MAT 301 Problem Set 2

[Posted: January 20, 2012. Due: January 30, 2012. Worth: 100 points]

Note: I value *succinct* and *clearly written* solutions *without unnecessary verbiage*. Such solutions will be rewarded with bonus points.

1. **(10 points)** Compute the multiplicative inverse of $a \pmod{n}$ for the following values of a and n (if it exists), using Extended Euclid. Show your work.
   - $a = 2011$, $n = 2012$.
   - $a = 5678$, $n = 8765$.
2. **(10 points)** Solve for x in the following equations. You may use any method you like, but you have to show your work.
   - $2x = 19 \pmod{127}$.
   - $111x = 4 \pmod{496}$.
3. **(10 points)** Find a multiple of 203 that ends with the digits 999. Show your work.
4. **(30 points)** You are in Strangeland, where the only official coins are the 7 cent coin and the 11 cent coin. How can you accomplish the following tasks?
   - **(10 points)** You go to a shop and buy an item for 1 cent. Assume that you and the shopkeeper have an unlimited supply of the 7 cent and 11 cent coins. You have to pay the shopkeeper 1 cent for the item.
   - **(20 points)** You have to pay a 59 cent parking charge for your strangemobile on a strangeland parking meter. The meter takes in coins, but does not dispense change. You have to pay exactly 59 cents at the meter. If this is possible, how would you do it? If not, prove that this is impossible.
5. **(10 points)** You have two drinking glasses, one that holds exactly 15ℓ. and the other that holds exactly 19ℓ. (The glasses are plain, and have no measuring scales). How do you measure 1ℓ of liquid using the two glasses?
6. **(30 points)** The Fibonacci sequence of numbers $F_0, F_1, F_2, \ldots$ is defined by the following recurrence: $F_0 = 0, F_1 = 1$ and $F_i = F_{i-1} + F_{i-2}$ for all $i > 1$. Thus, the first few Fibonacci numbers are

$$F_0 = 0, F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5, F_6 = 8, F_7 = 13, \ldots$$

   - **(10 points) Prove** that any two adjacent Fibonacci numbers F_i and F_{i+1} are relatively prime.

- **(20 points) Prove** the identity $F_{i+1}F_{i-1} - F_i^2 = (-1)^i$.
  [*Hint: Use Induction. Show your work.*]
  Using the identity, compute $F_i^{-1} \pmod{F_{i+1}}$.