PROBE-GK: Supporting Material

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This document contains supplemental material for our paper on predictive robust estimation for stereo visual odometry. This material is broken into two sections; in the first section, we provide the details necessary to implement the least-squares optimizer we employed in the paper. In the second, we derive the Q function used in our expectation maximization procedure. Please consult the paper for information on our notational conventions.

I. The Jacobian matrix of the reprojection error

The optimization procedure defined in the paper involves the calculation of the Jacobian matrix $\mathbf{J}_{i,t}$ of the reprojection error.

$$\mathbf{J}_{i,t} = \nabla \boldsymbol{e}_{i,t}(\mathcal{T}_t) = -\nabla f(\mathcal{T}_t f^{-1}(\boldsymbol{y}_{i,t})) \tag{1}$$

This Jacobian can be factored into two terms.

$$\mathbf{J}_{i,t} := \mathbf{J}^f_{i,t} (\mathcal{T}_t^{(n)} \bar{\boldsymbol{p}}_{i,t})^{\odot}. \tag{2}$$

The first term is defined as

$$\mathbf{J}^f_{i,t} := \left.\frac{\partial f(\boldsymbol{p})}{\partial \boldsymbol{p}}\right|_{\boldsymbol{p}=\mathcal{T}_t^{(n)}\bar{\boldsymbol{p}}_{i,t}}, \tag{3}$$

$$\frac{\partial f(\boldsymbol{p})}{\partial \boldsymbol{p}} = \mathbf{M}\frac{1}{p_3}\begin{bmatrix} 1 & 0 & \frac{-p_1}{p_3} & 0 \\ 0 & 1 & \frac{-p_2}{p_3} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-p_4}{p_3} & 1 \end{bmatrix}. \tag{4}$$

The camera matrix $\mathbf{M}$ is defined as in the paper.

$$\mathbf{M} := \begin{bmatrix} f_u & 0 & c_u & f_u\frac{b}{2} \\ 0 & f_v & c_v & 0 \\ f_u & 0 & c_u & -f_u\frac{b}{2} \\ 0 & f_b & c_v & 0 \end{bmatrix}. \tag{5}$$

The second term employs the operator $(\cdot)^{\odot}$ defined by Barfoot and Furgale [1] as the map $\mathbb{R}^4 \to \mathbb{R}^{4\times 6}$.

$$\begin{bmatrix} \boldsymbol{\phi} \\ \eta \end{bmatrix}^{\odot} := \begin{bmatrix} \eta\mathbf{1} & \boldsymbol{\phi}^{\wedge} \\ \mathbf{0}^\top & \mathbf{0}^\top \end{bmatrix} \tag{6}$$

with

$$\boldsymbol{\phi}^{\wedge} := \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}^{\wedge} := \begin{bmatrix} 0 & -\phi_3 & \phi_2 \\ \phi_3 & 0 & -\phi_1 \\ -\phi_2 & \phi_1 & 0 \end{bmatrix}. \tag{7}$$

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II. Derivation of the expected log-likelihood

The expectation-maximization procedure described in the paper sequentially maximizes the expected log-likelihood of the data.

$$Q(\mathcal{T}_{1:T}|\mathcal{T}^{(n)}_{1:T}) = \int \left(\prod_{i,t} \mathrm{d}\mathbf{R}_{i,t}\, p(\mathbf{R}_{i,t}|\mathcal{D}_{i,t}, \mathcal{T}^{(n)}_{1:T})\right) \times \log \prod_{i,t} p(\boldsymbol{y}'_{i,t}|\boldsymbol{y}_{i,t}, \mathcal{T}_t, \mathbf{R}_{i,t}) \tag{8}$$

Note that

$$p(\boldsymbol{y}'_{i,t}|\boldsymbol{y}_{i,t}, \mathcal{T}_t, \mathbf{R}_{i,t}) = \mathcal{N}\left(\boldsymbol{e}_{i,t}(\mathcal{T}_t); \mathbf{0}, \mathbf{R}_{i,t}\right) \tag{9}$$

$$p(\mathbf{R}_{i,t}|\mathcal{D}_{i,t}, \mathcal{T}^{(n)}_{1:T}) = \mathrm{IW}\left(\mathbf{R}_{i,t}; \boldsymbol{\Psi}^{(n)}_{i,t}, \nu^{(n)}_{i,t}\right) \tag{10}$$

We can compute the expected log-likelihood in closed form.

$$Q(\mathcal{T}_{1:T}|\mathcal{T}^{(n)}_{1:T}) =$$

$$= \int \prod_{t=1}^{T}\prod_{i=1}^{N_t} \mathrm{d}\mathbf{R}_{i,t}\mathrm{IW}\left(\mathbf{R}_{i,t}; \boldsymbol{\Psi}^{(n)}_{i,t}, \nu^{(n)}_{i,t}\right) \sum_{t=1}^{T}\sum_{i=1}^{N_t} \log \mathcal{N}\left(\boldsymbol{e}_{i,t}; \mathbf{0}, \mathbf{R} \tag{11}$$

$$= \sum_{t=1}^{T}\sum_{i=1}^{N_t} \int \mathrm{d}\mathbf{R}_{i,t}\mathrm{IW}\left(\mathbf{R}_{i,t}; \boldsymbol{\Psi}^{(n)}_{i,t}, \nu^{(n)}_{i,t}\right) \log \mathcal{N}\left(\boldsymbol{e}_{i,t}; \mathbf{0}, \mathbf{R}_{i,t}\right) \tag{12}$$

$$= \sum_{t=1}^{T}\sum_{i=1}^{N_t} \int \mathrm{d}\mathbf{R}_{i,t}\mathrm{IW}\left(\mathbf{R}_{i,t}; \boldsymbol{\Psi}^{(n)}_{i,t}, \nu^{(n)}_{i,t}\right) \left(-\frac{d}{2}\log(2\pi) - \frac{1}{2}\log|\mathbf{R}_{i,t}| - \frac{1}{2}\boldsymbol{e}_{i,t}{}^\top \mathbf{R}^{-1}_{i,t}\boldsymbol{e}_{i,t}\right) \tag{13}$$

$$= \sum_{t=1}^{T}\sum_{i=1}^{N_t} -\frac{d}{2}\log(2\pi) - \frac{1}{2}\left(-\psi_d(\frac{\nu^{(n)}_{i,t}}{2}) - d\log 2 + \log\left|\boldsymbol{\Psi}^{(n)}_{i,t}\right|\right) - \frac{\nu^{(n)}_{i,t}}{2}\boldsymbol{e}_{i,t}{}^\top\left(\boldsymbol{\Psi}^{(n)}_{i,t}\right)^{-1}\boldsymbol{e}_{i,t} \tag{14}$$

$$\cong -\frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{N_t} \boldsymbol{e}_{i,t}{}^\top \left(\frac{1}{\nu^{(n)}_{i,t}}\boldsymbol{\Psi}^{(n)}_{i,t}\right)^{-1}\boldsymbol{e}_{i,t} \tag{15}$$

References

[1] T. D. Barfoot and P. T. Furgale, "Associating uncertainty with three-dimensional poses for use in estimation problems," *IEEE Transactions on Robotics*, vol. 30, no. 3, pp. 679–693, 2014. DOI: 10.1109/TRO.2014.2298059.