HMMT Friday Night Event
Physics is *Phun*!

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1. (a) A cannon fires a ball with initial speed v at a vertical wall distance D away. The angle of the cannon can be freely adjusted. What's the maximum height that the ball can reach on the wall?

   (b) A cannon fires a ball with initial speed v towards a horizontal platform of height H. The angle of the cannon can be freely adjusted. How far can the platform be placed from the cannon such that the ball can still reach it?

2. (a) A ball of radius R is completely submerged in a liquid of density ρ. What is the buoyant force on the ball?

   (b) Suppose electric charge is distributed as $\sigma(\theta) = \sigma_0 \cos\theta$ on the surface of a ball or radius R, where $\sigma(\theta)$ denotes the surface charge density (i.e., charge per unit area) at an angle θ from the z-axis. What is the magnitude of the electric field at the center of the ball?

3. An ideal black body (perfect absorber of radiation) at a positive temperature emits radiation satisfying the Stefan-Boltzmann law. It says that the power radiated per unit area of the surface of a black body is σT^4, where T is the temperature and $\sigma \approx 5.67 \times 10^{-8}\,\mathrm{W/m^2\,K^4}$ is the Stefan-Boltzmann constant.

   (a) What is the total power radiated by a black-body sphere of radius R at temperature T?

   (b) Consider two concentric black-body spherical shells of radii R_1 and $R_2 > R_1$. What is the net power of radiation energy transferred from the inner shell to the outer shell if both shells are at temperature T?

   (c) Consider two concentric black-body spherical shells of radii R_1 and $R_2 > R_1$. What is the net power of radiation energy transferred from the inner shell to the outer shell if the inner shell has temperature T_1 and the outer shell has temperature T_2?

4. (a) Two masses m_1 and m_2 are held at x-coordinates x_1 and x_2 at the same height and are connected with a spring. The tension in the spring is a known positive function of its length. The masses are released from rest and enter a free fall. What is the x-coordinate at which they collide? Suppose the masses never hit the ground.

   (b) Two blocks of equal mass are connected by a straight horizontal massless rope that passes over a massless pulley, which is positioned at the edge of a vertical cliff and situated at the midpoint of the rope. Mass 1 sits on a horizontal frictionless surface and mass 2 is floating in air for now. The masses are released from rest. Which happens first: mass 1 hits the pulley or mass 2 hits the cliff? Why?

Answer key:

1. (a) $\frac{1}{2}D\left(\frac{v^2}{gD} - \frac{gD}{v^2}\right)$

   (b) $\sqrt{\frac{v^2}{g}\left(\frac{v^2}{g} - 2H\right)}$

2. (a) $\frac{4}{3}\pi R^3 \rho g$

   (b) $\frac{\sigma_0}{3\epsilon_0}$

3. (a) $4\pi R^2 \sigma T^4$

   (b) 0

   (c) $4\pi R_1^2 \sigma (T_1^4 - T_2^4)$

4. (a) $\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

   (b) Mass 1 hits the pulley first.

   Explanation 1: Applying force balance to the pulley, we get that the edge of the cliff applies a force directed away from the cliff. So the center of mass moves away from the cliff, and mass 1 must hit the pulley first.

   Explanation 2: The horizontal component of the force in either mass comes from the tension in the rope. But because the tension force on mass 2 is at an angle, it accelerates towards the cliff slower than mass 1 accelerates towards the cliff. So mass 1 will hit the pulley first.

Tiebreaker: Consider a tube of diameter 0.5 cm bent into a ring of radius r, resting on a horizontal surface with its axis vertical. Suppose your phone is modeled as a rectangular prism of dimensions 10 cm $\times$ 6 cm $\times$ 1 cm. Your answer should be a value of r with the property that you can place your phone on the ring such that tilting the phone by 0.01 radians will not cause the phone to fall off the ring. Suppose the coefficient of friction between the ring and phone is high enough that the phone would not slip. The team with the largest valid value of r wins the tiebreaker.

Answer to tiebreaker: $r = 34\,\mathrm{m}$