HMMT Friday Night Event
Physics is *Phun*!

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November 2025

Use $g = 9.8$ m/s^2.

1. **thing that i came up with in the shower (based on real values i measured)**

   Showering has long been appreciated as a refreshing and restorative daily ritual. Here are some things you can explore while in there: (Unit conversion: 1 tile = 20 cm)

   (a) You point the showerhead directly upwards and discover the water streams climb a maximum height of $h = 6.5$ tiles before falling back down. What's the speed at which the water streams exit the showerhead? (Assume you don't have a ceiling.)

   (b) You point the showerhead parallel to the floor and discover the water falls by about $h = 0.5$ tiles when the water hits the wall $d = 4$ tiles away. What's the speed at which the water streams exit the showerhead? (Assume you have a wall.)

   (c) You start turning the showerhead in random directions while keeping it in the same place. Luckily, you brought your phone with you into the shower so you were able to take a video of it. The envelope of water streams forms a parabola described (in tile units) by

   $$y = a - bx^2 = 6.5 - 0.039x^2$$

   with the origin set at the showerhead. What's the speed at which the water streams exit the showerhead? (Assume you don't have walls.)

2. **A Tough Fall**

   Your main and your back-up parachutes both failed to deploy during your first skydiving experience (on your birthday, might I add). Luckily, you learned from your late-night Youtube Shorts scrolling sessions that tree branches could catch your fall, so you aim for the big tree in the center of the desert. The tree has branches spaced vertically $h = 1.0$ m apart, and hitting each branch reduces your speed by $\Delta v = 1.0$ m/s. After some time, your motion reaches a repeating pattern. What is your long-term average speed?

3. **Scaling Sand**

   (a) Sand is gradually poured onto a surface, forming a perfectly circular cone of increasing size. The sand is granular and has coefficient of friction μ. How does the ratio of the cone's height to its radius, h/r, scale with μ? Fill in the blank: $h/r \propto$ ____.

   (b) Now you have sand spread out evenly all over the floor, so you decide to clean it up with a broom (after procrastinating for a while). You sweep the broom over the floor at a constant speed v, and at this speed, you need to push down on the broom with a force of at least F if you want to clean up everything in one sweep. How does F scale with v? Fill in the blank: $F \propto$ ____.

4. **GPS (Goofy Positioning Systems)**

An MIT student is devising a new way of determining their location based on the frequency of the "choo-choo" noises of their local trains. Suppose the MIT student approximates their school as a flat plane. It is known that a train traveling at constant speed v in the positive x-direction passes through $(0, 0)$ at a particular time. At that time, the train emits a "choo-choo" at frequency f_T and the stationary student perceives the "choo-choo" at frequency f_R. Let the speed of sound in the medium be $v_S > v$.

(a) In terms of f_T, v, and v_S, what are the possible values of f_R? Express your answer as an interval, $\_\_\_\_ \leq f_R \leq \_\_\_\_$.

(b) The MIT student obtained a measurement of f_R that indeed lies within the interval calculated in part (a). The student now knows their direction relative to the origin. What is $\angle POx$, the angle between the student's position vector $\overrightarrow{OP}$ and the positive x-axis? Provide your answer in terms of f_R, f_T, v, and v_S.

(c) A Harvard student with superior connections obtains access to a GPS satellite that continuously emits an EM wave in all directions at a fixed frequency. The satellite S follows a circular orbit of radius zR (with $z > 1$) with fixed angular velocity around Earth, which has radius R. At any given moment, the Harvard student wishes to find the point P on Earth at which they would perceive the maximum frequency of the wave emitted by the satellite (Figure 1). In terms of z, what is the measure of $\angle POS$?

You may make the following assumptions:

- The satellite is non-relativistic.
- Earth has no atmosphere and doesn't rotate.
- The Harvard student cannot receive a signal at locations without direct line-of-sight to the satellite.

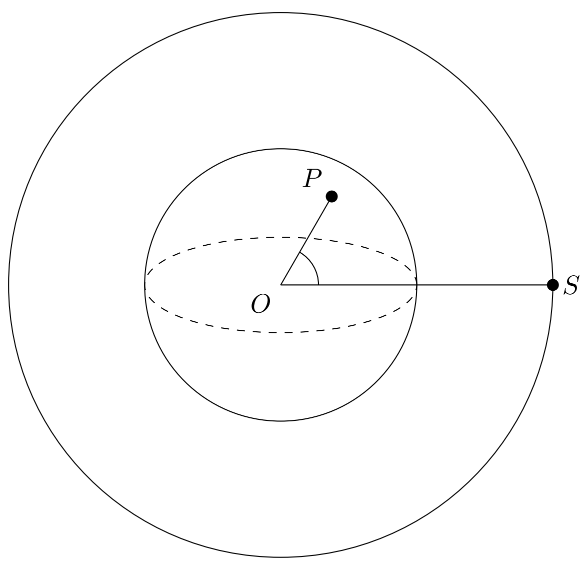

Figure 1: Satellite diagram

Problems by Zander (Q1–3), Jonathan (Q4) and Zed (tiebreaker)

Answer key:

1. (a) $\sqrt{2gh} = 5.0\,\text{m/s} = 25\,\text{tiles/s}$

   (b) $d\sqrt{\dfrac{g}{2h}} = 5.6\,\text{m/s} = 28\,\text{tiles/s}$

   (c) $\sqrt{2ga} = 5.0\,\text{m/s} = 25\,\text{tiles/s}$

2. $\dfrac{gh}{\Delta v} = 9.8\,\text{m/s}$

3. (a) $h/r \propto \mu$

   (b) $F \propto v^2$

4. (a) $\dfrac{v_S}{v_S + v} f_T \leq f_R \leq \dfrac{v_S}{v_S - v} f_T$

   (b) $\arccos\left(\dfrac{v_S}{v}\dfrac{f_R - f_T}{f_R}\right)$

   (c) $\arccos(1/z)$

***Tiebreaker:* Many colliding blocks**

2025 perfectly elastic blocks are placed stationary on a frictionless ground at coordinates $x = 1, 2, 2^2, \ldots, 2^{2024}$ meters. Their respective masses are $1, 3, 3^2, \ldots, 3^{2024}$ kilograms. At $x = 0$, there's a perfectly elastic vertical wall. Now, the rightmost block is given a leftward initial velocity v_0. Many collisions occur among the blocks and the wall, after which no more collisions occur ever again. Let N be the total number of collisions that occurred. What is $\log_{10} N$?

Your answer must be expressed as an explicit decimal number (e.g., write 1.33333 instead of 4/3 and 1.414 instead of $\sqrt{2}$). If your answer is a and the correct answer is b, the team with the smallest $|a - b|$ wins.

Answer to tiebreaker: 609.88677...