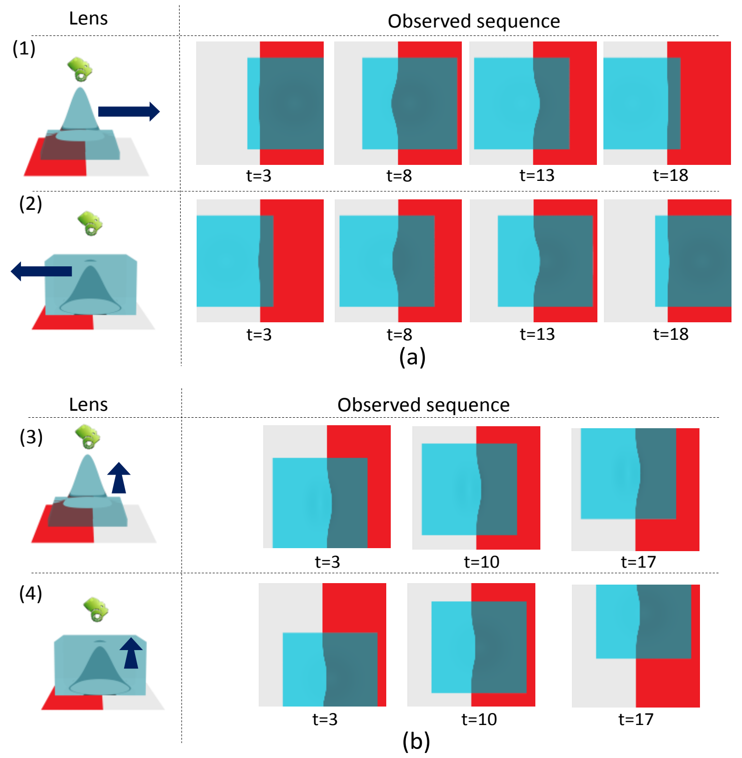
$$-\mathbf{n}_\perp^T \frac{d\mathbf{q}/dt}{2a} = \mathbf{n}_\perp^T \mathbf{u}$$

2. When A is a general 2×2 matrix, we get **the general refractive flow equation:**

$$-\mathbf{q}_\perp^T \frac{d\mathbf{q}/dt}{2} = (A\mathbf{q}_\perp)^T \mathbf{u}$$



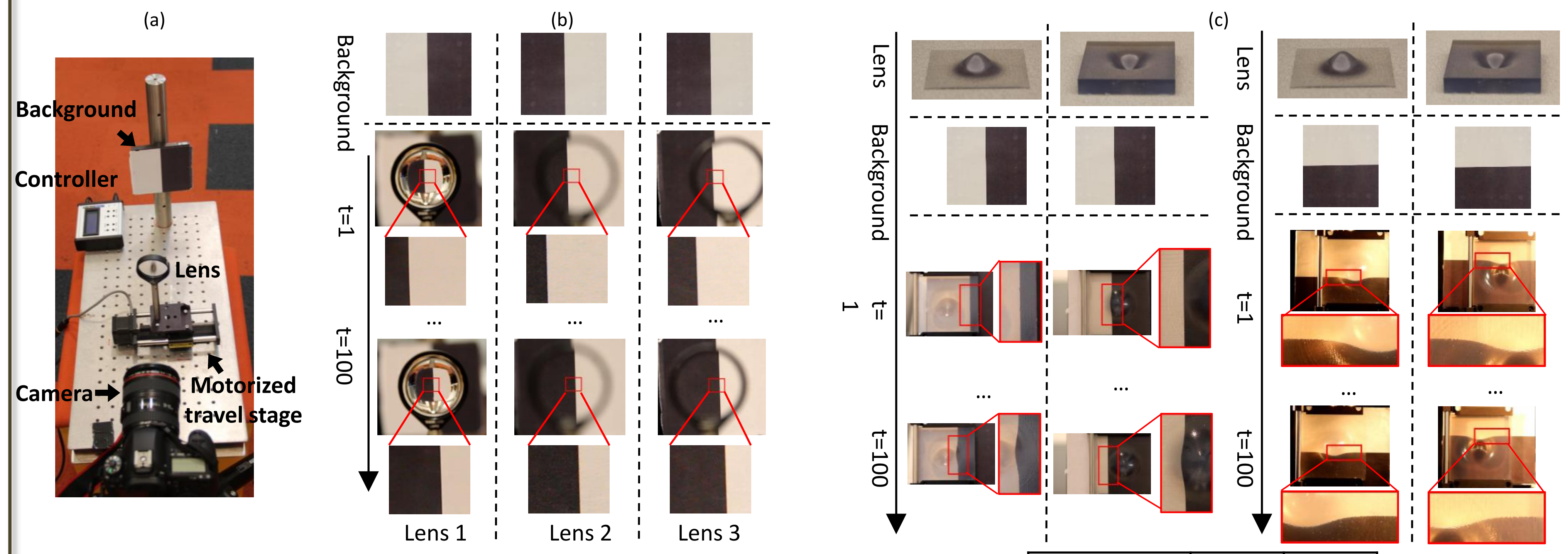
Example:



(a) When two glasses move perpendicularly to the background edge (in opposite directions), the same sequences are observed, meaning it is impossible to recover the component of the motion perpendicular to the background edge . (b) When they move parallel to the background edge, to generate the same sequence, they must move towards the same direction, meaning we can recover the component of the motion parallel to the background edge.

Experiments

Validating our theory with controlled experiments of lenses

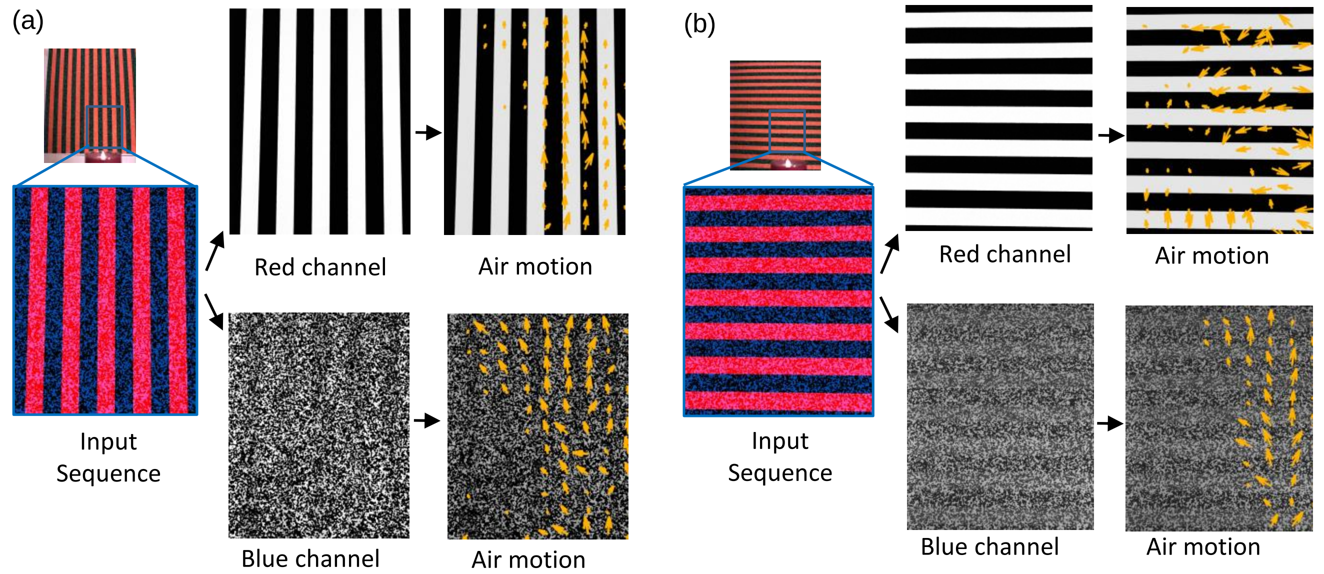


	Lens 1	Lens 2	Lens 3
Lens Speed (cm/s)	1.90	-1.78	-10.69

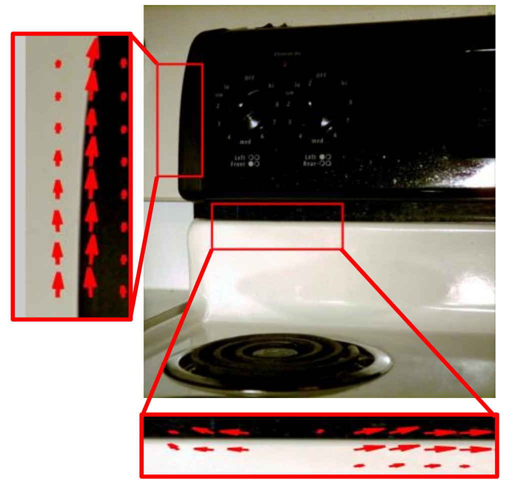
	Lens 1	Lens 2
Lens Speed	1.90	-1.78

(a) Setup. (b) Illustration of the ambiguity when observing a first order structure. The same sequences are observed with in the aperture (red region), although three lenses move in different speeds. Thus we cannot recover the motion of the lens just from local observation. (c) Illustration of the ambiguity when observing a second order structure.

Validating our theory with real videos of air flow



Hot air from a candle. Refractive motion estimation algorithm [4] fails if there is ambiguity. When the background structure is vertical (so that the air motion is parallel to the background), the recovered air motion is almost correct ((a) top). However, when the background structure is horizontal, the recovered air motion is incorrect ((b) top). The air motion estimated from fully-textured background serves a ground truth (bottom).



Hot air from a stove. The recovered air motion is correct where the background structure is vertical and incorrect where the background structure is horizontal.

References:

[1] E. Goldhahn and J. Seume. The background oriented schlieren technique: sensitivity, accuracy, resolution and application to a three-dimensional density field. Experiments in Fluids, 43(2-3):241–249, 2007
[2] M. J. Hargather and G. S. Settles. Natural-background-oriented schlieren imaging. Experiments in fluids, 48(1):59–68, 2010
[3] M. Alterman, Y. Y. Schechner, and Y. Swirski. Triangulation in random refractive distortions. ICCP, 2013
[4] T. Xue, M. Rubinstein, N. Wadhwa, A. Levin, F. Durand, and W. T. Freeman. Refraction wiggles for measuring fluid depth and velocity from video. ECCV, 2014