HMMT Friday Night Event
Physics is *Phun*!

Zander Jonathan Zed Alex

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Use $g = 9.8$ m/s^2.

1. **A Tough Fall**

   Your main and your back-up parachutes both failed to deploy during your first skydiving experience (on your birthday, might I add). Luckily, you learned from your late-night Youtube Shorts scrolling sessions that tree branches could catch your fall, so you aim for the big tree in the center of the desert. The tree has branches spaced vertically $h = 1.0\,\text{m}$ apart, and hitting each branch reduces your speed by $\Delta v = 1.0\,\text{m/s}$. After some time, your motion reaches a repeating pattern. What is your long-term average speed?

2. **Scaling Sand**

   (a) Sand is gradually poured onto a surface, forming a perfectly circular cone of increasing size. The sand is granular and has coefficient of friction μ. How does the ratio of the cone's height to its radius, h/r, scale with μ? Fill in the blank: $h/r \propto$ ____.

   (b) Now you have sand spread out evenly all over the floor, so you decide to clean it up with a broom (after procrastinating for a while). You sweep the broom over the floor at a constant speed v, and at this speed, you need to push down on the broom with a force of at least F if you want to clean up everything in one sweep. How does F scale with v? Fill in the blank: $F \propto$ ____.

3. **Minecraft Physics**

   In case you have been living under a rock, Minecraft is a 3D game. The world is made of cubes with side length of 1 m. Also, any valid approximation is fine.

   (a) If you fall off a cliff of height $h = 50$ blocks onto a block of water, somehow no fall damage is taken. The water induces a drag force proportional to your velocity: $F_{\text{drag}} = -\alpha v$. What is the minimum value of α such that your speed becomes zero before you hit the ground? Also, you weigh $m = 1\,\text{kg}$.

   (b) You can apparently row a boat faster on ice than on water. Say you are going around in a circle of radius $r = 1$ m on ice at speed $v = 1\,\text{m/s}$ and you push off the ice with your paddle every $\Delta t = 0.01\,\text{s}$ to accelerate yourself using the kinetic friction between your paddle and the ice. The ice decelerates you at $a = 0.01\,\text{m/s}^2$, and you can move your paddle at a speed of $u = 100\,\text{m/s}$. In your reference frame, what is the direction the paddle's movement when you push off the ice so that you keep going in the same circle? Write the angle relative to the radial direction in radians.

   (c) Redstone signals (the equivalent of electrical signals) lose their strength when you get too far, which is kinda accurate I guess. To simulate this, say you have a current source (i.e. a redstone block) of $I = 1$ A at the surface of the ground. The ground is made of material with resistivity $\rho = 100\ \Omega\cdot\text{m}$. What is the voltage at a distance r away in the ground?

4. **In Preparation for Building a Dyson Sphere,**

you place a black circular disk and a white sphere of equal radius in space at an equal distance from the Sun. What is the ratio between of the force due to radiation pressure experienced by the disk versus the sphere ($F_{\text{disk}}/F_{\text{sphere}}$)? Assume both objects are sufficiently far away so that the radiation is uniform, and the axis of the black disk points towards the Sun.

5. **You Thought This Wasn't the Integration Bee...**

Parts (a), (b), and (especially) (c) can be solved independently of parts (d) and (e).

(a) Compute the following integral:

$$\int_0^\infty te^{-t}dt$$

(b) Express the following ratio of integrals in terms of z:

$$\frac{A}{B} \text{ where } A = \int_0^\infty t^z e^{-t}dt, B = \int_0^\infty t^{z-1}e^{-t}dt$$

(c) In terms of the Riemann zeta function $\zeta(s) = \sum_{n=1}^\infty \frac{1}{n^s}$, compute the following function (the same function as in part (e)):

$$f(z) = \int_0^\infty \frac{x^z}{e^x - 1}dx$$

The Stefan–Boltzmann law states that the energy density of a blackbody is proportional to its temperature raised to the fourth power. We outline some key steps in the derivation of this law.

(d) We assume an oscillator at a fixed frequency ν can only have quantized energies $E_n = nh\nu$ for non-negative integers n. The probability that an oscillator has energy E_n is proportional to $e^{-E_n/kT}$ according to the Boltzmann distribution, where k is the Boltzmann constant and T is the temperature. In terms of h, ν, k, and T, what is the average energy of such an oscillator? (You may use the fact that $\sum_{n=0}^\infty ne^{-nx} = \frac{e^x}{(e^x-1)^2}$.)

(e) To find the energy density of a blackbody, one computes an integral over frequencies ν of the product $E(\nu)\rho(\nu)$, where $E(\nu)$ is the average energy of an oscillator (from the previous part) and $\rho(\nu)$ is (roughly) the number of modes of oscillation at frequency ν ($\rho(\nu)$ is also known as the density of states). Given that $\rho(\nu) = \frac{8\pi}{c^3}\nu^2$, what is the value of the energy density? Express your answer in terms of π, c, h, k, T, as well as the following function (the same function as in part (c)) evaluated at a particular value of z:

$$f(z) = \int_0^\infty \frac{x^z}{e^x - 1}dx.$$

Problems by Zander (Q1–4), Mojang (Q3), Jonathan (Q5) and Zed (nonexistent tiebreaker)

Answer key:

1. $\dfrac{gh}{\Delta v} = 9.8\,\mathrm{m/s}$
2. (a) $h/r \propto \mu$
   (b) $F \propto v^2$
3. (a) $\alpha = m\sqrt{2gh}/(1\text{ m}) = 31$ kg/s
   (b) $\theta = \frac{ar}{v^2} + \frac{v}{u} = 0.02$ rad, relative to the radius
   (c) $V = \frac{I\rho}{2\pi r}$
4. They experience the same force.
5. (a) 1
   (b) z
   (c) $f(z) = z! \cdot \zeta(z+1)$
   (d) $\frac{h\nu}{e^{h\nu/kT}-1}$
   (e) $\frac{8\pi(kT)^4}{(hc)^3} f(3)$

***Tiebreaker:* Many colliding blocks**

2025 perfectly elastic blocks are placed stationary on a frictionless ground at coordinates $x = 1, 2, 2^2, \ldots, 2^{2024}$ meters. Their respective masses are $1, 3, 3^2, \ldots, 3^{2024}$ kilograms. At $x = 0$, there's a perfectly elastic vertical wall. Now, the rightmost block is given a leftward initial velocity v_0. Many collisions occur among the blocks and the wall, after which no more collisions occur ever again. Let N be the total number of collisions that occurred. What is $\log_{10} N$?

Your answer must be expressed as an explicit decimal number (e.g., write 1.33333 instead of 4/3 and 1.414 instead of $\sqrt{2}$). If your answer is a and the correct answer is b, the team with the smallest $|a - b|$ wins.

Answer to tiebreaker: 609.88677...