# HMMT Friday Night Event
# Physics is *Phun*!

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1. **Michael Jordan's vertical jump.** Michael Jordan's vertical jump is $h = 1.20\,\mathrm{m}$. What's the speed at which he leaves the ground when he jumps? Use $g = 9.81\,\mathrm{m/s^2}$. (If you don't have a calculator, write your answer as a formula in terms of $h$ and $g$.)

2. **The Groove Tube.** The Groove Tube (fig. 1) is a toy from the 1980s consisting of a plastic circular groove track with groove width $d$ and a ball with radius $R$ that sits in the groove. To play with the toy, you place the toy flat on a table and move it in small circular motions to accelerate the ball to a speed $v$ around the groove track. You then lift the groove track off the table and the ball should continue to move in its circular trajectory of radius $r$ around the groove track, without falling out due to gravity. What's the minimum speed $v$ that is required for this to happen? Write a formula in terms of $d, R, r, g$.

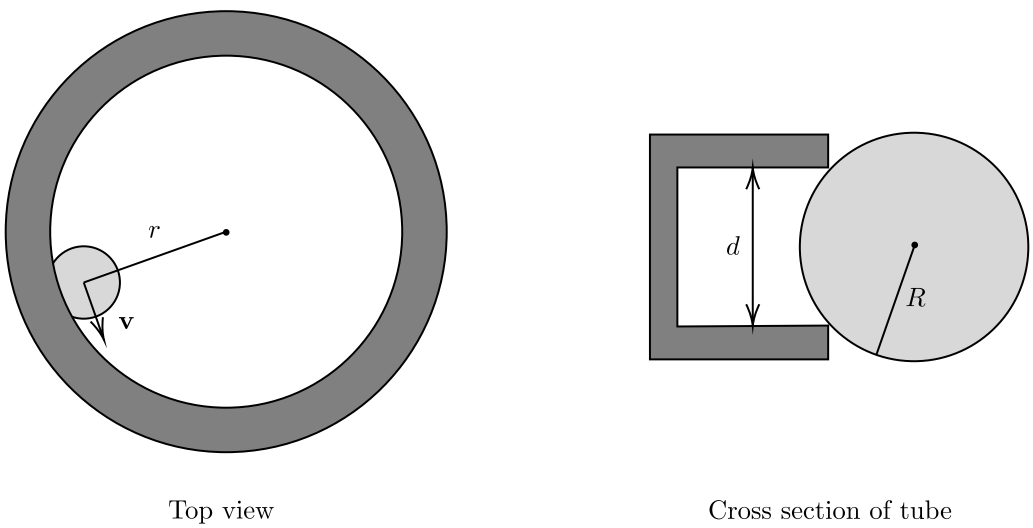



Figure 1: Schematic of the Groove Tube toy

3. **Rolling soda cans.** In an experiment, Quan releases a full soda can from rest at the top of a ramp, and measures the time it takes to roll (without slipping) to the bottom of the ramp to be $t_{\text{full}}$. Then, Zed got thirsty and, not knowing the soda can was being used in a physics experiment, drinks all the soda. Afterwards, Quan repeats his experiment, without realizing the soda can is now empty, measuring a time of $t_{\text{empty}}$. What is the ratio $t_{\text{full}}/t_{\text{empty}}$? You may assume the empty soda can is much lighter than the full one, and you're given the formula for the moment of inertia of a solid cylinder about its axis: $I = \frac{1}{2}mR^2$ where $m$ is its mass and $R$ its radius.

4. **Land warms more than the Earth's average.**[1] As the Earth warms due to climate change, the temperature increase in any land region is not the same as the averaged global temperature increase because the ocean has a much higher heat capacity. For a very simplified model, assume the land's temperature $T_l$ and the ocean's temperature $T_o$ evolve independently, with no heat exchange between the land and the ocean. The flux density $r(t)$ of heat arriving to any point on Earth depends on the sun's power output and greenhouse gas absorption, among other factors. For simplicity, you may assume that $r(t)$ is the same at all locations on Earth. The total heat capacity of the land is $C_l$, and the total heat capacity of the ocean is $C_o = 20C_l$. The ocean's area is twice the land's. The global averaged temperature is defined as the temperature averaged uniformly over Earth's surface.

   In 2023, the Earth has warmed about 1°C on average compared to pre-industrial times. Assume the Earth warms by 2°C from 2023 to 2100, what would be the averaged *land* temperature increase in 2100 compared to 2023?

5. **Two rotating charges.** On the ISS, Zed successfully puts two electric charges of the same chare $q$ and same mass $m$ into a constant-speed circular motion around each other with distance $d$ between them (i.e., the diameter of their common circular trajectory is $d$); see fig. 2. This motion is maintained by the presence of a uniform magnetic field $B$. What's the minimum value of $B$ required to maintain this motion? Write your answer as a formula in terms of $m, d, q$ and physical constants. You may ignore all forces other than the electrostatic interaction between the charges and the Lorentz force they experience from the magnetic field.

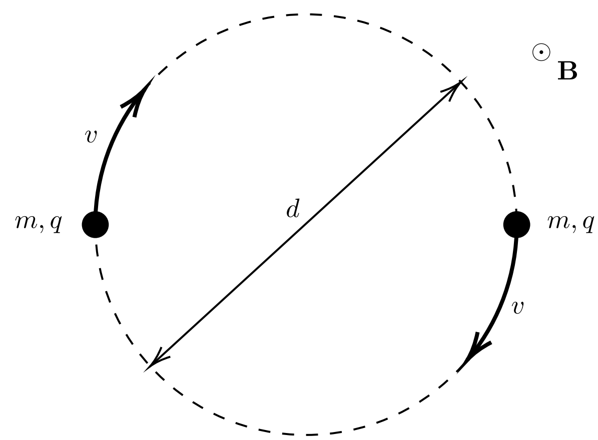



Figure 2: Two charges rotating around each other at constant speed $v$ and constant distance $d$ in a uniform magnetic field $\mathbf{B}$.

6. **Radio antennas.** Consider a rod-shaped radio antenna with length $L$. Estimate $\mathrm{Re}(Z)$, the real part of the antenna's complex impedance, when it's emitting radio waves of frequency $f$. Write a formula in terms of $L, f$ and physical constants, ignoring numerical constant factors.

   Recall that complex impedance $Z$ is like resistance but for AC circuits, and in particular the analog of $P = I^2R$ for AC circuits is $P = I^2\,\mathrm{Re}(Z)$.

   *Hint:* The Larmor formula says that a charge $q$ with acceleration $a$ emits electromagnetic radiation with power $P \propto q^2a^2/\varepsilon_0c^3$.

[1] You see this effect very clearly in climate models. MIT scientists in the "Bringing computation to the climate challenge" group is working on simple emulators of state-of-the-art climate models that show temperature changes with high spatial resolution as a function of net emissions (see http://eddies.mit.edu/bc3). Also check out the educational En-ROADS where you see how different policies impact global temperature changes. This work uses a lot of math, physics, and computer science!

7. **Fried ice cream.**

   (a) What is the capacitance of a capacitor consisting of two concentric spherical metal shells of radii $a < b$? In other words, if the charge on on the inner metal shell is $Q$ when a voltage of $V$ is applied between the inner and outer shells, what is $Q/V$? Write your answer as a formula in terms of $a, b, \varepsilon_0$.

   (b) Fried ice cream balls are a delicious dessert consisting of an ice cream ball coated in a layer of fried crust. Model the (uncooked) dessert as an ice cream ball of radius $r = 1.5\,\mathrm{cm}$ with latent heat of fusion $L = 3.4 \times 10^5\,\mathrm{J/kg}$, coated in a layer of batter of uniform thickness $d = 0.5\,\mathrm{cm}$ and thermal conductivity $\kappa = 0.6\,\mathrm{W/m\,K}$. To fry it, we put it in oil of temperature $T = 200°\mathrm{C}$. Find the rate at which the ice cream melts in units of grams per second (g/s), shortly after it starts melting in the fryer. You may make the approximation that the temperature profile of the batter/crust coating is in steady state.

   (If you don't have a calculator, write your answer as a formula in terms of $r, d, L, \kappa, \Delta T$ where $\Delta T = 200\,\mathrm{K}$ is the temperature difference between the oil and the ice cream when it starts melting.)

   *Hint:* Your answer from part (a) may be useful here.

8. **Chlorophyll.** Chlorophyll is the main chemical responsible for photosynthesis in plants. To estimate the wavelength of light that chlorophyll captures, consider the simplified model where the molecule's ring is modeled as 18 electrons that are constrained to move on a circular ring of radius $r = 0.4\,\mathrm{nm}$ (fig. 3). For simplicity, assume that the electrons do not interact, but they still must satisfy Pauli's exclusion principle.

   (a) Under the wave-particle duality, an electron with momentum $p$ has de Broglie wavelength $\lambda = h/p$ (zero momentum corresponds to an infinite wavelength). The fact that the electron's matter wave must go through an integer number of periods around the ring quantizes the energy levels. Draw the energy level diagram and fill in the 18 electrons to create a simplified version of the molecular orbital diagram for chlorophyll. Also draw the lowest two empty energy levels.

   (b) What is the wavelength of the photon with the minimum energy required to excite chlorophyll in this simplified model? Write your answer in nanometers (nm). The mass of an electron is $m = 9.1 \times 10^{-31}\,\mathrm{kg}$, the speed of light is $c = 3.0 \times 10^8\,\mathrm{m/s}$, and Planck's constant is $h = 6.6 \times 10^{-34}\,\mathrm{J\,s}$.

   (If you don't have a calculator, write your answer as a formula in terms of $r, m, c, h$.)

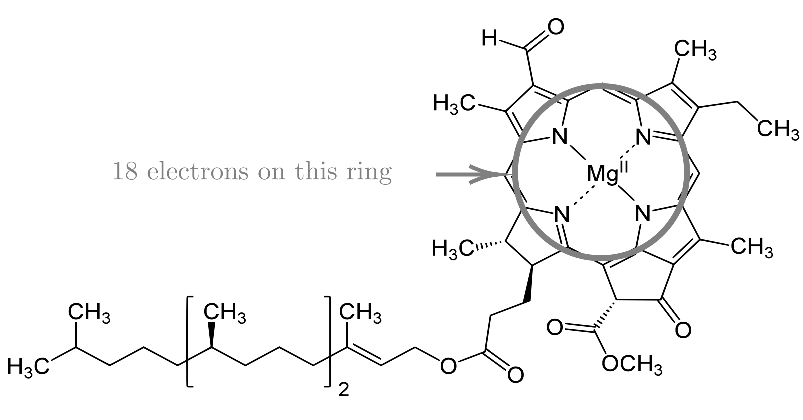



Figure 3: Chlorophyll modeled as 18 electrons moving around a ring (gray).

*Answer key:*

1. $\sqrt{2gh} = 4.85\,\text{m/s}$

2. $\sqrt{gr\sqrt{(2R/d)^2 - 1}} = \sqrt{gr\tan\theta}$ where $\theta = \cos^{-1}(d/2R)$

3. $\dfrac{\sqrt{3}}{2}$

4. $5°\text{C}$ (exactly)

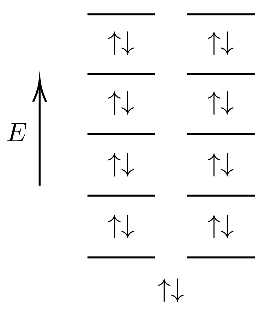
5. $\sqrt{\dfrac{2m}{\pi\varepsilon_0 d^3}} = \sqrt{8k_e\dfrac{m}{d^3}}$

6. $\dfrac{L^2 f^2}{\varepsilon_0 c^3}$

7. (a) $4\pi\varepsilon_0\left(\dfrac{1}{a} - \dfrac{1}{b}\right)^{-1} = 4\pi\varepsilon_0\dfrac{ab}{b-a}$

   (b) $4\pi\kappa\left(\dfrac{1}{r} - \dfrac{1}{r+d}\right)^{-1}\dfrac{\Delta T}{L} = 4\pi\kappa\dfrac{r(r+d)}{d}\dfrac{\Delta T}{L} = 0.27\,\text{g/s}$

8. (a)




   (b) $\dfrac{8\pi^2 cmr^2}{9h} = 580\,\text{nm}$

***Tiebreaker:*** **Simple model of a swing.** Consider a pendulum consisting of a massless rod and a bob of mass $m$ attached at the end. An additional mass $m$ can be programmatically controlled to move up or down the rod by a motor. Assume the motor is so powerful that it can accelerate the mass to very high speeds almost instantly. Now, the pendulum is released from rest at an angle of $\theta_0 = 1'$ (1 arcminute, i.e., 1/60 of a degree). Your task is to control the moving mass in a way that inverts the pendulum ($\theta = 180^\circ$) as fast as possible. What's the minimum number of full periods the pendulum must go through before it gets inverted?

*Answer to tiebreaker:* 6